Nützliche Formeln

$$(1) \quad \boxed{\operatorname{rot} \operatorname{grad} \phi = 0 \quad ; \quad \vec{\nabla} \times (\vec{\nabla} \phi) = 0}$$

Bew: $(\operatorname{rot} \operatorname{grad} \phi)_3 = \partial_1 \partial_2 \phi - \partial_2 \partial_1 \phi = 0$

usw.

$$(2) \quad \boxed{\operatorname{div} \operatorname{rot} \vec{A} = 0 \quad ; \quad \vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0}$$

Bew: $\partial_1 (\partial_2 A_3 - \partial_3 A_2) + \partial_2 (\partial_3 A_1 - \partial_1 A_3)$
$+ \partial_3 (\partial_2 A_1 - \partial_1 A_2) = 0$

$$(3) \quad \boxed{\operatorname{div} \operatorname{grad} \phi = \nabla^2 \phi = \Delta \phi}$$

$\Delta = \partial_1^2 + \partial_2^2 + \partial_3^2$ Laplace Operator

$$(4) \quad \boxed{\begin{aligned} \operatorname{rot} \operatorname{rot} \vec{A} &= \vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla} (\vec{\nabla} \cdot \vec{A}) - \Delta \vec{A} \\ &= \operatorname{div} \operatorname{grad} A - \Delta \vec{A} \end{aligned}}$$

Bew: $\varepsilon_{ijk} \varepsilon_{klm} \partial_j \partial_l A_m = (\delta_{il} \delta_{jm} - \delta_{im} \delta_{jl}) \partial_j \partial_l A_m$
$= \partial_i \partial_j A_j - \partial_j^2 A_i = [\vec{\nabla} (\vec{\nabla} \cdot \vec{A}) - \Delta \vec{A}]_i$

$$(5) \quad \boxed{\operatorname{div} (\vec{A} \times \vec{B}) = \vec{\nabla} (\vec{A} \times \vec{B}) = \vec{B} \operatorname{rot} \vec{A} - \vec{A} \operatorname{rot} \vec{B}}$$

(Vorzeichen ≙ zykl. Reihenfolge

Bew: $\partial_i \varepsilon_{ijk} A_j B_k \overset{\text{Produktregel}}{=} \varepsilon_{ijk} [(\partial_i A_j) B_k + A_j (\partial_i B_k)]$
$= \vec{B} \operatorname{rot} \vec{A} - \vec{A} \operatorname{rot} \vec{B}$.

$$(6) \quad \boxed{\vec{\nabla} (\phi \cdot \vec{A}) = \phi \, \vec{\nabla} \cdot \vec{A} + (\vec{\nabla} \phi) \vec{A}}$$

Bew: Produktregel