Non-Relativistic Quantum Gravity

Oriol Pujolàs

CERN

o9o6.3o46, JHEP o910 o29 o9o9.3525, PRL xxx xxx o912.0550, PLB xxx xxx + in progress

with **Diego Blas,** Sergey Sibiryakov Fundaments of Gravity Munich, April 14 2010

Motivation

Great interest/merit of Hořava's proposal:

explicit construction of NR gravity theories, possibly renormalizable

many possible applications/implications

In this talk, focus on application as QG for our 3+1 world: can any version of NR QG possibly be phenomenologically viable?

- no ghosts, no instabilities
- weakly coupled, if possible
- pass observational tests
- recovery of Lorentz invariance in IR

Plan

- Anisotropic Scaling.
- NR Gravity. A healthy extension.
- Stückelberg Formalism
- Phenomenology & bounds
- Open issues & applications

Hořava's proposal: 'Anisotropic Scaling'

In the UV,
$$w^2 \sim k^{2z}$$
 $z > 1$

$$G = \frac{1}{w^2 - k^2 - a k^{2z}} => \text{Loops are less divergent (\& no ghosts)}$$

2 quick ways to see how Anisotropic Scaling assists renormalizability

1) Dimensional analysis Free Kinetic term: $\int dt \, d^3x \, \left\{ (\stackrel{\bullet}{\phi})^2 + \frac{\phi \Delta^z \phi}{M^{2(z-1)}} \right\} \quad \begin{array}{c} \text{invariant} \\ \text{under} \end{array} \quad \begin{array}{c} x \mapsto b^{-1} \, x \\ t \mapsto b^{-z} \, t \\ \phi \mapsto b^{\frac{3-z}{2}} \, \phi \end{array}$

To this, we add interactions:

For z = 2, $[\phi] = \frac{1}{2} \Rightarrow \phi^{10}$ is marginal (renormalizable) For z = 3, ϕ^n relevant (super-renormalizable) $\phi^n \Delta^3 \phi$ marginal (renormalizable)

Note:

 $\phi \Delta \phi$ relevant => generated => chance to recover Lorentz Inv

1) Dimensional analysis Free Kinetic term: $\int dt \, d^3x \left\{ (\dot{\phi})^2 + \frac{\phi \Delta^z \phi}{M^{2(z-1)}} \right\} \quad \begin{array}{l} \text{invariant} \\ \text{under} \end{array} \quad \begin{array}{l} x \mapsto b^{-1} \, x \\ t \mapsto b^{-z} \, t \\ \phi \mapsto b^{\frac{3-z}{2}} \, \phi \end{array}$

To this, we add interactions:

For z = 2, $[\phi] = \frac{1}{2} \implies \phi^{10}$ is marginal (renormalizable) For z = 3, ϕ^n relevant (super-renormalizable) $\phi^n \Delta^3 \phi$ marginal (renormalizable)

E.g.,
$$L = \Lambda^2 \left[(\dot{\phi})^2 + c^2(\phi) \ \phi \Delta \phi + d^2(\phi) \ \frac{\phi \Delta^2 \phi}{M^2} + e^2(\phi) \ \frac{\phi \Delta^3 \phi}{M^4} \right]$$

 $c(\phi) = c_0 + c_1 \phi + \dots$

is renormalizable.

1) Dimensional analysis Free Kinetic term: $\int dt \, d^3x \left\{ (\stackrel{\bullet}{\phi})^2 + \frac{\phi \Delta^z \phi}{M^{2(z-1)}} \right\} \quad \text{invariant} \quad \begin{array}{l} x \mapsto b^{-1} \, x \\ \text{invariant} \\ \text{under} \end{array} \quad \begin{array}{l} t \mapsto b^{-z} \, t \\ \phi \mapsto b^{\frac{3-z}{2}} \, \phi \end{array}$

To this, we add interactions:

For z = 2, $[\phi] = \frac{1}{2} \Rightarrow \phi^{10}$ is marginal (renormalizable) For z = 3, ϕ^n relevant (super-renormalizable) $\phi^n \Delta^3 \phi$ marginal (renormalizable)

