A flavor superconductor from string theory

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Motivation

Holographic Superconductor from charged scalar in Einstein-Maxwell gravity

Gubser; Hartnoll, Herzog, Horowitz

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p-wave superconductor (current dual to gauge field condensing)

Gubser, Pufu

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(AdS_4 examples)

Motivation

Follow-up questions:

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Is there a holographic superconductor...

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1) for which the field theory is explicitly known ?

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1) for which the field theory is explicitly known ?

2) within ten-dimensional type IIB supergravity?

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A holographic superconductor with field theory in 3+1 dimensions for which

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A holographic superconductor with field theory in 3+1 dimensions for which

1. the dual field theory is explicitly known

2. there is a qualitative ten-dimensional string theory picture of condensation

Ammon, J.E., Kaminski, Kerner 0810.2316, 0903.1864

p-wave superconductor

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This is achieved in the context of

adding flavor to gauge/gravity duality

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$\phi \to e^{i\Lambda} \phi \, e^{-i\Lambda}$

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$q \to e^{i\Lambda} q$

Brane probes added on gravity side $\Rightarrow$ fundamental d.o.f. in the dual field theory

Additional hyperplanes within $AdS_5 \times S^5$ or deformed version thereof

Motivation

Fluctuations of brane probes $\Rightarrow$ Mesons

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Chemical potentials for baryon, isospin density:

From non-trivial A_t on gravity side

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Chemical potentials for baryon, isospin density:

