Improved Relativistic Hydrodynamics from the AdS/CFT

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based on M.L. and Edward Shuryak, arXiv:0905.4069; arXiv:0704.1647

Motivation: Experiments (RHIC) probe systems with finite gradients.
Phenomenologically observed low viscosity is an "effective" viscosity measured at momentum typical for a process in study.
New phenomena: Conical flows linear perturbations on top of global explosion.
These are small size perturbations sensitive to high gradients.

Main Idea:
Introduce all order dissipative terms in the gradient expansion of $T^{\mu\nu}$.

$$(\nabla\nabla\mathbf{u}) \quad \text{we keep} \qquad\qquad (\nabla\mathbf{u})^2 \quad \text{we neglect}$$

Extract momenta dependent viscosities by matching two-point correlation functions of stress tensor with correlation functions computed from BH AdS/CFT.

Outlook of the talk:

- Old Life on the boundary: relativistic hydro (NS and IS)
- Life in the bulk: gravity perspective
- New Life on the boundary: all order (linearized) hydro
- The bulk meets the boundary

Relativistic Hydrodynamics

Energy momentum tensor

$$\langle \mathbf{T}^{\mu\nu} \rangle \;=\; (\epsilon \;+\; \mathbf{P})\, \mathbf{u}^{\mu}\; \mathbf{u}^{\nu} \;+\; \mathbf{P}\; \mathbf{g}^{\mu\nu} \;+\; \mathbf{\Pi}^{\langle\mu\nu\rangle}$$

$\Pi^{\mu\nu}$ **- tensor of dissipations (ideal fluid:** $\Pi^{\mu\nu} \;=\; 0$**)**

$$\mathbf{\Pi}^{\langle\mu\nu\rangle} \;=\; \frac{\mathbf{1}}{\mathbf{2}}\, \mathbf{\Delta}^{\mu\alpha}\, \mathbf{\Delta}^{\mu\beta}\, (\mathbf{\Pi}_{\alpha\beta} \;+\; \mathbf{\Pi}_{\beta\alpha}) \;-\; \frac{\mathbf{1}}{\mathbf{3}}\, \mathbf{\Delta}^{\mu\nu}\, \mathbf{\Delta}^{\alpha\beta}\, \mathbf{\Pi}_{\alpha\beta}$$

$$\mathbf{\Delta}^{\mu\nu} \;=\; \mathbf{g}^{\mu\nu} \;+\; \mathbf{u}^{\mu}\, \mathbf{u}^{\nu}$$

Navier Stokes hydro (expanding in the velocity gradient)

$$\mathbf{\Pi}_{\alpha\beta} \;=\; -\,\eta\, \nabla_{\alpha}\, \mathbf{u}_{\beta}$$

$$\nabla_{\mu}\, \langle \mathbf{T}^{\mu\nu} \rangle \;=\; \mathbf{0} \qquad\qquad \longrightarrow \qquad\qquad \mathrm{Navier - Stokes\ Eq.}$$

Retarded Correlators

$$\mathbf{G}^{\mu\nu\alpha\beta}(\mathbf{k},\omega) \;=\; -\,\mathbf{i}\int_{\mathbf{0}}^{\infty}\mathbf{dt}\int\mathbf{d^3x}\,\mathbf{e}^{-\mathbf{i}\,\omega\,\mathbf{t}\,+\,\mathbf{ikz}}\,\langle[\mathbf{T}^{\mu\nu}(\mathbf{x},\mathbf{t}),\;\mathbf{T}^{\alpha\beta}(\mathbf{0})]\rangle$$

The sound:

$$\mathbf{G}^{\mathbf{S}}(\mathbf{k},\mathbf{w}) \;\equiv\; \mathbf{G}^{\mathbf{tztz}} \;=\; (\epsilon\,+\,\mathbf{P})\,\frac{\mathbf{k^2}\;-\;\mathbf{4\,i}\,\bar{\eta}\,\omega\,\mathbf{k^2}}{\mathbf{k^2}\;-\;\mathbf{3}\,\omega^{\mathbf{2}}\;-\;\mathbf{4\,i}\,\bar{\eta}\,\omega\,\mathbf{k^2}}$$

The shear:

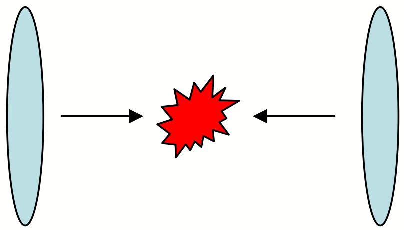
$$\mathbf{G}^{\mathbf{D}}(\mathbf{k},\mathbf{w}) \;\equiv\; \mathbf{G}^{\mathbf{txtx}} \;=\; (\epsilon\,+\,\mathbf{P})\,\frac{\bar{\eta}\,\mathbf{k^2}}{-\mathbf{i}\,\omega\;+\;\bar{\eta}\,\mathbf{k^2}}$$

The scalar:

$$\mathbf{G}^{\mathbf{T}}(\mathbf{k},\mathbf{w}) \;\equiv\; \mathbf{G}^{\mathbf{xyxy}} \;=\; -\,\mathbf{i}\,(\epsilon\,+\,\mathbf{P})\,\omega\,\bar{\eta}$$

$2\,\pi\,T \;=\; 1$ and $\bar{\eta} \;\equiv\; 2\,\pi\,\eta/s$

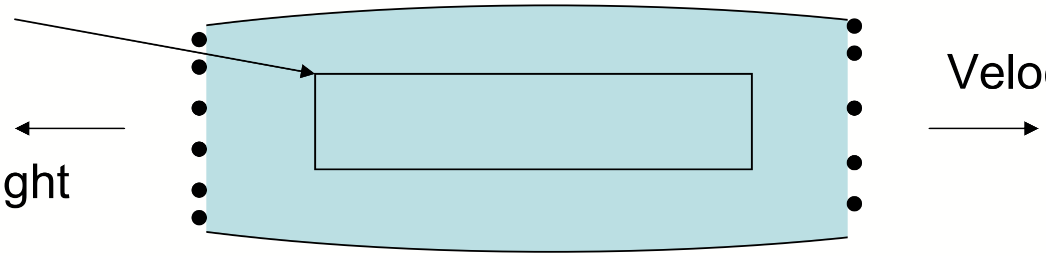
RHIC and Bjorken set up



Relativistically accelerated heavy nuclei

Central Rapidity Region (CRR)

