

Hawking radiation at strong coupling

Mukund Rangamani
Durham University

The Fluid-Gravity correspondence
Munich, Germany
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- Veronika Hubeny, Don Marolf, MR: 0908.2270
- Veronika Hubeny, Don Marolf, MR: 0909.0005
- Veronika Hubeny, Don Marolf, MR: to appear

Outline

- Motivation
- CFTs on black hole backgrounds
- Black funnels & black droplets
- Examples of black funnels
- Novel dynamics of plasma in black hole bgs
- Hawking radiation in AdS
- Summary and discussion

Motivation

Motivation

How do black holes radiate strongly coupled matter?

- Hawking radiation is conventionally discussed for perturbative fields.
- Quantify differences when matter is strongly coupled?
- Precise nature of the quantum stress tensor due to the radiation (matter)?
- How does the nature of the radiation change if the matter can be in distinct phases?
- Dynamical behaviour during collapse: how does the field theory state evolve. Relatedly, Unruh vacuum.

Motivation

Plasma dynamics in black hole backgrounds

- How does a fluid plasma react to the gravitational field of a black hole?
- Does the fluid generically drain away into the black hole or is it possible for a stationary equilibrium solution to exist?
- Do fluid plasmas exhibit qualitative differences as function of the background parameters, with regard to say the phase structure?
- Does a deconfined plasma always behave universally in a black hole background?

Motivation

Strong coupling dynamics in curved spacetime

- Characteristic behaviour of a strongly coupled field theory in a generic curved spacetime background.
- How does the matter react to the background curvature: differences in vacuum polarization effects, etc..
- Phase structure?

The basic framework:

- Attempt to use AdS / CFT to address some of these issues.

Motivation

Brane-world black holes

- Does one have a complete understanding of the potential set of brane-world black holes in induced gravity models?
- Are there new solutions apart from localized black holes and black strings?
- Why don't large localized black holes seem to exist?

Emparan, Fabbri, Kaloper

Fitzpatrick, Randall, Wiseman

- Induced gravity models will be treated as AdS/CFT with a UV cut-off.

**CFTs on black hole
backgrounds**

CFTs on black hole backgrounds

Perturbative fields

- Consider a conformally coupled scalar field in a black hole background such as Schwarzschild.

$$ds_{\partial}^2 = \gamma_{\mu\nu} dx^{\mu} dx^{\nu} = -{}^d f(r) dt^2 + \frac{dr^2}{{}^d f(r)} + r^2 d\Omega_{d-2}^2 .$$

$${}^d f_{\text{Schw}}(r) = 1 - \frac{r_+^{d-3}}{r^{d-3}} .$$

- One can also consider the Schwarzschild-AdS black hole:

$${}^d f_{\text{SAdS}}(r) = \frac{r^2}{\ell_d^2} + 1 - \frac{r_+^{d-3}}{r^{d-3}} \left(1 + \frac{r_+^2}{\ell_d^2} \right) .$$

CFTs on black hole backgrounds

Perturbative fields

- On these black hole backgrounds we want to study the dynamics of a conformally coupled scalar field with action

$$\mathcal{S}_{\text{scalar}} = \int d^d x \sqrt{-\gamma} \left(\frac{1}{2} \gamma^{\mu\nu} \nabla_{\mu} \phi \nabla_{\nu} \phi + \frac{d-2}{8(d-1)} R^{(\gamma)} \phi^2 \right)$$

- Of particular interest to us will be the nature of the quantum stress tensor, which encapsulates the physics of vacuum polarization, Hawking radiation etc..

CFTs on black hole backgrounds

Perturbative fields: Expectations

- ◎ Scalar radiation via the Hawking process at a temperature set by the black hole size.
- ◎ Stress tensor is thermal asymptotically ($r \rightarrow \infty$).

- ◎ Thermal stress tensor is not regular on the event horizon.
- ◎ Correct quantum stress tensor should be regular at the horizon.
- ◎ Nothing singles out the event horizon: were the quantum stress tensor not regular there, we would be able to tell where the horizon was!