From non-trivial A_t on gravity side

$\Rightarrow$ Rich phase structure

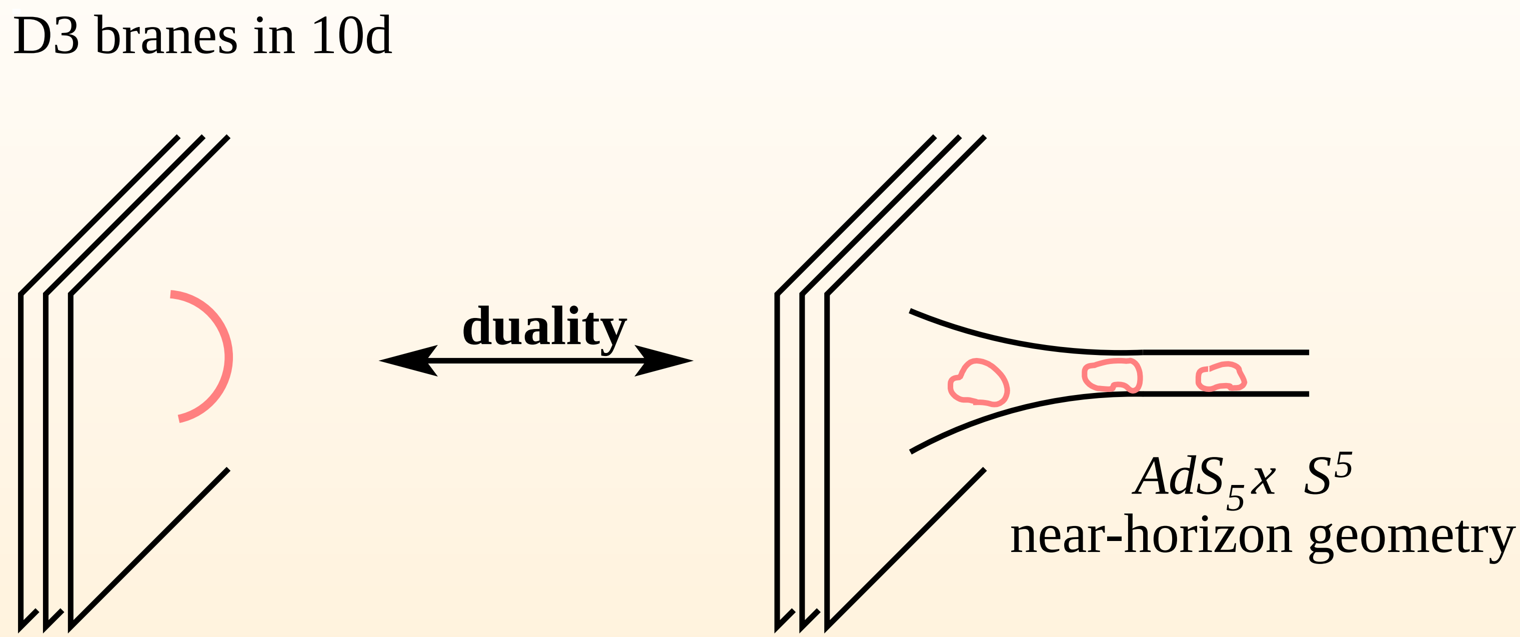
Outline

1. Adding Flavor to Gauge/Gravity Duality

2. Holographic Quarks at finite Temperature and Density

3. Superfluidity and Superconductivity

String theory origin of AdS/CFT correspondence



$\Downarrow$ Low-energy limit

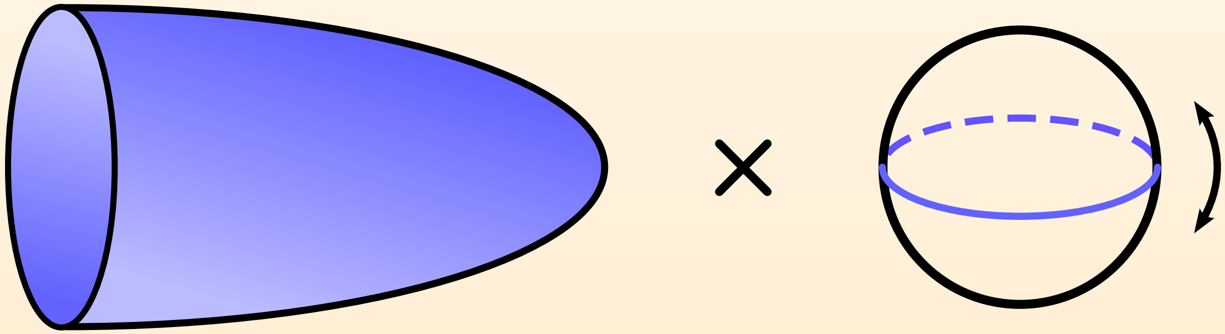
$\mathcal{N} = 4$ SUSY $SU(N)$ gauge theory in four dimensions ($N \to \infty$)

Supergravity on $AdS_5 \times S^5$

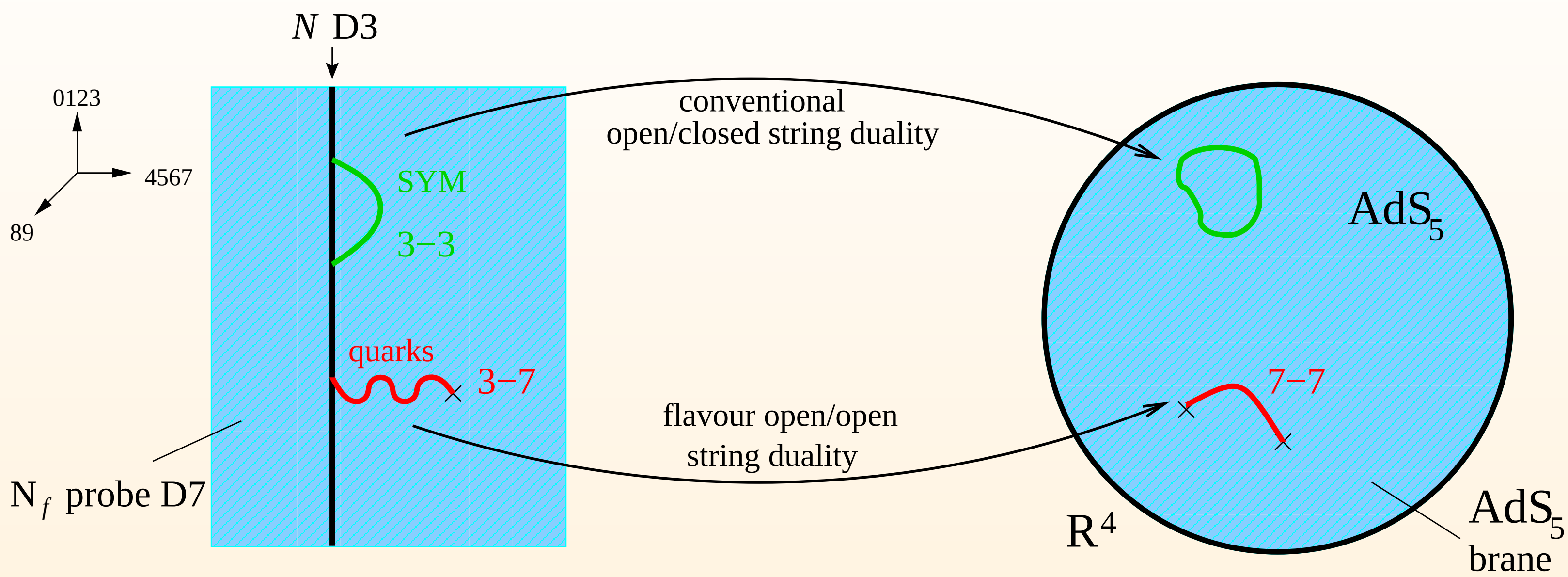
Quarks (fundamental fields) within the AdS/CFT correspondence

