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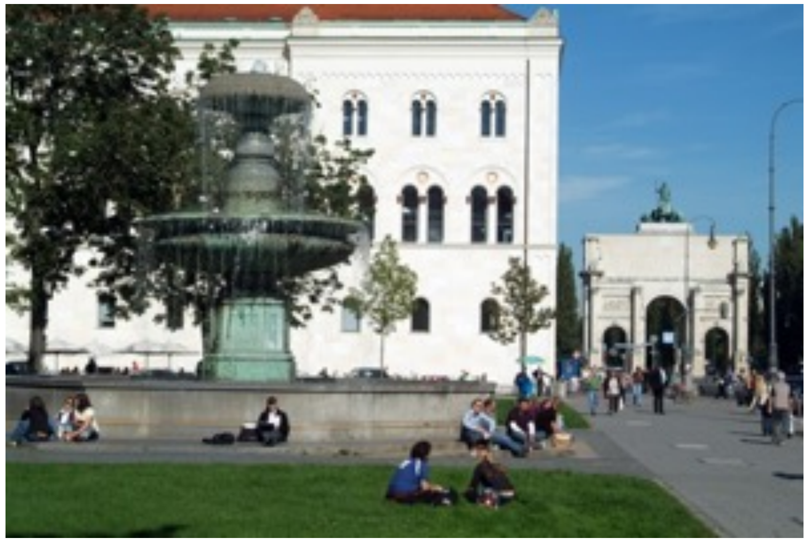

Special Geometry and Born-Infeld Attractors

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Summary of the talk

- *Nilpotent Superfields in Superspace*
- *Applications to Rigid and Local Supersymmetry*
- *Partial Supersymmetry Breaking in Rigid N=2 Theories*
- *Emergence of Volkov-Akulov and Born-Infeld Actions*
- *Symplectic Structure and Black-Hole Attractors: analogies and differences*
- *New $U(1)^n$ Born-Infeld Actions and Theory of Invariant Polynomials*

Some of the material of this presentation originates from some recent work with Antoniadis, Dudas, Sagnotti; Kallosh, Linde and some work in progress with Porrati, Sagnotti

The latter introduces a generalization of the Born-Infeld Action for an arbitrary $U(1)^n$ N=2 supergravity with N=2 self-interacting vector multiplets

NILPOTENCY CONSTRAINTS IN SPONTANEOUSLY BROKEN N=1 RIGID SUPERSYMMETRY

(Casalbuoni, De Curtis, Dominici, Ferruglio, Gatto; Komargodski, Seiberg; Rocek; Lindstrom, Rocek)

X chiral superfield $(\bar{\mathscr{D}}_{\dot\alpha} X = 0)$ nilpotency: $X^2 = 0$

solution: $X = \frac{GG}{F_G} + i\sqrt{2}\theta\, G + \theta^2\, F_G$ (G Weyl fermion)

Lagrangian: $\mathrm{Re}\, X\, \bar{X}\Big|_D + f\, X\Big|_F$

Equivalent to Volkov-Akulov goldstino action

More general constraints (Komargodski,Seiberg)

$X^2 = 0\,, \quad XY = 0$ independent fields $\psi_X = G\,, \quad \psi_Y$

$XW_\alpha = 0$ independent fields $\psi_X = G\,,\ F_{\mu\nu}\,,\ \lambda = f(\psi_X, F_{\mu\nu})$

NILPOTENT CONSTRAINTS IN LOCAL SUPERSYMMETRY (SUPERHIGGS EFFECT IN SUPERGRAVITY)

Pure supergravity coupled to the Goldstino

$$W(X) = f\,X + m$$

theory of a massive gravitino coupled to gravity
and cosmological constant: $\Lambda = |f|^2 - 3m^2$

(Deser, Zumino; Rocek; Antoniadis, Dudas, Sagnotti, S.F.)

