Black Holes as Bose-Einstein Condensates of GRAVITONS.

Work with Gia DVALI.

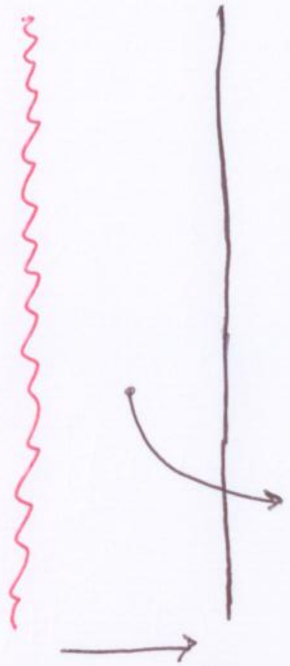
* Why black holes are mysterious ?

* Because when combined with Q.M lead to several paradoxical results. In particular to Hawking's Information paradox. (*)

* At the root of these paradoxes lies the Classical notion of HORIZON.

(*) And to the surprising prediction on the Non existence of Global continous symmetries.

unbroken

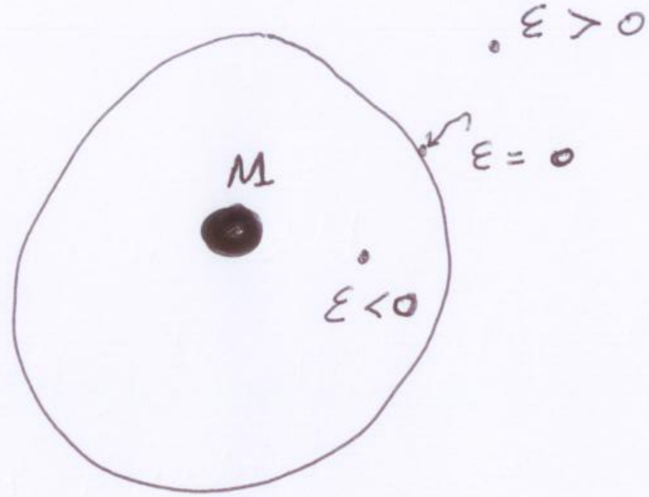


$\frac{\partial}{\partial t}$ timelike

$\frac{\partial}{\partial t}$ spacelike

i.e $\pm$ eigenvalues.

$R_g = M L_{pe}^2$



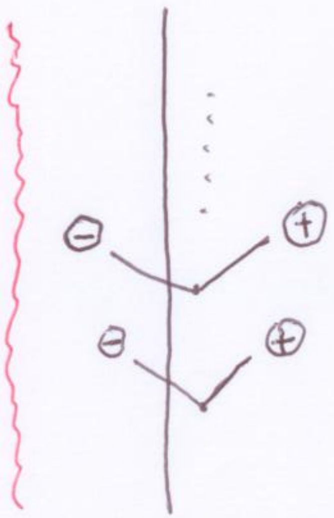
$$\varepsilon \sim m + \varepsilon_{gr} = m - \frac{M \cdot m L_{pe}^2}{r}$$

Hawking main insight:

Vacuum polarization near the classical horizon leads to BH radiation.

$\boxed{\text{Horizon Geometry transforms virtual pairs into REAL emission.}}$

Hawking's process:



