NORDITA

The University of Iceland

Workshop on Fundaments of Gravity, April 12-16, 2010

Quantum critical points via gravity

Lárus Thorlacius

U. Danielsson & LT: *"Black holes in asympotically Lifshitz spacetime"*, JHEP **0903** (2009) 070

E. Brynjolfsson, U. Danielsson, LT, & T. Zingg:

"Holographic superconductors with Lifshitz scaling" - J. Phys. A: Math. Theor. **43** (2010) 065401
"Black hole thermodynamics and heavy fermion metals" - arXiv:1003.5361
"Fermionic quantum critical points via gravity" - in preparation

Applied AdS-CFT

Investigate strongly coupled quantum field theories via classical gravity

- growing list of applications:

- hydrodynamics of quark gluon plasma
- jet quenching in heavy ion collisions
- quantum critical systems
- holographic superconductors
- cold atomic gases
- ....

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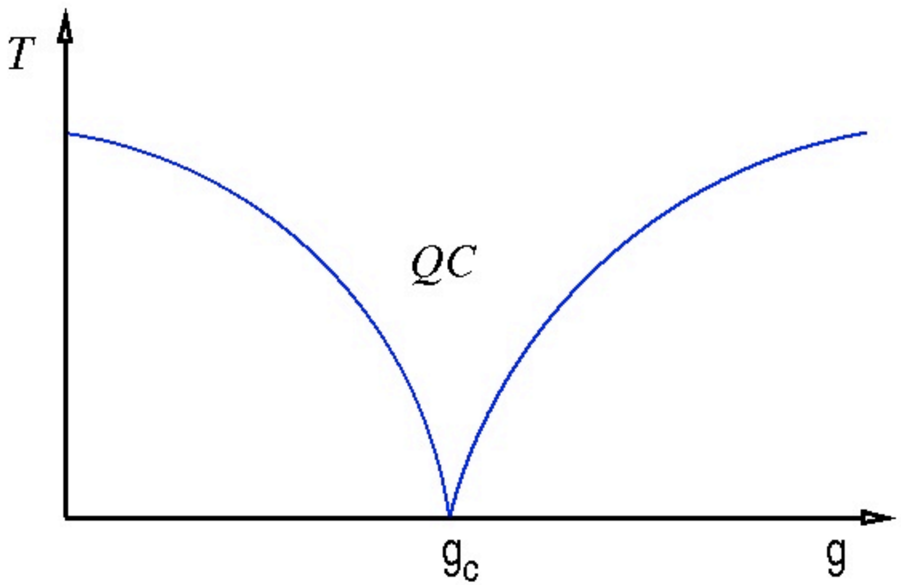
Some reviews

- D.T. Son and A. Starinets, *Viscosity, black holes, and quantum field theory*, Ann. Rev. Nucl. Part. Sci. **57** (2007) 95.
- M. Mueller and S. Sachdev, *Quantum criticality and black holes*, arXiv:0810.3005.
- C.P. Herzog, *Lectures on holographic superfluidity and superconductivity,* J. Phys. A: Math. Theor. **42** (2009) 343001.
- S.A. Hartnoll, *Lectures on holographic methods for condensed matter physics*, Class. Quant. Grav. **26** (2009) 224002.
- M. Rangamani, *Gravity and hydrodynamics: Lectures on the fluid-gravity correspondence*, Class. Quant. Grav. **26** (2009) 224003.
- J. McGreevy, *Holographic duality with a view toward many-body physics*, arXiv:0909.0518.
- G. Horowitz, *Introduction to holographic superconductors*, arXiv:1002.1722.
- S. Sachdev, *Condensed matter and AdS/CFT*, arXiv:1002.2947.

Quantum critical points



Typical behavior at $T = 0$

characteristic energy $\delta \sim (g - g_c)^{-z\nu}$

coherence length $\xi \sim (g - g_c)^{-\nu}$

$\delta \sim \xi^z$ z = dynamical scaling exponent

The quantum critical point at $g = g_c$ dominates the quantum critical region QC

$\delta < T$ inside *QC* region

Physical systems with z = 1, 2, and 3 are known

- non-integer values of *z* are also possible

If $z = 1$, then the scaling symmetry is part of SO(d+1,1) conformal group

= isometries of adS_{d+1}

Models with anisotropic scaling

$$t \to \lambda^z t, \qquad \mathbf{x} \to \lambda \mathbf{x}, \qquad z \geq 1 \qquad \mathbf{x} = (x_1, \ldots, x_d)$$

Example: Quantum Lifshitz model

$$L = \int d^2x dt \left((\partial_t \phi)^2 - K \left(\nabla^2 \phi\right)^2 \right)$$

$z = 2, \ d = 2$

Q: Can we give a gravity dual description of a strongly coupled system which exhibits anisotropic scaling?

Look for a gravity theory with spacetime metric of the form

$$ds^2 = L^2 \left(-r^{2z} dt^2 + r^2 d^2\mathbf{x} + \frac{dr^2}{r^2} \right)$$

which is invariant under

$$t \to \lambda^z t, \quad \mathbf{x} \to \lambda \mathbf{x}, \quad r \to \frac{r}{\lambda}$$

Dual gravity model

Kachru, Liu, & Mulligan '08; Taylor '08; Brynjolfsson et al. '09

$$S = S_{\text{Einstein-Maxwell}} + S_{\text{Lifshitz}} + S_{\text{matter}}$$

$$S_{\text{Einstein-Maxwell}} = \int \mathrm{d}^{d+2}x\sqrt{-g}\,\left(R - 2\Lambda - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right)$$

$$S_{\text{Lifshitz}} = -\int \mathrm{d}^{d+2}x\sqrt{-g}\,\left(\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \frac{c^2}{2}\mathcal{A}_\mu\mathcal{A}^\mu\right)$$

d = 3, 2, or 1 for CM applications

Field equations:
without matter

$$\begin{aligned} G_{\mu\nu} + \Lambda g_{\mu\nu} &= T^{\text{Maxwell}}_{\mu\nu} + T^{\text{Lifshitz}}_{\mu\nu} \\ \nabla_\nu F^{\nu\mu} &= 0 \\ \nabla_\nu \mathcal{F}^{\nu\mu} &= c^2\mathcal{A}^\mu \end{aligned}$$

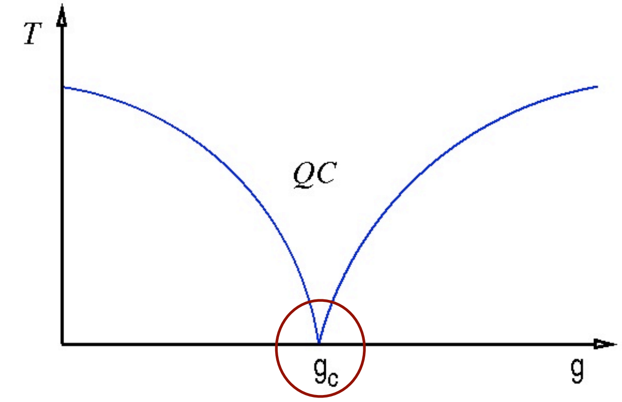
$$\begin{aligned} T^{\text{Maxwell}}_{\mu\nu} &= \frac{1}{2}\left(F_{\mu\lambda}F_\nu^{\ \lambda} - \frac{1}{4}g_{\mu\nu}F_{\lambda\sigma}F^{\lambda\sigma}\right) \\ T^{\text{Lifshitz}}_{\mu\nu} &= \frac{1}{2}\left(\mathcal{F}_{\mu\lambda}\mathcal{F}_\nu^{\ \lambda} - \frac{1}{4}g_{\mu\nu}\mathcal{F}_{\lambda\sigma}\mathcal{F}^{\lambda\sigma}\right) + \frac{c^2}{2}\left(\mathcal{A}_\mu\mathcal{A}_\nu - \frac{1}{2}g_{\mu\nu}\mathcal{A}_\lambda\mathcal{A}^\lambda\right) \end{aligned}$$