Volkov-Akulov-Starobinsky supergravity

(a linear (inflaton) multiplet T, and a non-linear Goldstino multiplet S, with $S^2=0$)

$$V = \frac{M^2}{12}\left(1 - e^{-\sqrt{\frac{2}{3}}\phi}\right)^2 + \frac{M^2}{18}\,e^{-2\sqrt{\frac{2}{3}}\phi}\,a^2$$

Starobinsky inflaton potential ... axion field

Recently, nilpotent super fields have been used in more general theories of inflation

(Kallosh, Linde, S.F.; Kallosh, Linde)

For a class of models with SUPERPOTENTIAL

$$W = S\, f(T)$$

(Kallosh, Linde, Roest; Kallosh, Linde, Rube)

the resulting potential is a no-scale type, and has a universal form

$$V_{\rm eff} = e^{K(T)}\, K^{S\bar{S}}\, |f(T)|^2 > 0 \qquad \text{goldstino} \quad \psi_S$$

with $T_{\theta=0} = \varphi + i a\,,$ inflaton: φ or a

depending which one is lighter during inflation

GENERALITIES ON PARTIAL SUPERSYMMETRY BREAKING
GLOBAL SUPERSYMMETRY

(Witten; Hughes, Polchinski; Cecotti, Girardello, Porrati, Maiani, S.F.;
Girardello, Porrati, S.F.; Antoniadis, Partouche, Taylor)

$a = 1 \ldots N$ $\quad \delta\chi^a$ fermion variations; $\quad J^a_{\mu\alpha}(X)$ susy Noether current

Current algebra relation (Polchinski)

$$\int d^3y \, \{\bar{J}_{0\dot{\alpha}b}(y), J^a_{\mu\alpha}(x)\} = 2\sigma^{\nu}_{\alpha\dot{\alpha}} T_{\mu\nu}(x)\delta^a_b + \sigma_{\mu\alpha\dot{\alpha}} C^a_b$$

Relation to the scalar potential:

$$\delta\chi^a \, \delta\bar{\chi}_b = V \, \delta^a_b + C^a_b$$

In N=2 (APT, FGP)

$$C^a_b = \sigma^{xa}{}_b \epsilon_{xyz} Q^y \wedge Q^z = 2\sigma^{xa}{}_b (\vec{E} \wedge \vec{M})_x \, ; \qquad Q^x = \begin{pmatrix} M^x \\ E^x \end{pmatrix}$$

In Supergravity (partial SuperHiggs) (CGP, FM, FGP)
there is an extra term in the potential which allows
supersymmetric anti-de-Sitter vacua

$$\delta\chi^a \, \delta\bar{\chi}_b = V \, \delta^a_b + 3\mathcal{M}^{ac} \bar{\mathcal{M}}_{bc}$$

$\mathcal{M}_{ab} = \mathcal{M}_{ba}$ *"gravitino mass" term*

$$\delta\psi^a_\mu = D_\mu \epsilon^a + \mathcal{M}^{ab} \gamma_\mu \bar{\epsilon}_b$$

N=2 *RIGID (SPECIAL) GEOMETRY*

$$R_{i\bar{j}k\bar{l}} = C_{ikp}\,\bar{C}_{\bar{j}\bar{l}\bar{p}}\,g^{p\bar{p}} \qquad\qquad V = \left(X^A\,,\,\frac{\partial\mathcal{U}}{\partial X^A}\right)$$

$$\partial_{\bar{\imath}}V = 0\,, \quad \mathscr{D}_j\partial_i V = C_{jik}\,g^{k\bar{k}}\,\partial_{\bar{k}}\,\bar{V}\,, \quad \partial_{\bar{\jmath}}\partial_i V = 0$$

$$C_{ikp} = \partial_i\partial_k\partial_p\mathcal{U} = \mathcal{U}_{ikp}$$

$$\mathcal{U} = X^2 + \frac{X^3}{M} + \ldots + \frac{X^{n+2}}{M^n}$$

for M large: $\mathcal{U} - X^2 \sim \dfrac{X^3}{M}$ $\qquad M\,\mathcal{U}_{ABC} = d_{ABC}$

N=2 rigid supersymmetric theory with Fayet-Iliopoulos terms (n vector multiplets, no hypermultiplets) in N=1 notations

Kahler potential: $K = i(X^a \bar{\mathcal{U}}_A - \bar{X}^A \mathcal{U}_A)\,, \quad \mathcal{U}_A = \frac{\partial \mathcal{U}_A}{\partial X^A}$

($\mathcal{U}$: N=2 prepotential)

Fayet-Iliopoulos terms:
triplet of (real) symplectic (Sp(2n)) constant vectors

$$\vec{Q} = (\vec{M}^A, \vec{E}_A) = (Q_c, Q_3) = \begin{pmatrix} m_1^A + i\,m_2^A\,, \; e_{1A} + i\,e_{2A} \\ m_3^A \quad , \quad e_3^A \end{pmatrix}$$