Velocity of light



Velocity of light

After collision

- **one-dimensional expansion**: **Boost Invariant Formulation**

Local rest frame $u = (1, 0, 0, 0)$ $\qquad$ $(x^0,\, x^1,\, x_\perp) \;\longrightarrow\; (\tau,\, y,\, x_\perp)$

τ - proper time, $\qquad$ y - spacetime rapidity

$$x^0 \;=\; \tau\; ch(y) \qquad\qquad x^1 \;=\; \tau\; sh(y)$$

The metric (1d Hubble expansion)

$$ds^2 \;=\; -\; d^2\tau \;+\; \tau^2\, d^2y \;+\; d^2x_\perp$$

Hydro eq. simplify dramatically:

$$\partial_\tau\, \epsilon(\tau) \;=\; -\,\frac{\mathbf{4}\,\epsilon}{\mathbf{3}\,\tau} \;+\; \frac{\mathbf{4}\,\eta}{\mathbf{3}\,\tau^{\mathbf{2}}}$$

Solution for $\eta \;=\; 0$: $\qquad$ Bjorken (1986)

$$\epsilon \;\sim\; \frac{1}{\tau^{4/3}} \qquad\qquad T \;\sim\; \frac{1}{\tau^{1/3}} \qquad\qquad \partial_\tau\,(s\,\tau) \;=\; 0$$

Solution for $\eta \;\neq\; 0$:

$$\partial_\tau\,(\mathrm{s}\,\tau) \;=\; \frac{\mathbf{4\,s}}{\mathbf{3}}\;\frac{\eta}{\mathbf{s}}\;\frac{\mathbf{1}}{\mathbf{T}\,\tau}$$

Israel-Stewart second order Hydrodynamics

Solves causality problems present in Navier-Stokes

Add extra term in the gradient expansion + non-linear terms in (∇u)

$$\mathbf{\Pi}^{\mu\nu} \;=\; (\mathbf{1} \;-\; \tau_{\mathbf{R}}\, \mathbf{u}_\lambda\, \nabla^\lambda)\; \mathbf{\Pi}^{\mu\nu}_{\mathbf{NS}}$$

Iterate the equation

$$(\mathbf{1} \;+\; \tau_{\mathbf{R}}\, \mathbf{u}_\lambda\, \nabla^\lambda)\; \mathbf{\Pi}^{\mu\nu} \;=\; \mathbf{\Pi}^{\mu\nu}_{\mathbf{NS}}$$

When thinking about small perturbations $\mathbf{u}_\lambda\, \nabla^\lambda \;\to\; \nabla_{\mathbf{t}} \;\to\; -\,\mathbf{i}\,\omega$

The IS second order hydro is equivalent (in the linear approximation) to

$$\eta \;\to\; \frac{\eta}{\mathbf{1} \;-\; \mathbf{i}\,\tau_{\mathbf{R}}\,\omega}$$

Sound dispersion

$$\omega \;=\; \mathbf{c}\,\mathbf{k}\,[\mathbf{1} \;+\; \bar{\eta}\,\mathbf{c}^2\,\mathbf{k}^2\,(\mathbf{2}\,\tau_{\mathbf{R}} \;-\; \bar{\eta})] \;-\; \mathbf{i}\,\mathbf{c}^2\,\bar{\eta}\,\mathbf{k}^2\,[\mathbf{1} \;+\; \mathbf{c}^2\,\mathbf{k}^2\,\bar{\eta}\,\tau_{\mathbf{R}}\,(\mathbf{2}\,\bar{\eta} \;-\; \tau_{\mathbf{R}})]$$

Retarded Correlators from gravity

P. Kovtun and A. Starinets, Phys.Rev.Lett.96:131601,2006

For three channels a=S,D,T

$$\frac{d^2}{dr^2}Z_a(r) + p_a(r)\frac{d}{dr}Z_a(r) + q_a(r)Z_a(r) = 0\,,$$

Absorptive boundary condition (incoming wave) at the horizon $r = 1$:

$$Z_a(r \to 1) \;\sim\; e^{-i\omega/2}$$

Two independent local solutions at $r = 0$,

$$Z_a(r) \;=\; \mathcal{A}_a\, Z_a^I(r) \;+\; \mathcal{B}_a\, Z_a^{II}(r)\,,$$

Z_a^I is irregular in the origin while Z_a^{II} is a regular solution.

$$\tilde{G}^a(\omega, k) \;=\; -\,8\,P\,\frac{\mathcal{B}_a(\omega, k)}{\mathcal{A}_a(\omega, k)}$$

The scalar channel

$$p_T(r) = -\frac{1+r^2}{rf}\ , \qquad\qquad q_T(r) = \frac{\omega^2 - k^2 f}{rf^2}\ ,$$

where $f = 1 - r^2$

The shear channel

$$p_D(r) = \frac{(\omega^2 - k^2 f)f + 2r^2\omega^2}{rf(k^2 f - \omega^2)}\,, \qquad\qquad q_D(r) = \frac{\omega^2 - k^2 f}{rf^2}\,.$$

The sound channel

$$p_S(r) = -\frac{3\omega^2(1+r^2) + k^2(2r^2 - 3r^4 - 3)}{rf(3\omega^2 + k^2(r^2-3))}\ ,$$

$$q_S(r) = \frac{3\omega^4 + k^4(3-4r^2+r^4) + k^2(4r^2\omega^2-6\omega^2-4r^3 f)}{rf^2(3\omega^2 + k^2(r^2-3))}\,.$$