Note:

 $(\phi)^2$ irrelevant => not generated => no ghosts generated

2) Power-counting argument

Count superficial degree of divergence of 1PI diagrams

$$\sim \left[\int dw d^3k\right]^L G(w,k)^I (k^{2z})^V$$

$$\delta \le (z+3)L - 2zI + 2zV = (3-z)L + 2z$$
$$(V - I + L = 1)$$

With z=3, all loop orders diverge equally -> Renormalizable

Hence,
$$L = \Lambda^2 \left[(\dot{\phi})^2 + c^2(\phi) \ \phi \Delta \phi + d^2(\phi) \ \frac{\phi \Delta^2 \phi}{M^2} + e^2(\phi) \ \frac{\phi \Delta^3 \phi}{M^4} \right]$$

is renormalizable. $c(\phi) = c_0 + c_1 \phi + \dots$

Not only that...



Tree level Unitarity bound

 $\begin{array}{c} \text{disp} \\ \text{relation} \end{array} E = \mathcal{E}(p) \end{array}$

Optical Thm:

$$2\text{Im}\,\mathcal{M}(2\to 2) = \sum_{n} \left(\prod_{i=1}^{n} \int \frac{d^3 p_i}{2\mathcal{E}(p_i)}\right) \left|\mathcal{M}(2\to n)\right|^2 \,\delta^4\left(\Sigma \, p_\mu\right)$$

$$\left|\mathcal{M}(2\to 2)\right| \leqslant 16\pi \mathcal{E}'(p_0) \frac{E_0^2}{p_0^2}$$

Tree level Unitarity bound

 \Rightarrow

 $\begin{array}{c} \text{disp} \\ \text{relation} \end{array} E = \mathcal{E}(p) \end{array}$

Optical Thm:

$$2\text{Im}\,\mathcal{M}(2\to 2) = \sum_{n} \left(\prod_{i=1}^{n} \int \frac{d^3 p_i}{2\mathcal{E}(p_i)}\right) \left|\mathcal{M}(2\to n)\right|^2 \,\delta^4\left(\Sigma \, p_\mu\right)$$

$$\left|\mathcal{M}(2 \to 2)\right| \leqslant \left(\frac{E_0}{M_*}\right)^{3\frac{z-1}{z}}$$

Does the trick work for gravity ??

Lorentz Invariance => (part of the) gauge group broken

=> additional degrees of freedom

One needs to be extra-careful, or else extra d.o.f. pathological

Non-Relativistic Gravity

Hořava '09

Introduce preferred time coordinate t (back to "Absolute time")

Globally defined *Foliation* by spatial 3D surfaces:



Non-Relativistic Gravity

Hořava '09

Introduce preferred time coordinate t

3+1 split:
(ADM)
$$g_{\mu\nu} = \begin{pmatrix} N^2 - N^i N_i & N^i \\ N^j & \gamma_{ij} \end{pmatrix}$$

Foliation- $t \mapsto \hat{t}(t)$ preserving diffs $x \mapsto \hat{x}(t,x)$

covariant
$$\begin{cases} K_{ij} \equiv \frac{1}{2N} \left(\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i \right) \\ \text{objects:} \end{cases} \quad a_i \equiv \frac{\partial_i N}{N} , \quad R_{ijkl}^{(3)}[\gamma_{ij}] \dots \end{cases}$$

Hořava '09

Action:
$$S = M_P^2 \int d^3x dt N \sqrt{\gamma} \left[K_{ij} K^{ij} - \lambda (K_i^i)^2 - V(\gamma_{ij}, N) \right]$$

$$V(\gamma_{ij}, N) = R_{(3)} + \frac{R_{(3)}^2 + \dots}{M_P^2} + \frac{R_{(3)}^3 + \dots}{M_P^4}.$$
 (z = 3)

covariant
$$\begin{cases} K_{ij} \equiv \frac{1}{2N} \left(\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i \right) \\ \text{objects:} \end{cases} \quad a_i \equiv \frac{\partial_i N}{N} , \quad R_{ijkl}^{(3)}[\gamma_{ij}] \dots \end{cases}$$

