Geometry as Graviton Bose-Einstein Condensate

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In this talk we shall take:

speed of light $\equiv c = 1$

but keep $\hbar$ explicit.

$$[\hbar] = [\text{mass} \times \text{length}]$$

$$[m] = [E] = [p] = [L]^{-1} = [t]^{-1}$$

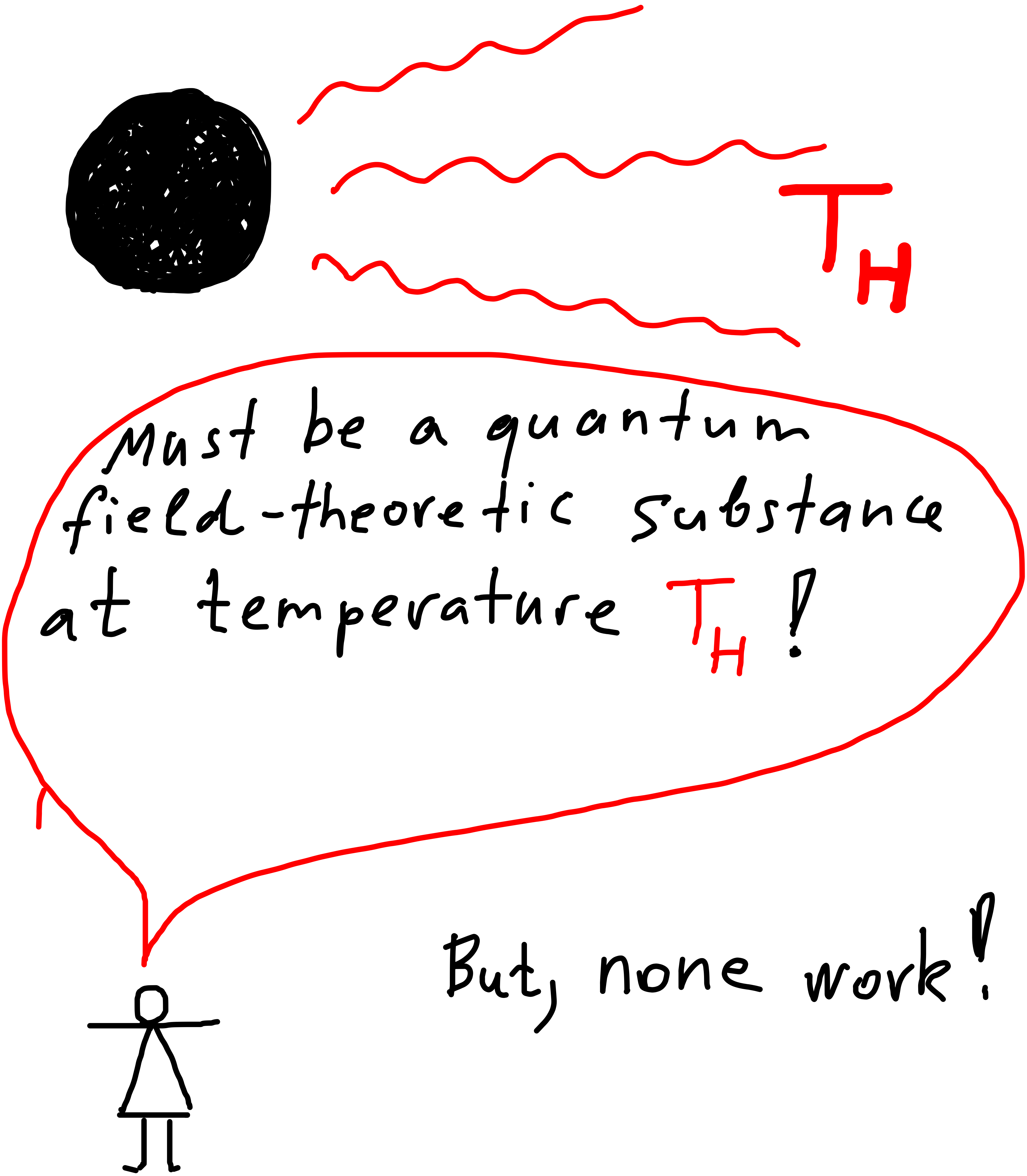
Black Hole Mysteries (semi-classically):

*) Absence of hair;

*) Exact thermality of Hawking radiation and negative heat;

*) Bekenstein entropy;

*) . . . . . .


T_H
Must be a quantum field-theoretic substance at temperature T_H!
But, none work!

Absence of hair and exact thermality

+

A small logical gap filled with a seemgly-logical assumption

=

* "Folk theorems" about no global charges (e.g. baryon and lepton numbers).
* "Information Paradox".

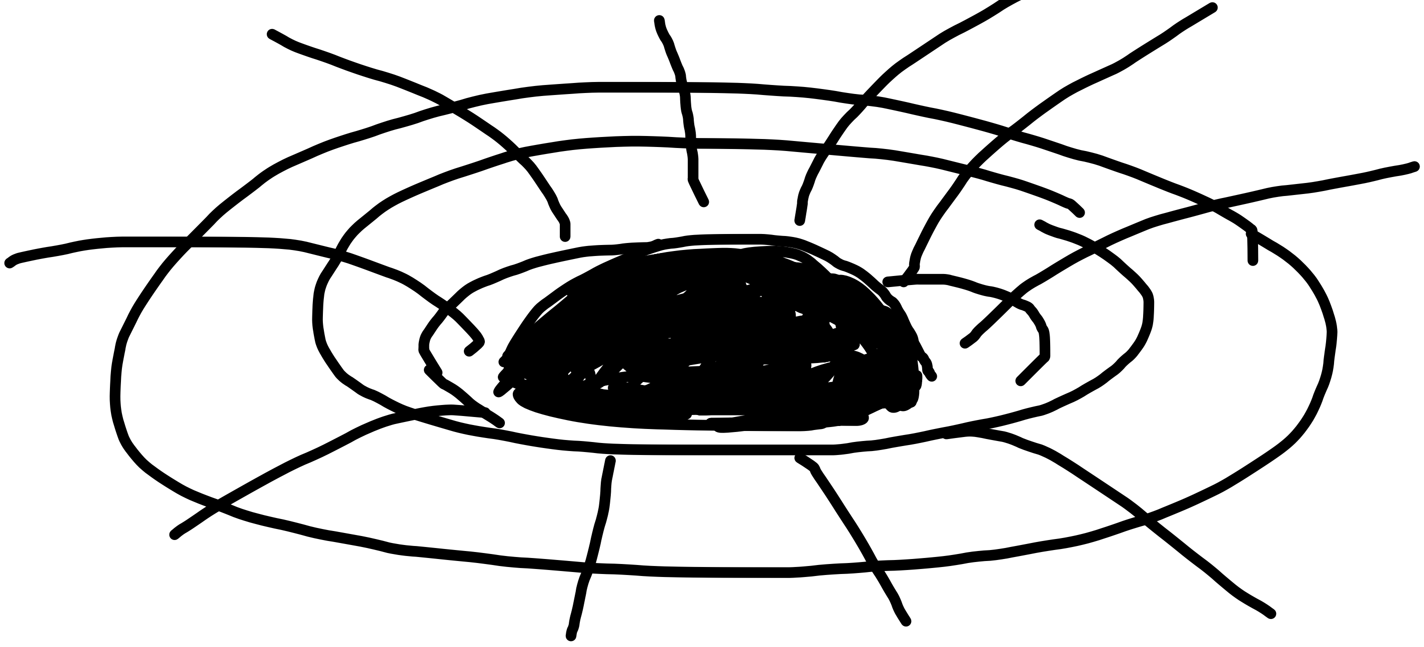
To resolve these "paradoxes", and to close the logical gap, we need a microscopic quantum theory.

In this talk we shall provide such a theory and show how it demystifies semi-classical black hole properties.

We shall see:

Black holes <u>do carry</u> - hair under global charges (baryonic and leptonic numbers), which can be of 100% astrophysical importance.

Recall:
Schwarzschild black hole is a solution in GR



$g_{\mu\nu}(x)$

Intrinsically-classical concept!

In quantum field-theory the building blocks are particles:

$$a^\dagger |0\rangle = |1\rangle$$

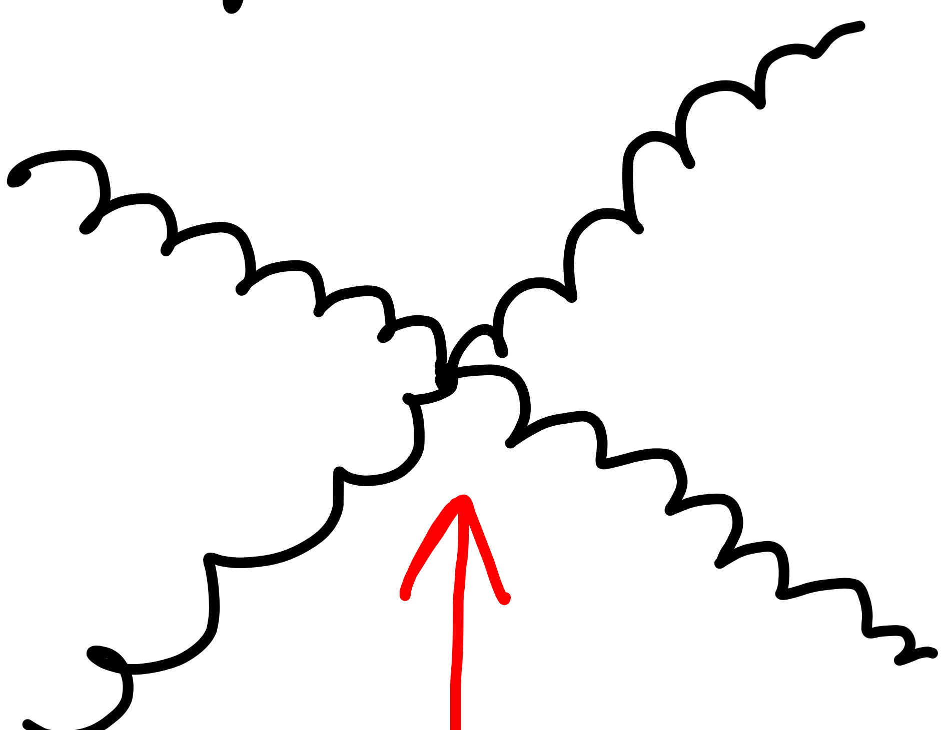
There is nothing else.

Our main concept:

Geometry is a quantum Bose-Einstein condensate of gravitons.

$$g_{\mu\nu} \longrightarrow \left(a^{+}_{\lambda}\right)^{N_\lambda} |0\rangle$$

Gravity is a quantum theory of a particle (graviton) of $m = 0$ and spin $= 2$



$$\alpha_{gr} \equiv \hbar G_N \bar{\lambda}^{2}$$

Quantum entities:
Planck length and Mass

$$L_p^2 \equiv \hbar G_N, \quad M_p \equiv \frac{\hbar}{L_p}$$

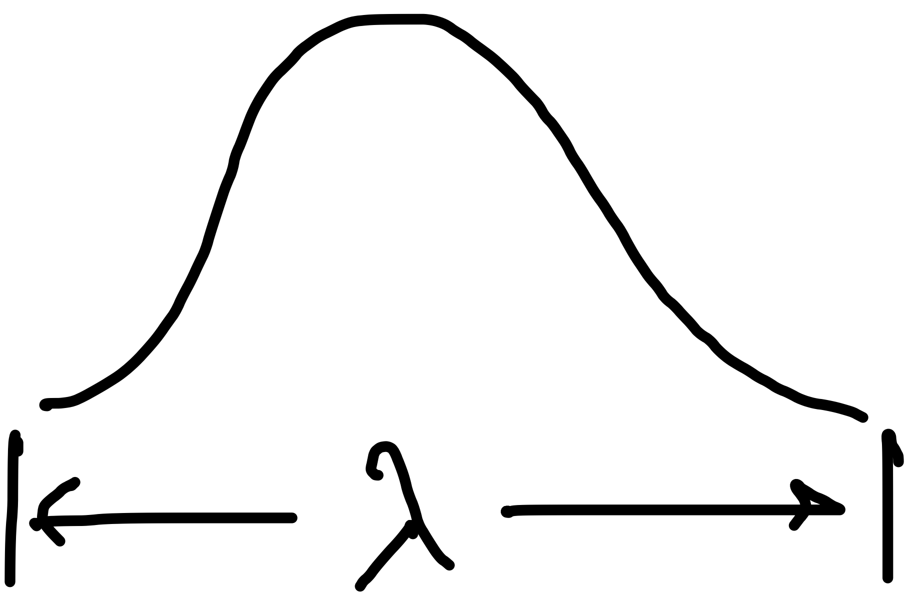
$$\alpha_{gr} = \frac{L_p^2}{\lambda^2}$$

In classical limit ($\hbar \to 0$)

$$L_p \to 0$$

$$\alpha_{gr} \longrightarrow 0$$

Now, try to form a graviton wave packet.