Small momenta perturbation theory

Extending R. Baier, P. Romatschke, D.T. Son, A.O. Starinets, M.A. Stephanov, JHEP 0804:100,2008

$$\frac{1}{(\epsilon+\mathbf{P})}\mathbf{G^T} = -\,\mathbf{i}\,\frac{\mathbf{1}}{\mathbf{2}}\,\omega - \frac{\mathbf{1}}{\mathbf{2}}\mathbf{k}^2 - \frac{\mathbf{1}}{\mathbf{2}}(\ln\mathbf{2}-\mathbf{1})\,\omega^2 - \frac{\mathbf{1}}{\mathbf{4}}\,(\mathbf{3}\ -\ \mathbf{4}\ \ln\mathbf{2})\ \mathbf{k}^4 + \mathbf{i}\ \ln\mathbf{2}\,\omega\,\mathbf{k}^2 - \ln^2\mathbf{2}\,\mathbf{k}^6\ \ldots$$

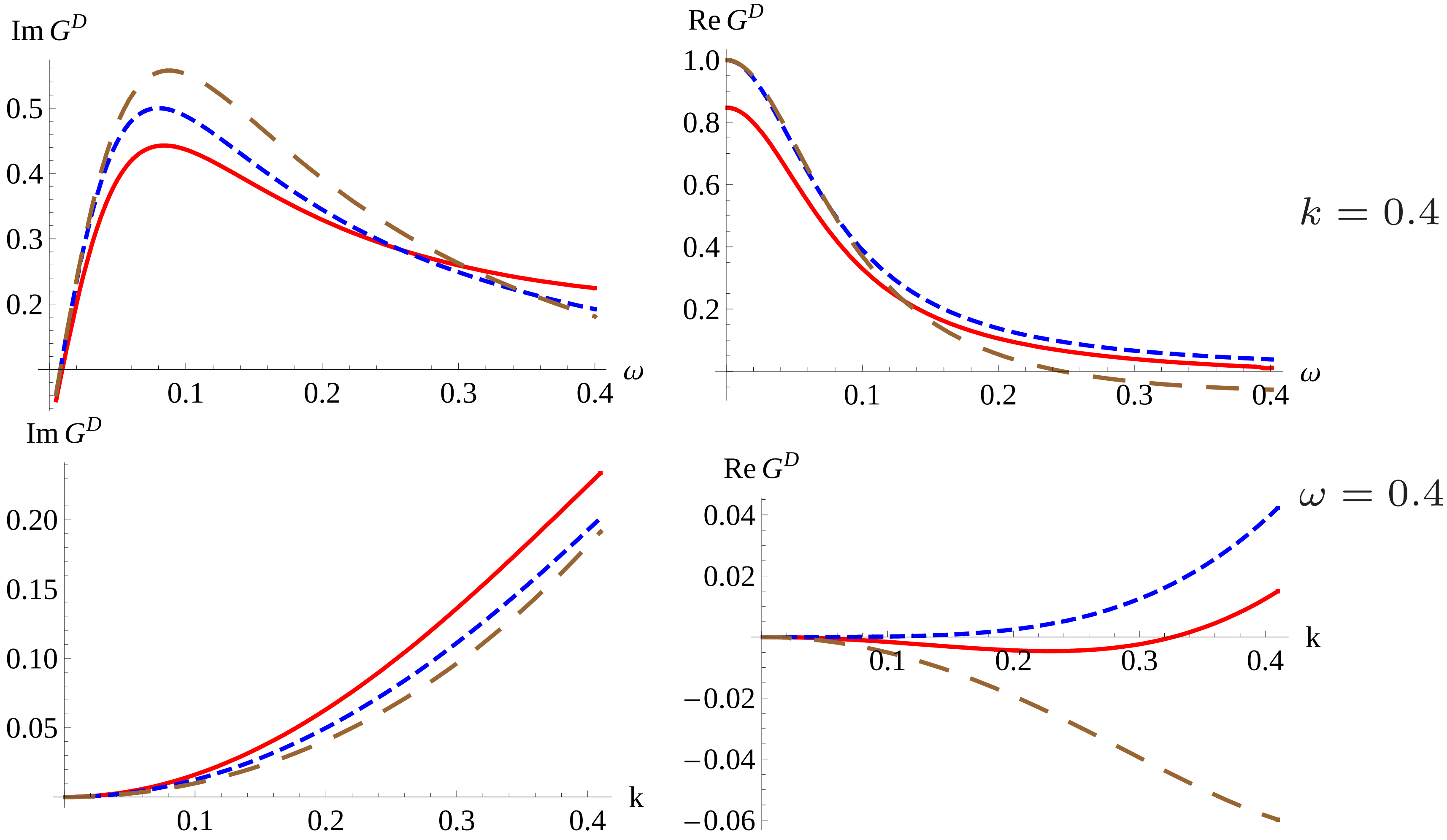
$$\frac{1}{(\epsilon+\mathbf{P})}\mathbf{G^D} = \frac{\mathbf{i}\,\mathbf{k}^2/\mathbf{2}\,[\mathbf{1}\ +\ \mathbf{i}\,(\mathbf{2}\ -\ \ln\mathbf{2})\,\omega\ -\ \mathbf{k}^2/\mathbf{2}\ \ldots]\ +\ \omega\,\mathbf{k}^2/\mathbf{2}\ +\cdots}{\omega\ +\ \mathbf{i}\,\mathbf{k}^2/\mathbf{2}\,[\mathbf{1}\ +\ \mathbf{i}\,(\mathbf{2}\ -\ \ln\mathbf{2})\,\omega\ -\ \mathbf{k}^2/\mathbf{2}+\ \cdots]}$$

$$\frac{1}{(\epsilon+\mathbf{P})}\mathbf{G^S} = \frac{-\,\mathbf{k}^2\ +\ \mathbf{i}\,\mathbf{2}\,[\mathbf{1}\ -\ \mathbf{i}\,\omega\,(\ln\mathbf{2}\ -\ \mathbf{2})\ +\ \cdots]\,\omega\,\mathbf{k}^2\ +\ \mathbf{2}\,\omega^2\,\mathbf{k}^2\ \ldots}{\mathbf{3}\,\omega^2\ -\ \mathbf{k}^2\ +\ \mathbf{i}\,\mathbf{2}\,\omega\,\mathbf{k}^2\,[\mathbf{1}\ -\ \mathbf{i}\,\omega\,(\ln\mathbf{2}\ -\ \mathbf{2})\ +\ \cdots]}$$