Lifshitz geometry

The Lifshitz metric

$\mathbf{x} = (x_1, \ldots, x_d)$

$$ds^2 = L^2 \left(-r^{2z} dt^2 + r^2 d\mathbf{x}^2 + \frac{dr^2}{r^2} \right)$$



is a solution for a particular choice of couplings

$$c = \frac{\sqrt{z\,d}}{L}, \qquad \Lambda = \frac{z^2 + (d-1)z + d^2}{2L^2}$$

and background fields

$$\mathcal{A}_t = \sqrt{\frac{2(z-1)}{z}} L\, r^z, \quad \mathcal{A}_{x_i} = \mathcal{A}_r = 0 \qquad\qquad A_\mu = 0$$

Gravity duals at finite temperature

periodic Euclidean time: $\tau \simeq \tau + \beta, \quad \beta = \frac{1}{T}$

ß introduces an energy scale: scale symmetry is broken

thermal state in field theory: black hole with $T_{\rm Hawking} = T_{\rm qft}$

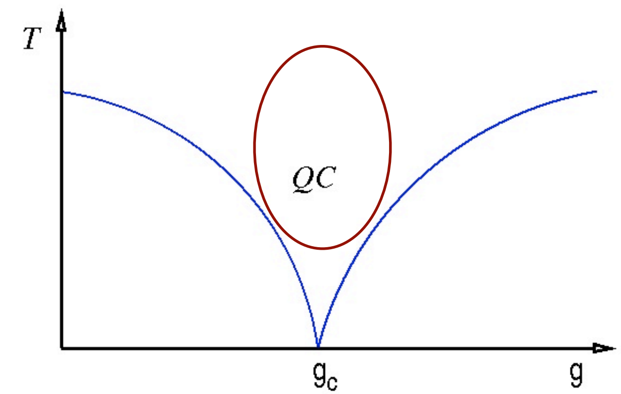
finite charge density in dual field theory: electric charge on BH

magnetic effects in dual field theory: dyonic BH

$z = 1$: consider Reissner-Nordström BH in d+2 dimensions

$z > 1$: look for charged BH's in d+2 dimensional $z > 1$ gravity model

Quantum critical region



Look for black hole solutions of gravity model

Metric: $$ds^2 = L^2\left(-r^{2z}f(r)^2dt^2 + r^2 d\mathbf{x}^2 + \frac{g(r)^2}{r^2}dr^2\right)$$

black brane

Vierbein: $$e_t^0 = Lr^z f(r)\,, \quad e_{x_i}^i = Lr\,, \quad e_r^{d+1} = L\frac{g(r)}{r}$$

Vector fields: $$A_M = (\alpha(r),0,0,0)\,, \qquad \mathcal{A}_M = \sqrt{\frac{2(z-1)}{z}}(a(r),0,0,0)$$

Field equations:

$d=2$

$$\begin{aligned}
\dot\alpha + \alpha\frac{\dot f}{f} &= -z\alpha + \tilde\rho e^{-2u} g \\
\dot a + a\frac{\dot f}{f} &= -za + zgb \\
\dot b &= -2b + 2ga \\
\dot g + \frac{\dot f}{f} g &= (z-1)\left(g^2a^2 - 1\right)g \\
\frac{\dot f}{f} &= \frac{g^2}{2}\left((z-1)a^2 - \frac{z(z-1)}{2}b^2 + \frac{(z^2+z+4)}{2} - \frac{\tilde\rho^2}{4}e^{-4u}\right) - \frac{(2z+1)}{2}
\end{aligned}$$

$$u \equiv \log\left(\frac{r}{r_0}\right), \quad \dot f \equiv \frac{df}{du}$$

$$\tilde\rho = \frac{\rho}{r_0^2}$$ black hole charge

Lifshitz geometry: $$f = g = a = b = 1, \qquad \tilde\rho = \alpha = 0$$

Black brane solutions

event horizon: $u = 0$

asymptotic region: $u \to \infty$

U.Danielsson & L.T. '09
E.Brynjolfsson, U.Danielsson, L.T., T. Zingg '09
R.Mann '09; G.Bertoldi, B.Burrington, & A.Peet '09

Known exact solutions:

AdS-Reissner-Nordström: $d = 2, \; z = 1$

$$f^2 = \frac{1}{g^2} = (1 - e^{-u})(1 + e^{-u} + e^{-2u} - \frac{\tilde{\rho}^2}{4} e^{-3u}), \quad A_t = L r_0 \tilde{\rho}\,(1 - e^{-u}), \quad \mathcal{A}_\mu = 0$$

$d = 2, \; z = 4, \; \tilde{\rho} = \pm\sqrt{8}$:

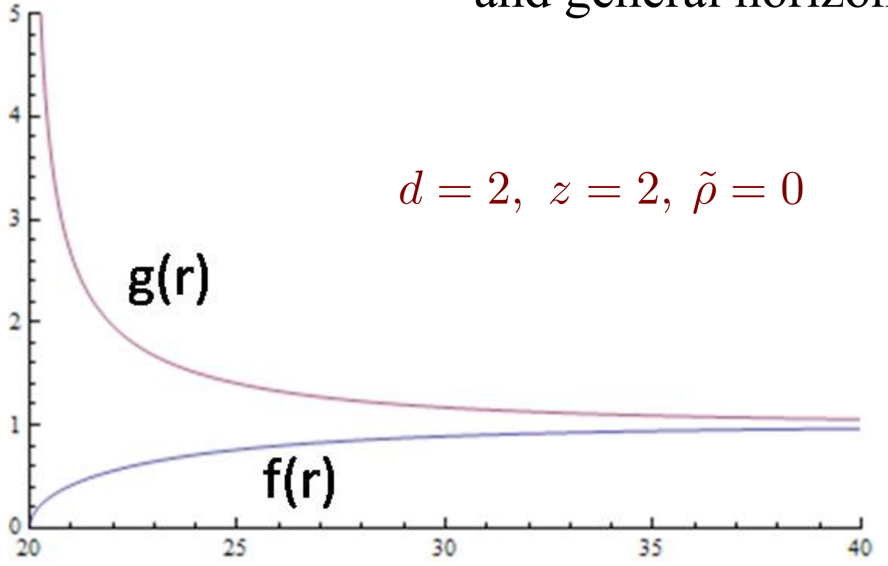
$$f^2 = \frac{1}{g^2} = a^2 = 1 - e^{-4u}, \quad b = 1, \quad A_t = \pm\sqrt{2} L r_0^4\,(e^{2u} - 1)$$

can be generalized to black hole solutions of spherical or hyperbolic horizon topology

can be generalized to solution with $z = 2d$
D.Pang '09

AdS-RN solution exists for any integer $d \geq 1$

Numerical solutions can be found for any d and z and general horizon topology



Near horizon behavior

Smooth horizon requires

$$
\begin{aligned}
f(u) &= f_0\sqrt{u}(1+f_1u+f_2u^2+\ldots)\\
g(u) &= \frac{g_0}{\sqrt{u}}(1+g_1u+g_2u^2+\ldots)\\
\alpha(u) &= \alpha_0\sqrt{u}(1+\alpha_1u+\alpha_2u^2+\ldots)\\
a(u) &= a_0\sqrt{u}(1+a_1u+a_2u^2+\ldots)\\
b(u) &= b_0(1+b_1u+b_2u^2+\ldots)
\end{aligned}
$$