For $\lambda \gg L_P$

$\alpha_{gr} \ll 1$

A typical Hartree situation:

Eech graviton sees a collective potential.

Collective binding potential for $r \sim \lambda$

$$V = -N\alpha_{gr}\frac{\hbar}{\lambda}$$

and kinetic energy

$$E_k = \frac{\hbar}{\lambda}$$

The boundstate condition

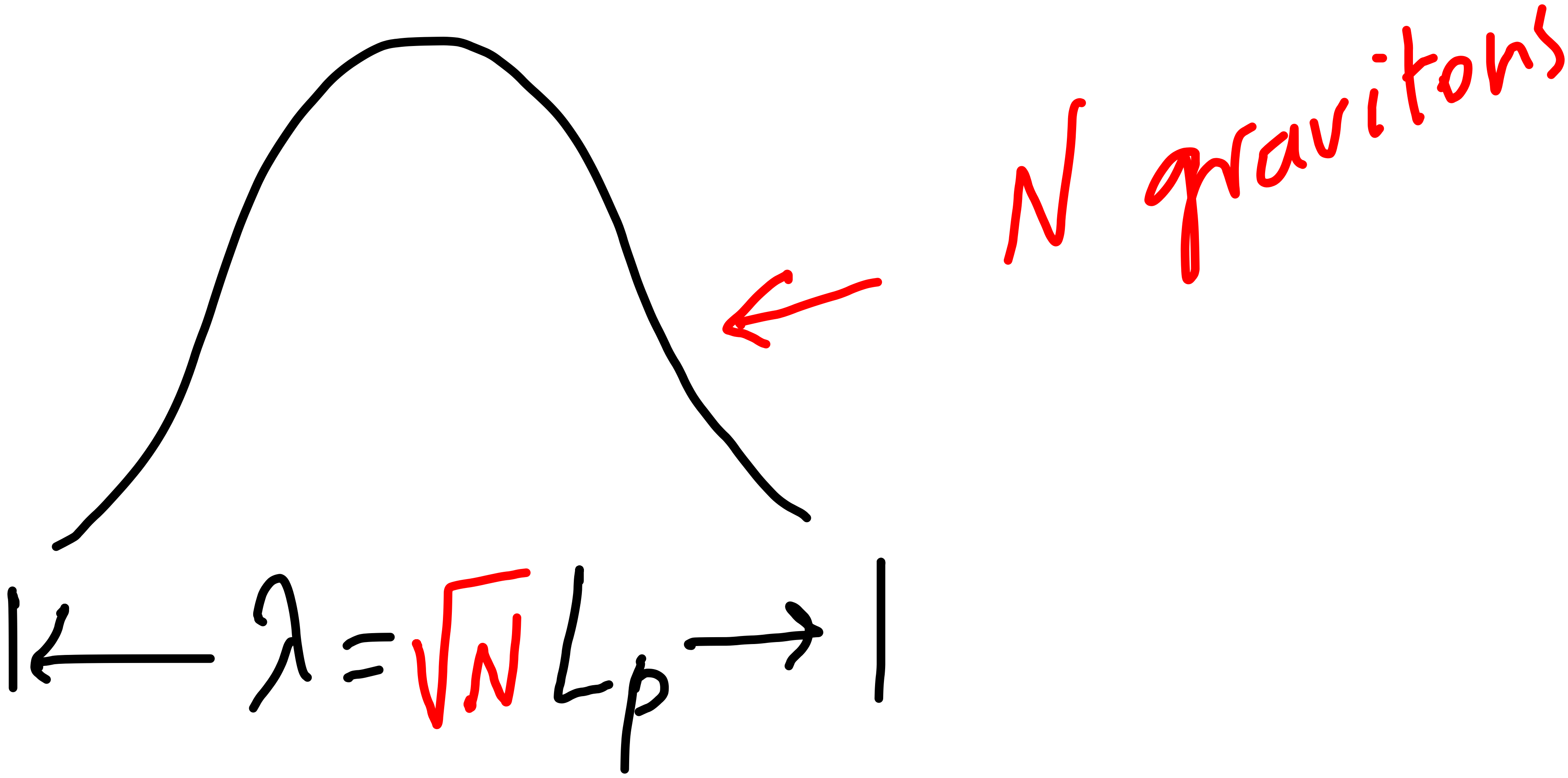
$$E_k + V = 0$$

$$\downarrow$$

$$\left(1 - N\alpha_{gr}\right)\frac{\hbar}{\lambda} = 0$$

A self-sustained boundstate is formed for

$$\alpha_{gr} = 1/N \; !$$



This self-sustained boundstate is a black hole

$$\lambda = \sqrt{N} L_p , \quad \alpha_{gr} = \frac{1}{N}$$

Black hole quantum physics is remarkably simple, with a singe parameter N:

$$M = \sqrt{N}, \quad \lambda = \sqrt{N},$$

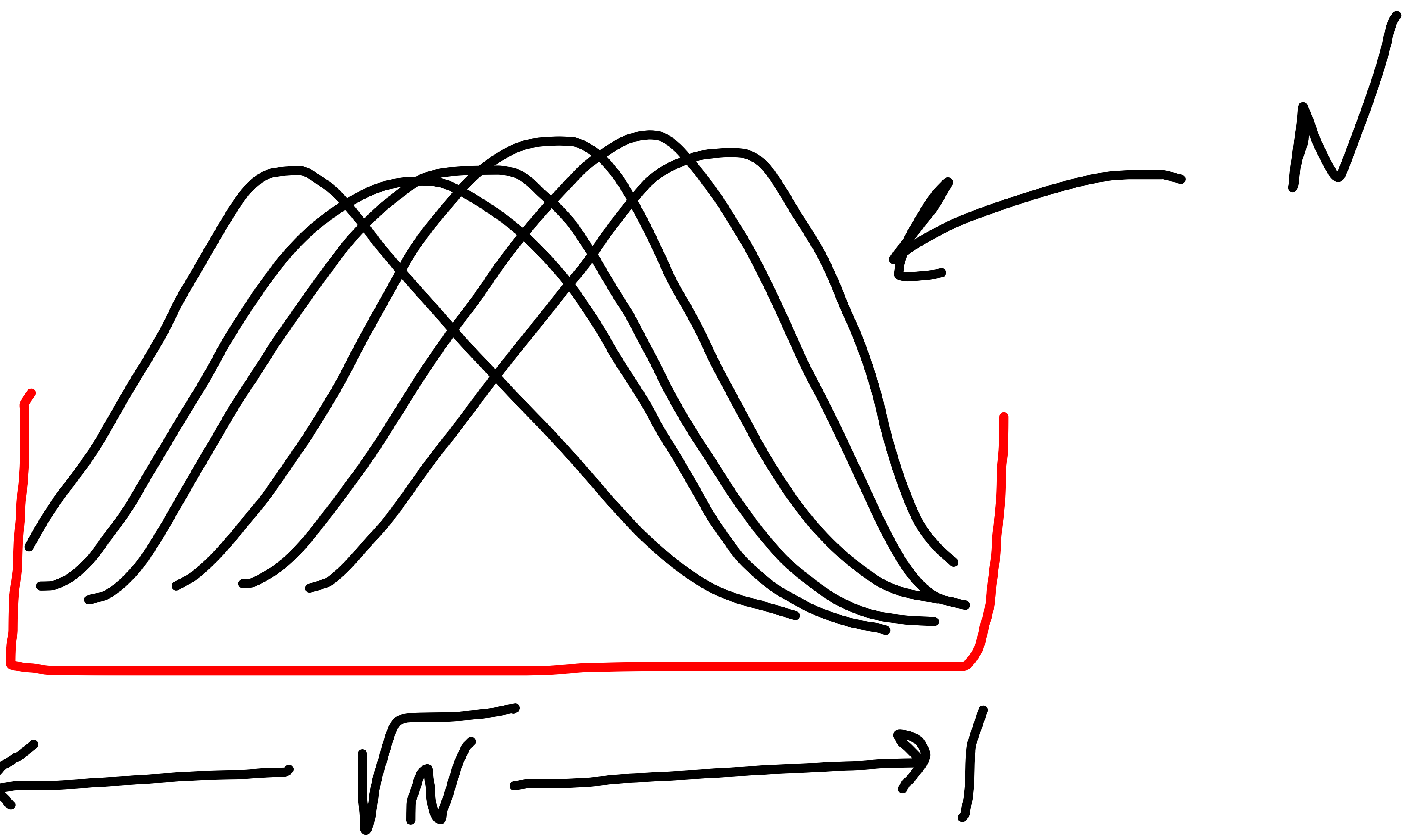
$$\alpha_{gr} = \frac{1}{N}$$

It is a large-N physics (in 't Hooft's sence) and is a result of maximal overpacking.

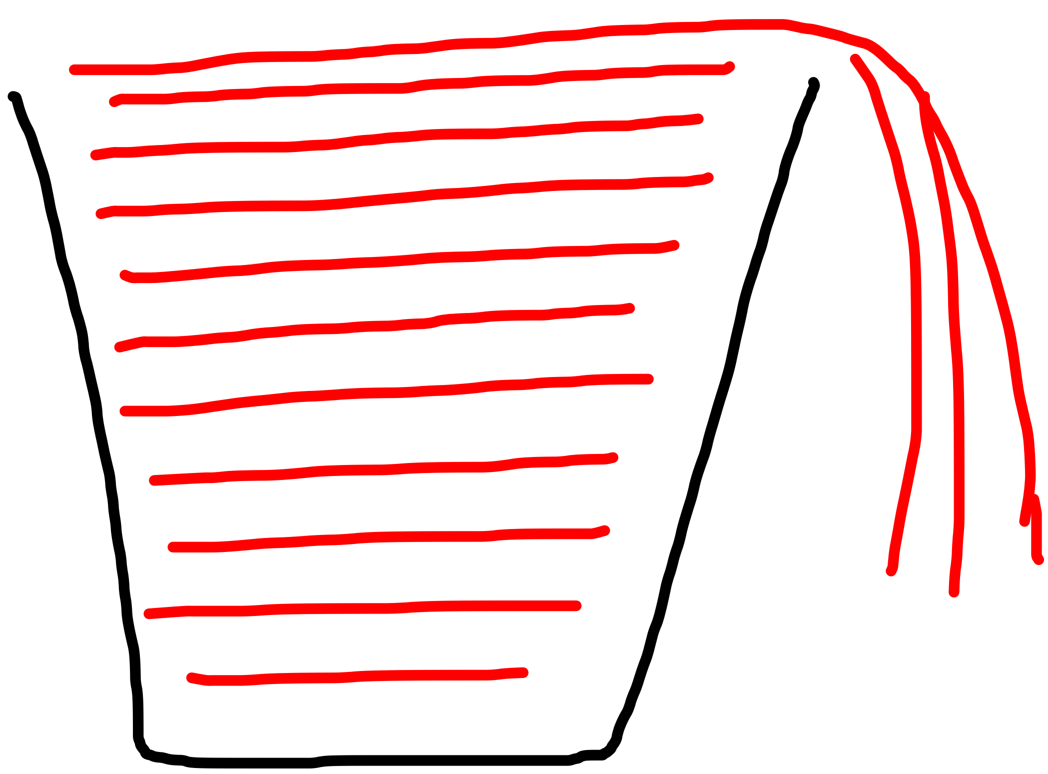
Black hole is a most overpacket quantum system of nature

