Conformal Hydrodynamics and rotating Blacholes in AdS

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References : arXiv [hep-th] : 0801.3701,0809.2596,0809.4272
Collaborators : Nabamita Banerjee, Jyotirmoy Bhattacharya, Sayantani Bhattacharyya , Suvankar Dutta, Ipsita Mandal, Shiraz Minwalla, Ankit Sharma, P. Surówka

Workshop on Fluid Gravity Correspondence (ASC,LMU)
3rd' September'09

Two (related) themes in fluid-gravity correspondence

Questions in Hydro_d

- Two (related) themes in fluid-gravity correspondence : Questions in Hydro_d vs Questions in Gravity_{d+1}
- For most questions in Hydro_d, oserveables of primary interest are Energy momentum tensor/Charge current/Entropy current/Green functions at the boundary
- Once a dual metric has been obtained, No need to study the detailed form of the bulk metric except for extracting the quantities above.

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Questions in Gravity$_{d+1}$

- Second theme : What new insights does fluid-gravity correspondence bring to gravity ?
- For these questions, one is interested in much more details of the bulk field configurations than before.
- In this talk : will focus mainly on a question in the second set - exact solutions in AdS$_{d+1}$ dual to Hydro$_d$.
- But, will also learn some interesting phenomena in Hydro$_d$ along the way.

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- A quick review of fluid gravity correspondence : Conformal hydrodynamics and their metric duals
- Hydrodynamical form for some exact solutions in AdS_{d+1}
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Hydrodynamics - A tractable description at finite T

- **Basic Variables** : Velocity u^μ(Energy frame/Charge frame), Temperature T
  and Charge density n or Chemical potential μ.
- Energy/Momentum transport : $T^{\mu\nu}$ with $\nabla_\mu T^{\mu\nu} = 0$.
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Scaling in Hydrodynamics

- Scaling is a powerful concept in hydrodynamics - especially if there is an underlying CFT$_d$.
- Weyl transformation in hydrodynamics : $g_{\mu\nu} = e^{2\phi}\tilde{g}_{\mu\nu}$ and $u^\mu = e^{-\phi}\tilde{u}^\mu$.
- Energy scales scale as $T = e^{-\phi}\tilde{T}$ and $\mu = e^{-\phi}\tilde{\mu}$.
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Weyl Covariance is useful

- Introduce in CFT$_d$ hydrodynamics

$$\mathcal{A}_\nu \equiv u^\lambda \nabla_\lambda u_\nu - \frac{\nabla_\lambda u^\lambda}{d-1} u_\nu = \tilde{\mathcal{A}}_\nu + \partial_\nu \phi .$$

R. Loganayagam . arXiv:0801.3701 [hep-th]

- Helps in 'Weyl-covariantly' differentiating tensors

$$\text{If} \quad Q^{\mu\ldots}_{\nu\ldots} = e^{-w\phi} \widetilde{Q}^{\mu\ldots}_{\nu\ldots} \quad \text{then} \quad \mathcal{D}_\lambda Q^{\mu\ldots}_{\nu\ldots} = e^{-w\phi} \tilde{\mathcal{D}}_\lambda \tilde{Q}^{\mu\ldots}_{\nu\ldots}$$

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Weyl-Covariantised Curvature Tensors

Weyl covariantised Riemann tensor can be obtained from

$$\begin{aligned}
[\mathcal{D}_\mu, \mathcal{D}_\nu] V_\lambda &= w\, \mathcal{F}_{\mu\nu}\ V_\lambda + \mathcal{R}_{\mu\nu\lambda}{}^{\alpha}\ V_\alpha \qquad \text{with} \\
\mathcal{F}_{\mu\nu} &\equiv \nabla_\mu \mathcal{A}_\nu - \nabla_\nu \mathcal{A}_\mu \\
\mathcal{R}_{\mu\nu\lambda\sigma} &\equiv R_{\mu\nu\lambda\sigma} + \mathcal{F}_{\mu\nu} g_{\lambda\sigma} \\
&\quad - \delta^{\alpha}_{[\mu} g_{\nu][\lambda} \delta^{\beta}_{\sigma]} \left(\nabla_\alpha \mathcal{A}_\beta + \mathcal{A}_\alpha \mathcal{A}_\beta - \frac{\mathcal{A}^2}{2} g_{\alpha\beta} \right)
\end{aligned}$$

where $B_{[\mu\nu]} \equiv B_{\mu\nu} - B_{\nu\mu}$ indicates antisymmetrisation. Other related tensors defined similarly - will later need

$$\mathcal{S}_{\mu\nu} \equiv \frac{1}{d-2} \left(\mathcal{R}_{\mu\nu} - \frac{\mathcal{R} g_{\mu\nu}}{2(d-1)} \right)$$

