Black Hole Formation and Classicalization in Ultra-Planckian Scattering

based on hep-th 1409.7405
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Einstein Gravity

Einstein gravity ($m = 0$, $s = 2$) is a well-studied theory of gravitation,

- Many interesting features (Geometry, Black Holes, Relation to Yang-Mills,...),
- Supersymmetric extensions,
- Well-tested experimentally,
- ...

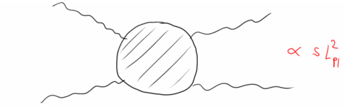
Many problems and properties still not completely understood.

- UV completion, i.e. perturbative unitarity at tree level?
- Quantum understanding of BH?
- Renormalizability?
- ...

Unitarity in Gravity

Known: Gravity scattering amplitudes grow like s (center of mass energy)

$\Rightarrow$ **violation of (perturbative) unitarity at $s = M_{pl}^2$.**



Wilsonian UV completion: interactions at higher and higher energies regulated by integrating-in weakly-coupled degrees of freedom of shorter and shorter wave-lengths.

Consequences for gravity: at energies $s > M_{Pl}^2$ the UV-completion achieved by new quantum degrees of freedom of wavelength much shorter than the Planck length.

UV Completion and Classicalization

But: Gravity has a smallest length scale – **the Planck length**. Cannot go beyond this length since **black holes** will inevitably form, i.e. Wilsonian UV completion does not make sense anymore.

Based on this [Dvali, Gómez] argued that gravity is UV complete by itself through classical black hole formation – called **classicalization**.

Basic idea of UV completion by classicalization is that

short-scale UV physics → long-scale IR physics

by formation of classical object at large energies – **black holes dominate**

In other words: gravity protects itself at high energies by BH formation.

Without doubt: better quantum understanding of black holes needed.

Black Hole N Portrait

Developments towards this in a program of work entitled **Quantum Black Hole corpuscular N-portrait**.

[Dvali, Gómez], [Dvali, Gómez, Kehagias], [Dvali, Gómez, Lüst]

$\rightarrow$ See also Gia's talk

Quantum black hole

=

collection of N self-bound gravitons at quantum critical point
(Bose-Einstein condensate)

- interaction strength of gravitons $\alpha = \frac{1}{N}$ at this point
- BH fully characterized by the number N
- BH mass $M_{BH} = \sqrt{N} M_P$, BH radius $R_{BH} = \sqrt{N} L_P$, entropy $S = N$
- Black hole physics $\rightarrow$ condensed matter physics

Black Hole N Portrait

Reproduce **semi-classical behavior** via mean-field approximation

$$N \to \infty \quad \text{and} \quad L_p \to 0 \quad \text{with} \quad \hbar \neq 0$$

Used to pinpoint **quantum origin** of semi-classical properties:

- Bekenstein entropy $\leftrightarrow$ quantum degeneracy of states at critical point
- Hawking radiation $\leftrightarrow$ quantum depletion and leakage of condensate

Can think about classicalization as **large N quantum physics**.

UV Completion, Classicalization, and the N portrait

Consequently: there are **two interconnected claims**:

- Einstein gravity is UV complete by classicalization (i.e. black hole formation)
- Black holes are a Bose-Einstein graviton condensate at a quantum critical point

Hence, in the language of classicalization and N portrait:

- Black hole formation process should correspond to graviton scattering

$$2 \to N \quad \text{with} \quad p_{in} \sim \sqrt{s} \quad \text{and} \quad p_{out} \sim \sqrt{s}/N \quad \text{with} \quad N \gg 1$$

- Moreover, black hole formation should be dominating

This Talk

Investigate the question of **UV completion** and **black hole formation** in (Einstein) gravity.

More precisely: Take **input from classicalization** and the N-**portrait**. Investigate high energy behavior of scattering amplitudes given this input using **recent amplitude developments**. $\rightarrow$ See also Massimo's talk

Questions of this talk:

- What does the N-portrait say about unitarity?
- How do FT and ST amplitudes behave at high energies?
- What do they know about unitarity? How is it implemented?
- What do amplitudes reveal regarding the N-portrait?
- What can we learn with respect to black hole formation?
- ...