CFTs on black hole backgrounds

Perturbative fields: Results

- The general expectation is borne out by an explicit computation of quantum stress tensor.
- The computation was done by Page using the so called **optical metric** for static spacetimes:

Page '82

$$\overline{ds}_\partial^2 = \overline{\gamma}_{\mu\nu} dx^\mu dx^\nu = -dt^2 + \frac{dr^2}{f(r)^2} + \frac{r^2}{f(r)} d\Omega_{d-2}^2 .$$

- One finds a regular stress tensor using the optical metric using heat kernel techniques. The result is given as:

$$T_\nu^\mu = \frac{\pi^2 T^4}{90} \left[A \left(\frac{r_+}{r} \right) (\delta_\nu^\mu - 4 \delta_0^\mu \delta_\nu^0) + B \left(\frac{r_+}{r} \right) (3 \delta_0^\mu \delta_\nu^0 + \delta_1^\mu \delta_\nu^1) \right]$$

$$A(x) = \frac{1 - (4 - 3x)^2 x^6}{(1 - x)^2} , \quad B(x) = 24 x^6$$

CFTs on black hole backgrounds

Perturbative fields: Results

- At large distances $r \gg r_+$ one has thermal radiation at

$$T_{\text{local}} = \frac{1}{\sqrt{d f(r)}} T_{\text{BH}}$$

- On the black hole horizon the quantum stress tensor evaluates to:

$$T_{\mu}^{\nu} = \frac{2\pi^4}{15} T_{\text{BH}}^4 \text{diag} \{3, 3, 1, 1\}$$

- Note that vacuum polarization renders local energy density negative at the horizon.

Fluids on black hole backgrounds

Stationary fluids

- In a static black hole background one can try to solve the relativistic Navier-Stokes equations:

$$\nabla_{\mu} T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (P + \rho) u^{\mu} u^{\nu} + P g^{\mu\nu}$$

- It is a-priori clear that the only stationary solution should have the fluid velocity aligned with the asymptotic time-translational Killing field.
- However, noting that a black hole solution is not globally static, we realize that the fluid is classically going to drain away down the black hole.

CFTs on black hole backgrounds

Conformal fluids in Schwarzschild

- The naive solution to viscous, conformal fluids is

$$u^\mu = \frac{1}{\sqrt{d\mathfrak{f}(r)}} (\partial_t)^\mu, \quad T = T_H \frac{1}{\sqrt{d\mathfrak{f}(r)}}$$

- This data can be used to attempt to find a solution to the fluid-gravity equations for a bulk solution.
- Use second order fluid dynamics in curved background to ask where the bulk horizon would be.
- See corrections away from thermal behaviour at $\mathcal{O}(r^{-1})$.
- However, the fluid approximation breaks down well before the horizon.

CFTs on black hole backgrounds

Beyond perturbation theory: Expectations

- Even when the matter fields are strongly coupled one should obtain a stress tensor that is regular on the horizon and is thermal asymptotically.
- The interpolation between the two regimes will be sensitive to the details of the system.
- Also take into account the phase structure of the QFT.

- To understand these issues we are going to embed the problem within the AdS/CFT correspondence, which is the cleanest framework to discuss these issues.

**The AdS/CFT story:
black funnels and droplets**

The AdS / CFT story

QFT + Gravity \rightsquigarrow Classical gravity

- Consider a situation in the AdS / CFT framework where the boundary field theory is on a non-dynamical black hole background \mathcal{B}_d with metric $\gamma_{\mu\nu}$.
- Dynamics of these field theories is governed at strong coupling by “asymptotically locally AdS” geometries which asymptote to \mathcal{B}_d .
- This involves finding bulk spacetimes \mathcal{M}_{d+1} which have as their conformal boundary the preferred manifold \mathcal{B}_d .
- One has to therefore find solutions to Einstein’s equations with a negative cosmological constant which has the correct boundary conditions.

The AdS / CFT story

Known solutions for boundary black holes

- Consider the case where the boundary is the asymptotically flat Schwarzschild black hole.
- One solution which satisfies the field equations is the AdS black string

$$ds^2 = z^2 ds_{\partial, \text{Schw}}^2 + \frac{dz^2}{z^2} .$$

- This solution is singular on the Poincare horizon (due to the black hole singularity which extends along z).
- Ignoring this issue we can ask what the induced stress tensor is on the boundary.
- The result is simply:

$$T_{\mu}^{\nu} = 0$$

The AdS / CFT story

Schwarzschild boundary conundrums:

- The vanishing stress tensor for the AdS black string is at odds with expectations of thermal Hawking radiation.
- Anticipate strong vacuum polarization effects due to the strongly interacting matter.
- Rather curious that the conventional Hawking thermal spectrum is not reproduced.
- A similar result can be derived for the boundary Schwarzschild-AdS black hole.
- Note that this result in particular implies that if we make the boundary gravity dynamical then it would see no back-reaction from the strongly coupled matter sector!