$$|M\rangle \to$$

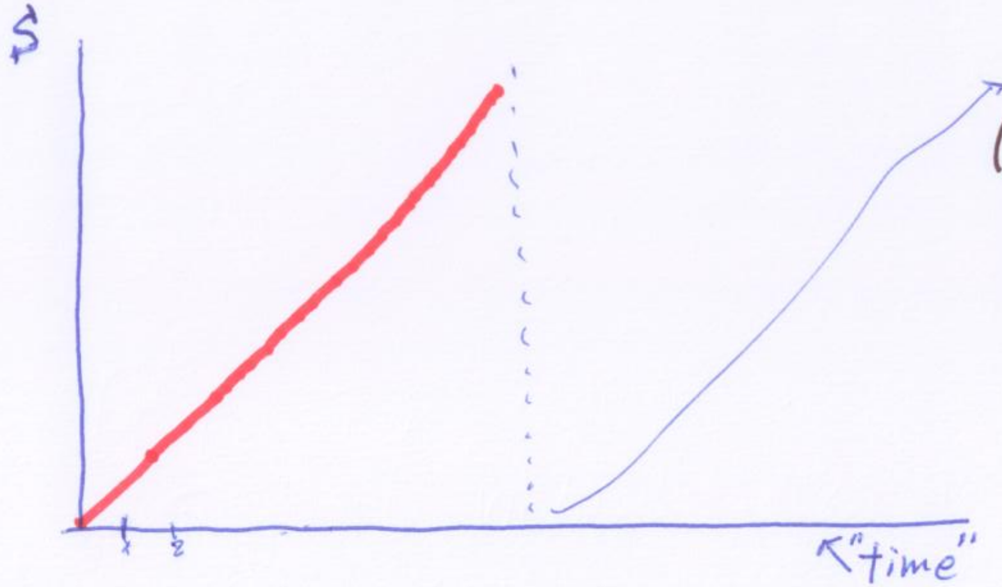
$$|M\rangle \otimes \underbrace{\left(\frac{1}{\sqrt{2}}|0\rangle_-|0\rangle_+ + \frac{1}{\sqrt{2}}|1\rangle_-|1\rangle_+\right)}_{\text{entangled pair}} \to$$

$$\to \quad \cdots\cdots$$

Trace over states inside the horizon

$\Rightarrow$ entanglement entropy

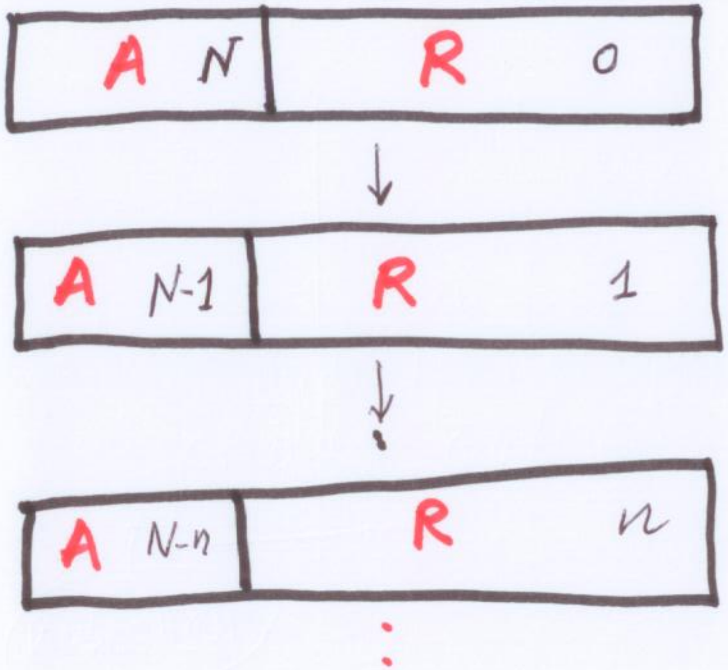
$S \sim \ln(2)$ (in each step)



"Q. Gravity scale"

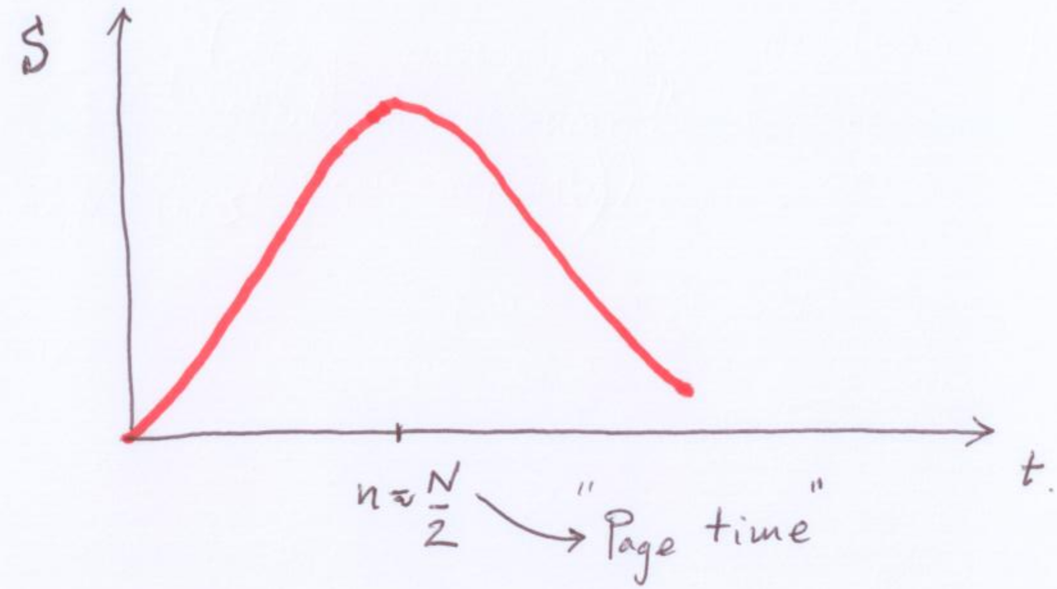
(strong gravitational effects in the near horizon region)