This Talk

Plan of the talk:

1.) Non-perturbative input from the N-portrait

2.) Scattering amplitudes in FT and ST at high energies

3.) Interpretation of High Energy Behavior in light of N-portrait

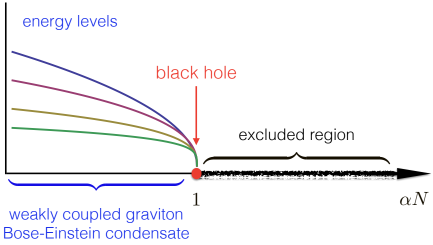
4.) Some Further Observations

1.) Non-perturbative Input from the N-portrait

Black Hole N Portrait: Regimes of αN

Different regimes of αN (i.e. the self coupling of the graviton condensate)

- $\alpha N = 1$ black hole formation: **exponential degeneracy of states** (N Bogolyubov modes become gapless) $\sim \exp\{cN\}$ with $c > 0$.
- $\alpha N < 1$ free graviton Bose gas: can be approximated by perturbative methods. **No exponential degeneracy.**
- $\alpha N > 1$ **unphysical region**: Excluded, not a viable $S - matrix$ state (Bogolyubov frequencies complex $\rightarrow$ positive Lyapunov exponents). Region where unitarity *would* be violated.



2.) Scattering amplitudes in FT and ST at high energies

Goal: Analyze graviton amplitudes in the kinematics presented in the previous section. For this talk: assume $D = 4$ and MHV configuration, i.e. two particles of negative helicity, rest positive. Can / will be dropped later.

High Energy Regimes

There are several high energy regimes for the Mandelstam $s_{ij} = (p_i + p_j)^2$. Each interesting in their own right. E.g. for four-point kinematics

$$s = (p_1 + p_2)^2 \quad t = (p_1 + p_3)^2 \quad u = (p_1 + p_4)^2$$

with $s + t + u = 0$.

- Regge Regime: $s \gg t \gg \Lambda$ with $|s/t| \gg 1$.
- Hard Scattering: $s, t \to \infty$ with $|s/t| \sim |s/u| \sim |u/t| = const$.
- Can be simplified further using so-called Eikonal constraints, i.e. setting as many Mandelstams to zero as possible.

Classicalization Regime

Energy regime in $2 \to N-2$ scattering according to classicalization corresponds to

$$p_{in} \sim \sqrt{s} \qquad \text{and} \qquad p_{out} \sim \frac{\sqrt{s}}{N-2}$$

$$\Rightarrow \quad s_{ij} = (p_i + p_j)^2 \sim \begin{cases} s, & \{i,j\} \in \{1,N\} \\ -\frac{s}{N-2}, & i \in \{1,N\},\ j \notin \{1,N\} \\ \frac{s}{(N-2)^2}, & \{i,j\} \notin \{1,N\} \end{cases}$$

Defined particles 1 and N incoming, $2, \ldots, N-1$ outgoing.

Graviton Scattering Amplitudes in Field Theory

N-point gravity scattering tree amplitudes accessible via Kawai-Lewellen-Tye (KLT) relations as "square of Yang-Mills amplitudes".

$$M_N = \left(-\frac{\kappa}{2}\right)^{N-2} \sum_{\sigma,\gamma \in S_{(N-3)}} A_N(1, \sigma(2, ..., N-2), N-1, N)$$
$$S[\gamma(2, ..., N-2), \sigma(2, ..., N-2)]_{N-1} A_N(1, N-1, \sigma(2, ..., N-2), N)$$

[Bern, Dixon, Perelstein, Rozowsky], [Bjerrum-Bohr, Damgaard, Sondergaard, Vanhove]

- $S[..., ...]$ called *momentum kernel*. Roughly $S \sim s_{ij}^{N-3}$
- $A_N(...)$ *color-ordered* Yang-Mills amplitude (here in $D = 4$).