and

because of this it is maximally simple

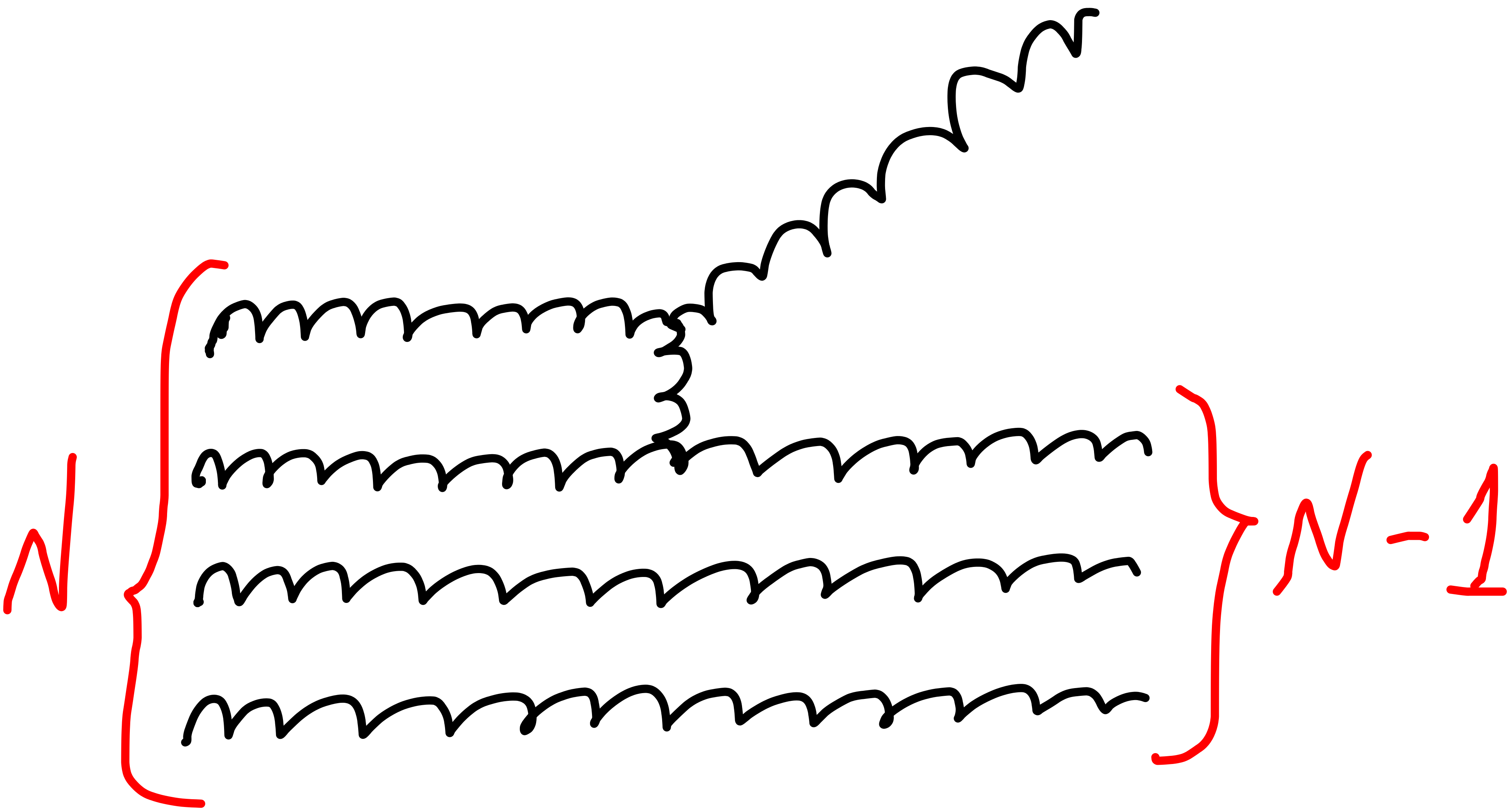


This self-sustained Bose-condensate exists for any N and for any N it is leaky



The condensate depletes self-similarly

$$N \to N-1$$



depletion law

$$\dot{N} = -\frac{1}{\sqrt{N}\,L_P} + O\left(\frac{1}{N^{3/2}}\right)$$

$$\dot{N} = -\frac{1}{\sqrt{N}\,L_p}$$

Defining $T \equiv \frac{\hbar}{\sqrt{N}\,L_p}$,

in the semi-classical limit

$$N \to \infty, \quad L_p \to 0, \quad \sqrt{N}\,L_p = \text{fixed}$$

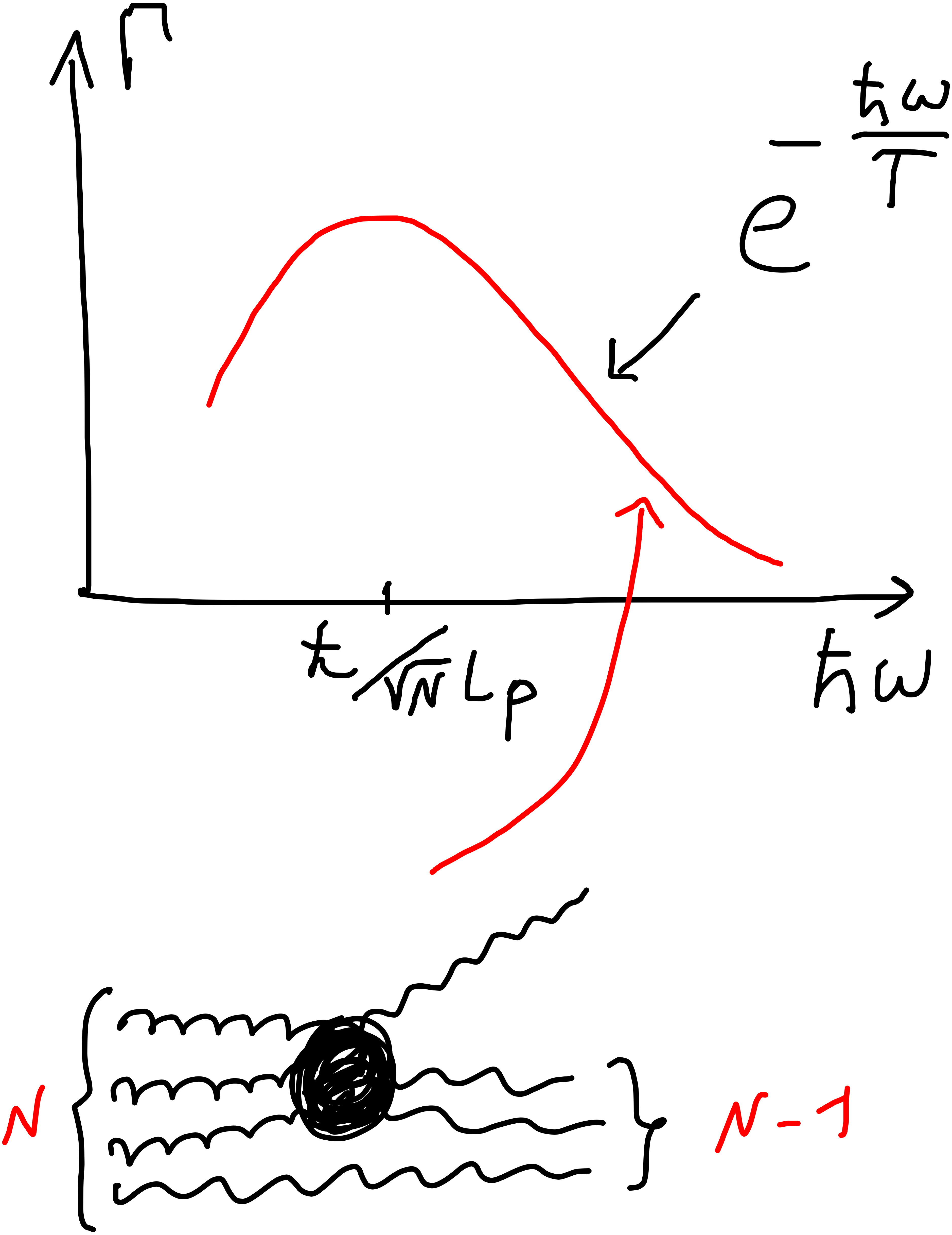
We get Stefan-Boltzmann law for Hawking evaporation

$$\dot{M} = -T^2/\hbar$$

We discover that thermality is an "optical illusion":

Spectrum is thermal because of the self-similarity of depletion, not because the source is hot.

The graviton condensate is cold!



Γ
$e^{-\frac{\hbar\omega}{T}}$
$\hbar/\sqrt{N}L_p$
$\hbar\omega$
N
N-1

We see:

*) Thermality of the source is an "optical illusion".

*) Deviations are $\sim \frac{1}{N}$, not e^{-N}.

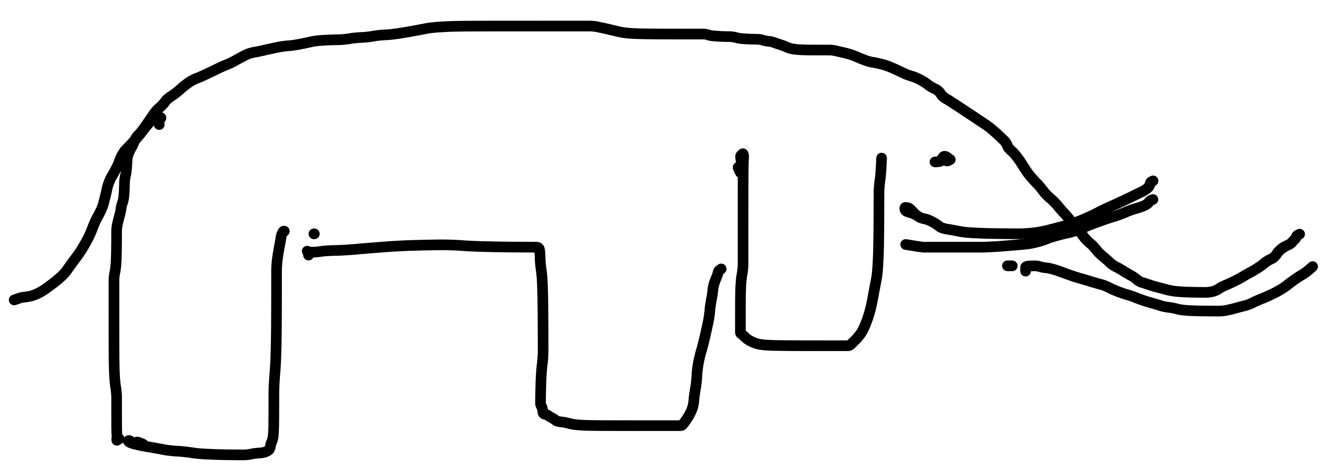
Thus, quantum effects are 100% important on scales $\sim N$!

All the black hole "paradoxes" are result of <u>semi-classical treatment</u>.

But, how can quantum effects be important for macroscopic objects?