What is the difference with standard evaporation?



$2^N = \dim \mathcal{H}_{tot}$.

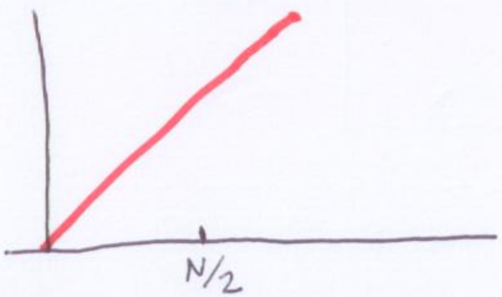
At each step: $S \sim \ln d$ dimension of smaller system



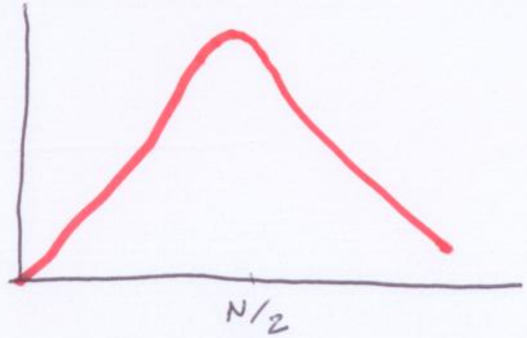
Black Hole problem:

What kind of Quantum Corrections transform:

"Hawking"



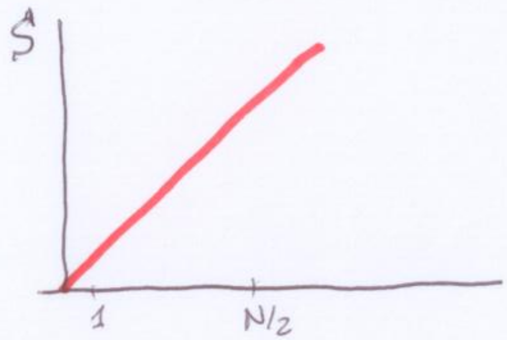
into



?

Assuming Bekenstein Entropy: $S_{BH} = \frac{1}{4}\frac{A}{L_{pl}^2}$ $\boxed{N \equiv S_{BH}}$

It is clear that what we need to take into account to transform:



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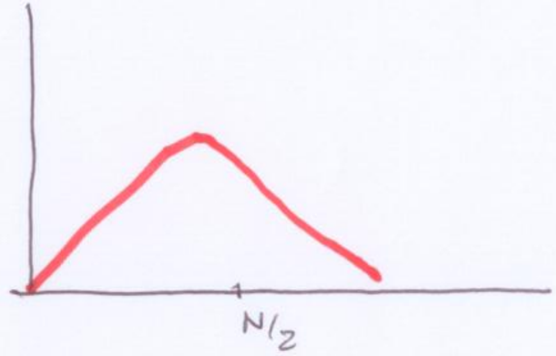
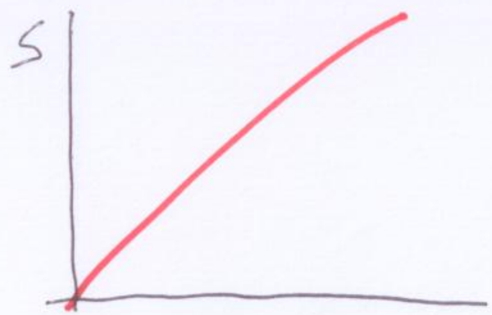


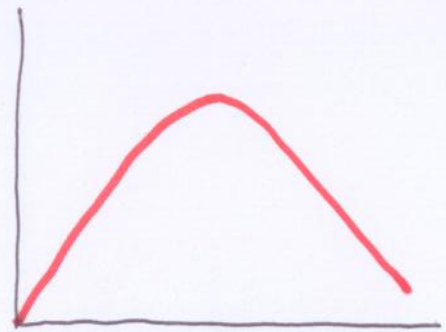
are "back reaction" $1/N$ effects:

$N \to N-1$ in the evaporation process.