$$\begin{split} \text{Non-Relativistic Gravity} \\ \text{Product} \\ \text{Action:} \quad & S = M_p^2 \int d^3x \, dt \, N \sqrt{\gamma} \Big[K_{ij} K^{ij} - \hat{\lambda} (K_i^i)^2 - V \big(\gamma_{ij}, N \big) \Big] \quad \text{Sibiryakov 'og} \\ & V \big(\gamma_{ij}, N \big) = \quad R_{(3)} + \hat{\alpha} a_i a^i + \frac{R_{(3)}^2 + a^4 + \dots}{M_p^2} + \frac{R_{(3)}^3 + a^6 + \dots}{M_p^4} . \end{split}$$

$$Potava 'og$$

$$Potava 'og$$

$$Potava 'og$$

$$Potava 'ng$$

$$Pot$$

$$\begin{aligned} \text{Non-Relativistic Gravity} \\ \text{Bias OP} \\ \text{Action:} \quad S = M_p^2 \int d^3x \, dt \, N \sqrt{\gamma} \left[K_{ij} K^{ij} - \lambda (K_i^i)^2 - V \left(\gamma_{ij}, N \right) \right] \quad \overset{\text{Sibiryakov 'og}}{\text{Sibiryakov 'og}} \\ V \left(\gamma_{ij}, N \right) = \quad R_{(3)} + \alpha \, a_i a^i + \frac{R_{(3)}^2 + a^4 + \dots}{M_p^2} + \frac{R_{(3)}^3 + a^6 + \dots}{M_p^4} \\ \text{tensor modes:} \quad L_2^{(2)} = M_p^2 \left[\left(h_{TT}^{\bullet} \right)^2 + h_{TT} \left(1 + \tilde{A}_1 \frac{\Delta}{M^2} + \tilde{A}_2 \frac{\Delta^2}{M^4} \right) h_{TT} \right] \end{aligned}$$

∃ choice of parameters for which z = 3 in the UV good UV behaviour *not* spoiled

Massive QED:

$$L = \frac{1}{4} F_{\mu\nu}^2 - m^2 A_{\mu} A^{\mu} \qquad 2 + 1 \text{ d.o.f.}$$

Gauge invariance *restored* by introducing the Stückelberg field ϕ

$$A_{\mu} = \hat{A}_{\mu} - \partial_{\mu}\phi$$

$$L = \frac{1}{4}\hat{F}_{\mu\nu}^{2} - m^{2}(\partial_{\mu}\phi - \hat{A}_{\mu})(\partial^{\mu}\phi - \hat{A}^{\mu})$$

The Lagrangian is invariant under

 $\hat{A}_{\mu} \to \hat{A}_{\mu} + \partial_{\mu} \varepsilon$ $\phi \to \phi + \varepsilon$

Isolating the scalar mode: $S = S_{GR} + S[\phi; \alpha, \lambda...]$ in covariant form ϕ defines the Foliation structure: $\langle \phi \rangle = t$

Shortcut: unbroken $t \rightarrow \hat{t}(t)$ symmetry implies

 $\phi \rightarrow f(\phi)$ internal symmetry

$$\begin{split} \phi &\to f(\phi) \quad \text{internal symmetry} \\ \hline \text{Outanics (OCD) !!} \\ \hline \phi &\to f(\phi) \quad \text{internal symmetry} \\ \hline \text{Invariants:} \quad u_{\mu} &\equiv \frac{\partial_{\mu} \phi}{\sqrt{(\partial \phi)^2}} \quad \text{and its derivatives} \\ S[\phi] &= M_P^2 \int d^4x \sqrt{-g} \left\{ (\lambda - 1) \Big(\nabla_{\!\!\mu} \, u^{\mu} \Big)^2 + \alpha \, (u^v \nabla_{\!\!v} u_{\mu})^2 + ... \right\} \end{split}$$