The field equations imply

$d=2 \longrightarrow$

$$
g_0=\frac{1}{\sqrt{D}},\qquad \alpha_0=\frac{\tilde\rho}{\sqrt{D}},\qquad a_0=\frac{zb_0}{\sqrt{D}},
$$

$$
b_1=\frac{2z}{D}-2,\qquad f_1=\frac{1-4z}{4}+\frac{2z(z-1)+\tilde\rho^2}{4D}+\frac{z^2(z-1)}{4D^2},
$$

$$
\vdots
$$

$\downarrow$

with

$$
D=\frac{z^2+z+4}{2}-\frac{\tilde\rho^2}{4}-\frac{z(z-1)}{2}b_0^2
$$

Two parameter family of black brane solutions? $(\tilde\rho\,,\ b_0)$

b_0 turns out to be determined by global considerations $\longrightarrow$ one parameter family $\tilde\rho$

Asymptotic behavior $d=2$

Linearize e.o.m.'s around Lifshitz point: $g = 1 + \delta g \quad b = 1 + \delta b \quad a = 1 + \delta a$

$$\frac{d}{du}\begin{bmatrix}\delta g\\ \delta b\\ \delta a\end{bmatrix} = M(z)\begin{bmatrix}\delta g\\ \delta b\\ \delta a\end{bmatrix} + \frac{\tilde\rho^2}{8}e^{-4u}\begin{bmatrix}1\\0\\1\end{bmatrix}$$

reduced system involving (g,b,a)

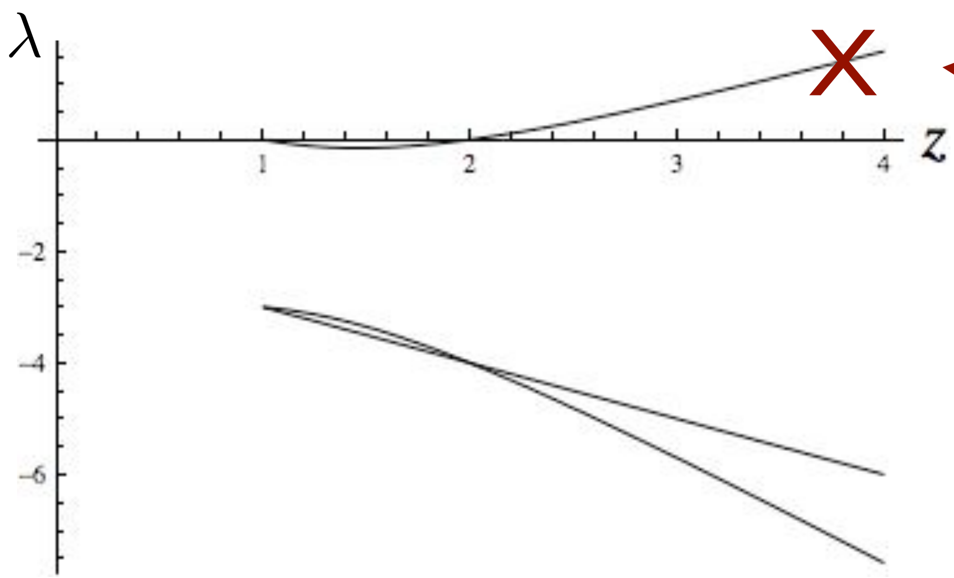
$$\mathcal{M}(z) = \begin{bmatrix} -z-1 & \frac{z(z+1)}{2} & -2z+1 \\ 2 & -2 & 2 \\ -3 & \frac{z(z-1)}{2} & z-1 \end{bmatrix}$$

$$\lambda = \begin{cases} -2-z \\ \frac{1}{2}\left(-2-z \pm \sqrt{9z^2-20z+20}\right) \end{cases}$$

Solve eigenvalue problem: $M(z)\,\vec{F}_i = \lambda_i\,\vec{F}_i$

$$\begin{bmatrix}\delta g\\ \delta b\\ \delta a\end{bmatrix} \approx \sum_{i=1}^{3}\alpha_i e^{\lambda_i u}\vec{F}_i + \frac{\tilde\rho^2}{8}e^{-4u}\vec{F}_0$$

eigenmodes universal mode



Asymptotically Lifshitz spacetime U.Danielsson &LT '09

Finite energy G.Bertoldi et al. '09

S.Ross & O.Saremi '09

Asymptotic behavior of Maxwell field:

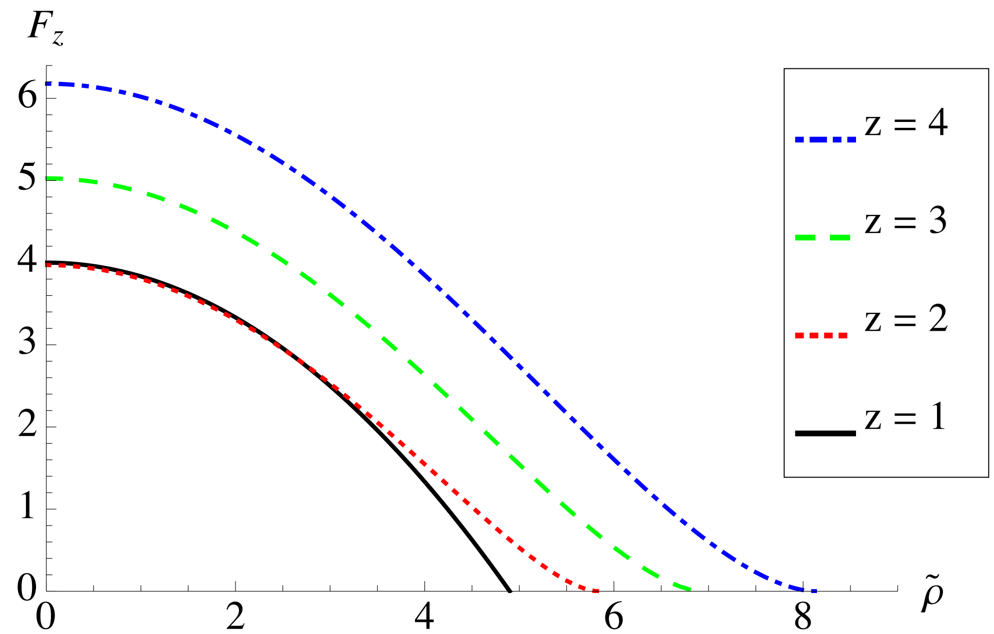
$$\alpha \approx \begin{cases} \tilde\mu\, e^{-zu} + \frac{\tilde\rho}{(z-2)}e^{-2u} & \text{if} \quad z \neq 2 \\ \tilde\mu\, e^{-2u} + \tilde\rho\, u\, e^{-2u} & \text{if} \quad z = 2 \end{cases}$$

Black hole thermodynamics $d = 3$

Hawking temperature:

$$T_H = \frac{r_0^z}{4\pi}\frac{f_0}{g_0} \equiv \frac{r_0^z}{4\pi} F_z(\tilde{\rho})$$

$$\begin{aligned} f(u) &= \sqrt{u}\,(f_0 + \ldots) \\ g(u) &= \frac{1}{\sqrt{u}}\,(g_0 + \ldots) \end{aligned}$$



Heat capacity: $C_V = T\,\frac{dS}{dT}\Big|_V$ Black hole entropy: $S = \frac{\pi^2}{2} r_0^3$