(These effects become $O(1)$ at Page's time $n \approx N/2$)



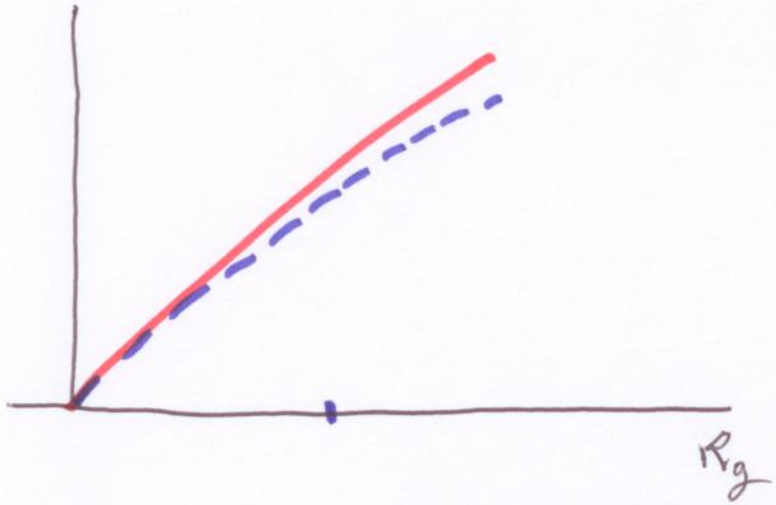
$N = \infty$



N - finite.

* Why Hawking cannot see these $1/N$ effects?

Hawking can track the change of R_g during evaporation process BUT nothing special, takes place for R_g (at Page's time) (regarding vacuum polarization)



However Hawking cannot track the change $N \to N-1$

Actually the semiclassical analysis is done in the "planar" LIMIT.

$$\boxed{\left.\begin{array}{l} N \to \infty \\ L_{pe} \to 0 \end{array}\right\} \; N L_{pe}^2 \equiv R_g \;\; \text{finite.}}$$

Key Idea: To model gravitational self energy in terms of quantum distributions of <u>GRAVITONS</u>.

M

$$E_{gr} \sim \frac{M r_g}{R} = N_{gr} \cdot \frac{1}{R}.$$

$$\boxed{N_{gr} = M r_g}$$

Quantum Mechanically we can describe this system of gravitons in terms of the hamiltonian:

$$\boxed{\hat{H} = \sum_{i=1}^{N_{gr}} \left[\nabla_i + V(r_i) \right] + \sum_{i<j} v(r_i - r_j)}$$

$V(r_i)$: externae potential created by the source.

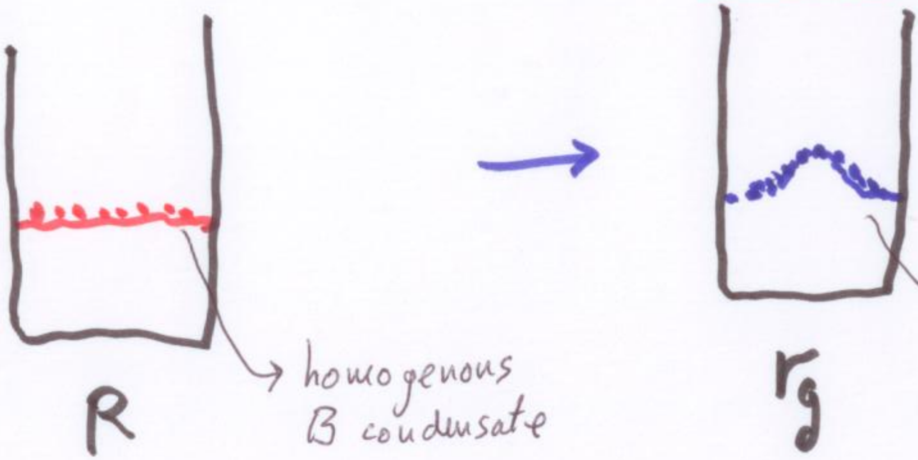
$v(r_i - r_j)$: graviton-graviton potential interaction.