$$A(1, 2, ..., i^-, ..., j^-, ..., N) = \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle ... \langle N-1N \rangle \langle N1 \rangle}$$

[Parke, Taylor]

- Spinor helicity brackets: $\langle ab \rangle = \sqrt{|s_{ab}|} \exp(i\phi_{ab})$

Graviton Scattering Amplitudes in Classicalization Regime

- Yang-Mills building blocks scale like (focus on one helicity config)

$$A_N(i^-, j^-) = f(\phi) \times s^{\frac{4-N}{2}} \times (N-2)^{N-4}$$

- Entries of momentum kernel scale $\sim \left(\frac{s}{(N-2)^2}\right)^{N-3}$.
- Thus gravity amplitude scales via KLT as

$$M_N(i^-, j^-) \sim \kappa^{N-2} \tilde{C}(N)\, s \times (N-2)^{-2}$$

with $\tilde{C}(N)$ double sum over phase factors. Hard to compute directly here.

Better way:

Use recent representation of gravity amplitude based on **scattering equations** in D-dim. Gives same result and fixes $\tilde{C} = (N-1)!$.

Graviton Scattering Amplitudes in Classicalization Regime

To obtain the physical probability i.e. the S-matrix element, have to consider

$$d|\langle 2|S|N-2\rangle|^2 \sim \frac{1}{(N-2)!} \prod_{i=2}^{N-1} dp_i^4 |M_N|^2 \delta^4(P_{total})$$

(Full cross section by integrating over momenta and summing over helicities)

Plugging in classicalization regime gives (taking $N \gg 1$, $\kappa = L_P$, and Stirling's formula)

$$\boxed{|\langle 2|S|N\rangle|^2 \sim \left(\frac{L_P^2 s}{N^2}\right)^N N! \sim \exp(-N)\ \lambda^N}$$

Define $\lambda = \frac{L_P^2 s}{N}$ for later convenience (collective coupling).

Closed String Amplitudes

Known: High Energy behavior of open and / or closed string amplitudes given by exponential fall-off. [Veneziano], [Gross, Mende], [Gross, Manes]
Thus no problem with unitarity at transplanckian energies.

Example: 4-point closed string amplitude for $\alpha' \to \infty$

$$\mathcal{M}_4 \sim \kappa^2 |A_4|^2 \times 4\pi\alpha' \, \frac{st}{u} \, \exp\left\{ \frac{\alpha'}{2} (s \ln |s| + t \ln t + u \ln u) \right\}$$

Note: State-of-the-art until our paper came out!

- Computation via Laplace's *saddle point method* on world-sheet integrals.
- High energy limit connected to recent work by [Cachazo, He, Yuan] and others on *scattering equations* in FT.

High Energy Behavior of N-point Closed-String Amplitudes

State-of-the-art way of writing a closed string amplitude in a KLT-like fashion presented by [Stieberger], [Stieberger, Taylor] recently.

$$\mathcal{M}_N = (-1)^{N-3}\kappa^{N-2}A^t_{YM}S_0 sv(\mathcal{A})$$

- $\mathcal{A}$ is an $(N-3)!$-dim. vector of the indep. open string amplitudes

$$\mathcal{A}_N(1,\pi(2,\ldots,N-2),N-1,N) = g^2_{YM}\sum_{\sigma\in S_{N-3}} F_{\pi\sigma}A_{YM}(\sigma)$$

  $\pi \in S_{N-3}$ and F is a function involving momentum kernel S and the world-sheet integrals.
- S_0 is a $(N-3)! \times (N-3)!$ matrix.
- sv - single-valued map. See [Stieberger] for more details.