Phase structure of the CFT

Resolution: Confinement vs. Deconfinement

- The vanishing stress tensor for the AdS black string seems to suggest that the dual field theory is actually in the confined phase, i.e.,

$$0 \sim \mathcal{O}(1)$$

- Typically in the AdS/CFT context, the boundary field theories on Minkowski space doesn't undergo a confinement-deconfinement transition.
- At any non-zero temperature, the preferred phase is the deconfined phase with $\mathcal{O}(N^2)$ free energy.
- It is somewhat reassuring that the black string geometry actually is classically unstable, indicating that it is not a local minimum of the free energy.

Phase structure of the CFT

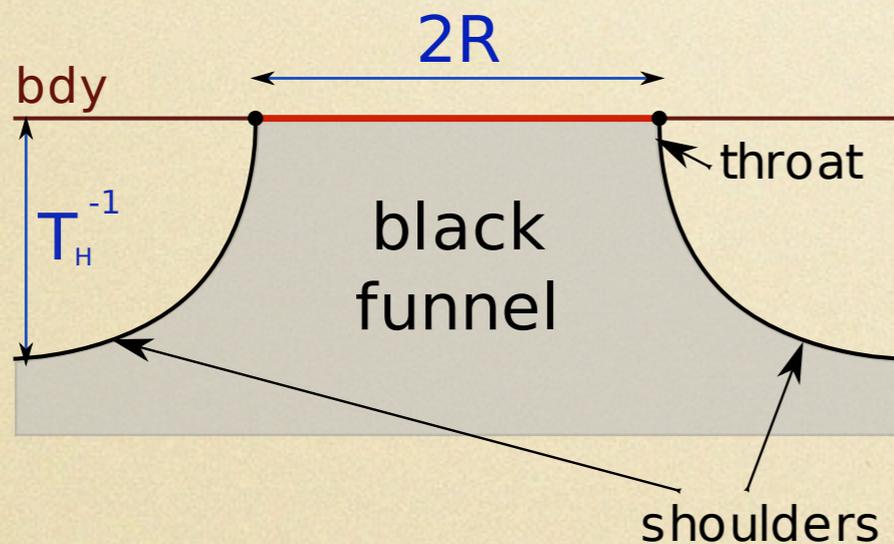
Whither the Deconfined phase?

- The stable phase at any finite temperature ought to be the deconfined phase.
- This phase has a thermal stress tensor asymptotically and should give rise to a strongly coupled version of the Page stress tensor.
- In AdS/CFT this should just amount to a multiplicative renormalization of the free field result (the famous $3/4$).
- Question then remains: what is the bulk solution which encapsulates the physics of a deconfined field theory plasma in equilibrium with the thermal Hawking radiation of the boundary black hole?
- Relatedly, is there a unique candidate bulk geometry or can there be phase transitions?

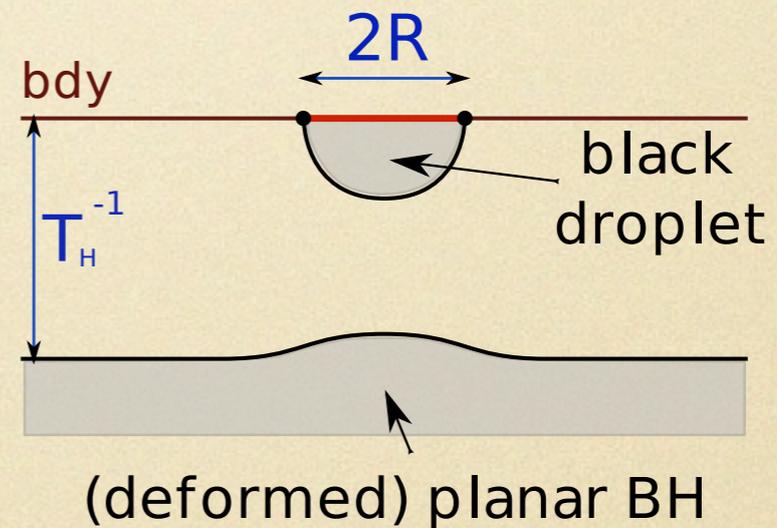
Black Funnels, Black Droplets in AdS

Motivating new classes of solutions

- ◎ The deconfined phase is dual to either
 - a **Black Funnel** geometry.
 - a **Black Droplet** together with a bulk black hole (disconnected horizons).