 $\partial_{\mu}\phi$

In the *unitary gauge* ($\phi = t$) the action coincides with previous form

Eg, $(\lambda - 1)$ term is: $S[\phi] = (\lambda - 1)M_P^2 \int d^4x \sqrt{-g} \frac{1}{(\partial \phi)^2} \left(\Box \phi - \frac{\partial^{\mu} \phi \, \partial^{\nu} \phi}{(\partial \phi)^2} \, \nabla_{\!\mu} \nabla_{\!\nu} \phi\right)^2 + \dots$

Subtlety: higher covariant derivatives, yet no ghosts!

the e.o.m. contains, eg: $\left[g^{\mu\nu} - u^{\mu}u^{\nu} \right] \left[g^{\rho\sigma} - u^{\rho}u^{\sigma} \right] \nabla_{\!\mu} \nabla_{\!\nu} \nabla_{\!\rho} \nabla_{\!\sigma} \phi + \dots = 0$

Always reduce to purely spatial derivatives in the preferred frame defined by $u_{\mu} \equiv \frac{\partial_{\mu}\phi}{\sqrt{(\partial\phi)^2}}$

Cauchy problem well posed with less initial data in the preferred frame

Similarly, pc-renormalizability not explicit (but present)

Around flat space and $\phi = t$, let us study the scalar mode $\phi = t + \chi(t, x)$

$$S[\chi] = M_P^2 \int d^4x \left[\alpha \left(\partial_i \dot{\chi} \right)^2 - (\lambda - 1) \left(\Delta \chi \right)^2 \right]$$

Disp. relation at low energies

$$v^2 = \frac{\lambda - 1}{\alpha}k^2 + \dots$$

Around flat space and $\phi = t$, let us study the scalar mode $\phi = t + \chi(t, x)$

$$S[\chi] = M_P^2 \int d^4x \left[\alpha \left(\partial_i \dot{\chi} \right)^2 - (\lambda - 1) \left(\Delta \chi \right)^2 - (\lambda - 1) \dot{\chi} \left(\Delta \chi \right)^2 + \dots \right]$$

Disp. relation at low energies $w^2 = \frac{\lambda - 1}{\alpha}k^2 + ...$

Naive strong coupling scale $\Lambda \approx \sqrt{|\lambda - 1|} M_P$

If High. deriv. terms suppressed by $M_* \leq \Lambda \implies$ NO strong coupling!

Projectable Horava (
$$\alpha \to \infty$$
): $c^2 < 0$
slow instabilities $\Rightarrow \Lambda \simeq (100m)^{-1} \Rightarrow$ either $\begin{cases} \text{strong coupl} \\ \text{or ruled out} \end{cases}$

Reparametrization symmetry $\phi \rightarrow f(\phi)$

The conserved current $\nabla^{\mu}J_{\mu} = 0$ is purely spatial $u^{\mu}J_{\mu} = 0$ (no charge density)

Infinitessimal form of the symmetry is $\chi \rightarrow \chi + f(t)$

At least one power of momentum in front of χ everywhere

Coupling to matter

Chronon field, $\chi = \phi - t$, couples to matter through $L^{\chi-matter} = \chi \nabla^i j_i$ any vector constructed from matter (Dictated by reparametrization invariance, $\chi \to \chi + f(t)$)

In principle, non-Universal coupling => violations of Equivalence Pple However, Fr suppressed because derivative coupling

Chronon-force

If chronon couples Universally, then $j_i = T_{0i}$

(because matter couples to effective metric $\tilde{g}_{\mu\nu} = g_{\mu\nu} + \beta u_{\mu}u_{\nu}$)

Velocity-dependent force,

$$V = \beta^2 \frac{M_1 M_2}{r_{12}} \left[v_1 \cdot v_2 - (v_1 \cdot \hat{r}_{12})(v_2 \cdot \hat{r}_{12}) \right]$$



Phenomenology

Phenomenology

$$S = \int d^4x \sqrt{-g} \left(M_P^2 R + \Lambda^2 L^{(\phi)} \left[\partial_\mu \phi , \ldots \right] + L^{(matter)} \left[\psi, g_{\mu\nu}, \partial_\mu \phi \right] \right)$$

-> Both lower and upper bounds on Λ

Note: $\Lambda \ll M_P$ **not** a fine-tuning

Phenomenology is very similar to

ghost condensation Einstein-aether (gauged gh condens)