Sound mode: $$\omega = \frac{k}{\sqrt{3}} - i\,\frac{k^2}{3} + \frac{3\ -\ \ln 4}{6\,\sqrt{3}}\,k^3 + \cdots$$

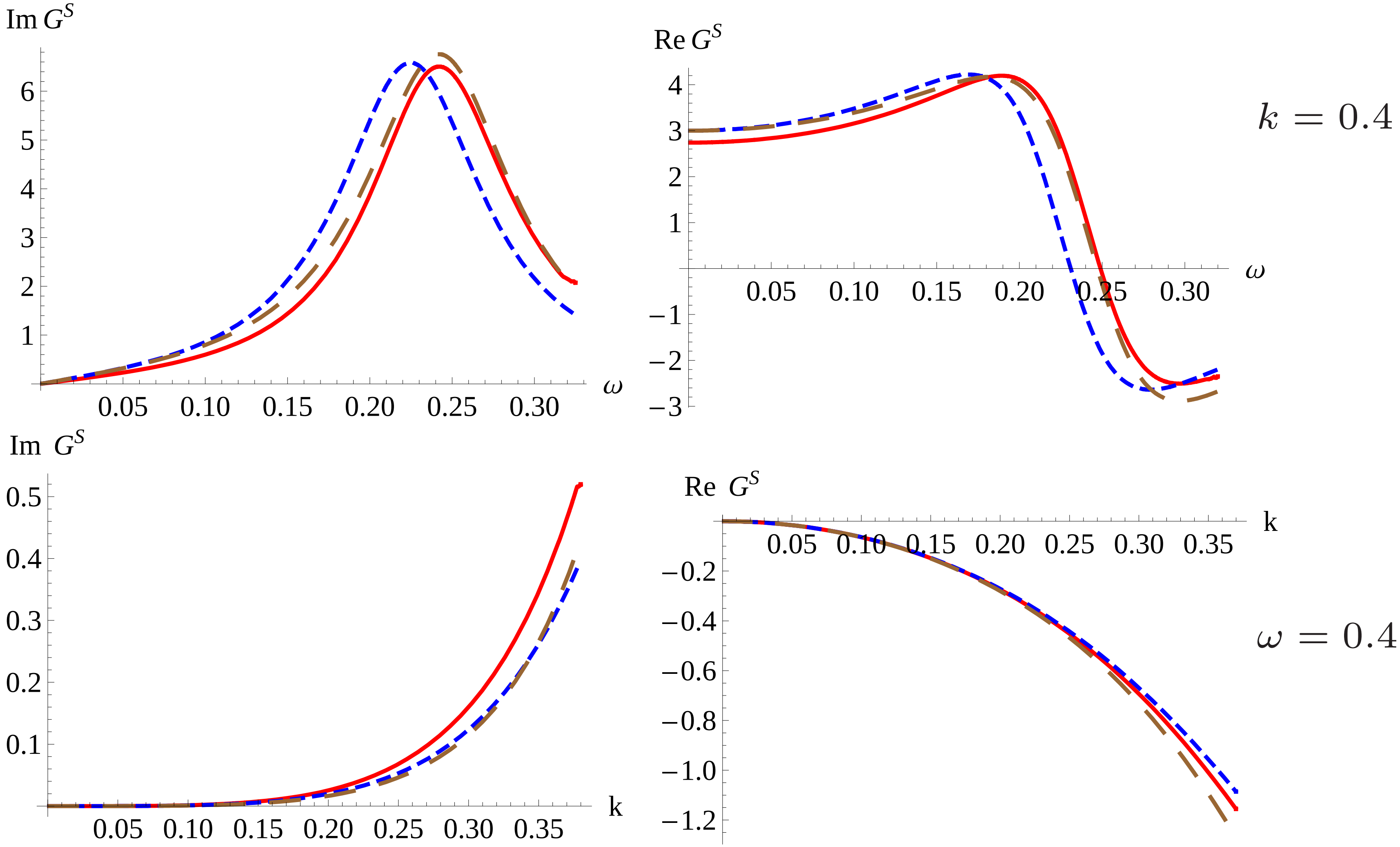
Shear mode: $$\omega = -\,i\,\frac{k^2}{2} - i\,\frac{1\ -\ \ln 2}{4}\,k^4 + \cdots$$

$\tau_R = 2 - \ln 2$ S. Bhattacharyya, V. E Hubeny, S. Minwalla, M. Rangamani, JHEP 0802:045,2008



red - BH AdS/CFT P. Kovtun, A. Starinets, Phys.Rev.Lett.96:131601,2006

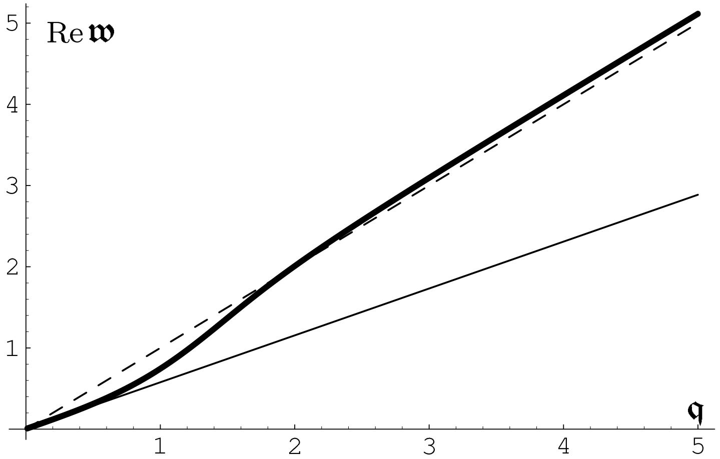
blue dash - Navier Stokes brown dash - IS second order hydro.