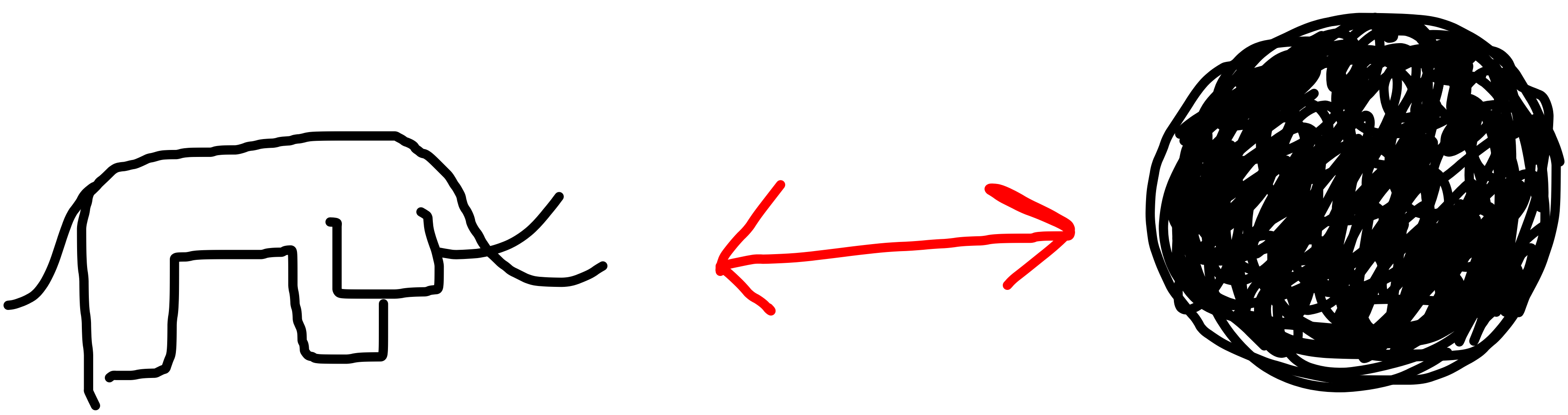
After all, treating semi-classically stars, planets, elephants, ... is fine.

Naively, this contradicts to usual intuition that macroscopic objects are (almost) classical



quantum gravity $\sim e^{-M_E L_E}$

The answer is that Black Holes are macroscopic, but <u>quantum</u>!



$$\sim e^{-N} \longleftrightarrow \sim 1$$

Notice, Bekenstein entropy

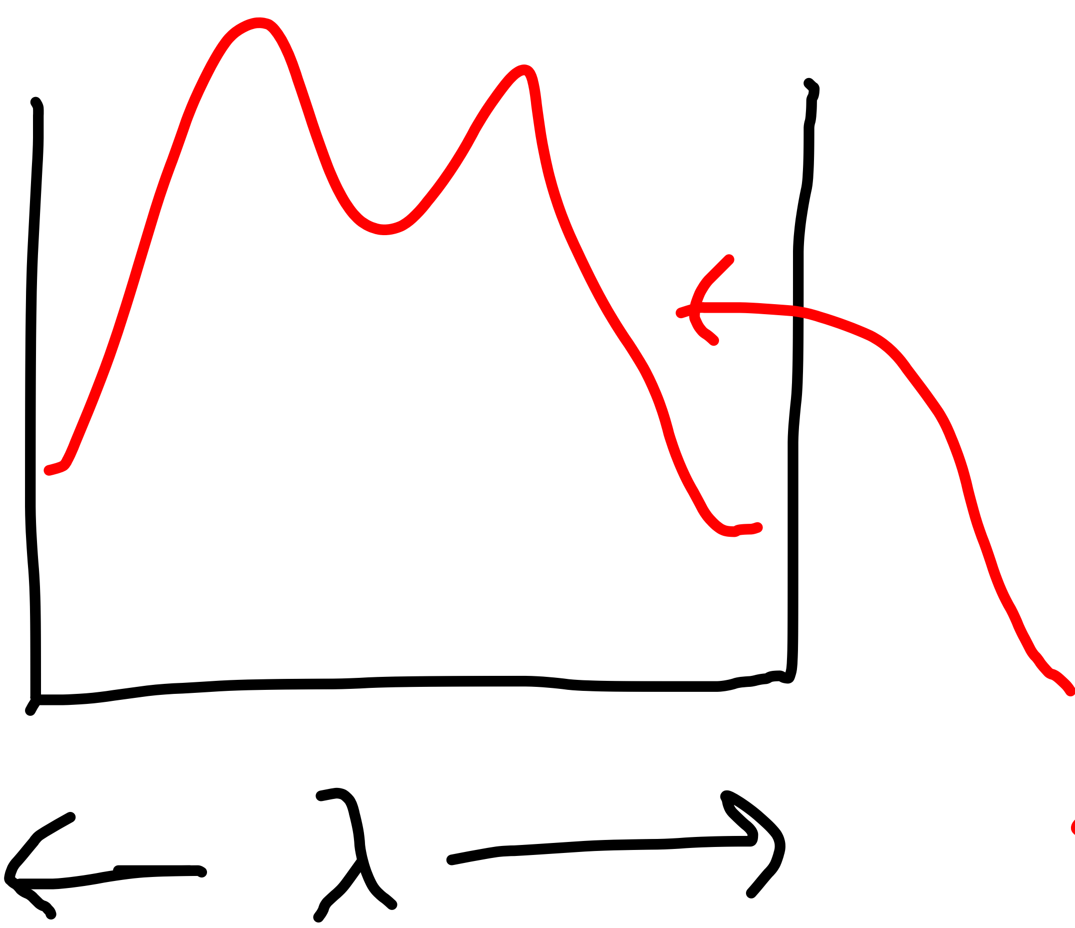
$$S = N = \frac{\lambda^2}{L_p^2} \to \infty$$

for $L_p \to 0$, $\lambda = \text{fixed}$
$\hbar = \text{fixed}$

Semi-classical limit of Hawking.

Why is this non-trivial?

Because, usually a box of size λ takes energy $\Delta E \gtrsim \hbar/\lambda$ to store one bit of information



$$\Delta E \gtrsim \frac{\hbar}{E}$$

Instead, it appears that a fixed size black hole can store unlimited information as long as $N \to \infty$.

What is micro-physics behind this phenomenon?

First, what is <u>classicality</u>?

Nature is quantum $\hbar \neq 0$.

Classicality implies many particles.

For example, earth's gravitational field is classical because it contains $N \sim 10^{66}$ gravitons!

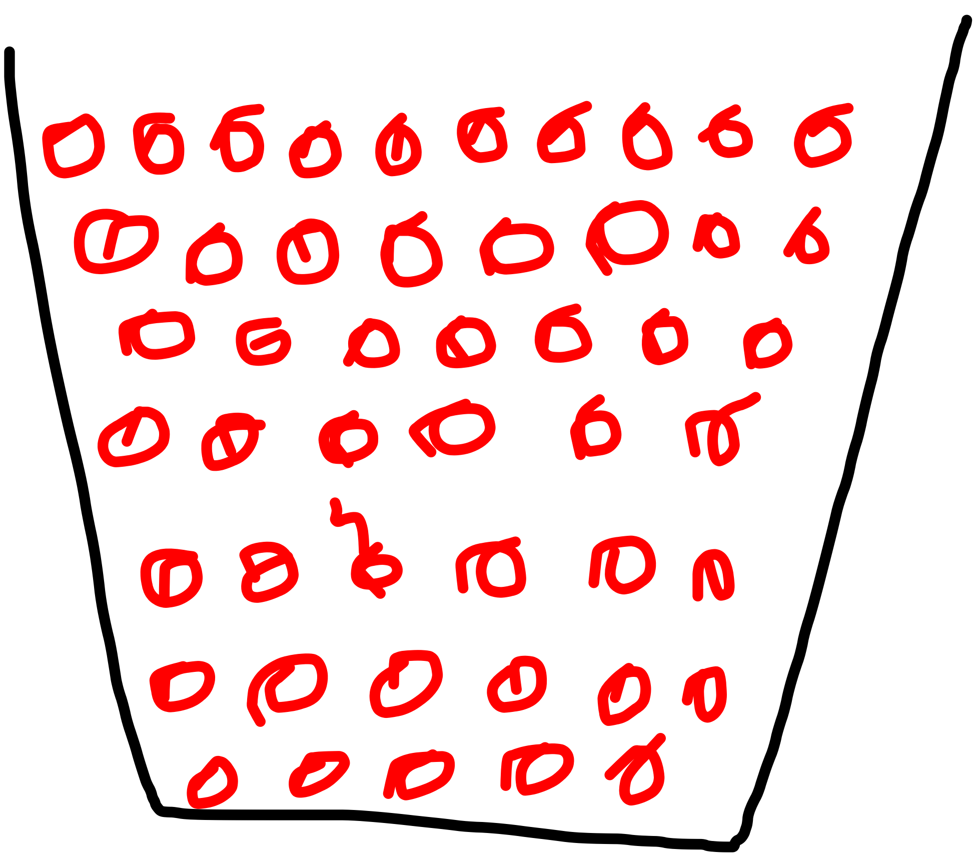
In contrast, gravitational field created by a single electron contains only

$$N \sim 10^{-44}$$

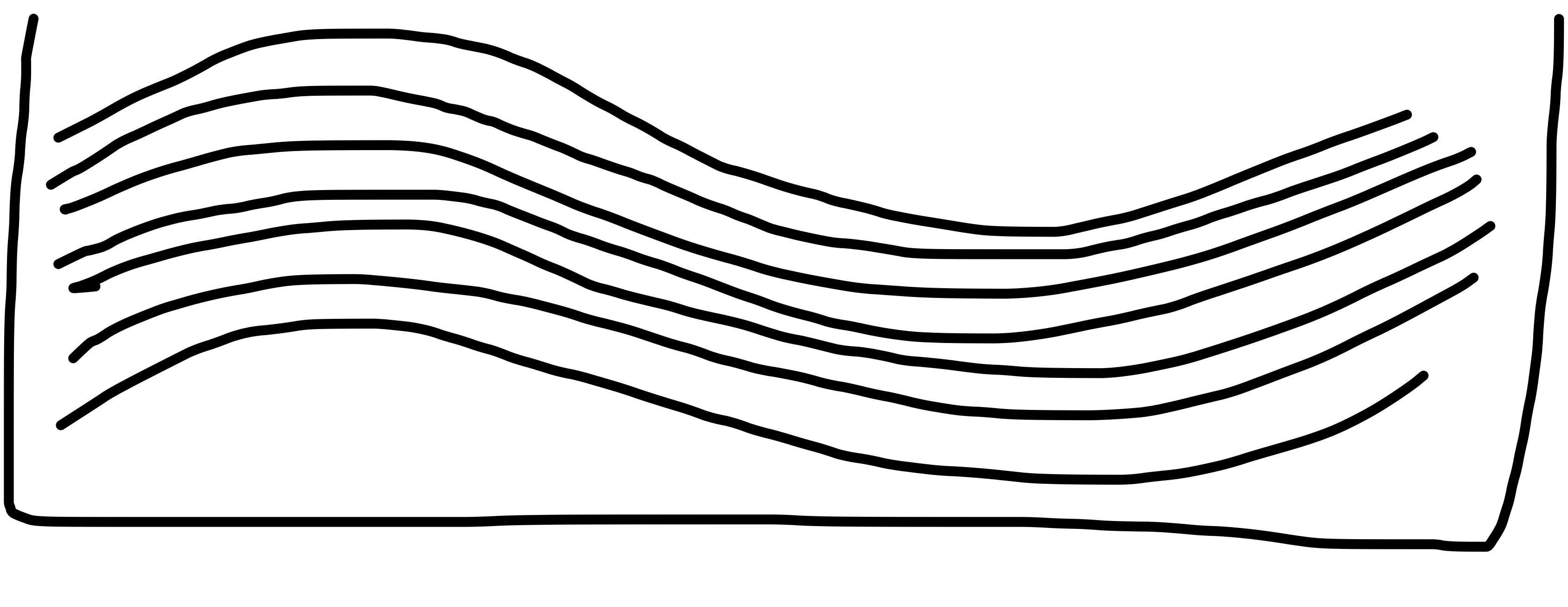
gravitons!

(This is why the electron is not a black hole.)

Marcoscopic objects are characterized by number of constituents N, their coupling strength α, ...



However α has an universal meaning in the systems in which everybody talks to each other at a same strength, such as Bose-Einstein condensates.