Blas OP Sibiryakov '09 Jacobson '10

Newtonian potential vs cosmology

By mixing with graviton, ϕ couples to matter so as to renormalize G_N

$$\phi_N = -G_N \frac{M}{r}; \qquad G_N = \frac{1}{8\pi M_P^2} \frac{1}{1 - \alpha/2}$$
$$H^2 = \frac{8\pi}{3} G_{\text{cosmo}} \rho; \qquad G_{\text{cosmo}} = \frac{1}{8\pi M_P^2} \frac{1}{1 - 3(\lambda - 1)/2}$$

Observational bound (BBN):

$$\left|\frac{G_{\text{cosmo}}}{G_{N}}-1\right| < 0.13 \qquad \Rightarrow \qquad \alpha, (\lambda-1) \lesssim 0.1$$

Limits from PPN expansion

(PPN = "Parameterized Post-Newtonian" expansion)

 $\alpha_1^{PPN}, \alpha_2^{PPN}$ parameterize preferred-frame effects

velocity with repect to preferred frame



Observational bounds:

 $lpha_2^{PPN} \le 10^{-7}$ (alignment of solar spin axis w.r.t. ecliptic) $lpha_1^{PPN} \le 10^{-4}$ (Lunar ranging, binary pulsars)

Limits from PPN expansion

(PPN = "Parameterized Post-Newtonian" expansion)

 $\alpha_1^{PPN}, \alpha_2^{PPN}$ parameterize preferred-frame effects

Limits from PPN expansion

(PPN = "Parameterized Post-Newtonian" expansion)

 $\alpha_1^{PPN}, \alpha_2^{PPN}$ parameterize preferred-frame effects

In our model,
$$\alpha_1^{PPN} = -4\alpha$$
 $\alpha_2^{PPN} = \frac{\alpha^2/2}{\lambda - 1} \sim \alpha$

$$\Rightarrow \quad \alpha, (\lambda - 1) \lesssim 10^{-7} \quad \Rightarrow \quad \Lambda \lesssim 10^{15} GeV$$

since
$$\Lambda = \sqrt{\alpha} M_P$$

Lower bounds on Λ

Within gravitational sector:

Table-top tests of Newton's law ->
$$\Lambda > 0.1 eV$$

Assuming that UV scale M_* is *the same* in the Matter sector:

$$w^{2} = c^{2}k^{2} + \underbrace{\frac{k^{3}}{M_{*3}}}_{M_{*3}} + \frac{k^{4}}{M_{*4}^{2}} + \dots$$

From GRB and AGN observations, $M_* \ge 10^{10} \div 10^{11} GeV$

$$\Rightarrow \quad 10^{10 \div 11} GeV \lesssim \Lambda \lesssim 10^{15} GeV$$

in the simplest model

So far:

∃ 1 formulation that is free from instabilities, strong coupling or other basic pathologies

At low energies:

Lorentz-breaking scalar-tensor theory

Deviations from GR are small, theory can be weakly coupled

Observational bounds leave a window for UV scale

 $10^{10 \div 11} GeV \lesssim \Lambda \lesssim 10^{15} GeV$

So far:

∃ 1 formulation that is free from instabilities, strong coupling or other basic pathologies

Response to criticism:

- Papazoglou & Sotiriou '09 1) strong coupling claimed

$$\begin{array}{ll} \text{absent if} & M_{\substack{high \\ deriv}} \leq \Lambda_{\substack{naive \\ strong \ c}} \end{array}$$

2) fine tuning? Not so.

(marginal/irrelevant operators)

So far:

∃ 1 formulation that is free from instabilities, strong coupling or other basic pathologies

Response to criticism:

- Papazoglou & Sotiriou '09 🛛 🖌
- Kimpton & Padilla '10 Strong coupling re-claimed
- " \exists a decoupling limit that spoils UV scaling "
 - \exists other dec. lims. that do not spoil scaling.