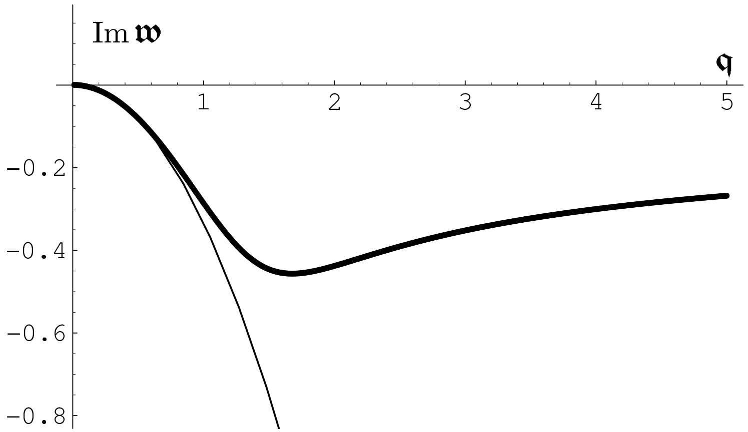

Im G^S
ω
Re G^S
ω
$k = 0.4$
Im G^S
k
Re G^S
k
$\omega = 0.4$

Sound and Holography

P. Kovtun and A. Starinets, Phys.Rev.D72:086009,2005

Quasi-normal mode analysis in the AdS BH background - the sound channel





$$\Re e[\omega] = c\,k + \sum_{n=1}^{\infty} r_n\, k^{2n+1} \qquad\qquad \Im m[\omega] = -\bar{\eta}\left[c^2\,k^2 + \sum_{n=2}^{\infty}\beta_n\, k^{2n}\right]$$

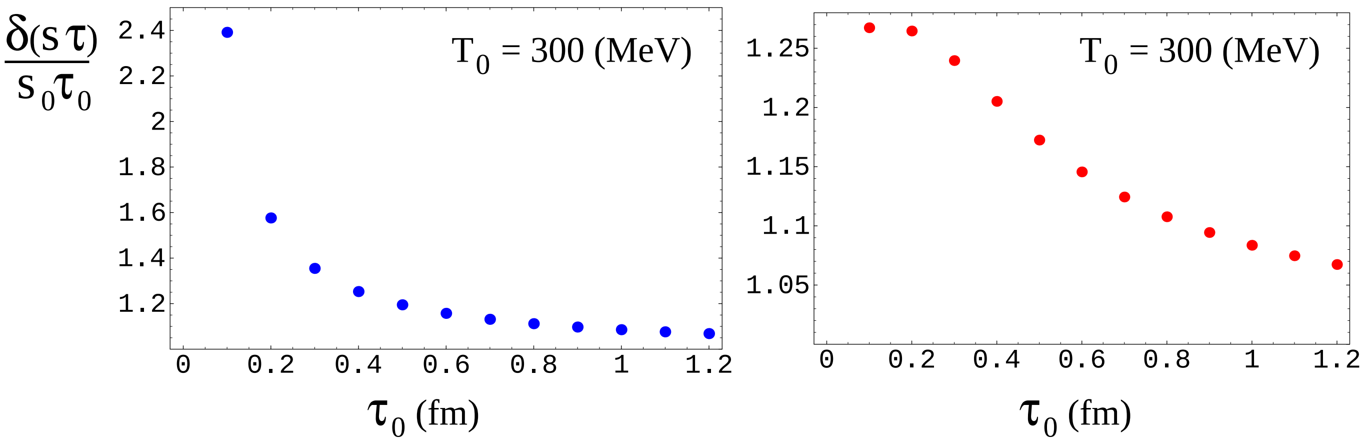
$\beta_2 < 0$ while the IS second order hydro leads to $\beta_2 > 0$

Momenta dependent viscosity (naive)

M.L. and E. Shuryak, Phys.Rev.C76:021901,2007.

Effective viscosity
$$\eta(k) \;=\; \bar{\eta}\left[1 + c^{-2}\sum_{n=1}^{\infty}\beta_{n+1}\,k^{2n}\right]$$

Entropy production in the Bjorken Hydro:



Too naive: Bjorken hydro is very sensitive to the non-linear effects (so far neglected)

Life on the boundary: Linearized Hydro to all orders

Invariance under the local Weyl transformation $\quad g_{\mu\nu} \;\longrightarrow\; e^{-2\,\Omega(x,t)}\, g_{\mu\nu}$

$$T^{\mu\nu} \;\longrightarrow\; e^{6\,\Omega(x,t)}\, T^{\mu\nu}\,; \qquad u^{\mu} \;\longrightarrow\; e^{\Omega(x,t)}\, u^{\mu} \qquad C^{\mu}_{\alpha\nu\beta} \;\longrightarrow\; C^{\mu}_{\alpha\nu\beta}$$

$$C^{\lambda}_{\mu\nu\alpha} \;=\; R^{\lambda}{}_{\mu\nu\alpha} - \frac{1}{2}\,(g^{\lambda}_{\nu}\, R_{\mu\alpha} - g^{\lambda}_{\alpha}\, R_{\mu\nu} - g_{\mu\nu}\, R^{\lambda}{}_{\alpha} + g_{\mu\alpha}\, R^{\lambda}{}_{\nu}) + \frac{1}{6}\, R\,(g^{\lambda}_{\nu}\, g_{\mu\alpha} - g^{\lambda}_{\alpha}\, g_{\mu\nu}),$$

Introduce all order gradient expansion of $T^{\mu\nu}$:

$$\boxed{\boldsymbol{\Pi}^{\mu\nu} \;=\; -\,\mathbf{2}\,\eta\,\nabla^{\mu}\,\mathbf{u}^{\nu} \;+\; \mathbf{2}\,\kappa\,\mathbf{u}_{\alpha}\,\mathbf{u}_{\beta}\,\mathbf{C}^{\mu\alpha\nu\beta} \;+\; \rho\,(\mathbf{u}_{\alpha}\,\nabla_{\beta} \;+\; \mathbf{u}_{\beta}\,\nabla_{\alpha})\,\mathbf{C}^{\mu\alpha\nu\beta} \;+\; \xi\,\nabla_{\alpha}\,\nabla_{\beta}\,\mathbf{C}^{\mu\alpha\nu\beta}}$$

$$\eta \;=\; \eta[\nabla^2, (u\nabla)]\,; \qquad \kappa \;=\; \kappa[\nabla^2, (u\nabla)]\,; \qquad \rho \;=\; \rho[\nabla^2, (u\nabla)]\,; \qquad \xi \;=\; \xi[\nabla^2, (u\nabla)]\,;$$