(a)



(b)

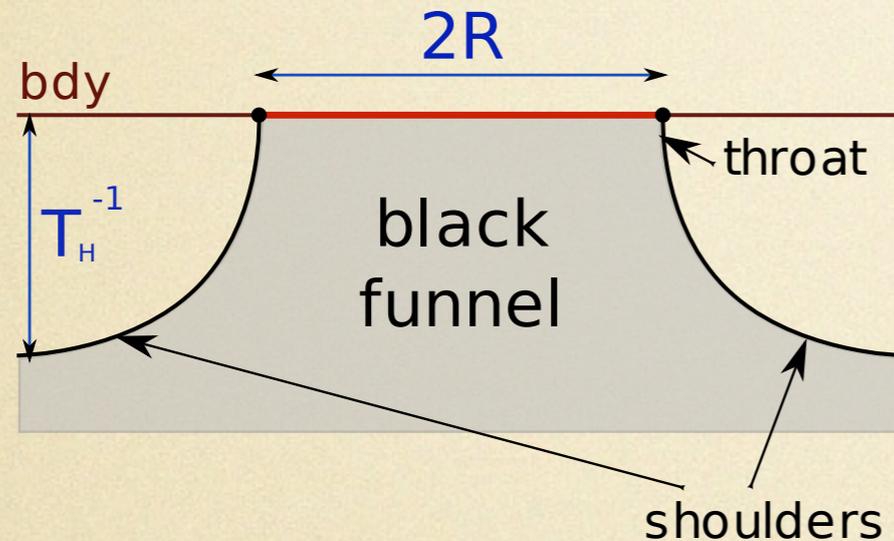
Black Funnels

Motivating the solutions

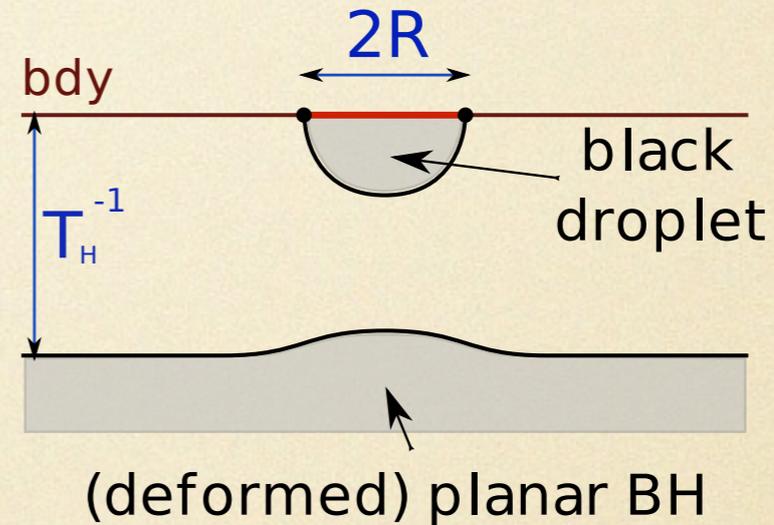
- Without the boundary black hole the spacetime dual of the deconfined phase is the Schwarzschild AdS black hole.
- Induces the correct thermal stress tensor on the boundary.
- Expect the bulk spacetime to behave similarly when we are asymptotically far away from the black hole on the boundary.
- Denote boundary radial coordinate as r and the bulk direction as z .
- Then as $r \rightarrow \infty$ expect that the bulk solution for all z behaves like the Schwarzschild AdS black hole.
- In addition the bulk horizon could smoothly connect to the horizon on the boundary (black funnel) or there could be two disconnected horizons (a droplet and a planar bh).