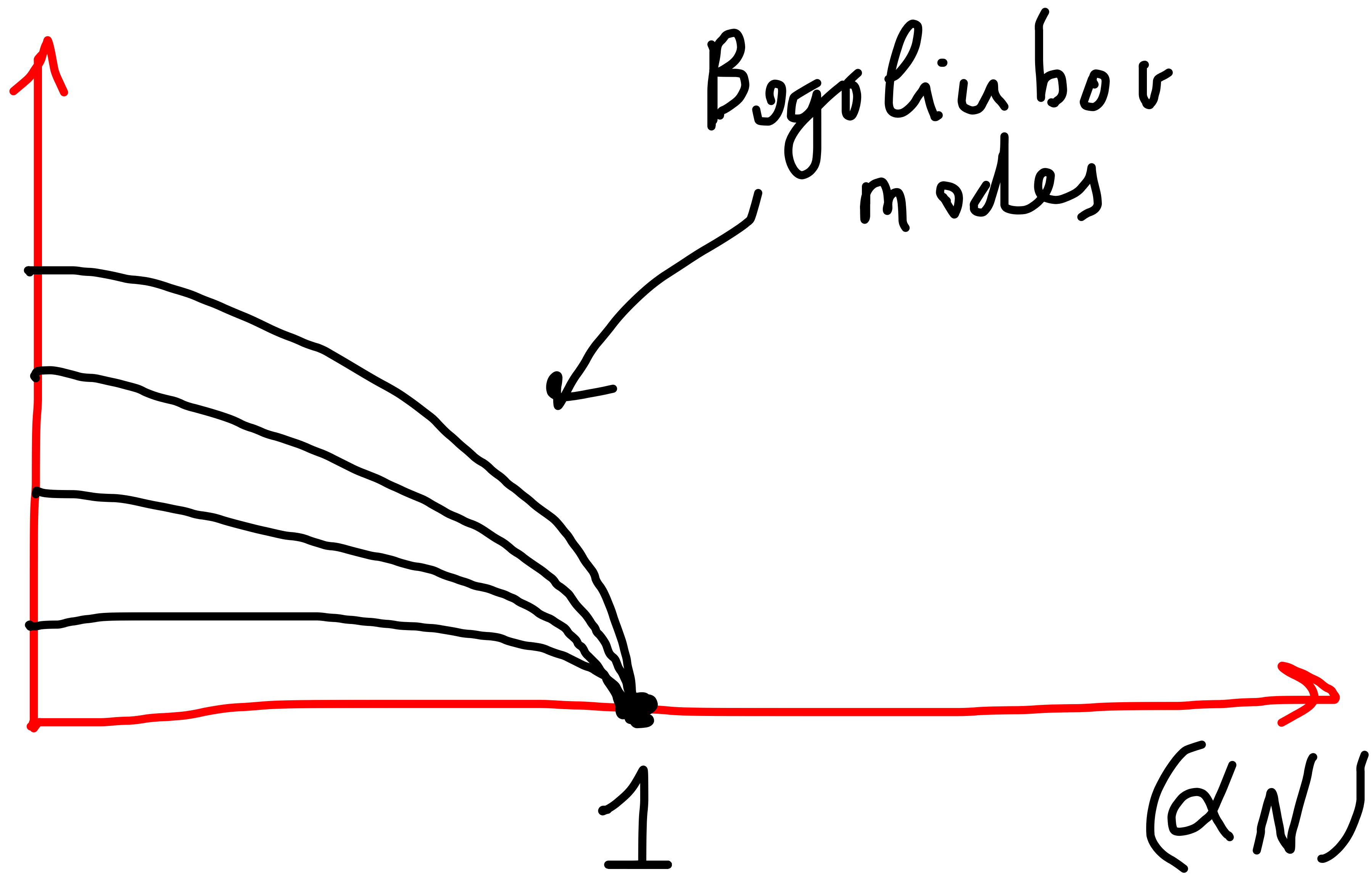
For such systems we can define a quantity

$$(N\alpha)$$

Something very special takes place at

$$N\alpha = 1$$

Critical point of quatum phase transition.



Such a system although multi-particle in reality is fully quantum.

Black hole reduced to bare essentials.

Bose-gas order parameter $\equiv \psi$

$$n(x) = \langle \psi(x)\, \psi(x) \rangle$$

Hamiltonian

$$H = \int_V -\hbar L_0 \psi^\dagger \Delta \psi - \hbar L_p^2 \psi^\dagger \psi^\dagger \psi \psi$$

Normalization

$$\int_V \psi^\dagger \psi = N$$

$$\Psi = \sum_k \frac{a_k}{\sqrt{V}} e^{i\frac{kx}{\hbar}}$$

$$[a_k a^+_{k'}] = \delta_{kk'}$$

$$\mathcal{H} = \sum_k k^2 a^+_k a_k - \frac{\alpha}{4} a^+_{k+p} a^+_{k'-p} a_k a_{k'}$$

Bogoliubov replacement:

$$a^+_0 = a_0 = \sqrt{N_0} \simeq \sqrt{N}$$

$$a^+_0 a_0 + \sum_{k\neq 0} a^+_k a_k = N$$

$$\mathcal{H} = \sum_{k\neq 0} \left(k^2 + \frac{\alpha N}{2}\right) a_k^+ a_k -$$

$$- \frac{1}{4} (\alpha N) \left(a_k^+ a_{-k}^+ + a_k a_{-k} \right)$$

Bogoliubov transform

$$a_k = u_k b_k + v_k^* b_k^+$$

$$u, v = \pm \frac{1}{2} \left(\frac{k^2 - \alpha N/2}{\varepsilon(k)} \pm 1 \right)$$

$$\varepsilon(k) = \sqrt{k^2 (k^2 - \alpha N)}$$

$$\mathcal{H} = \sum_k \varepsilon(k)\, b_k^+ b_k$$

collapses to $\frac{1}{N}$ at the critical point. Depletion sets in

$$n_k = |\gamma_k|^2$$

$$\Delta N \sim n_1 = \left(\frac{1 - \frac{\alpha N}{2}}{\sqrt{1 - \alpha N}} - 1 \right) \simeq \sqrt{N}$$

Energy gap

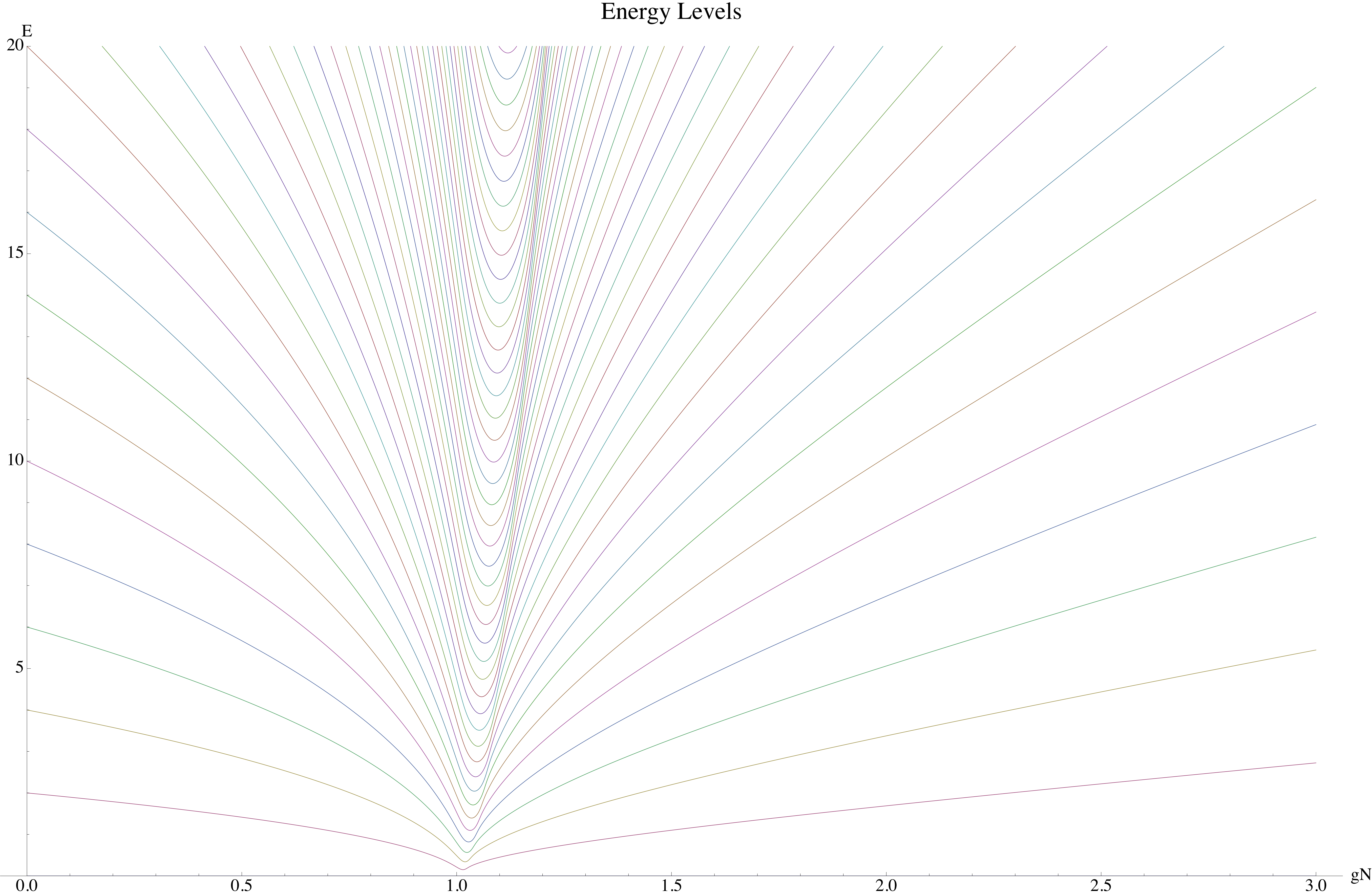
$$\varepsilon_1 = \frac{\hbar}{\lambda\sqrt{N}} = \frac{1}{N}\frac{\hbar}{L_P} \,!$$

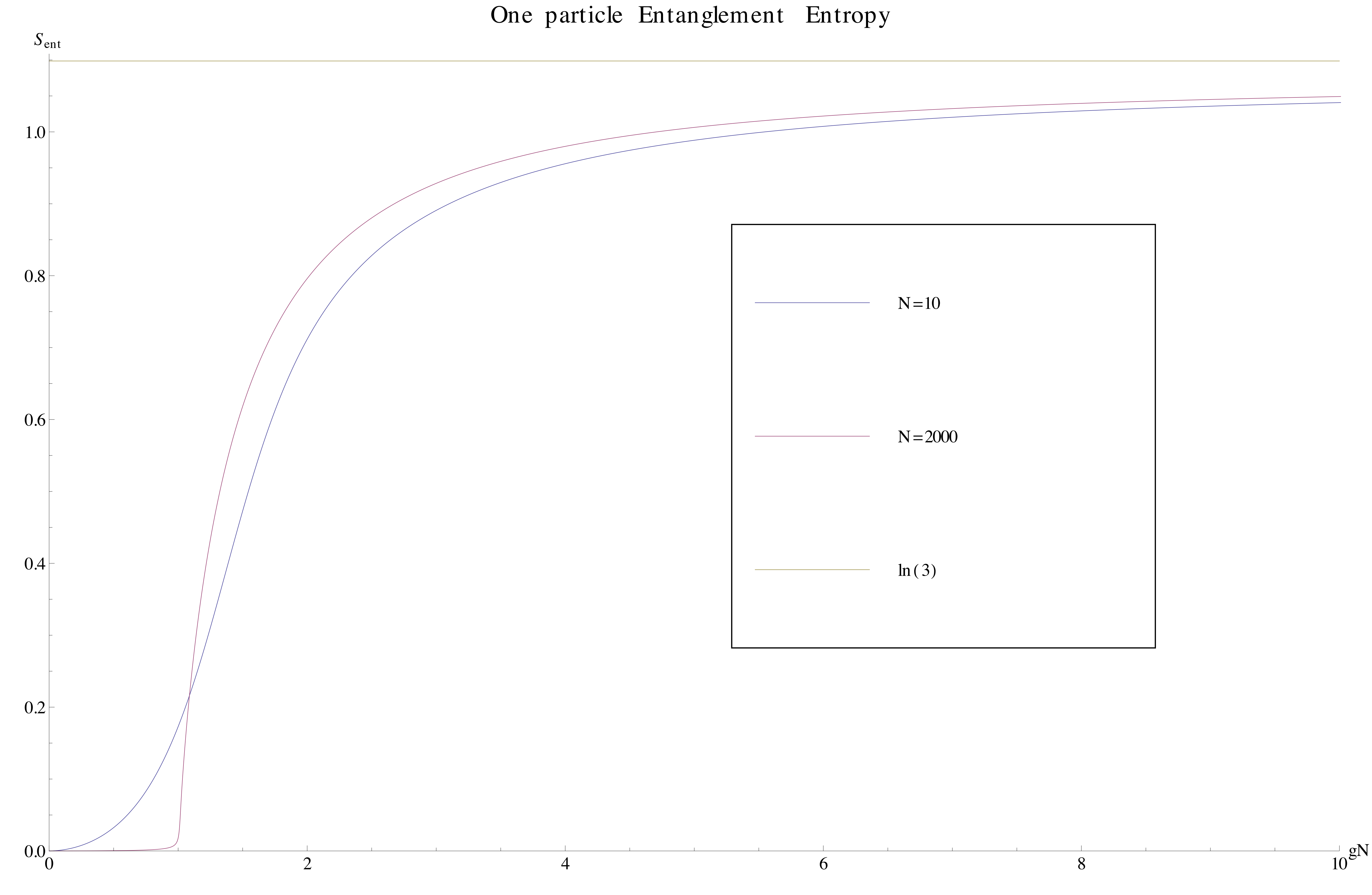
These Bogoliubov modes are quantum ("holographic") degrees of freedom responsible for

Bekenstein entropy.