Weyl-covariant Hydrodynamics

- By construction, $\mathcal{D}_\mu u_\nu$ is traceless and transverse to the velocity.
- Split $\mathcal{D}_\mu u_\nu = \sigma_{\mu\nu} + \omega_{\mu\nu}$ where
- $\sigma_{\mu\nu}$ is the shear strain rate (symmetric,traceless,transverse) tensor which in viscous fluids leads to dissipation.
- $\omega_{\mu\nu}$ is the vorticity (antisymmetric,transverse) tensor which measures local rotation of the fluid element.
- The hydrodynamic equations can be written in a manifestly Weyl-covariant form($\mathcal{W}$ is the Weyl anomaly)

$$\mathcal{D}_\mu T^{\mu\nu} \equiv \nabla_\mu T^{\mu\nu} + \mathcal{A}^\nu (T^\mu{}_\mu - \mathcal{W}) = 0$$
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Encoding hydrodynamics around Ingoing geodesics

- In fluid-gravity correspondence, this hydrodynamic data in the particular patch of the boundary encoded in the ‘tube’ around the ingoing null geodesic.
- Within these tubes, the metric is approximately that of a static black-brane metric which has been appropriately ‘boosted’.
- Starting from this picture, we can systematically calculate order by order in the boundary derivative expansion how the metric deviates from the locally boosted black-brane metric. (See Prof.Shiraz Minwalla’s talk before).

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Dual metric in d dimensions

- This procedure was implemented in AdS_{d+1} for arbitrary d by

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- Since we are eventually interested in stationary blackhole configurations we will specialise to stationary fluid configurations without any dissipation.
- Further, since the Hydro_d is conformal, the metric should just depend on the conformal data in the boundary
- This implies that we expect the metric to be invariant under boundary Weyl transformations (along with the scaling of the radius)

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- Choose a gauge in which the bulk metric is of the form

$$ds^2 = -2u_\mu(x)dx^\mu(dr + \mathcal{V}_\nu(r,x)dx^\nu) + \mathfrak{G}_{\mu\nu}(r,x)dx^\mu dx^\nu$$

where $\mathfrak{G}_{\mu\nu}$ is transverse, i.e., $u^\mu \mathfrak{G}_{\mu\nu} = 0$.

- Boundary Weyl transformation should induce a bulk-diffeomorphism of the form $r = e^{-\phi(x)}\tilde{r}$ along with a scaling in the temperature of the form $b = e^{\phi}\tilde{b}$.
- Under this, the metric components transform as

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& + \frac{1}{(br)^d}\left(r^2 - \frac{1}{2}\omega_{\alpha\beta}\omega^{\alpha\beta}\right)u_\mu u_\nu dx^\mu dx^\nu \\
& + \text{dissipative/higher derivative terms}
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AdS$_{d+1}$ Kerr metric

- In ‘altered’ Boyer-Lindquist co-ordinates, AdS Kerr metric is

G.W. Gibbons, H. Lu, D.N. Page, C.N. Pope [hep-th/0404008]

$$
\begin{aligned}
ds^2 = & -W(1+r^2)d\hat{t}^2 + \frac{\mathfrak{F}dr^2}{1-2M/V} \\
& + \frac{2M}{V\mathfrak{F}}\left(Wd\hat{t} - \sum_{i=1}^{n}\frac{a_i\hat{\mu}_i^2 d\hat{\varphi}_i}{1-a_i^2}\right)^2 \\
& + \sum_{i=1}^{n+\epsilon}\frac{r^2+a_i^2}{1-a_i^2}\left[d\hat{\mu}_i^2 + \hat{\mu}_i^2 d\hat{\varphi}_i^2\right] \\
& - \frac{1}{W(1+r^2)}\left(\sum_{i=1}^{n+\epsilon}\frac{r^2+a_i^2}{1-a_i^2}\hat{\mu}_i d\hat{\mu}_i\right)^2
\end{aligned}
$$