We want to track the quantum ground state of this system when we move R but keeping constant N_{gr}.

In the large $N_{gr} \gg 1$ limit we discover a

quantum phase transition

at $R = r_g$



self sustained "soliton" condensate. $\Bigg\} \equiv$ Black Hole of

$M \approx \sqrt{N_{gr}}\, M_{pl}$

$r_g \approx 2\sqrt{N_{gr}}\, L_{pl}$.

$$\hat{H}\,\Psi^0(x_1 \cdots x_N) = E_Q^{(0)}\,\Psi^0(x_1 \cdots x_N) \qquad (1)$$

In $N_{gr} \gg 1$ we can solve (1) using M.F.A.

- master scalar field: $\phi(x)$
- Energy functional: $\boxed{\mathcal{E}(\phi) = N_{gr} \int d^3x\, |\nabla\phi|^2 + V|\phi|^2 + N_{gr}|a|\,|\phi|^4} \equiv N_{gr}\,\mathcal{E}_1(\phi)$

with $|a|$ the scattering length of v.

$$\phi_{cl}(x): \qquad \left.\frac{\partial \mathcal{E}(\phi)}{\partial \phi}\right|_{\phi_{cl}} = 0 \qquad \int |\phi|^2 d^3x = 1$$

"'t Hooft's" planar limit

$$\boxed{\lim_{\substack{N_{gr} \to \infty \\ |a| \to 0 \\ N_{gr}\cdot|a| = \lambda}} \frac{E_Q^{(0)}}{N_{gr}} = \mathcal{E}_1(\phi_{cl};\lambda)}$$

"t'Hooft" coupling:

$$N_{gr} \to \infty \qquad N_{gr}\cdot|a| \equiv \lambda$$
$$|a| \to 0$$

$$\boxed{E_Q^{(0)} = N_{gr}\Big(\underbrace{\mathcal{E}_1(\phi_{cl};\lambda)}_{\substack{\text{"classical"}\\ \text{planar}}} + \underbrace{\mathcal{O}\left(\tfrac{1}{N}\right)\Big)}_{\substack{\text{"quantum"}\\ \text{non planar.}}}}$$

For the particular case of gravitons:

$$|a| = \frac{L_{pe}^2}{R^2} \qquad \lambda = N_{gr}|a| = \frac{L_{pe}^2 N_{gr}}{R^2}$$

planar limit: $\quad N_{gr} \to \infty \quad L_{pe} \to 0 \quad N_{gr} L_{pe}^2 = R_g.$

$$\boxed{\begin{aligned} \lambda &= \frac{R_g^2}{R^2} \\ g^2 = |a| &= \frac{R_g^2}{R^2}\frac{1}{N_{gr}} \end{aligned}}$$

$\lambda = 1 \qquad \boxed{R_g = R}$

$\boxed{g^2 = \frac{1}{N_{gr}}}$

i.e. we have **TWO** couplings: $\quad \boldsymbol{\lambda} \longrightarrow$ planar (semiclassical physics)

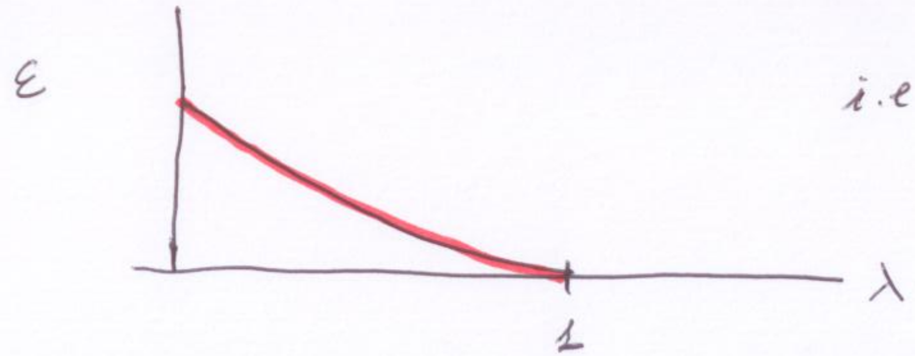
$\boldsymbol{g^2} \longrightarrow$ Non planar ("quantum" $\frac{1}{N}$ effects)