Some numerical studies by:

Daniel Flassig,
Alex Pritzel,
Nico Wintergerst


Energy Levels
E
20
15
10
5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
gN



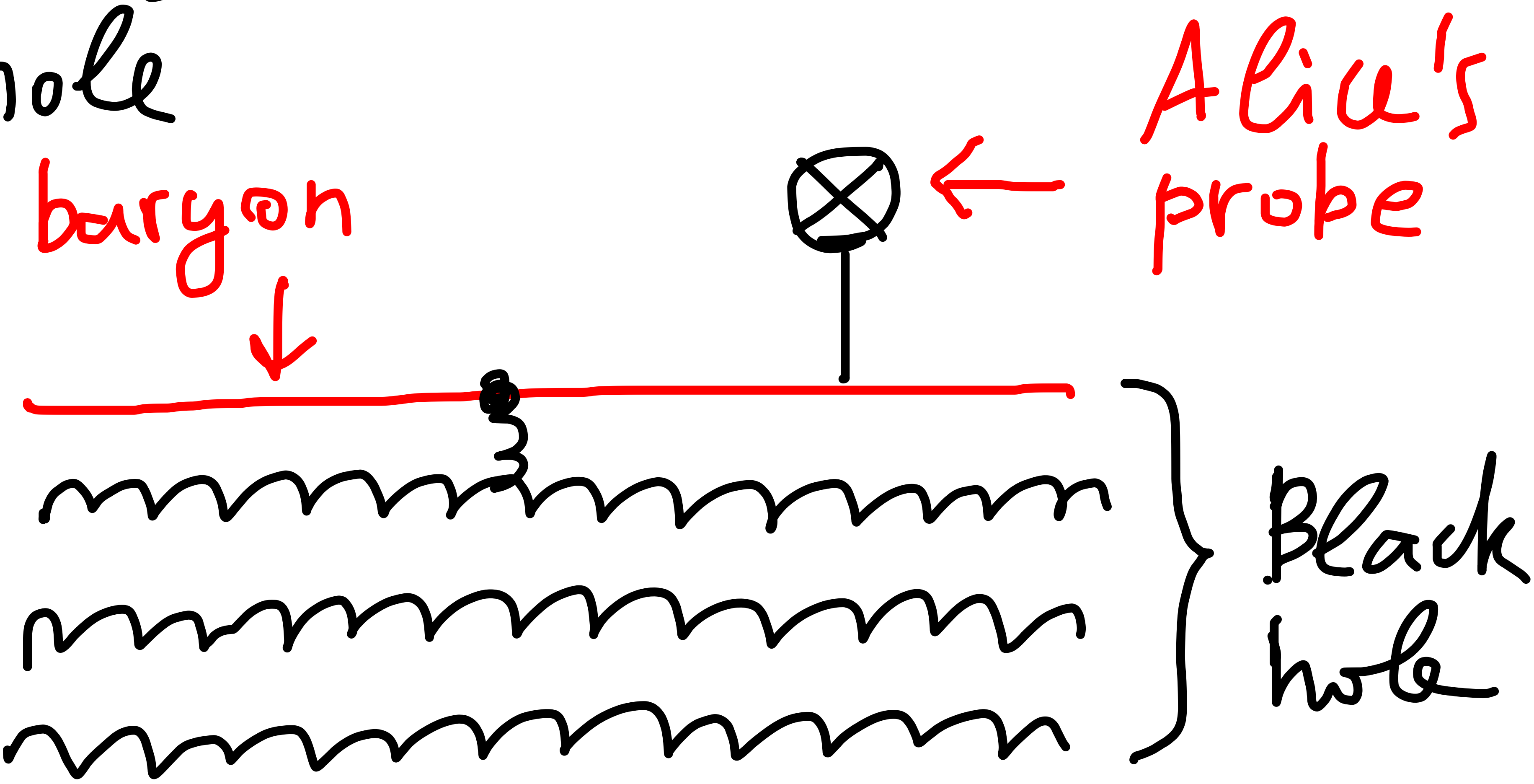
One particle Entanglement Entropy
S_{ent}
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10
gN
N=10
N=2000
ln(3)

Another (false) artifact of semi-classical limit is the absence of hair.

In reality black holes carry a detectable hair as

$$\frac{N_B}{N} - \text{effect}$$

How Alice detect a baryonic hair of a black hole



$$\Gamma_{hair} = \frac{1}{\sqrt{N}\,L_p}\left(\frac{N_B}{N}\right)$$

For Astrophysical black holes (that carry large baryonic or leptonic charges) the hair can be an observable effect.

Depletion law for a global charge for $N \gg N_B \gg 1$:

$$\dot{N}_B = -\frac{1}{\sqrt{N}\, L_P} \frac{N_B}{N} + \dots$$

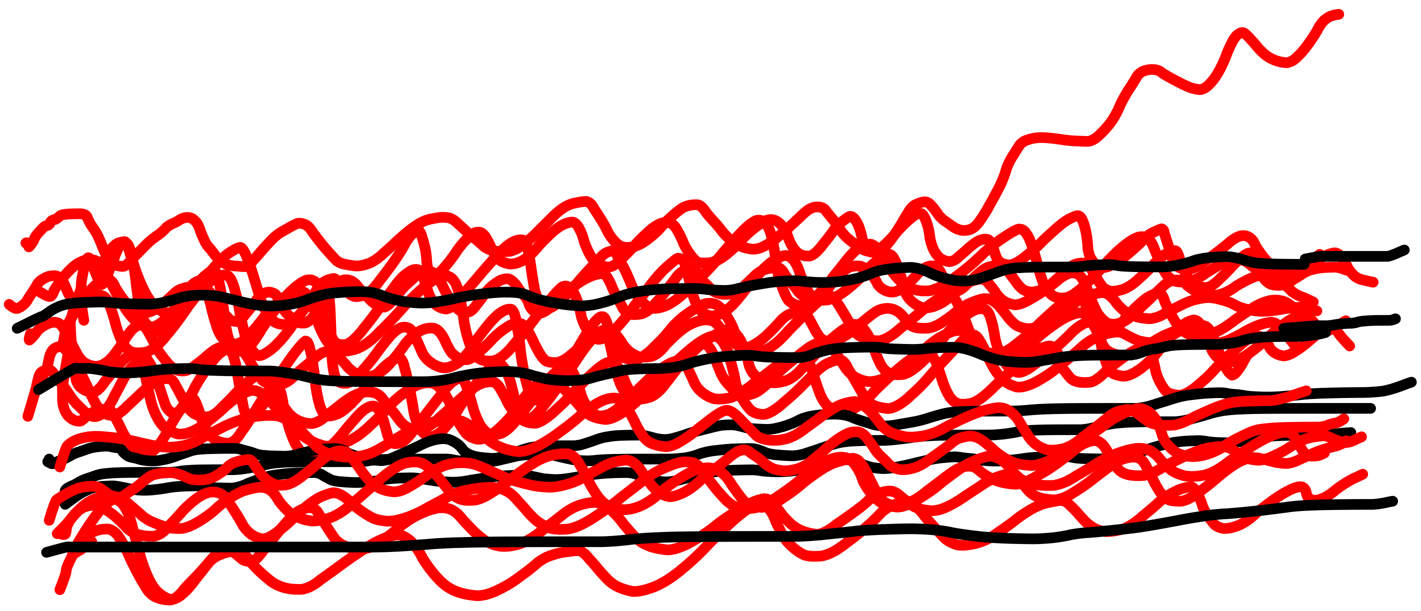
$$\boxed{\tau = \frac{2}{3} \frac{t}{L_P}}$$

$$\Big\downarrow$$

$$N(\tau) = (\tau_* - \tau)^{\frac{2}{3}}$$

$$N_B(\tau) = \left(1 - \frac{\tau}{\tau_*}\right)^{\frac{2}{3}} N_B(0)$$

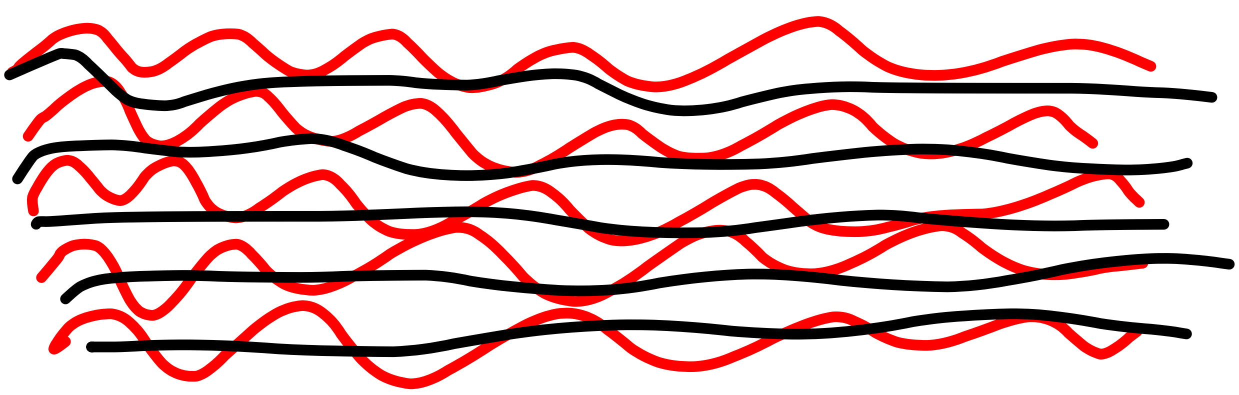
(microscopic reason for Page's information retrieval)



For $N_B \ll N$ the depletion and leakage continues untill non-gravitational interaction between "baryons" becomes important!

For example for "baryons" interacting with gravitational strength, this will happen when

$$N_B \sim N$$



what happens after?

Depends on a delicate balance between gravity and non-gravitational forces.

One thing is certain beyond this point evolution of a macroscopic black hole is nothing like we thought before.

The interesting case for Dark Matter is when short-range "baryonic" forces balance gravity.