$\nabla^2 \;\rightarrow\; \omega^2 \;-\; k^2$ **and** $(u\,\nabla) \;\rightarrow\; -\,i\,\omega$**.**

$$\boxed{\mathbf{\Pi}^{\mu\nu} \;=\; -\,\mathbf{2}\,\eta\,\nabla^{\mu}\,\mathbf{u}^{\nu} \;+\; \mathbf{2}\,\kappa\,\mathbf{u}_{\alpha}\,\mathbf{u}_{\beta}\,\mathbf{C}^{\mu\alpha\nu\beta} \;+\; \rho\,(\mathbf{u}_{\alpha}\,\nabla_{\beta} \;+\; \mathbf{u}_{\beta}\,\nabla_{\alpha})\,\mathbf{C}^{\mu\alpha\nu\beta} \;+\; \xi\,\nabla_{\alpha}\,\nabla_{\beta}\,\mathbf{C}^{\mu\alpha\nu\beta}}$$

We keep the nonlinear dispersion to all orders, but

We neglect nonlinear interactions (though some terms could be recovered).

We postulated a constitutive relation between $\langle T^{ij} \rangle$ and three-velocity v^i.

The first term generalizes the usual shear viscosity coefficient η_0 defined at zero frequency and momentum. It also contains the relaxation time τ_R.

What is the physical role of κ, ρ and ξ (we call them Gravitational Susceptibilities of Fluid (GSF))? GSFs are absent in Minkowski space. GSFs contribute directly to two-point functions of stress tensors.

The correlators of $T^{\mu\nu}$ contain not only "thermal" physics but in addition get contaminated by the vacuum or zero temperature contributions due to pair production.

It is tempting to identify the viscosity term with pure hydrodynamic ("thermal") physics associated with the matter flow, and the GSFs with non-matter effects and the interference thereof.

Retarded Correlators from Hydrodynamics

Linear response

$$G^{\alpha\beta\mu\nu} \;=\; \frac{\delta T^{\alpha\beta}}{\delta h^{\mu\nu}}\Big|_{h=0}$$

The scalar (h^{xy}):

$$\mathbf{G^T(k,\omega) \;=\; -\,i\,\omega\,\eta \;-\; \kappa\,\frac{1}{2}\,(\omega^2 \;+\; k^2) \;-\; \rho\,\frac{i\,\omega}{2}\,(\omega^2 \;-\; k^2) \;+\; \xi\,\frac{1}{4}\,(\omega^2 \;-\; k^2)^2}$$

The shear (h^{tx}):

$$\mathbf{G^D(k,\omega) \;=\; (\epsilon + P)\,\frac{\bar\eta\,k^2 \;-\; i\bar\kappa\,\omega\,k^2/2 \;-\; \bar\rho\,k^2\,(k^2 \;-\; 2\,\omega^2)/4 \;+\; i\,\bar\xi\,\omega\,k^2\,(\omega^2 \;-\; k^2)/4}{-i\,\omega \;+\; \bar\eta\,k^2}}$$

The sound (h^{tz}):

$$\mathbf{G^S(k,\omega) \;=\; (\epsilon \;+\; P)\,\frac{k^2 \;-\; 4\,i\,\bar\eta\,\omega\,k^2 \;-\; 2\,\bar\kappa\,\omega^2\,k^2 \;-\; 2\,i\,\bar\rho\,\omega^3\,k^2 \;+\; \bar\xi\,\omega^4\,k^2}{k^2 \;-\; 3\,\omega^2 \;-\; 4\,i\,\bar\eta\,\omega\,k^2}}$$

On quasinormal modes

The entire information about quasinormal modes is coded in viscosity η.

GSFs do not have any poles. If this were not true, we would observe appearance of identical quasinormal modes in all three channels.

$$\eta(\mathbf{k^2}, \omega) = \sum_{\mathbf{n=0}}^{\infty} \frac{\eta_{\mathbf{n}}(\mathbf{k^2}, \omega)}{\omega - \omega_{\mathbf{n}}(\mathbf{k^2})}$$

ω_n **coincide with the quasinormal modes of the scalar channel (poles of G^T).**

Quasinormal modes of the shear and sound channels

$$-i\,\omega + \eta(k^2, \omega)\,k^2 = 0; \qquad\qquad -3\,\omega^2 + k^2 - 4\,i\,\eta(k^2, \omega)\,\omega\,k^2 = 0$$

4 vs 3 Puzzle

There should be one to one correspondence between linearized $T^{\mu\nu}$ and the full set of its correlators.

Our program is to equate the "hydro" correlators with the correlators computed from the bulk gravity. The goal is to invert these equations in order to determine the four transport coefficient functions.

We end up having only 3 equations for 4 unknown functions!

This system does not seem to have a unique solution. Despite our failure to simultaneously determine all transport coefficient functions, we are able to extract them perturbatively in the long-wave limit approximation.

In the long-wave limit all coefficient functions are expandable in power series

$$\eta \;=\; \eta_0\,(1+i\eta_{0,1}\,\omega+\eta_{2,0}\,k^2+\eta_{0,2}\,w^2+i\,\eta_{2,1}\,\omega\,k^2+i\,\eta_{0,3}\,\omega^3+\eta_{4,0}\,k^4+\eta_{2,2}\,\omega^2\,k^2+\eta_{0,4}\,\omega^4+\cdots);$$

$$\kappa \;=\; \kappa_0\,(1\;+\;i\,\kappa_{0,1}\,\omega\;+\;\kappa_{2,0}\,k^2\;+\;\kappa_{0,2}\,w^2\;+i\,\kappa_{2,1}\,\omega\,k^2\;+\;i\,\kappa_{0,3}\,\omega^3\;+\;\cdots)\,;$$

$$\rho \;=\; \rho_0\,(1\;+\;i\,\rho_{0,1}\,\omega\;+\;\rho_{2,0}\,k^2\;+\;\rho_{0,2}\,w^2\;+\;\cdots)$$

$$\xi \;=\; \xi_0\,(1\;+\;i\,\xi_{0,1}\,\omega\;+\;\cdots)$$

1st and 2nd order hydro

$$\eta_0 \;=\; 1/2; \qquad \tau_R \equiv \eta_{0,1} \;=\; 2\;-\;\ln 2\,; \qquad \kappa_0 \;=\; 2\,\eta_0$$