To Know what is special about $\lambda = 1$ (i.e black hole formation) we need to work out the quantum fluctuations around the saddle point ϕ_{cl}.

$$\boxed{\mathcal{E}(\phi;\lambda) = \mathcal{E}(\phi_{cl};\lambda) + \left.\frac{\partial^2 \mathcal{E}}{\delta\phi\,\delta\phi}\right|_{\phi_{cl}} \delta\phi\,\delta\phi + \cdots}$$

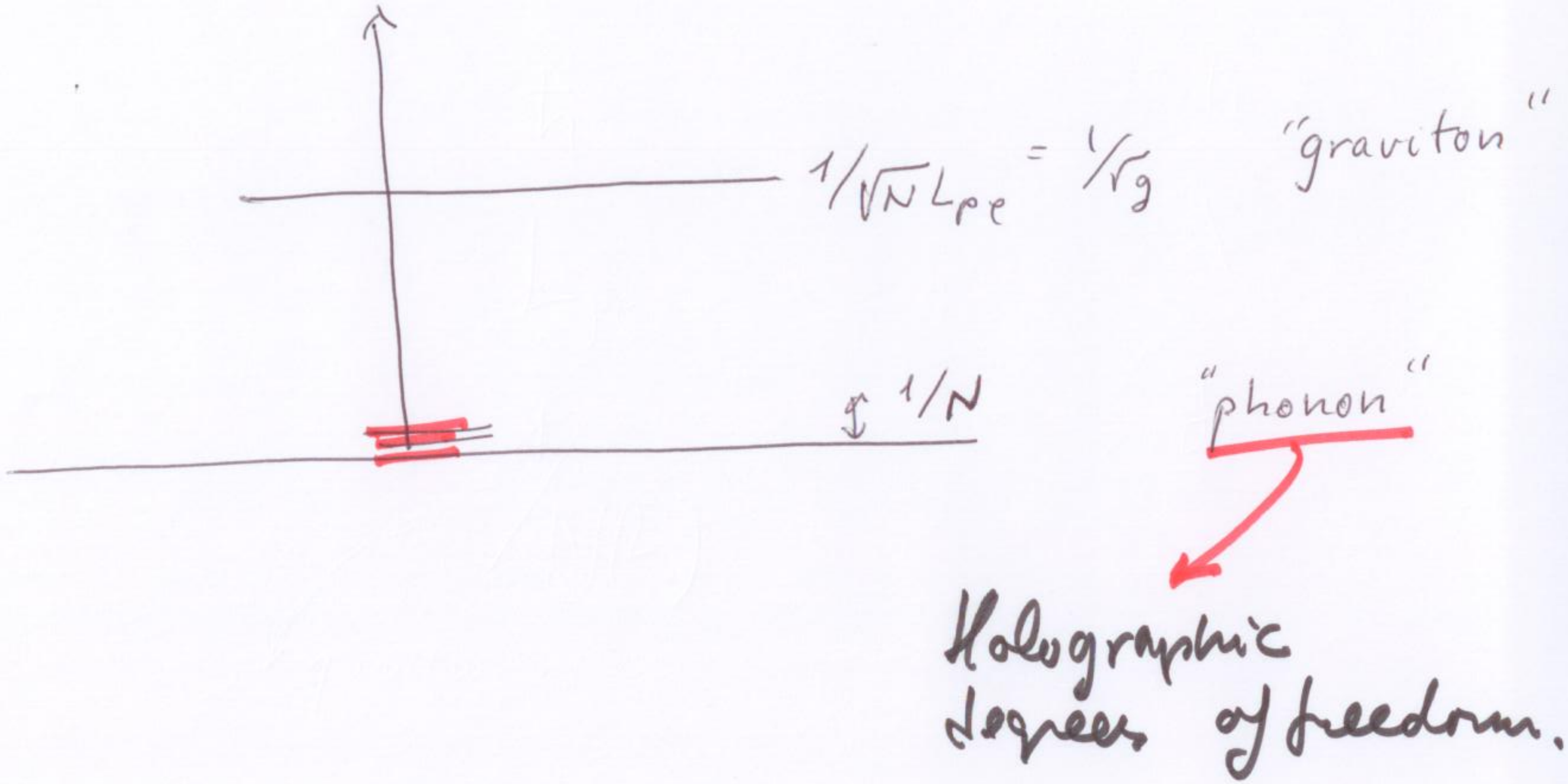
$$\underbrace{\mathcal{E}(k)\, b_k^{+} b_k}$$ Bogoliubov modes.