$(A = 1, \ldots, n)$

Superpotential: $W = (\mathcal{U}_A\, m^A - X^A\, e_A)$ $Q_c = (m^A, e_A)$

D term N=1 F-I *magnetic and electric charges:* $Q_3 = (\epsilon^A, \xi_A)$

Due to the underlying symplectic structure of N=2 rigid special geometry, we can rewrite all expressions by using the symplectic metric

$\Omega = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ *and symplectic sections* $V = (X^A, \mathcal{U}_A)$

$$W = (V, Q) = V\,\Omega\,Q\,, \qquad K = -i(V, \bar{V})$$

$$V_F = (\mathrm{Im}\,\mathcal{U}^{-1}_{AB})\,\frac{\partial W}{\partial X^A}\,\frac{\partial \bar{W}}{\partial \bar{X}^B} = \bar{Q}_c^T\,\mathcal{M}\,Q_c + i\,(\bar{Q}_c, Q_c)$$

$$V_D = Q_3^T \, \mathcal{M} \, Q_3 \quad \textit{so that}$$

$$V = V_F + V_D = \bar{Q}_c^T \, (\mathcal{M} + i\Omega) \, Q_c + Q_3^T \, \mathcal{M} \, Q_3$$

The matrix $\mathcal{M}$ is a real (positive definite) symmetric and symplectic $2n \times 2n$ matrix

$$\mathcal{M}^T = \mathcal{M} \qquad \mathcal{M} \, \Omega \mathcal{M} = \Omega \qquad \mathcal{M} > 0$$

It is related to the F^2 , $F \tilde{F}$ terms in the Lagrangian

$$\mathscr{L} = g_{AB}(X) F^A_{\mu\nu} F^{B\,\mu\nu} + \theta_{AB}(X) \, F^A_{\mu\nu} \tilde{F}^{B\,\mu\nu}$$

$$\mathcal{M} = \begin{pmatrix} g + \theta g^{-1} \theta & -\theta g^{-1} \\ -g^{-1} \theta & g^{-1} \end{pmatrix}$$

N=1 supersymmetric vacua require (Porrati, Sagnotti, S.F.)

$$\frac{\partial W}{\partial X^A} = 0\,, \qquad V_D = 0 \quad \text{since} \quad \mathcal{M} > 0$$

The $V_D = 0$ condition requires $Q_3 = 0$

The first equation implies

$$(\mathcal{M} + i\Omega)\, Q_c = 0 \quad \text{which requires} \quad i\bar{Q}_c\, \Omega\, Q_c < 0$$

Since $\bar{Q}_c\, \mathcal{M}\, Q_c$ is positive definite, and at the attractor point we have

$$\bar{Q}_c\, \mathcal{M}_{\text{crit}}\, Q_c = -i\bar{Q}_c\, \Omega Q_c$$

So it is crucial that Q_c is complex. To simplify, we will later consider m^A real and e_A complex, so that the previous condition is $m_1^A\, e_{2A} < 0$

The theory here considered is the generalization to n vector multiplets of the theory considered by Antoniadis, Partouche, Taylor (1995)

Later, this theory (n=1) was shown to reproduce in some limit (Rocek, Tseytlin, 1998) the supersymmetric Born-Infeld action (Cecotti, S.F., 1986). The latter was shown (Bagger, Galperin, 1996) to be the Goldstone action for N=2 partially broken to N=1 where the gauging $\lambda = W_\alpha\Big|_{\theta=0}$ is the Goldstone fermion of the second broken supersymmetry.

EXTREMAL BLACK HOLE ANALOGIES

In the case of asymptotically flat black holes, the so-called Black-Hole Potential for an extremal (single-center) black-hole solution is (Kallosh, S.F., 1996)

$$V_{\rm BH} = \frac{1}{2} Q^T \mathcal{M} Q \qquad\qquad Q = (m^A, e_A)$$

is the asymptotic black-hole charge

The theory of BH attractors (Kallosh, Strominger, S.F., 1995) tells that

$$V_{\rm BH}\Big|_{\rm crit} = \frac{1}{2} Q^c \mathcal{M}(X_{\rm crit})\, Q = \frac{1}{\pi} S(Q) = \frac{1}{4\pi} A_H$$

where $S(Q)$ is the BH entropy (Bekenstein-Hawking area formula)

In the N=2 case (Ceresole, D'Auria, S.F.; Gibbons, Kallosh, S.F.)

$$V_{\rm BH} = g^{i\bar{\jmath}}\, \mathscr{D}_i Z\, \mathscr{D}_{\bar{\jmath}} \bar{Z} + |Z|^2$$

so that at the BPS attractor point ($\mathscr{D}_i Z = 0$)

$$V_{\rm BH} = |Z_{\rm crit}|^2 = \frac{1}{\pi} S(Q)$$

In analogy to the N=2 partial breaking of the rigid case, the BPS black hole breaks N=2 down to N=1, and the central charge is the quantity replacing the super potential W.