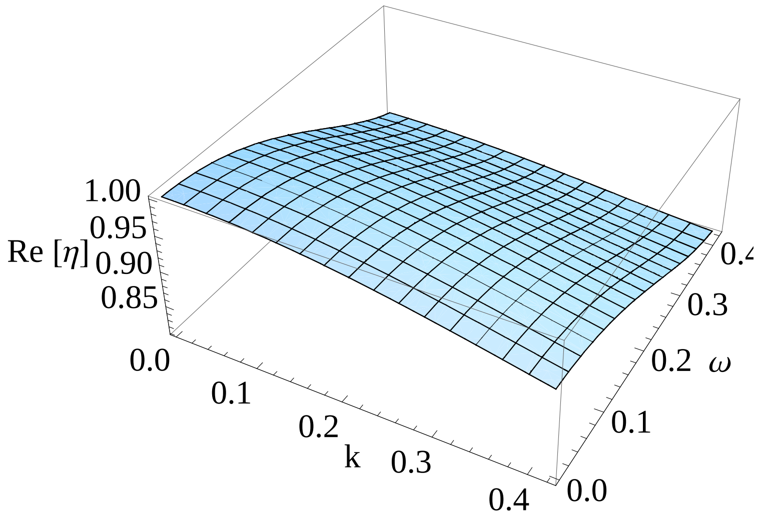
3rd order hydro

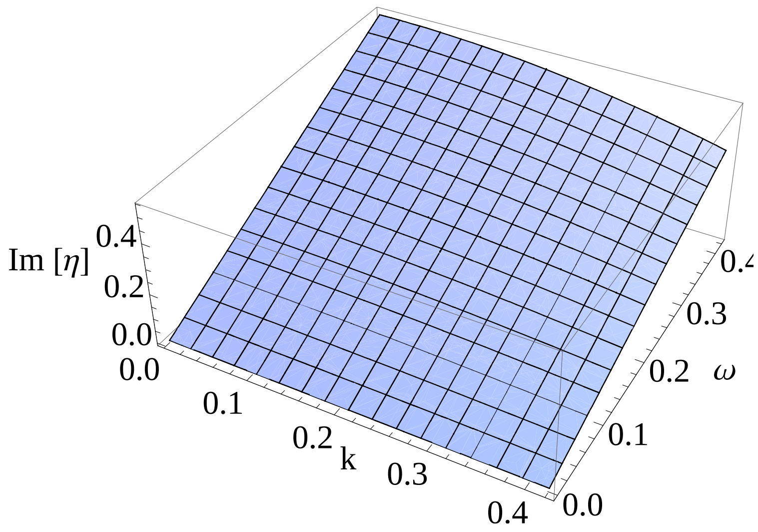
$$\lambda \;\equiv\; \eta_{2,0} \;=\; -\,1/2; \qquad \eta_{0,2} \;\simeq\; -\,1.379\;\pm\;0.001\;\simeq\;-\,\frac{3}{2}\;+\;\frac{\ln^2 2}{4}$$

$$\kappa_{0,1} \;=\; 5/2\;-\;2\,\ln 2\,; \qquad \rho_0 \;=\; 4\,\eta_0$$

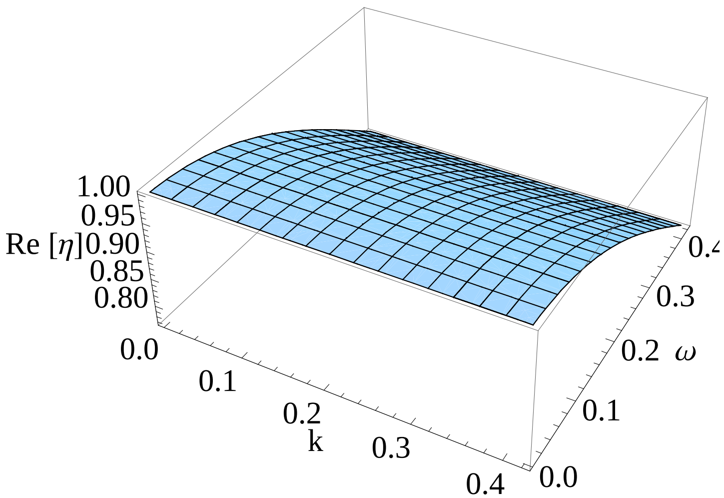
4th order hydro

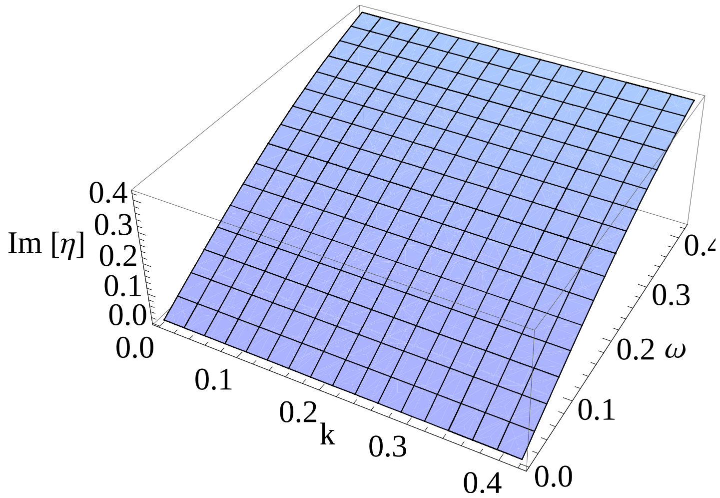
$$\eta_{2,1} \;=\; -\,2.275\;\pm\;0.005\,; \qquad \eta_{0,3} \;=\; -\,0.082\;\pm\;0.003$$





$$\eta \;=\; 1 + i\,\tau_R\,\omega + \lambda\,k^2 + \gamma\,w^2\;\ldots \qquad\qquad \tau_R \;=\; 2 - \ln[2]\,, \qquad \lambda \;=\; -1/2\,, \qquad \gamma \;\simeq\; -1.38$$





$$\bar{\eta} \;=\; 1 \;+\; i\,\tau_R\,\omega \;-\; \tau_R^2\,\omega^2\;\ldots$$

In order to illustrate the qualitative difference between the IS model and the AdS-based viscosity, we compute sound dispersion and focus on the first correction to its width in both cases.