There is an indication that for

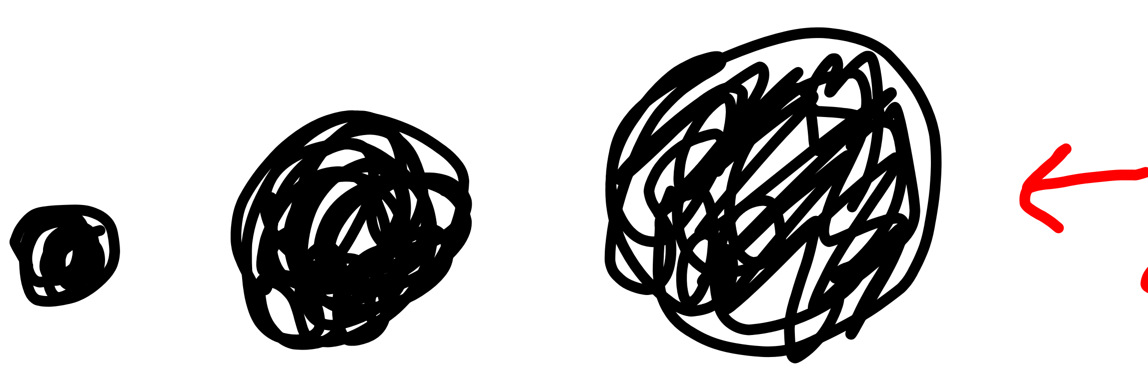
$$R_{BH} < L_{QCD}$$

this is the case for

$$N_B \sim N$$

If true, such black holes can be intersting Dark Matter candidates with masses in new range

$$M_p < M_{DM} < 10^{17} g$$



DM Species

Implications?

Many:

*) Global symmetries are OK with Black Holes (B, L, family, Axion ...)

*) Self-Completion of gravity by classicalization.

Our picture suggests the following quantum foundation of holography:

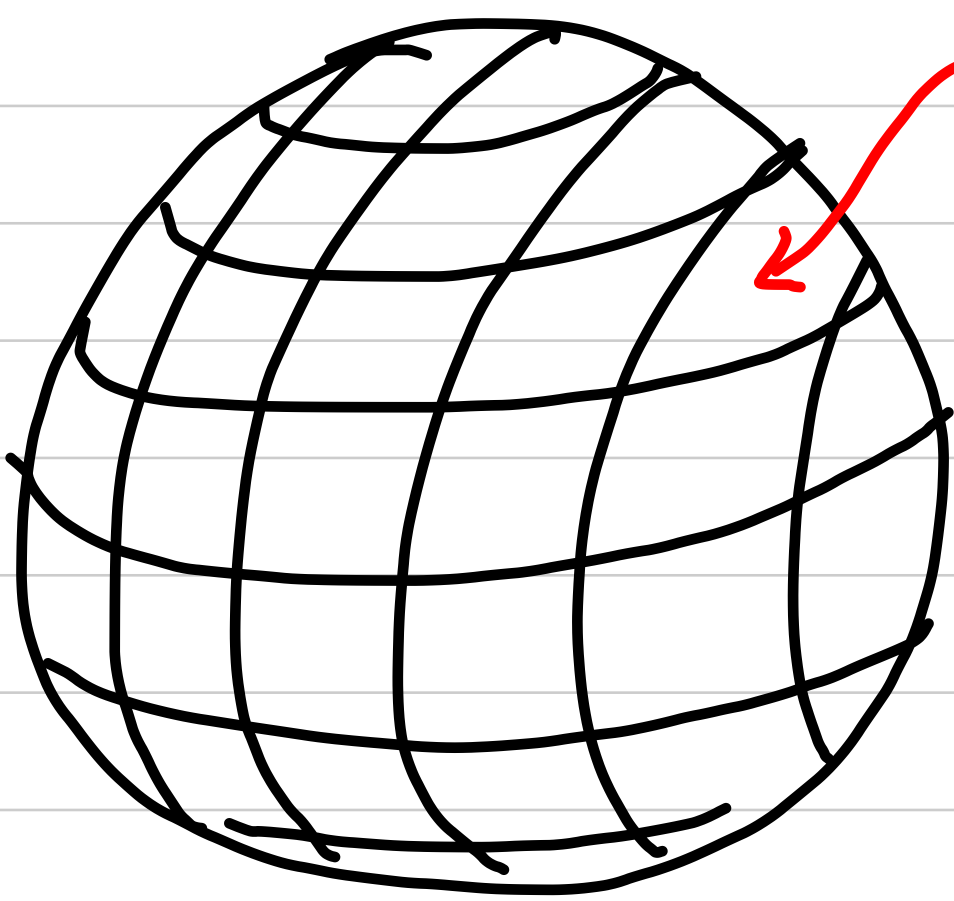
Gravitational systems that exhibit holography are Bose-Einstein condensates at the critical point of quantum phase transition.

The "holographic" degrees of freedom then are nearly gapless (and conformal) Bogoliubov modes.

We are learning that overpacked systems get oversimplified.

Origin of holography

L_p^2-pixel



$$N = \frac{r_g^2}{L_p^2}$$

It is interesting that generalizing our idea to AdS/dS-geometry, we get the same N-portrait:

In D-dimensions:

$$N = R^{D-2} / L_D^{D-2}$$

$$\lambda = N^{\frac{1}{D-2}} L_D$$

$$\alpha_D = \frac{1}{N}$$

Notice, that N coincides with the central charge of CFT

$$N_{CFT} = N = \left(\frac{R}{L_D}\right)^{D-2}$$

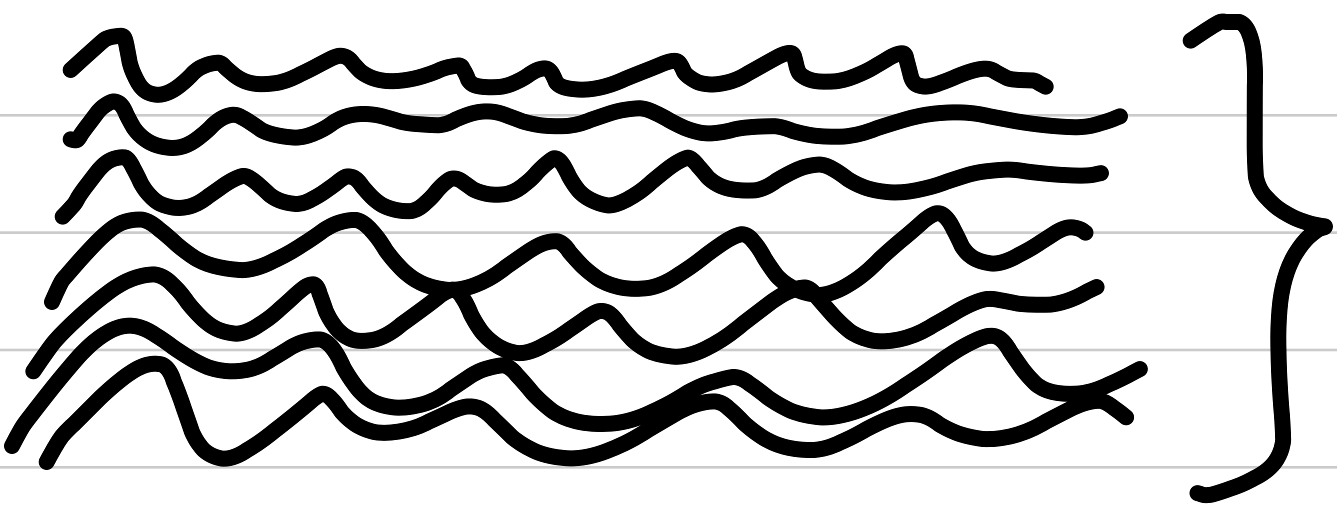
Let us apply this to AdS 5.

We can think of it as of a Bose-condensate of gravitons of wave length

$$\lambda = R$$

and occupation number

$$N = R^3 / L_5^3$$

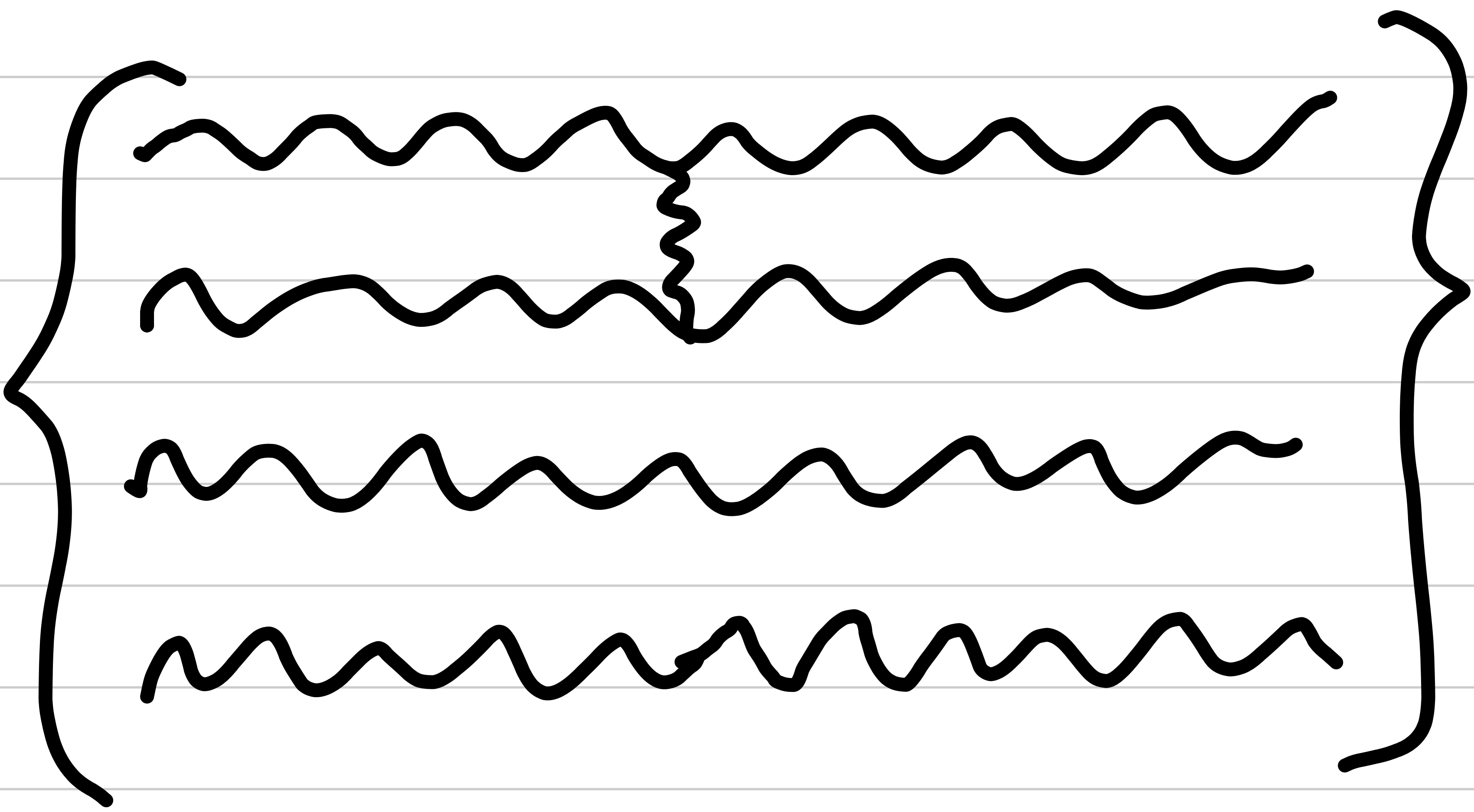


N

-

These gravitons interact with the strength

$$\alpha_{gr} = L_5^{2} R^{-2}$$



N

But, unlike the black hole case, the AdS Bose-condensate <u>cannot</u> deplete!

This is because N is fixed by the value of the 5D cosmological term in the action!

Rewriting everything in terms of N, we get the same quantum N-portrait for the AdS as for black holes, with the only difference being depletion

Large-N quantum portrait of AdS:

N-graviton condensate with

$$\lambda = N^{\frac{1}{3}} L_5$$

$$\alpha_{gravity} = \frac{1}{N}$$

The quantum origin of holography in this language is clear;

The system is maximaly packed. So the only characteristics is

$$N$$

which coincides with the central charge of CFT!