$\mathcal{E}(k;\lambda)$ energy low lying modes.



i.e <u>q. Ph. transition 2º order.</u>

Spectrum of fluctuations.



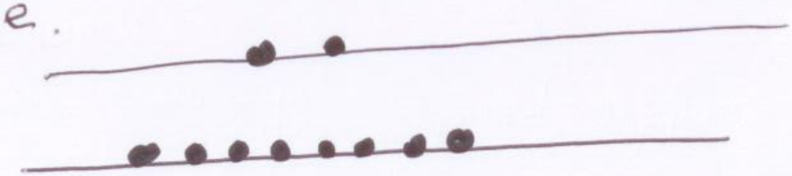
We are looking for $1/N_{gr}$ effects in evaporation process. But first we need to know how the BH evaporates in this picture.

Evaporation = Quantum Depletion.

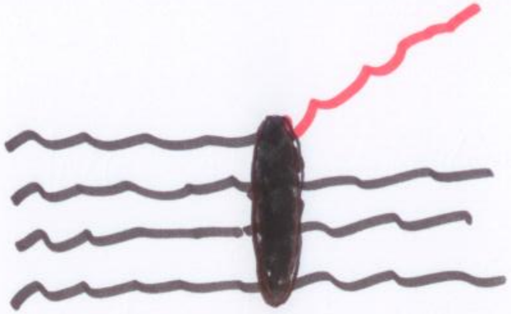
Two contributions:

* Planar
* $1/N_{gr}$ corrections.

Depletion means that some fraction of bosons are out of the condensate state.



If the first excited state is in the continuum (**Leaky**) depletion $\Rightarrow$ evaporation i.e instability from master field point of view.