$$\omega_{\mathbf{IS}} = \pm \frac{\mathbf{k}}{\sqrt{\mathbf{3}}} \left(\mathbf{1} + \left(\frac{\mathbf{1}}{\mathbf{2}} - \frac{\ln[\mathbf{2}]}{\mathbf{3}}\right) \mathbf{k}^2\right) - \mathbf{i}\,\frac{\mathbf{k}^2}{\mathbf{3}} \left(\mathbf{1} + \frac{\mathbf{k}^2}{\mathbf{3}} \ln \mathbf{2}\,(\mathbf{2} - \ln \mathbf{2})\right)$$

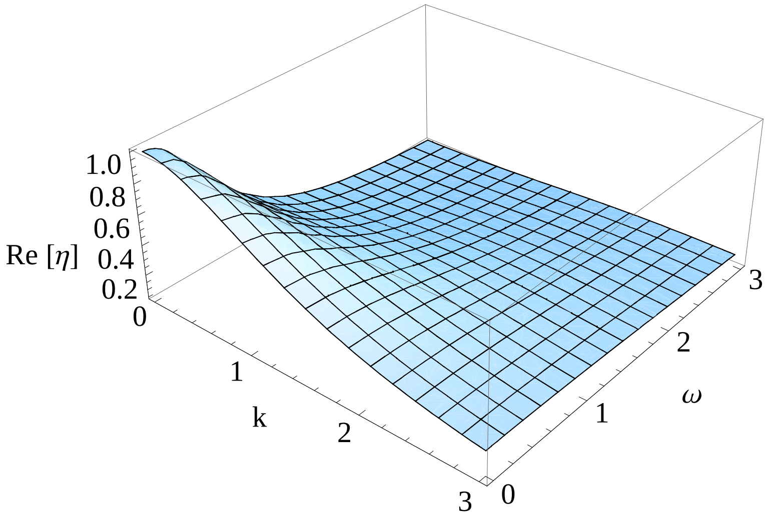
The k^4 correction to the sound width is positive, in contrast to the AdS result which is negative

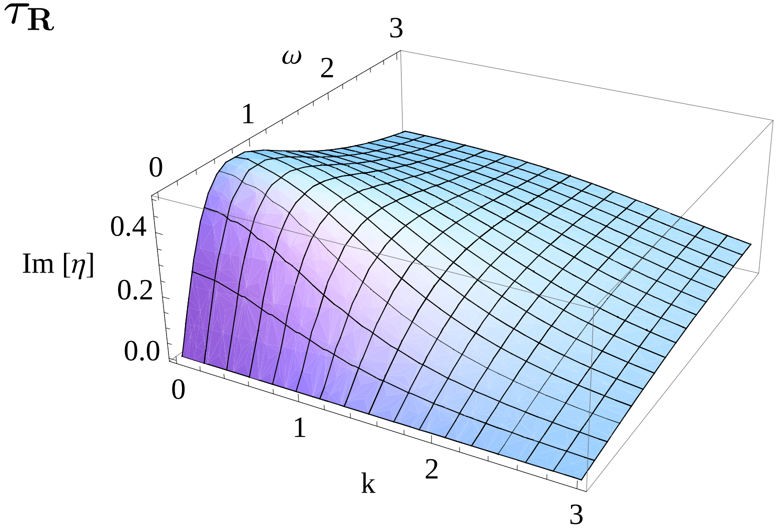
$$\omega_{\mathbf{AdS}} = \pm \frac{\mathbf{k}}{\sqrt{\mathbf{3}}} \left(\mathbf{1} + \left(\frac{\mathbf{1}}{\mathbf{2}} - \frac{\ln[\mathbf{2}]}{\mathbf{3}}\right) \mathbf{k}^2\right) - \mathbf{i}\,\frac{\mathbf{k}^2}{\mathbf{3}} \left(\mathbf{1} - \frac{\mathbf{k}^2}{\mathbf{12}} (\mathbf{4} - \mathbf{8}\,\ln \mathbf{2} + \ln^2 \mathbf{2})\right)$$

The IS predicts an increase of the width while AdS-based result leads to the opposite effect.

Improved Causal Hydrodynamics

$$\eta_{\mathrm{model}} \;=\; \frac{\eta_0}{1 \,-\, \lambda\,k^2 \,-\, i\,\omega\,\tau_R}$$





Introducing a memory function

$$D(x,t) \;=\; \int d\omega\, d^3k\, e^{-i\,\omega\,t\,+\,i\,k\,x}\,\eta(k^2,\omega) \;=\; \frac{1}{2\sqrt{2}}\,\frac{\eta_0}{\tau_R}\left(\frac{\tau_R}{-\lambda\,t}\right)^{3/2} e^{-\,t\,/\,\tau_R}\, e^{-\,x^2\,\tau_R\,/\,(-\lambda\,t)}$$

which leads to the following expression for the dissipation tensor Π:

$$\Pi^{\mu\nu}(x,t) \;=\; -\,2\int_0^t dt' \int d^3x'\, D(x-x',t-t')\;\nabla'^{\,\mu}\,u^{\nu}(x',t')$$

Conclusions

- We have initiated study of all order (linearized) hydrodynamics.
- The 4 vs 3 puzzle remains unsolved.
  Possible solutions may involve either the membrane paradigm approach or prove that the iterative procedure works to any order
- We have determined few new transport coefficients
- We cautiously suggest that the results based on IS might be less reliable than it was previously thought. We have proposed an improved phenomenological model.
- The effective viscosity is a decreasing function both of frequency and momentum. This behavior might be the reason behind the low viscosity observed at RHIC. It may also explain the exceptionally good survival of various hydrodynamic flows, particularly the sound waves.