The entropy and BI action are both expressed through W.

In our problem, the attractors occur at $V_{\text{crit}} = 0$,

(because of unbroken space time supersymmetry)

and this is only possible because the Fayet-Iliopoulos charge is an SU(2) triplet (charged in the F-term direction) and thus allows the attractor equation

$(\mathcal{M}_{\text{crit}} + i\Omega)\, Q_c = 0$ being satisfied.

We call these vacua Born-Infeld attractors,

for reasons will become soon evident

The superspace action of the theory in question is

$$\mathcal{L} = \mathrm{Im}\, \left(\mathcal{U}_{AB}\, W_\alpha^A\, W_\beta^B\, \epsilon^{\alpha\beta} + W(X) \right)\Big|_F + \left(X^A\, \bar{\mathcal{U}}_A - \bar{X}^A\, \mathcal{U}_A \right)\Big|_D$$

The Euler-Lagrange equations for X^A are

$$\mathcal{U}_{ABC}\, W^B\, W^C + \mathcal{U}_{AB}\, (m^B - \bar{D}^2\, \bar{X}^B) - e_A + \bar{D}^2\, \bar{\mathcal{U}}_A = 0$$

These are the complete X^A equations for the theory in question.
The first thing to note is that our attractor
gives a mass to the N=1 chiral multiplet X^A,
but not to the N=1 vector multiplets W_α.
So N=2 is broken.

Indeed, our action is invariant under a second supersymmetry η^{α}, which acts on the N=1 chiral super fields (X^A, W^A_α) (Bagger, Galperin n=1)

$$\delta X^A = \eta^\alpha W^A_\alpha\,,$$
$$\delta W^A_\alpha = \eta_\alpha(m^A - \bar{D}^2 \bar{X}^A) - i\partial_{\alpha\dot{\alpha}} X^A \bar{\eta}^{\dot{\alpha}}$$

and because of the m^A parameter, the second supersymmetry is spontaneously broken.

Note that the m^A, e_{2A} parameters are those which allow the equations

$$\frac{\partial W}{\partial Z^A} = \left(\mathcal{U}_{AB} m^B - e_A\right) = 0$$ to have solutions

Expanding the fields around their "classical" value cancel the linear terms in the action and we obtain (Porrati, Sagnotti, S.F.)

$$\frac{\delta \mathscr{L}}{\delta X^A} = 0 \quad \Rightarrow \quad d_{ABC}\left[W^B W^C + X^B(m^C - \bar{D}^2\,\bar{X}^C)\right] + \bar{D}^2\,\bar{\mathcal{U}}_A = 0$$

The BI approximation corresponds to have $\bar{D}^2\bar{\mathcal{U}}_A = 0$, (which we solve letting $\mathcal{U}_A = 0$ as operator condition)

The N=2 Born-Infeld generalized lagrangian turns out to be (Porrati, Sagnotti, S.F.)

$$\mathscr{L}_{\rm BI}^{N=2} = -{\rm Im}\, W(X)\Big|_{\theta^2} = {\rm Re}\, F^A e_{2A} + {\rm Im}\, F^A e_{1A}$$

with the chiral superfields X^A solutions of the above constraints

The super field BI constraint allows to write the n chiral fermions in terms of the n gauginos, so λ_g is a linear combination of the n (dressed) gauginos

In Black-Hole physics, the superpotential is the N=2 central charge, and

$$S_{\text{entropy}} = \pi |Z|^2_{\text{crit}}$$

SOLUTION OF THE BORN-INFELD ATTRACTOR EQUATIONS

For n=1, the above equation is the BI constraint

$$X = \frac{-W^2}{m - \bar{D}^2 \bar{X}}$$

(electric magnetic self-dual BG inherited from the linear theory)

which also implies the nilpotency
(Komargodsky, Seiberg; Casalbuoni et al)

Type of constraints $$X\, W_\alpha = 0\,, \qquad X^2 = 0$$

This type of constraints have been recently used in inflationary supergravity dynamics to simplify and to provide a more general supergravity breaking sector

(Kallosh, Linde, S.F.; Kallosh, Linde; Antoniadis, Dudas, Sagnotti, S.F.)