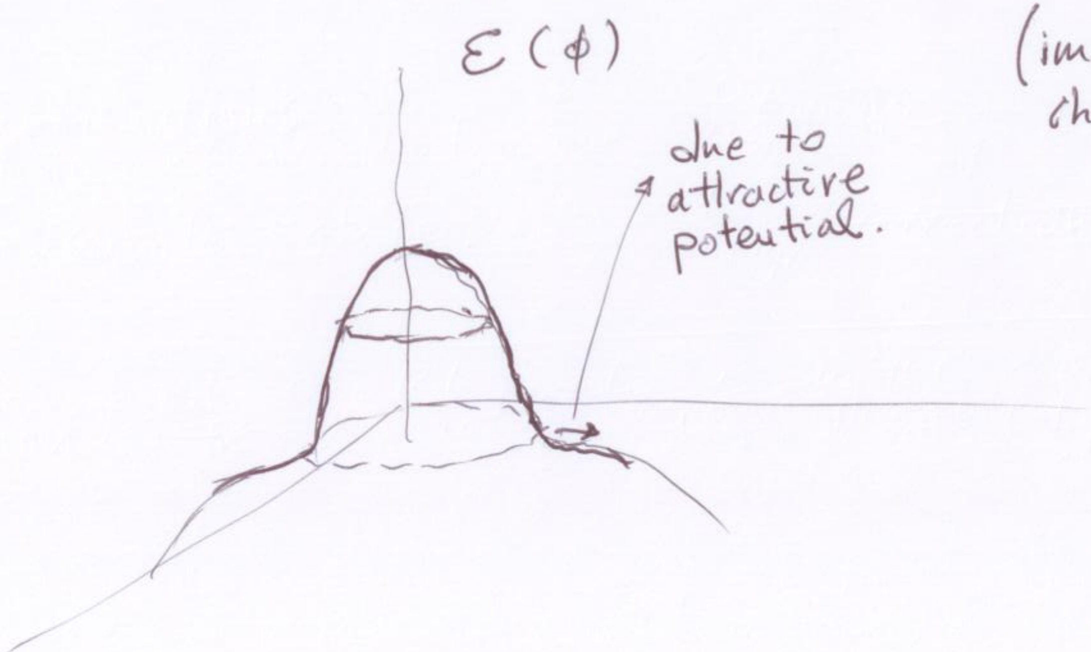
In <u>planar limit</u>

$$\Gamma \sim N^2 \frac{1}{N^2} \frac{1}{\sqrt{N} L_{pe}} \sim \frac{1}{r_g}$$

$$\boxed{\frac{dN}{dt} \approx \frac{1}{\sqrt{N} L_{pe}}.}$$

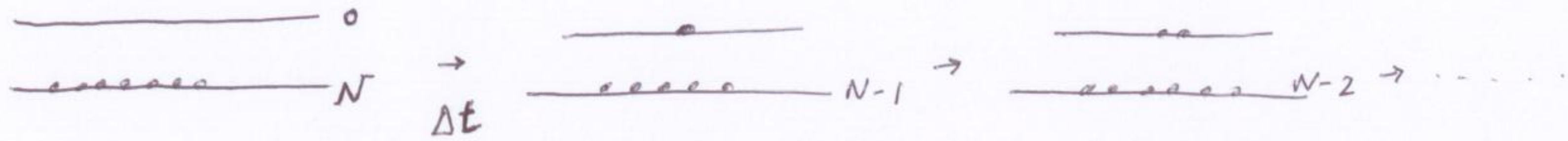
The master field analog.

(imaginary contribution to chemical potential)



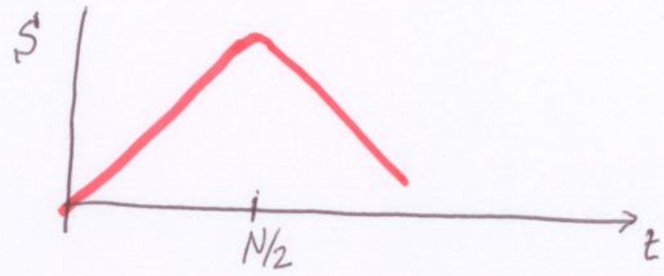
$1/N$ - effects in evaporation process.

fix N



$\hat{\rho}_1$ - one particle density matrix.

$S(\hat{\rho}_1) = Tr(\hat{\rho}_1 \ln \hat{\rho}_1)$

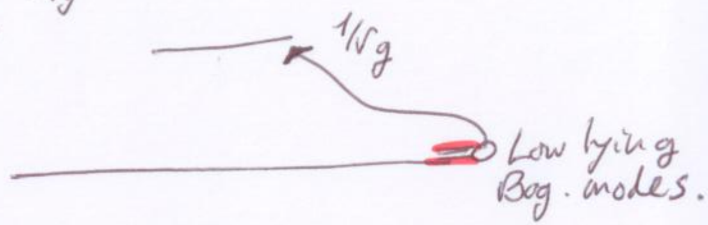


Δt is time needed for one particle to get maximally entangled.

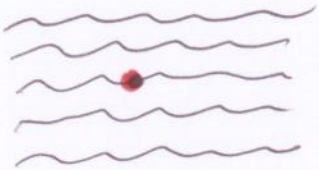
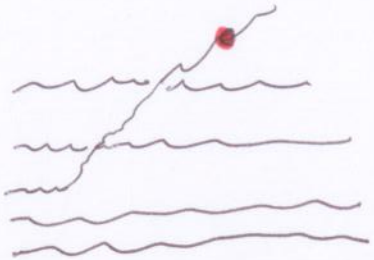
instability $\Rightarrow$ $\delta\rho(t) \sim e^{\frac{1}{\sqrt{N} L_{pe}} t} = e^{\frac{1}{r_g} t}$

$t \sim r_g \ln N$

(scrambling time.)



$1/N$ - effects are also responsible for black hole hair.

 ; 

$$N\frac{1}{N^2}\frac{1}{\sqrt{N}L_{pl}} \sim \frac{1}{N}\frac{1}{\sqrt{N}L_{pl}}$$

$$\boxed{\frac{dB}{dt} = \frac{1}{\sqrt{N}L_{pl}}\frac{B}{N}}$$

as well as for the BH ability to emit information.

In summary once we resolve

Geometry into Graviton distributions

we set

$$\boxed{\begin{array}{c} \hbar \\ L_{P\ell} \\ 1/N(E) \end{array}}$$

"quantum noise" in the distribution.

These are the effects that solve Hawking Paradox.