Our results show:
Einstein's gravity cannot be UV-completed in a Wilsonian way!

Instead, it is self-complete by classicalization!

WHY?

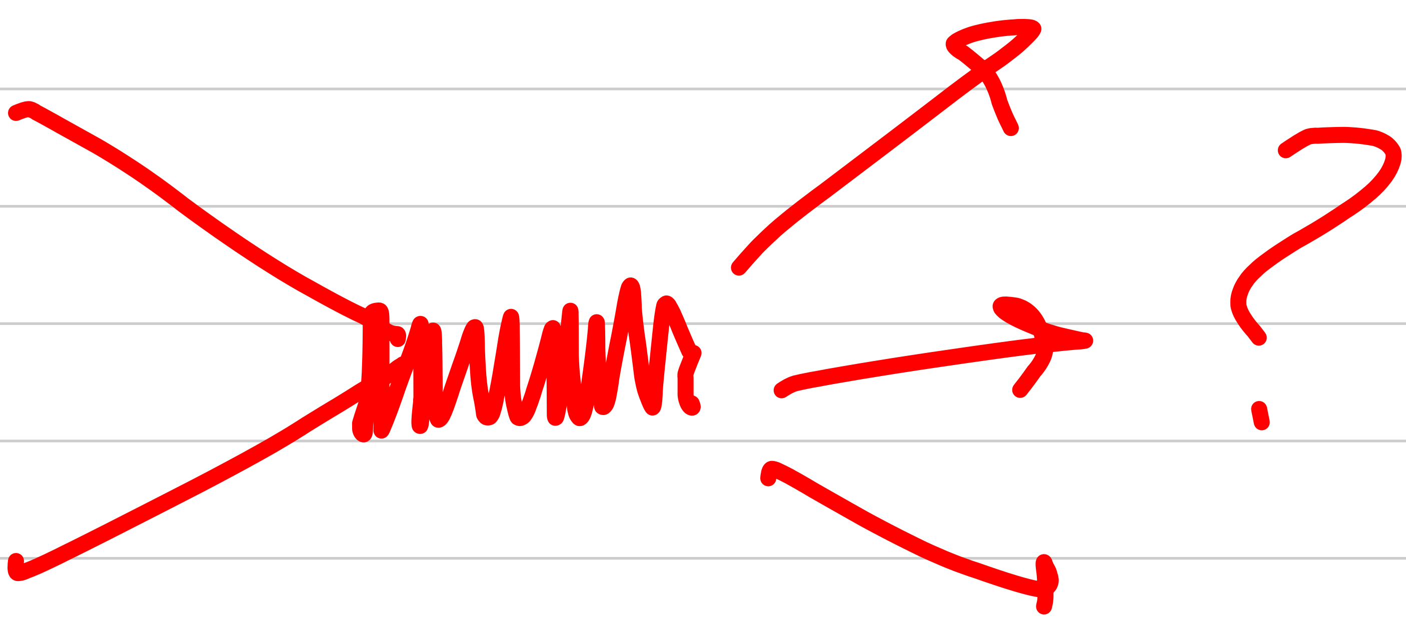
Because Wilsonian UV-completion implies that you can probe arbitrarily-short distances.

In particular,

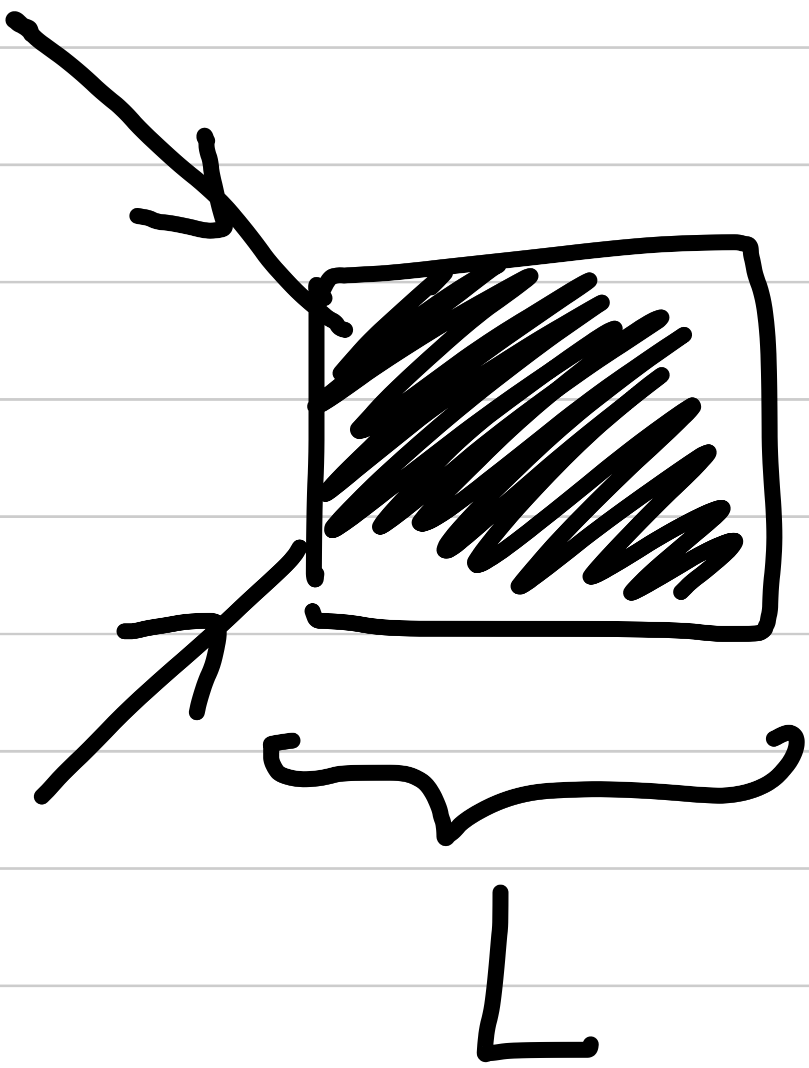
$$L \ll L_p$$

But, in our picture this is impossible!

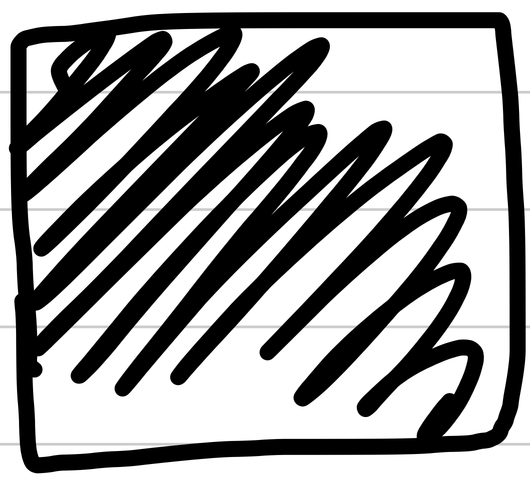
Let us assume that I ask you to probe distance L by scattering two particles



to do this you have to bring these two particles within the box of size L :



But, for $L \ll L_p$,



$\Big\} \; L \ll L_p$

We have

$$N = \frac{L_p^2}{L^2} \gg 1$$

and the box becomes an N-particle state!

In gravity there are no two-particle states with $\frac{1}{L} \gg \frac{1}{L_P}$!

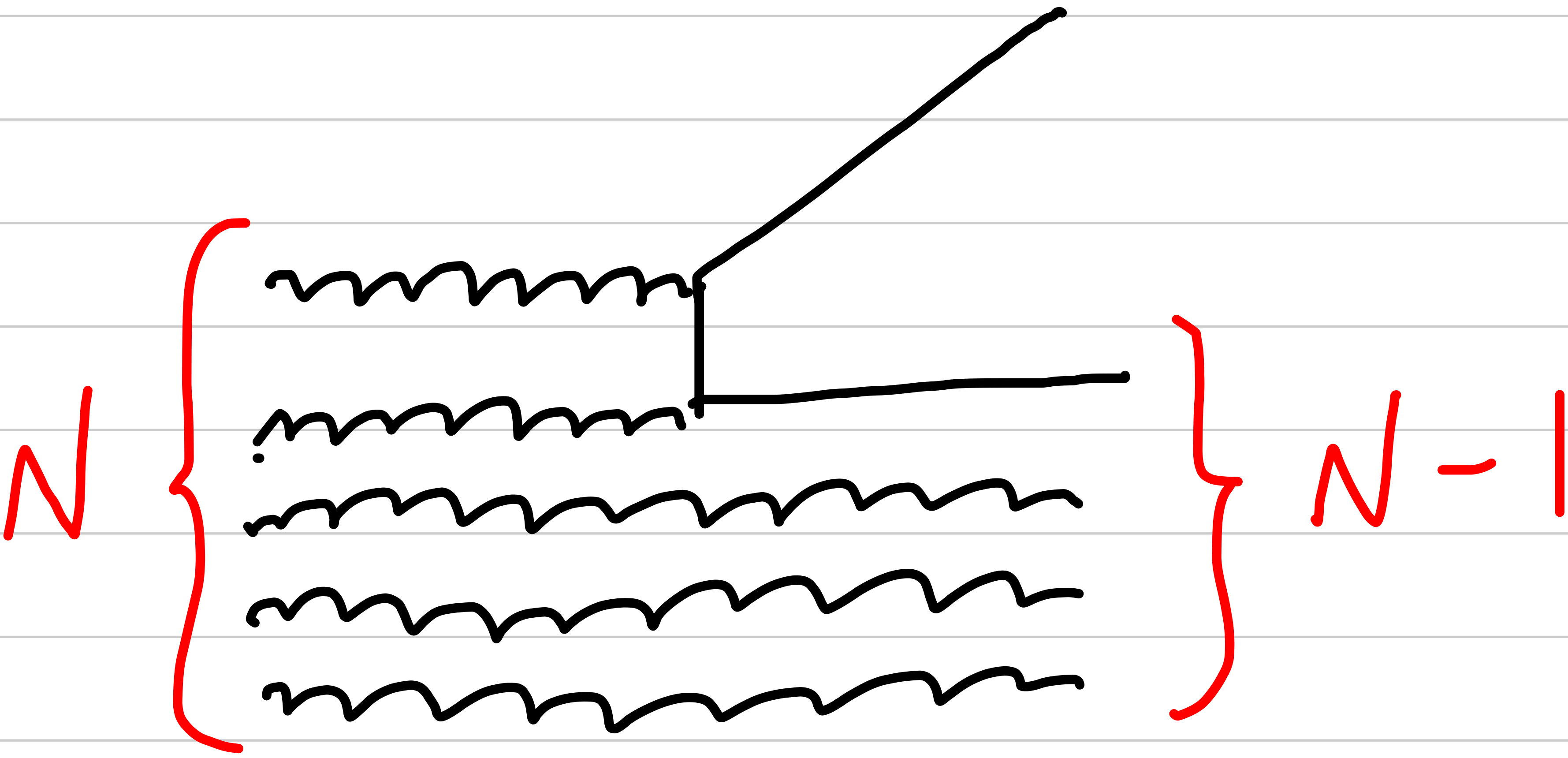
Instead in your scattering experiment the process will be dominated by

$$2 \to N$$

putting it differently,
we shall end-up
with N-particle states.

The system
<u>classicalizes!</u>

If there are extra species



$$\Gamma = \frac{\hbar}{\sqrt{N}\, L_p} N_{species}$$

Species bound:
Black holes cannot exist
for $N < N_{species}$!

Semi-classically this bound translates to the bound on r_g:

$$N > N_{species}$$

$$\Downarrow$$

$$r_g > L_N \equiv \sqrt{N_{species}}\, L_P$$

Outlook

Black hole's quantum portrait is a microscopic framework which allows to address questions that in the conventional treatment cannot even be formulated.

It demystifies the known semi-classical puzzles in black hole physics.

Among many potential applications is Cosmology:

The Universe is the largest black hole we know.

It's a graviton Condensate with

$$N \sim 10^{120}$$

Why self-completion by classicalization?

Because in gravity there are no small boxes with high energy and few particles!

$$2 \to 2$$

$$2 \to N$$