where $d = 2n + \epsilon$ with $\epsilon = d \bmod 2$ and

$$
W \equiv \sum_{i=1}^{n+\epsilon}\frac{\hat{\mu}_i^2}{1-a_i^2} \quad ; \quad V \equiv r^d\,(1+\frac{1}{r^2})\prod_{i=1}^{n}(1+\frac{a_i^2}{r^2})
$$

and $\mathfrak{F} \equiv \frac{1}{1+r^2}\sum_{i=1}^{n+\epsilon}\frac{r^2\hat{\mu}_i^2}{r^2+a_i^2}$

AdS_{d+1} Kerr metric II

- After a series of co-ordinate transformations, we can bring this complicated metric to a simple "hydrodynamic" form

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\begin{aligned}
ds^2 &= -2u_\mu dx^\mu \left(dr + r\,\mathcal{A}_\nu dx^\nu\right) \\
&\quad + \left[r^2 g_{\mu\nu} + u_{(\mu}\mathcal{S}_{\nu)\lambda}u^\lambda - \omega_\mu{}^\lambda\omega_{\lambda\nu}\right] dx^\mu dx^\nu \\
&\quad + \frac{r^2 u_\mu u_\nu}{b^d \det\left[r\,\delta^\mu_\nu - \omega^\mu{}_\nu\right]} dx^\mu dx^\nu
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- This agrees with the metric derived via boundary derivative expansion
- Further, in this hydrodynamic form, the AdS Kerr meric is manifestly invariant under boundary diffeomorphisms/ Weyl transformation !
- This is very useful. It is almost trivial in this form to go to a rotating co-ordinates in the boundary, for example.

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Charged Rotating AdS_5 Blackholes

- Focus on two-derivative theory of gravity in five dimensions with asymptotically AdS boundary conditions
- Interested in matter content that allows consistent truncation to Einstein-Maxwell Chern-Simons system
- E.g. IIB SUGRA in $AdS_5 \times S^5$ the equal R-charge sector.
- Truncated action is

$$S = \frac{1}{16\pi G_{\text{AdS}}} \int \left[\sqrt{-g_5}(R + 12) - \frac{1}{2}\mathbf{F} \wedge *_5 \mathbf{F} + \frac{2\kappa}{3}\mathbf{A} \wedge \mathbf{F} \wedge \mathbf{F} \right]$$

with $\kappa = (2\sqrt{3})^{-1}$ for the IIB SUGRA.

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Charged Blackhole Solution I

$$
\begin{aligned}
ds^2 = & -\frac{\left(r^2+1\right)\Delta_\Theta dt_1{}^2}{\left(1-\omega_1{}^2\right)\left(1-\omega_2{}^2\right)} + \frac{2(m-q\omega_1\omega_2)}{\rho^2} - \frac{q^2}{\rho^4} \\
& + \frac{(\mathrm{d}\psi_1 + dt_1\omega_2)^2\left(r^2+\omega_2{}^2\right)\cos^2\Theta}{1-\omega_2{}^2} \\
& + \frac{(\mathrm{d}\phi_1 + dt_1\omega_1)^2\left(r^2+\omega_1{}^2\right)\sin^2\Theta}{1-\omega_1{}^2} + \frac{\rho^2\mathrm{d}\Theta^2}{\Delta_\Theta} \\
& + \frac{\rho^2\mathrm{d}r^2 r^2}{q^2-2\omega_1\omega_2 q-2mr^2+\left(r^2+1\right)\left(r^2+\omega_1{}^2\right)\left(r^2+\omega_2{}^2\right)} \cdots
\end{aligned}
$$