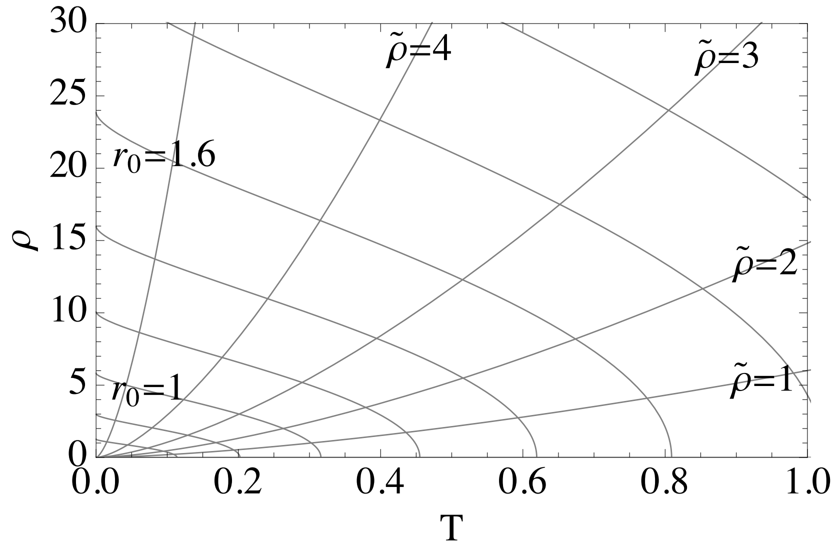
fixed $\rho = \tilde{\rho}\, r_0^3$

Write $\frac{dS}{dT} = \frac{(dS/dr_0)}{(dT/dr_0)}$ and use $\frac{dT}{dr_0} = \frac{\partial T}{\partial r_0} + \frac{d\tilde{\rho}}{dr_0}\frac{\partial T}{\partial \tilde{\rho}} = \frac{r_0^{z-1}}{4\pi}\left(z\,F_z - 3\tilde{\rho}\frac{\partial F_z}{\partial \tilde{\rho}}\right)$

Then $\frac{c}{T} = \frac{6\pi^3 r_0^{3-z}}{zF_z(\tilde{\rho}) - 3\tilde{\rho}F_z'(\tilde{\rho})}$ ⟵ $r_0 = r_0(\rho, T), \quad \tilde{\rho} = \tilde{\rho}(\rho, T)$

Black hole thermodynamics (contd.)

Numerically invert $T_H = \frac{r_0^z}{4\pi} F_z(\tilde\rho), \; \rho = \tilde\rho \, r_0^3$ to obtain $r_0 = r_0(\rho, T), \; \tilde\rho = \tilde\rho(\rho, T)$



$z = 2$

- High temperature behavior: $c \sim T^{3/z}$

- agrees with statistical mechanics system with $\omega \approx k^z$ dispersion Bertoldi et al. '09

- AdS-RN solution: $F_1(\tilde\rho) = 4 - \frac{\tilde\rho^2}{6} \quad \Longrightarrow \quad \frac{c}{T} = \frac{36\pi^3 r_0^2}{24 + 5\tilde\rho^2} \longrightarrow \frac{\pi^3 \rho^{2/3}}{8 \cdot 3^{1/3}}$ at low T

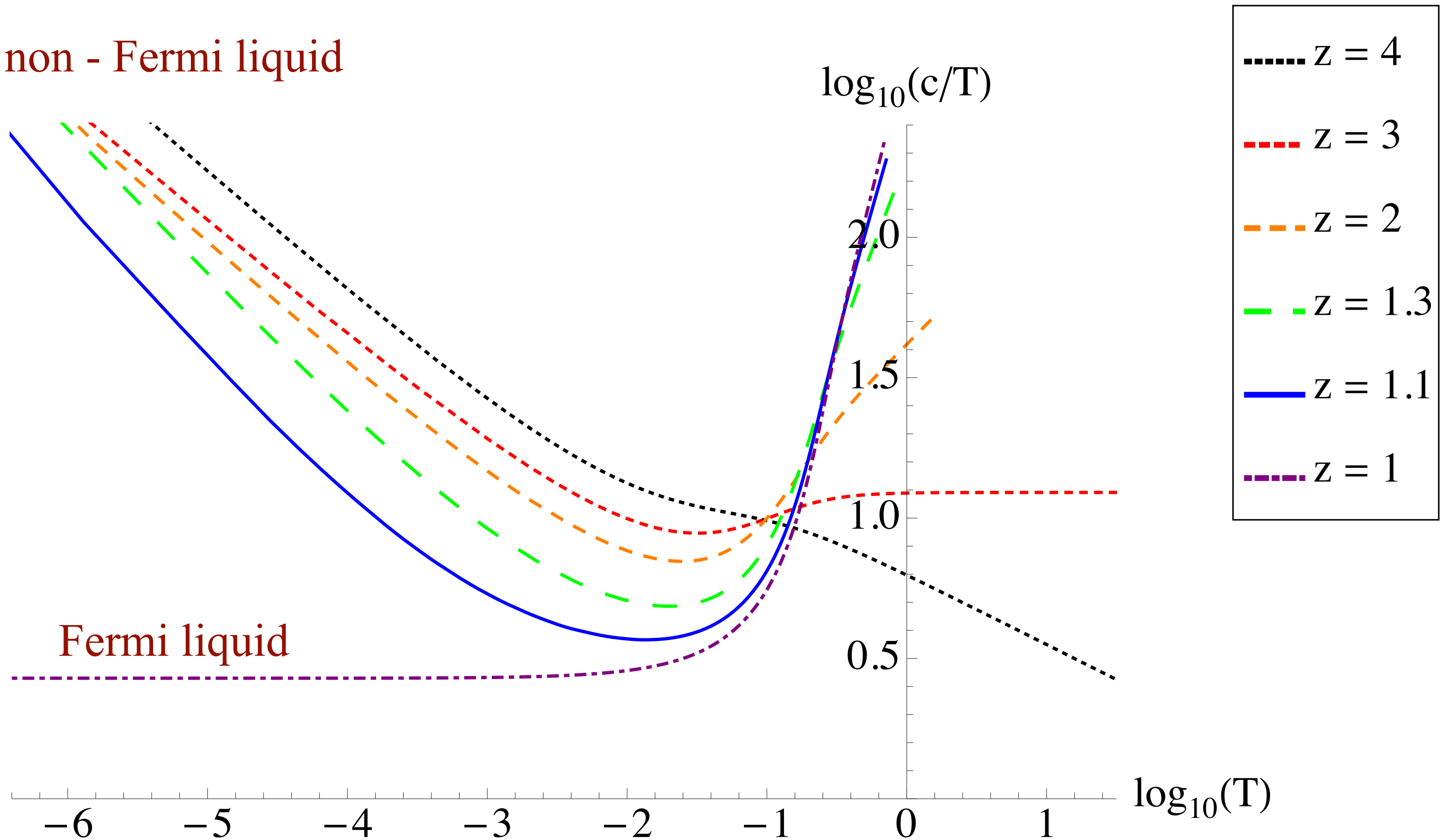
Fermi liquid

- Numerical z > 1 solutions:

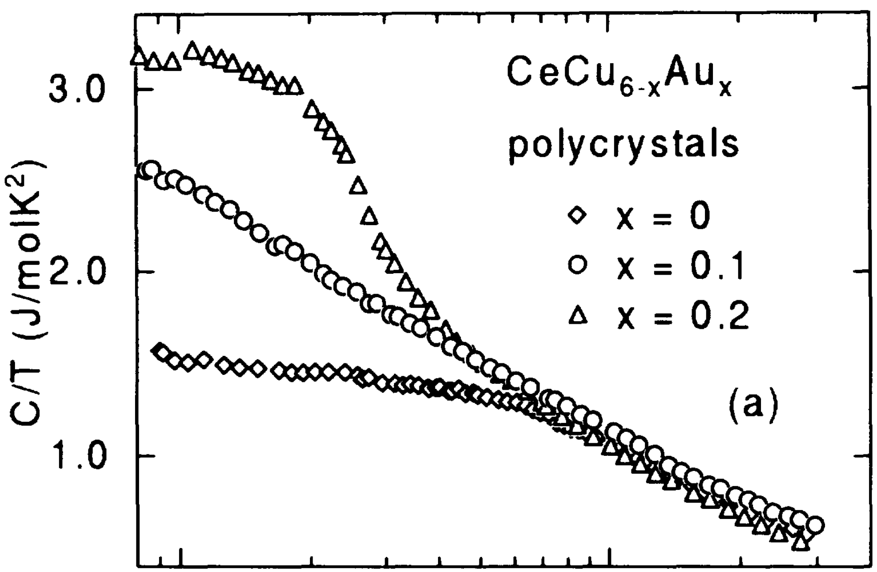
→ non - Fermi liquid behavior

S-J.Rey, Prog.Theor.Suppl. **177** (2009) 128

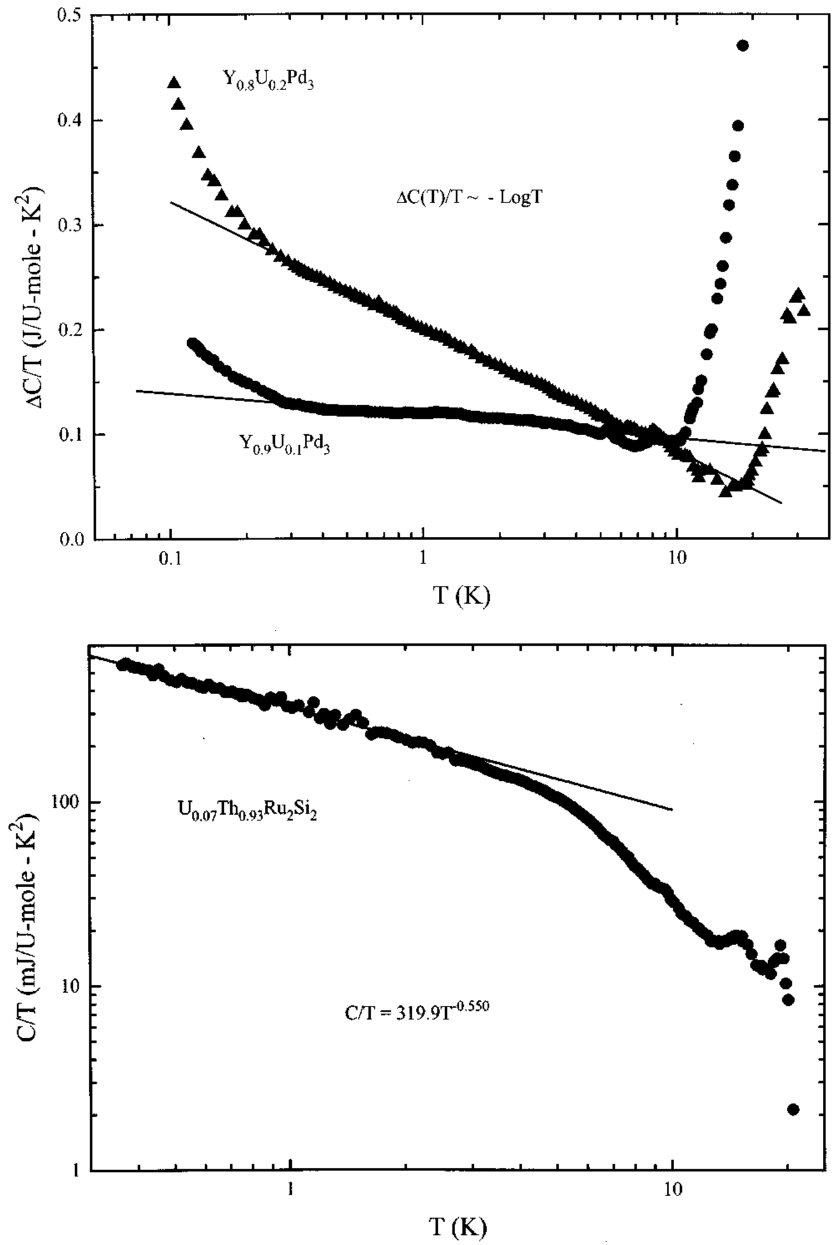
<u>Sommerfeld ratio *vs*. temperature</u>



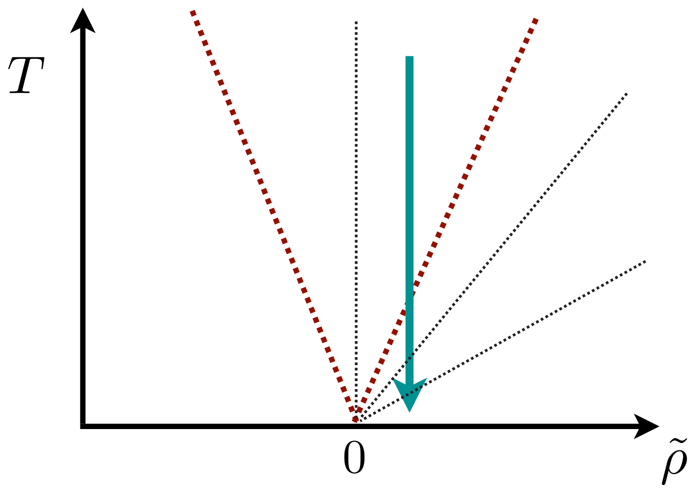
Some measured c/T values in heavy fermion metals



H. Lohneysen et al. *PRL* **72** (1994) 3262.



From G.Stewart, *Rev.Mod.Phys*. **73** (2001) 797.



Fermion probe calculations $d = 2$

$z = 1$ H.Liu, J.McGreevy, D.Vegh, arXiv:0903.2477
M.Cubrovic, J.Zaanen, K.Schalm, Science 325 (2009) 439

$z > 1$ E.Brynjolfsson, U.Danielsson, L.T., T.Zingg, in preparation

Include charged fermions: $$S_{\text{matter}} = -\int \mathrm{d}^4x\sqrt{-g}\ \{\bar{\Psi}\not{D}\Psi + m\bar{\Psi}\Psi\}$$

Dirac equation: $$(\not{D} + m)\Psi = 0$$

Boundary fermions: $$\psi_\pm(t,\vec{x}) = \lim_{r\to\infty}\Psi_\pm(t,\vec{x},r) \qquad \Gamma^3\Psi_\pm = \pm\Psi_\pm$$

$$\Psi_\pm(t,\vec{x},r) = \frac{1}{(2\pi)^3}\int d\omega\, d^2k\,\tilde{\Psi}_\pm(\omega,\vec{k},r)e^{-i\omega t+i\vec{k}\cdot\vec{x}}$$

1st order ODE's: $$\left(r\frac{\partial}{\partial r} + A^\pm\right)\tilde{\Psi}_\pm = \mp\not{T}\,\tilde{\Psi}_\mp$$

$$A^\pm = \frac{1}{2}\left(z + 2 + r\frac{f'}{f}\right) \pm Lmg \qquad \not{T} = i(T_0\sigma^0 + T_1\sigma^1 + T_2\sigma^2)$$

$$T_0 = -g\left(q\alpha + \frac{\omega}{r^z f}\right),\quad T_1 = \frac{g}{r}k_x\,,\quad T_2 = \frac{g}{r}k_y$$

Fermion spectral functions

Single fermion spectral function $A(\omega, k) = \frac{1}{\pi}\mathrm{Im}\left(\mathrm{Tr}\left[i\sigma^3 G_R(\omega, k)\right]\right)$ can be directly compared to ARPES data.