High Energy Behavior of N-point Closed-String Amplitudes

To simplify use Eikonal constraints (all $s_{ij} = 0$ when $j \neq i + 1$ and except when one index is 1 or N). Then the gravity amplitude becomes just a one-term expression:

$$\mathcal{M}_N = \kappa^{N-2} |A_{YM}(1, 2, ..., N)|^2 M_N$$

- M_N is the string form factor. Comprises all "stringy" physics.

  $$M_N = (-1)^{N-3} \sigma_N sv(F_N)$$

  with σ_N rational function of Mandelstams of degree $N - 3$ and

  $$F_N = \frac{\Gamma(1 + \alpha' s_{12})\Gamma(1 + \alpha' s_{23})}{\Gamma(1 + \alpha' s_{12} + \alpha' s_{23})} \prod_{l=1}^{N-4} \frac{\Gamma(1 + \alpha' x_l)\Gamma(1 + \alpha' y_l)}{\Gamma(1 + \alpha' x_l + \alpha' y_l)}$$

  with x_l, y_l sum over Mandelstams.

Γ function in F_N can be expanded using Stirling's approximation

$$\Gamma(az+b) \sim \sqrt{2\pi} \exp\left\{(az+b-1/2)\, ln(az) - az\right\}$$

for $|arg(z)| < \pi, a > 0$.

String form factor at high energies then given by

$$M_N \sim (4\pi\alpha')^{N-3} \frac{s_{12}s_{23}}{s_{2N}} \exp\left\{\frac{\alpha'}{2}(s_{12}\ln s_{12} + s_{23}\ln s_{23} + s_{2N}\ln s_{2N})\right\}$$
$$\times \prod_{l=1}^{N-4} \frac{x_l y_l}{-x_l - y_l} \exp\left\{\frac{\alpha'}{2}(x_l \ln x_L + y_l \ln y_l - (x_l + z_l)\ln|x_l + y_l|)\right\}$$

NB: Similar computations done for the open N-point string amplitude in the paper.

3.) Interpretation of High Energy Behavior

Interpretation of High Energy Behavior in light of N-portrait & Classicalization

Field theory result:

$$|\langle 2|S|N\rangle|^2 \sim \left(\frac{\lambda}{N}\right)^N N! \sim \exp(-N)\ \lambda^N \quad , \qquad \lambda \equiv \alpha N = \frac{L_P^2 s}{N}$$

- Remember that in N-portrait: $\lambda = \alpha N$ and $\alpha N > 1$ not allowed (unitarity violation).
- At $\lambda = 1$, amplitude $\sim \exp\{-N\}$ but has to be supplemented by bh degeneracy of states $\Rightarrow$ compensation

$$|\langle 2|S|N\rangle|^2 \sim \left(\frac{1}{N}\right)^N N! \times \exp cN \sim \mathcal{O}(1)$$

- Close to $\lambda \lesssim 1$, degeneracy of states still countable, but another suppression $\sim \lambda^N$ factor which is not compensated for.
- $\Rightarrow$ dominance of black hole final states over other possible multi-particle final states.

Interpretation of High Energy Behavior in light of N-portrait & Classicalization

Field theory result:

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- Behavior of large $\sqrt{s}$ smoothened out if N increases appropriately $\Rightarrow$ core idea of classicalization.
- Smoothing out starts at $N = sL_P^2$. "Unitarity threshold for given s".
- In N-Portrait this is exactly entropy of a BH of mass $\sqrt{s}$
- Everything above unitarity threshold excluded by corpuscular picture (by black hole formation)

Interpretation of High Energy Behavior in light of N-portrait & Classicalization

- $\frac{\sqrt{s}}{N} > M_S$ (stringy regime): Exponential fall-off

$$\mathcal{M}_N \sim \kappa^{N-2}\alpha'^{N-3} s \exp\{\frac{\alpha'}{2}(N-3)s\ln(\alpha' s)\}$$

- $\frac{\sqrt{s}}{N} < M_S$ (FT regime): String and Field amplitudes become identical

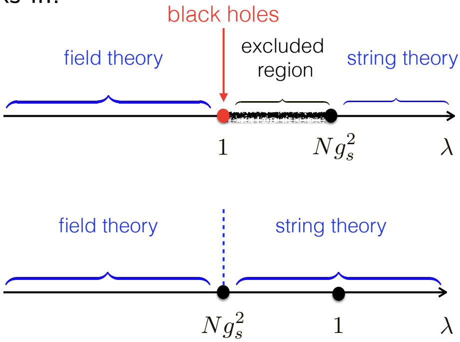
$$\mathcal{M}_N = M_N^{FT}$$

conjectured by [O'Connell, Wecht] for MHV configuration up to 5 points.