Charged Blackhole Solution II

$$\ldots - \frac{2\mathbf{A}}{\sqrt{3}}\left(\omega_1(\mathrm{d}\psi_1 + dt_1\omega_2)\cos^2\Theta + (\mathrm{d}\phi_1 + dt_1\omega_1)\omega_2\sin^2\Theta\right)$$

$$\mathbf{A} = -\frac{\sqrt{3}q}{\rho^2}\left[\frac{\Delta_\Theta dt_1}{\left(1-{\omega_1}^2\right)\left(1-{\omega_2}^2\right)} - \frac{\omega_2(\mathrm{d}\psi_1 + dt_1\omega_2)\cos^2\Theta}{1-{\omega_2}^2} - \frac{\omega_1(\mathrm{d}\phi_1 + dt_1\omega_1)\sin^2\Theta}{1-{\omega_1}^2}\right]$$

with

$$\rho^2 \equiv r^2 + {\omega_1}^2\cos^2\Theta + {\omega_2}^2\sin^2\Theta$$
$$\Delta_\Theta \equiv 1 - {\omega_1}^2\cos^2\Theta - {\omega_2}^2\sin^2\Theta$$

The fluid dynamical form

$$ds^2 = -2u_\mu dx^\mu \left(dr + r\,\mathcal{A}_\nu dx^\nu\right) + \left[r^2 g_{\mu\nu} + u_{(\mu}\mathcal{S}_{\nu)\lambda}u^\lambda - \omega_\mu{}^\lambda \omega_{\lambda\nu}\right] dx^\mu dx^\nu + \left[\left(\frac{2m}{\rho^2} - \frac{q^2}{\rho^4}\right) u_\mu u_\nu + \frac{q}{2\rho^2} u_{(\mu} l_{\nu)}\right] dx^\mu dx^\nu$$

$$\mathbf{A} = \frac{\sqrt{3}q}{\rho^2} u_\mu dx^\mu \quad ; \quad \rho^2 \equiv r^2 + \frac{1}{2}\omega_{\alpha\beta}\omega^{\alpha\beta} \quad ; \quad l_\mu \equiv \epsilon_{\mu\nu\lambda\sigma} u^\nu \omega^{\lambda\sigma}$$

Again the hydrodynamic form of the metric is surprisingly simple ! Can be reproduced in order by order derivative expansion - checked upto first order against the solution of

N. Banerjee, J. Bhattacharya, S. Bhattacharyya, S. Dutta, R. Loganayagam and P. Surowka arXiv:0809.2596 [hep-th]

J. Erdmenger, M. Haack, M. Kaminski and A. Yarom arXiv:0809.2488 [hep-th]

The Anomolous transport

- We now turn to the stress tensor dual to these BHs

$$T_{\mu\nu} = p(g_{\mu\nu} + 4u_\mu u_\nu) + 2\kappa l_{(\mu} J_{\nu)} + \frac{1}{64\pi G_{\text{AdS}}}\left(R^{\alpha\beta}R_{\alpha\mu\beta\nu} - \frac{R^2}{12}g_{\mu\nu}\right)$$

$$J_\mu = n u_\mu \qquad \text{where} \quad l_\mu \equiv \epsilon_{\mu\nu\lambda\sigma}u^\nu\omega^{\lambda\sigma} \quad ;$$

$$p \equiv \frac{m}{8\pi G_{\text{AdS}}} \quad \text{and} \qquad n \equiv \frac{\sqrt{3}q}{8\pi G_{\text{AdS}}}$$

- There is an anomolous energy momentum transport arising from the bulk Chern-Simons coupling !
- This term is crucial for the correct thermodynamics

S. Bhattacharyya, S. Lahiri, R. Loganayagam and S. Minwalla JHEP **0809**, 054 (2008) [arXiv:0708.1770 [hep-th]]

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Anomolous transport II

- In AdS/CFT, the bulk gauge theory induces a global symmetry in the boundary.
- A Chern-Simons coupling in the bulk translates into a global anomaly in the boundary theory.

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\begin{aligned}
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Anomolous transport III

- The standard anomaly is turned off if $\mathbf{F} = 0$. But, the anomolous transport survives this limit.
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Weyl Connections and All that - I

A Mathematical Aside

- More precisely, the 1-form $\mathcal{A}_\nu$ defines a natural Weyl Connection.
- Take a spacetime manifold $\mathcal{M}$ is with the conformal class of metrics $\mathcal{C}$
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- A fluid background provides an additional mathematical structure : a unit time-like vector field with conformal weight $w = 1$.
- Fluid background leads to a natural Weyl-Connection
- $\mathcal{A}_\mu$ is uniquely determined by requiring that $u^\lambda \mathcal{D}_\lambda u^\mu = 0$ and $\mathcal{D}_\lambda u^\lambda = 0$. Back