Adapt AdS/CFT prescription to compute $G_R(\omega, k)$: Liu et al. 2009; Cubrovic et al. 2009

- Work out near horizon expansion of fermions with ingoing modes only
  D.Son, A.Starinets '02
- Integrate Dirac equation numerically from $u = \epsilon \ll 1$ to $u = u_0 \gg 1$
- Construct 2 x 2 matrices

$$\begin{aligned}\mathcal{F}_+(u) &= \left[\Psi_+^{\uparrow}(u), \Psi_+^{\downarrow}(u)\right] \\ \mathcal{F}_-(u) &= \left[\Psi_-^{\uparrow}(u), \Psi_-^{\downarrow}(u)\right]\end{aligned}$$

- GKP-W prescription then gives $G_R(\omega, k) = \frac{1}{\mathcal{N}}\mathcal{F}_-(u_0)\mathcal{F}_+^{-1}(u_0)$

Large *u* behavior:

$$\begin{aligned}\Psi_+ &= c_+ e^{-(\frac{z+3}{2} - |m-\frac{1}{2}|)u} + d_+ e^{-(\frac{z+3}{2} + |m-\frac{1}{2}|)u} + \ldots \\ \Psi_- &= c_- e^{-(\frac{z+3}{2} - |m+\frac{1}{2}|)u} + d_- e^{-(\frac{z+3}{2} + |m+\frac{1}{2}|)u} + \ldots\end{aligned}$$

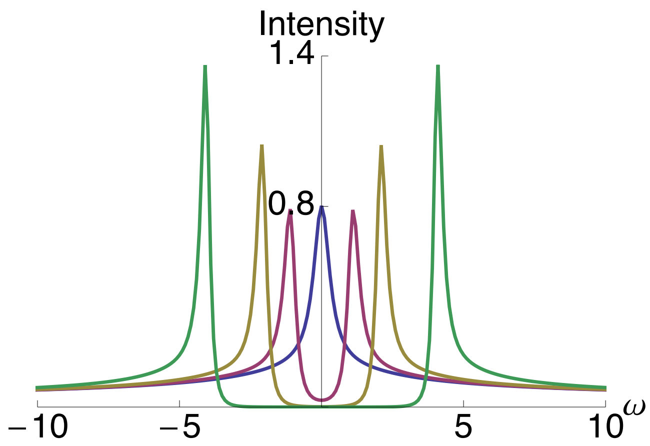
For $-\frac{1}{2} < m < \frac{1}{2}$: $G_R = \frac{1}{\mathcal{N}}\left((\frac{c_-}{c_+} + \ldots)e^{-2mu} + (\frac{d_-}{c_+} + \ldots)e^{-u}\right)$

$\frac{1}{\mathcal{N}} \sim e^{-2mu}$

Numerical results

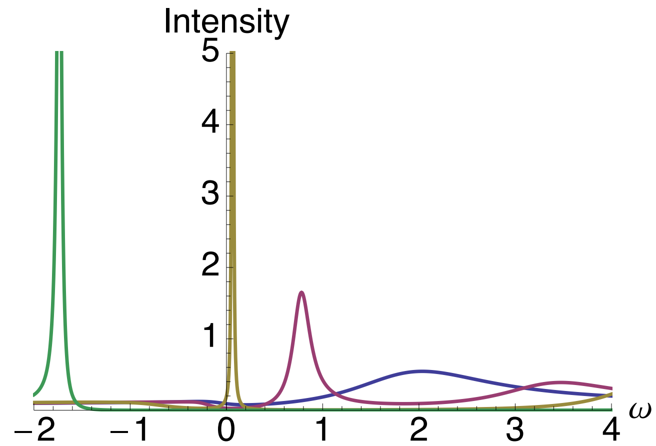
Spectral function at fixed k for $z = 1$

high T



$$\omega \approx k$$

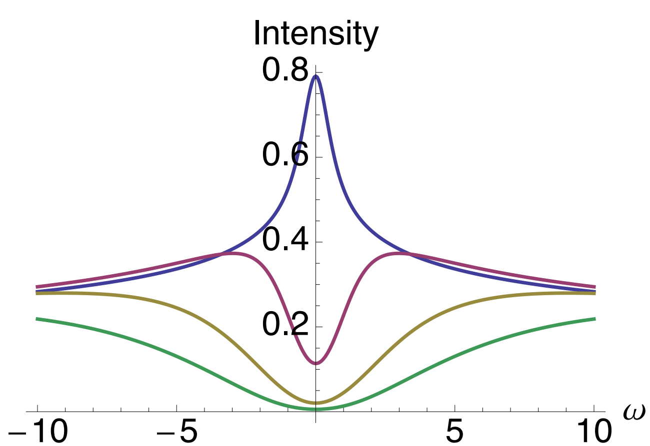
low T



$$\omega \approx \frac{k_F^2}{2m_F} + v_F(k - k_F) + \ldots$$

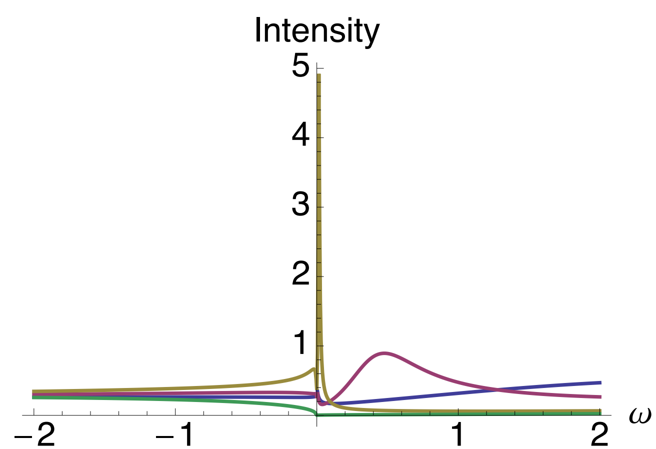
Spectral function at fixed k for $z = 2$