The Born-Infeld action comes by solving the θ^2 component of the chiral superfield equation

$$\left.\frac{\partial \mathscr{L}}{\partial X^A}\right|_{\theta^2} \quad \Rightarrow \quad d_{ABC}\left[G_+^B G_+^C + F^B(m^C - \bar{F}^C)\right] = 0$$

$$G_\pm^A = F_{\mu\nu}^A \pm \frac{i}{2}\tilde{F}_{\mu\nu}^A$$ and we have set $D^A = 0$

by taking the real and imaginary parts of this equation we have

$$d_{ABC}\left(H^B + \frac{m^B}{2}\right)\left(\frac{m^C}{2} - H^C\right) = d_{ABC}\left(-G^B G^C + \mathrm{Im}\, F^B\, \mathrm{Im} F^C\right)$$

$$d_{ABC}\, \mathrm{Im}\, F^B\, m^C = -d_{ABC}\, G^B\, \tilde{G}^C$$

and we have set $\mathrm{Re}\, F^A = \dfrac{m^A}{2} - H^A$

CLASSIFICATION OF d_{ABC}.

THEORY OF INVARIANT POLYNOMIALS

(Mumford, Gelfand, Dieudonné, et al)

$$\mathcal{U}(X) = \frac{1}{3!} d_{ABC} X^A X^B X^C$$

n=2 case: d_{ABC} (4 entries) spin 3/2 representation of SL(2)

It has a unique (quartic) invariant which is also the discriminant of the cubic (Cayley hyperdeterminant)

(Duff, q-bit entanglement in quantum information theory)

$$I_4 = -27\, d_{222}^2\, d_{111}^2 + d_{221}^2\, d_{112}^2 + 18 d_{222} d_{111} d_{211} d_{221} - 4 d_{111} d_{122}^3 - 4 d_{222} d_{211}^3$$

CLASSIFICATION OF d_{ABC}.

THEORY OF INVARIANT POLYNOMIALS

(Mumford, Gelfand, Dieudonné, et al)

$$I_4 = -27\, d_{222}^2\, d_{111}^2 + d_{221}^2\, d_{112}^2 + 18 d_{222} d_{111} d_{211} d_{221} - 4 d_{111} d_{122}^3 - 4 d_{222} d_{211}^3$$

Four orbits $I_4 > 0\,,\ I_4 < 0\,,\ I_4 = 0\,,\ \partial I_4 = 0$

Two of them ($I_4 < 0\,,\ \partial I_4 = 0$) give trivial models.

The other two ($I_4 \geq 0$) give non-trivial $U(1)^2$ BI theories

Extremal black-hole analogy:

the model in question corresponds to the T^3 model,

$\sqrt{|I_4|}$ is the BH Berenstein-Hawking entropy and the four orbits correspond to large BPS and non-BPS as well as small BH

An example: Explicit solution of the $I_4>0$ theory

$$\mathcal{U} = \frac{1}{3!}X^3 - \frac{1}{2}XY^2 \qquad (d_{111} = 1\,,\; d_{122} = -1)$$

$$\mathrm{Im}\,F_X = (m_X^2 + m_Y^2)^{-1}(m_X R_X + m_Y R_Y) \qquad R_X = -2G^X\tilde{G}^Y$$

$$\mathrm{Im}\,F_Y = (m_X^2 + m_Y^2)^{-1}(-m_Y R_X + m_X R_Y) \qquad R_Y = -G^X\tilde{G}^X + G^Y\tilde{G}^Y$$

$$H_X = \frac{1}{\sqrt{2}}\left(\sqrt{S_X^2 + S_Y^2} - S_X\right)^{1/2} \qquad S_X = T_X - \frac{m_X^2}{4} + \frac{m_Y^2}{4}$$

$$H_Y = \frac{1}{\sqrt{2}}\left(\sqrt{S_X^2 + S_Y^2} + S_X\right)^{1/2} \qquad S_Y = T_Y + \frac{m_X m_Y}{2}$$

$$-H_X^2 + H_Y^2 = S_X\,, \qquad 2H_X H_Y = S_Y \qquad (H^A \text{ equation})$$

$$T_X = -G^X G^X + \mathrm{Im}\,F^X\,\mathrm{Im}\,F^X + G^Y G^Y - \mathrm{Im}\,F^Y\,\mathrm{Im}\,F^Y$$

$$T_Y = 2(G^X G^Y + \mathrm{Im}\,F^X\,\mathrm{Im}\,F^Y)$$

Note: for $G_{\mu\nu}^{X,Y} = 0 \rightarrow F_X = F_Y = 0$