Interpretation of High Energy Behavior in light of *N*-portrait & Classicalization

Furthermore: Planck and string length related. Identify two more regimes:

- $\lambda = g_s^2 N > 1$: string effects relevant where outgoing gravitons strongly coupled. Does it tame unitarity violation in FT?
- $\lambda = g_s^2 N < 1$: string effects become relevant *before* black hole formation kicks in.



4.) Some further observations (and speculations...)

On the point $g_s^2 N = 1$

Threshold of string effects matches field theoretical critical point of black hole formation.

$$g_s = \frac{1}{\sqrt{N}}$$

- point where string coupling of constituent quanta becomes equally important as gravitational coupling
- corresponds to string-black hole correspondence,i.e.
  black hole state $\sim$ state of strings and D-branes with same charges

[Horowitz, Polchinski], [Dvali, Gómez], [Dvali, Lüst]

On $GR = YM^2$

Gravity amplitudes can be expressed as sum over Yang Mills amplitudes squared. Known for a long time, basis for many developments like recent study of UV properties of $\mathcal{N} = 8$ by [Bern et al] up to 5 loops.

- But: never used at any point information about color of Yang-Mills N_c
- Connection closed string open string coupling:

$$g_s = g_{open}^2$$

- At point of string-bh correspondence:

$$g_s = \frac{1}{\sqrt{N}}$$

- ['t Hooft]:

$$g_{open}^2 = \frac{1}{N_c}$$

Thus naively: $N = N_c^2$ Interpretation?

Summary

- Studied high energy behavior of graviton amplitudes at tree level.
- Established connection between transplanckian scattering amplitudes in FT and ST and unitarization by BH formation (classicalization).
- Used classicalization and the BH corpuscular N portrait as a guide.

Findings in Field theory:

- Identify microscopic reason of BH dominance over other final states.
- Find that high-energy behavior of graviton FT amplitudes becomes smoothened out when number N of produced gravitons is increased.
- Unitarity threshold at $N = sL_P^2$ for given s. Corresponds in N portrait to BH of mass $\sqrt{s}$.
- Strong coupling regime excluded by corpuscular arguments.

Summary

- Studied high energy behavior of graviton amplitudes at tree level.
- Established connection between transplanckian scattering amplitudes in FT and ST and unitarization by BH formation (classicalization).
- Used classicalization and the BH corpuscular N portrait as a guide.

Findings in String theory:

- Closed expressions for tree-level N-point open and closed string scattering at high energies.
- Beautiful connection to recent developments in FT (*scattering equations*) – more in the paper.
- Could identify two regimes
  - $\frac{\sqrt{s}}{N} < M_s$: String amplitudes agree with FT amplitudes
  - $\frac{\sqrt{s}}{N} > M_s$: String effects become important

Summary

- Could identify interplay between corpuscular physics, black hole formation, field theory regime and string regime.

- Amplitudes reveal key features of the N portrait; perturbative amplitudes already seem to know about non-perturbative physics.

Outlook

- Make more precise the role of color in "$GR = YM^2$"?
- Further: implications for kinematic group-theoretic structures in gravity? Some very preliminary steps already in [Boels, RSI].
- Implications along the lines of AdS/CFT?
- Beyond tree level in light of classicalization and N-portrait? First steps in [Kuhnel, Sandborg].
- Next: High energy behavior of amplitudes including gluons? Take as inspiration [Dvali, Gómez, Lüst] and [Stieberger] (ST) or [Cachazo, He, Yuan] in (FT) – *work in progress.*

Your Questions Here?