high T



$$\omega \approx k^z$$

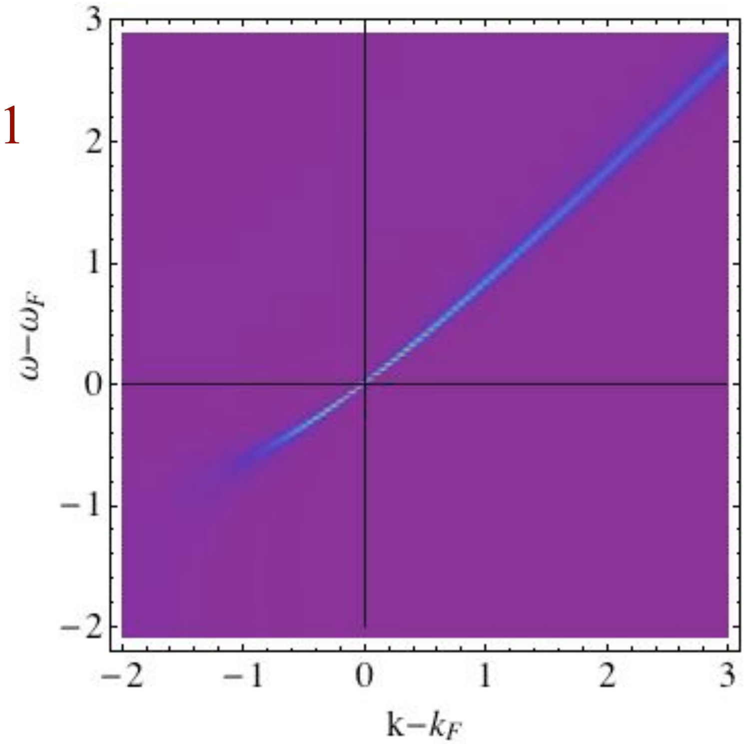
low T

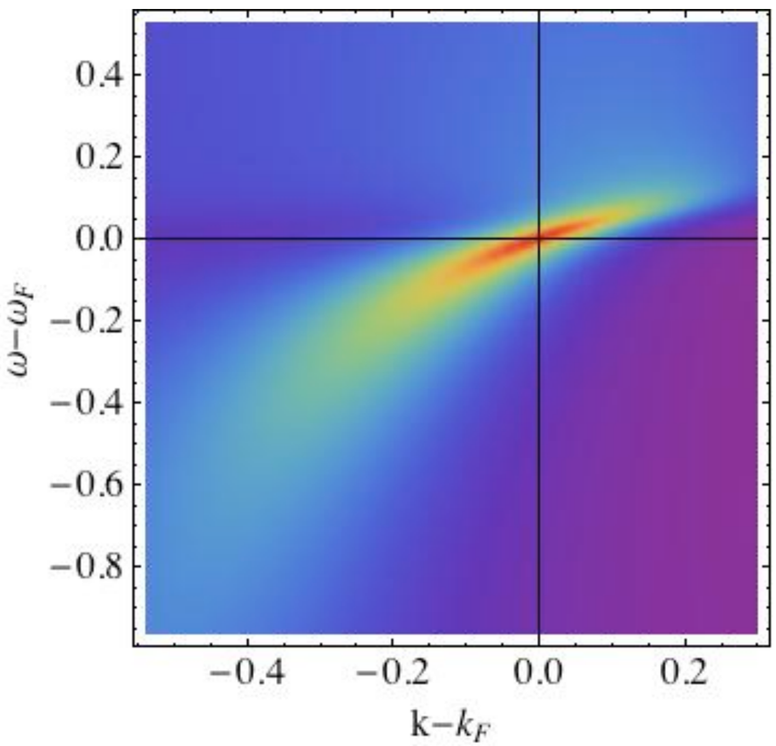


non-fermi liquid

Spectral function peak location at low T

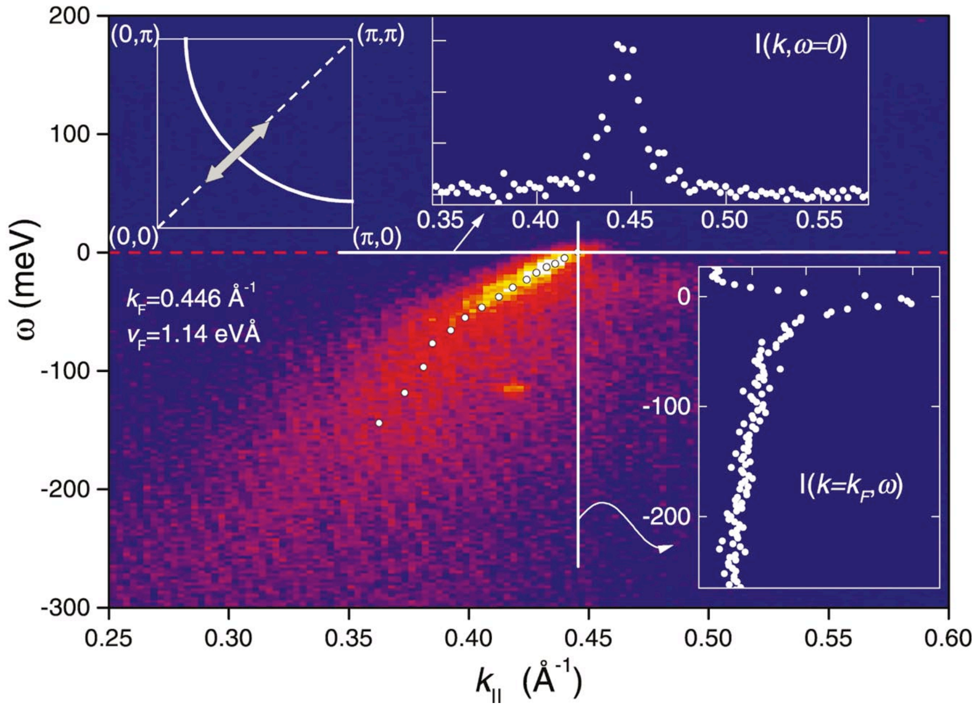
z = 1





z = 2

ARPES data from
Bi2Sr2CaCu2O8+δ



From Damascelli et al.,
Rev.Mod.Phys. **75** (2003) 473.

Holographic superconductors

S.Gubser, *Phys. Rev.* D**78** (2008) 065034
S.A. Hartnoll, C.P. Herzog, G.T. Horowitz, *Phys. Rev. Lett.* **101** (2008) 031601

Add charged scalar field to gravitational system

instability at low T: black brane with scalar "hair"

AdS/CFT prescription: hair corresponds to sc condensate

transport properties: solve classical wave equation in bh bacground

add magnetic field: dyonic black hole -- holographic sc is type II

conformal system: start from AdS-RN exact solution

z > 1 systems: work with numerical Lifshitz black branes

E.Brynjolfsson, U.Danielsson, L.T., T.Zingg,
J. Phys. A: Math. Theor. **43** (2010) 065401

Holographic superconductors with Lifshitz scaling

Add a charged scalar field

$$S_\psi = -\frac{1}{2}\int d^4x\sqrt{-g}\,\left(g^{\mu\nu}(\partial_\mu\psi^* + iq\mathcal{A}_\mu\psi^*)(\partial_\nu\psi - iq\mathcal{A}_\nu\psi) + m^2\psi^*\psi\right)$$

Two independent solutions: $\psi(x^\mu) = c_+\psi_+(x^\mu) + c_-\psi_-(x^\mu)$

Asymptotic behavior: $\psi_\pm(x^\mu) \to r^{-\Delta_\pm}\tilde{\psi}_\pm(\tau,\theta,\varphi) + \ldots$ $\quad \Delta_\pm = \frac{z+2}{2} \pm \sqrt{\left(\frac{z+2}{2}\right)^2 + m^2L^2}$

Finite Euclidean action: $L^2\,m^2 > -\frac{(z+2)^2}{4}$ analog of BF bound

Only ψ_+ falls off sufficiently rapidly as $r\to\infty$ if $L^2\,m^2 > -\frac{(z+2)^2}{4} + 1$