Black Funnels, Black Droplets in AdS

Funnel vs droplets



(a)



(b)

- ⊙ Note that the boundary black hole can have two scales
 - characteristic size R .
 - Hawking temperature T_H .
- ⊙ A-priori these can be unrelated and the dominant saddle point depends on RT_H .

Black Funnel & the phase structure of the CFT

A Conjecture: field theory on Schwarzschild bg

- The stable deconfined phase is dual to a Black Funnel geometry.
- Corresponds to the phase of the CFT in equilibrium with the thermal Hawking radiation.
- The unstable deconfined phase is dual to the AdS black string.

Extensions:

- Can be extended to situations where the field theory has a confining phase and to boundary metrics which have different asymptotics.

Examples

Black Funnel: Examples

Black funnels in AdS_3

- The simplest explicit example of black funnels can be obtained when the bulk is three dimensional.
- The non-dynamical boundary metric can be taken to be the two dimensional black hole with metric

$$ds^2 = -\tanh^2 r dt^2 + dr^2$$

- Einstein's equations (with -ve cc) can be easily solved.
- Work in Fefferman-Graham coordinates for simplicity and take the metric ansatz to be

$$ds^2 = \frac{1}{z^2} (-f(r, z) dt^2 + g(r, z) dr^2 + dz^2)$$

Black Funnel: Examples

Black funnels in AdS_3

● The metric functions turn out to be:

$$f(r, z) = \frac{1}{16} \left(4 \tanh r + z^2 \frac{1 - 2r_+ \cosh^4 r}{\sinh r \cosh^3 r} \right)^2$$

$$g(r, z) = \left(1 + z^2 \frac{2r_+ \cosh^4 r - 1 - 4 \sinh^2 r}{4 \sinh^2 r \cosh^2 r} \right)^2$$

● Naively we have a one-parameter family of solutions, but the parameter r_+ is fixed by demanding regularity:

$$T_{\text{bdy}} = \frac{1}{2\pi} \implies r_+ = \frac{1}{2}$$

Black Funnel: Examples

Black funnels in AdS_3

● There is a bulk event horizon located at

$$z_H(r) = \frac{2 \cosh r}{\sqrt{\cosh^2 r + 1}}, \quad \& \quad r = 0$$

Black Funnel: Examples

Black funnels / droplets in AdS_4

- One can also find black funnel like solutions within the AdS C-metric family.
- These turn out to correspond to boundary black holes which are asymptotically $\mathbf{R} \times \mathbf{H}^2$.
- Once again the expectations regarding the stress tensor are borne out by explicit computation.
- There are also black droplets within the C-metric family.
- Curiously, except for a degenerate case the solutions tend not to represent equilibrium states (temperatures don't match between the droplet and the planar black hole).
- There also exist funnels and droplets for spatially compact boundaries.

**Novel dynamics of the
deconfined plasma**

Funnel vs droplets

A novel phase transition

- ◎ Consider CFT on \mathcal{B}_d and imagine varying the parameters so that RT_H goes from large to small values.
- ◎ We expect that as RT_H passes through unity to see an exchanged in dominance of the two solutions.
- The black funnel throat could become thin and be subject to a Gregory-Laflamme type instability.
- As envisaged in the early days of the brane-world black holes perhaps this GL instability could end up in the droplet solution.

Funnel vs droplets

Novel dynamics in the deconfined phase

- ◎ Black funnels represent deconfined plasma in equilibrium with the bdy black hole.
- ◎ Perturbations of this system relax as expected in the deconfined regime.

- ◎ Black droplets + planar bhs are more exotic.
- ◎ While the planar bh ensures that the system is in deconfined phase, perturbations of the system relax as if it were confined!
- ◎ This is because any change on one horizon has to be transferred to the other through bulk Hawking radiation which is a very slow process.

Funnel vs droplets

Black holes and finite size excitations

- Slow equilibration of droplets is curious from the field theory.
- Suggests that the intrinsic excitations of the field theory are very weakly coupled to the boundary black hole.
- Natural interpretation: field theory excitations have finite size (cf. size of hadrons in string theory).
- If $R_e \gg R$ then the field theory quanta just fly past the bdy black hole without much ado (modulo tidal effects).
- In the bulk one again sees this by the small distortion on the droplet caused by a gravitational perturbation deep inside the bulk.