(In the appropriate limit, mixing with metric is important)

Non-Relativistic Gravity

Action:
$$S = M_P^2 \int d^3x dt N \sqrt{\gamma} \left[K_{ij} K^{ij} - \lambda (K_i^i)^2 - V(\gamma_{ij}, N) \right]$$

$$V(\gamma_{ij},N) = R_{(3)} + \alpha a_i a^i + \frac{R_{(3)}^2 + a^4 + \dots}{M_P^2} + \frac{R_{(3)}^3 + a^6 + \dots}{M_P^4}$$

scalar mode:

$$L_{0}^{(2)} = M_{P}^{2} \left[\frac{3\lambda - 1}{\lambda - 1} \left(\dot{\psi} \right)^{2} + \psi c_{0}^{2} \Delta \left(\frac{1 + A_{1} \frac{\Delta}{M} + \dots + A_{1} \frac{\Delta^{4}}{M^{4}}}{1 + B_{1} \frac{\Delta}{M} + \dots + B_{2} \frac{\Delta^{2}}{M^{2}}} \right) \psi \right]$$

So far:

∃ 1 formulation that is free from instabilities, strong coupling or other basic pathologies

Response to criticism:

- Papazoglou & Sotiriou '09 🛛 🖌

- Kimpton & Padilla '10 🛛 🖌

-Henneaux, Kleinschmidt & Lucena-Gomez'09

(also Pons & Talavera'10)

does not apply to extended model (nonlinear in lapse, N)

Many open questions :

- Recovery of Lorentz Invariance (matter sector): fine tuning avoidance
- Is it really renormalizable / UV complete ??

Is the $\phi \rightarrow f(\phi)$ symmetry non-anomalous? Absence of Landau poles? Generalized 2nd law of thermodynamics obeyed?

- Is it consistent with all observations ?

Any stronger bounds?

"Lorentz Fine-Tuning Problem":

c is species- dependent Generically, RG flow generates $c_i - c_j \neq 0$ But $c_i - c_j$ experimentally constrained $\leq 10^{-20}$ Severe fine-tuning

Collins Perez Sudarsky Urrutia Vucetich 04

```
lengo Russo
Serone og
```

Would be trivial in a single species theory

$$L = (\dot{\phi})^2 + c^2 \phi \Delta \phi + \frac{\phi \Delta^2 \phi}{M^2} + \dots$$

relevant operator

"Lorentz Fine-Tuning Problem":

Possible Way out: SUSY

No Dim ≤ 4 Lorentz operators in Lorentz MSSM !

Groot-Nibbelink Pospelov '04

$$\left\{Q,\overline{Q}\right\} = \sigma^0 E + c \sigma^i P_i$$

$$\Rightarrow c_i - c_j \propto \frac{M_{SUSY}^2}{M_*^2} \qquad \Rightarrow M_{SUSY} \le 10^{-10} M_* \text{ is enough}$$

 \Rightarrow SUSY at $1GeV - 10^2 TeV$??

Renormalizability?

Second 'kosher' property (in addition to power-counting)

Detailed Balance:
$$V(\gamma_{ij}) = \left| \frac{\delta W}{\delta \gamma_{ij}} \right|$$

(broken softly, otherwise $\Lambda < 0$)

 $W[\gamma_{ij}] = {\begin{array}{*{20}c} \operatorname{action} \operatorname{for} (\operatorname{euclidean}) _{3}D \\ \operatorname{Topologically} \operatorname{Massive} \operatorname{Gravity} (TMG) \end{array}}$

`Quantum inheritance' + TMG is renormalizable ->

actual renormalizability?

- -> Supersymmetrizable ?
- -> Generalization to scalar-tensor TMG?
- -> Parity broken ??

Applications

Applications 1) Black Holes



BH notion really **not present** (no light-cone structure)

Still, approximate notion @ low energies should exist

-> meaning of BH entropy?

Theory contains instantaneous propagation of signals

-> access to BH interior ?

-> enough for info. paradox ?

Kiritsis' talk

BHs in Einstein Aether

Applications 2) Cosmology

bouncing cosmologies

Kiritsis' talk

inflation

generation of scale-invariant perturbations

alignment between CMB and preferred frame

Armendariz-Picon, Farina & Garriga

3) 'Resolution' of singularities

Thank you!