ψ is then dual to an operator O_+ of dimension Δ_+ in the dual field theory

Two choices if $-\frac{(z+2)^2}{4} + 1 > L^2\,m^2 > -\frac{(z+2)^2}{4}$

$\psi = \psi_+$ dual to O_+ of dim Δ_+ OR $\psi = \psi_-$ dual to O_- of dim Δ_-

Superconducting phase

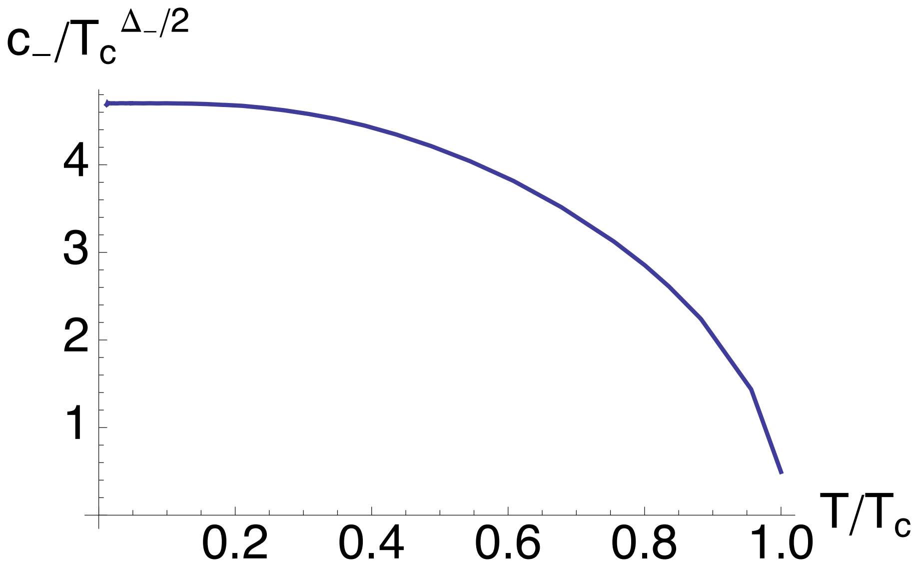
We work with $L^2\, m^2 = -\frac{(z+2)^2}{4} + \frac{1}{4}$ so that $\Delta_\pm = \frac{z+2}{2} \pm \frac{1}{2}$

A holographic superconductor in the superconducting phase is then dual to a hairy black hole with either

$$c_+ = 0, \quad \langle O_- \rangle \propto c_- \qquad \text{or} \qquad c_- = 0, \quad \langle O_+ \rangle \propto c_+$$

Numerical results for superconducting condensate:

$z = 2, \quad L^2 q^2 = 1$



Transport coefficients

- work in progress

E.Brynjolfsson, U.Danielsson, L.T. and T.Zingg

Zero temperature entropy

Low temperature limit is described by a near extremal black brane

$z = 1:$ Extremal RN black brane has non-vanishing entropy

BUT

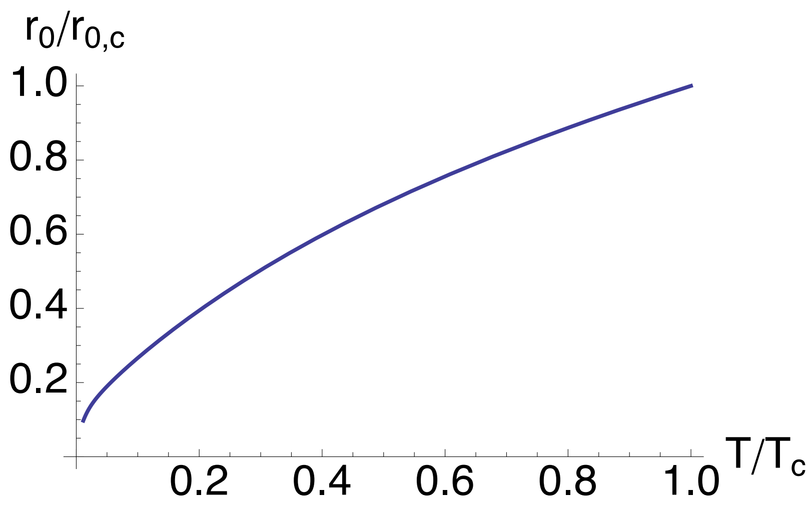
black hole with charged scalar hair has vanishing entropy in extremal limit

G.Horowitz and M.Roberts, JHEP **0911** (2009) 015

$z > 1:$ Lifshitz black brane without hair has non-vanishing entropy in extremal limit

near-horizon expansion + numerical integration

$z = 2$ Lifshitz black hole with charged scalar hair:



Global geometry of Lifshitz black holes

Consider exact charged black hole solution at $z = 4$

$$f(r) = \frac{1}{g(r)} = \sqrt{1 + \frac{k}{10r^2} - \frac{3k^2}{400r^4} - \frac{Q^2}{2r^4}} \qquad k = -1, 0, +1$$

generalizes the k = 1, z = 4 exact solution of Bertoldi et al. '09

Introduce a tortoise coordinate such that

$$ds^2 = L^2 \left(-r^{2z} f(r)^2 \left(dt^2 + dr_*{}^2 \right) + r^2 \left(d\theta^2 + \chi(\theta)^2 d\varphi^2 \right) \right)$$

$$\frac{dr_*}{dr} = \frac{1}{r^{z+1}} \frac{g(r)}{f(r)}$$

$$r_* - r_*^\infty = \frac{1}{2b_1(b_1 + b_2)} \log\left(1 - \frac{b_1}{r^2}\right) + \frac{1}{2b_2(b_1 + b_2)} \log\left(1 + \frac{b_2}{r^2}\right)$$

$$b_1 = \sqrt{\frac{k^2}{100} + \frac{Q^2}{2}} - \frac{k}{20} \qquad b_2 = \sqrt{\frac{k^2}{100} + \frac{Q^2}{2}} + \frac{k}{20}$$

Kruskal extension

Null coordinates $v = t + r_*$ $u = t - r_*$

Kruskal coordinates $V = \exp\left[b_1(b_1+b_2)(v - r_*^\infty)\right], \quad U = -\exp\left[-b_1(b_1+b_2)(u + r_*^\infty)\right]$

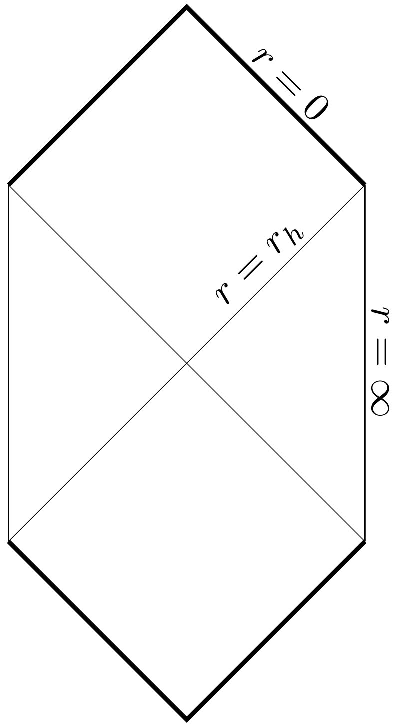
Metric is now non-singular at the horizon:

$$ds^2 = L^2 \left(-\frac{r^8}{\kappa^2}\left(1+\frac{b_2}{r^2}\right)^{1-\frac{b_1}{b_2}} dU\, dV + r^2(d\theta^2 + \chi^2(\theta)d\varphi^2) \right)$$

There is a null curvature singularity at $r = 0$

...but no inner horizon at finite r

This global structure is generic for Lifshitz black holes at $z > 1$



Global diagram of a Lifshitz black hole

Summary

- Black branes in asymptotically Lifshitz spacetime provide a window onto finite temperature effects in strongly coupled models with anisotropic scaling
- Black hole thermodynamics indicates that the $z = 1$ system behaves like a Fermi liquid at low T but that $z > 1$ systems do not
- This is supported by numerical computation of spectral functions for probe fermions
- A Lifshitz black brane with scalar hair is dual to the superconducting phase of a holographic superconductor at $z > 1$
- The back-reaction due to charged hair leads to vanishing zero temperature entropy in the extremal limit
- The global extension of a Lifshitz black hole has a null curvature singularity inside the horizon, but no smooth inner horizon (for charged Lifshitz black holes)

Remaining problems include:

- improved understanding of non-Fermi liquid low T behavior found at $z > 1$
- semi-analytic analysis of low T physics based on extremal Lifshitz black branes
- computing transport coefficients (electrical + thermal conductivities) for holographic superconductors with Lifshitz scaling
- probing the low T phase of the holographic superconductors with fermions