Summary

Summary & Discussion

New exotic AdS black holes

- Holographic techniques allow us for the first time to investigate the Hawking radiation of strongly coupled quanta.
- Analysis of the dual gravitational solutions leads to a new class of solutions which we've dubbed black funnels.
- Explicit solutions exist when the boundary is 2 dimensional, or BTZ or an exotic hyperbolic black hole.
- Not only can such solutions be constructed in a limited class of examples, but also they confirm our intuition regarding the quantum stress tensor due to curved spacetime effects.

Summary & Discussion

Realistic Black funnels?

- ◎ Black funnels with Schwarzschild boundary should exist (numerical relativists please step up)!
- ◎ Would be interesting to explore the explicit behaviour of these solutions and figure out the quantum stress tensor induced by strongly coupled $\mathcal{N} = 4$ SYM in this background.

There are important implications for:

- ❖ Plasma balls in confining theories
- ❖ Brane-world and induced gravity models.

**Hawking radiation and AdS
black holes**

Hawking radiation in AdS

Black funnels in AdS_4 with BTZ boundary

- The simplest non-trivial example where we can find explicit solution and in fact examine the full phase structure is when the boundary is a BTZ black hole.

$$ds^2 = -(r^2 - r_+^2) dt^2 + \frac{dr^2}{r^2 - r_+^2} + r^2 d\varphi^2$$

- Obtain all the requisite solutions via a set of solution generating tricks.

Black Funnel: Examples

Black funnels in AdS_3

- Given the solution we can use holographic techniques to find the induced stress tensor on the boundary:

$$T_{tt} = \alpha \tanh^2 r \left(\frac{\cosh^2 r - 3}{2 \cosh^2 r} \right), \quad T_{rr} = \alpha \frac{\cosh^2 r + 1}{2 \cosh^2 r}$$

- α is a normalization constant measuring the effective number of degrees of freedom.

$$\alpha \sim \frac{\ell}{\ell_p}$$

- This stress tensor is thermal asymptotically far from the black hole and regular on the event horizon as expected.

Black Funnel in AdS

Black funnels in AdS_4 with BTZ boundary

- ◎ Recall that for the global AdS_4 spacetime with the standard Einstein Static Universe $\mathbb{R} \times S^2$ boundary one has the following solutions dual the various phases:
 - ❖ Confined phase: Thermal AdS spacetime
 - ❖ Deconfined phase: Large Schwarzschild AdS black hole
 - ❖ Unstable phase: Small Schwarzschild AdS black hole
- ◎ By an analytic continuation + conformal transformation one can map the ESU to BTZ.
- ◎ Applying the same trick to the bulk solutions, we recover the desired black funnel geometries.

Black Funnel in AdS

Black funnels in AdS_4 with BTZ boundary

In particular, starting with the known class of solutions

$$ds^2 = -f(\rho) d\tilde{t}^2 + \frac{d\rho^2}{f(\rho)} + \rho^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

the following set of transformations

$$\tilde{t} = i\chi, \quad \varphi = it, \quad \tilde{r} = \frac{1}{w}$$

takes us to the full set of solutions dual to BTZ on the boundary:

$$ds^2 = \frac{\rho^2}{w^2} \left[-(w^2 - 1) dt^2 + \frac{dw^2}{w^2 - 1} + w^2 \frac{f(\rho)}{\rho^2} d\chi^2 \right] + \frac{d\rho^2}{f(\rho)}$$

Black Funnel in AdS

Black funnels in AdS_4 with BTZ boundary

While the “funnelness” of the solution is not manifest, one can compute the boundary stress tensor having fixed the non-normalizable part to coincide with BTZ metric.

$$f(\rho) = 1 + \rho^2 - \frac{M}{\rho}, \quad M = \rho_+ (1 + \rho_+^2)$$

$$T_{\mu}^{\nu} = \frac{M}{3r^3} \text{diag}\{1, 1, -2\}$$

- The stress tensor is regular on the horizon, but seems to behave somewhat strangely as $r \rightarrow \infty$.
- But this in fact turns out to be the precise form computed for perturbative fields.

Black Funnel in AdS

Black funnels in AdS_4 with BTZ boundary

For the BTZ geometry

$$ds^2 = -(r^2 - r_+^2) dt^2 + \frac{dr^2}{r^2 - r_+^2} + r^2 d\varphi^2$$

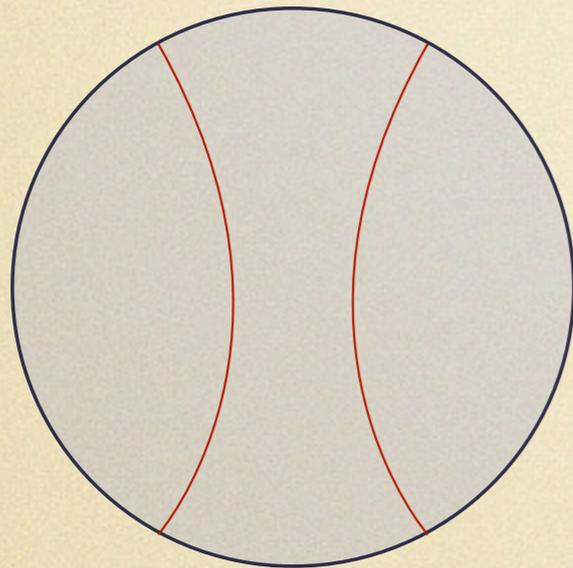
the calculation of the quantum stress tensor for scalar fields leads to the result:

$$T_{\mu}^{\nu} = \frac{A(r_+)}{3 r^3} \text{diag}\{1, 1, -2\} ,$$
$$A(r_+) = \frac{\sqrt{2}}{32\pi} \sum_{n=1}^{\infty} \frac{\cosh(2\pi n r_+) + 3}{(\cosh(2\pi n r_+) - 1)^{\frac{3}{2}}}$$

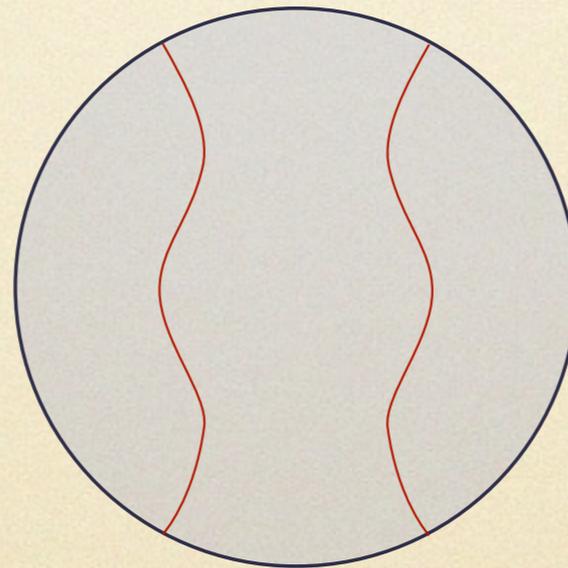
whose form coincides with the holographic computation.

Black Funnels in AdS

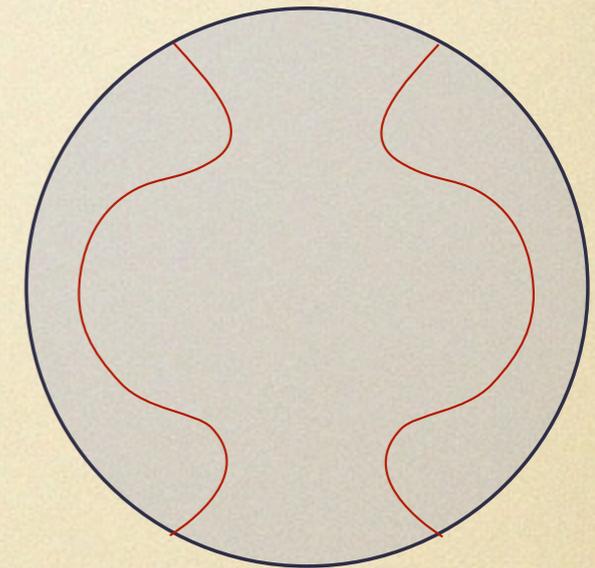
Black funnels in AdS_4 with BTZ boundary



AdS black string
Confined phase



Small Black Funnel
Unstable phase



Large Black Funnel
Deconfined phase