Adding D7 brane probe:

	0	1	2	3	4	5	6	7	8	9
D3	X	X	X	X						
D7	X	X	X	X	X	X	X	X		



Quarks (fundamental fields) from brane probes



$N \to \infty$ (standard Maldacena limit), N_f small (probe approximation)

duality acts twice:

$\mathcal{N} = 4$ SU(N) Super Yang-Mills theory coupled to $\mathcal{N} = 2$ fundamental hypermultiplet	$\longleftrightarrow$	IIB supergravity on $AdS_5 \times S^5$ + Probe brane action on $AdS_5 \times S^3$

Karch, Katz 2002

Dirac-Born-Infeld action

DBI (Dirac-Born-Infeld) action:

$$S_{DBI} = -T_7 \int d^8\xi \, \mathrm{tr}\sqrt{det(-P[G] + 2\pi\alpha' F)}$$

Contributions of order N_f/N_c

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Contributions of order N_f/N_c

Field theory involves fundamental fermions and scalars

Gauge/Gravity Duality at Finite Temperature

$\mathcal{N} = 4$ Super Yang-Mills theory at finite temperature is dual to AdS black hole

Witten 1998

$$ds^2 = \frac{1}{2}\left(\frac{\varrho}{R}\right)^2\left(-\frac{f^2}{\tilde{f}}dt^2 + \tilde{f}d\vec{x}^2\right) + \left(\frac{R}{\varrho}\right)^2(d\varrho^2 + \varrho^2 d\Omega_5^2)$$

$$f(\varrho) = 1 - \frac{\varrho_H^4}{\varrho^4}, \quad \tilde{f}(\varrho) = 1 + \frac{\varrho_H^4}{\varrho^4}$$

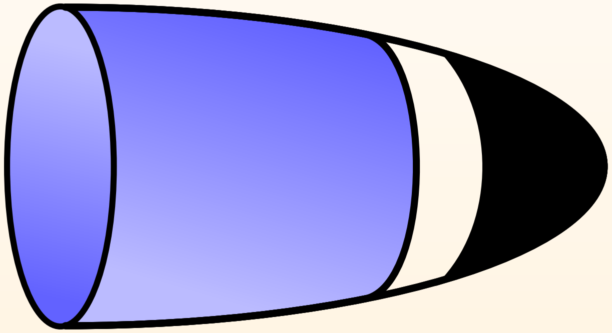
Temperature and horizon related by

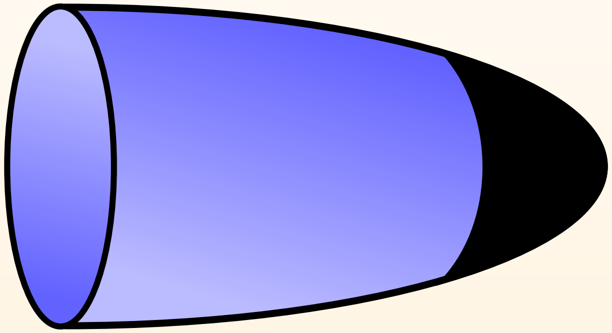
$$T = \frac{\varrho_H}{\pi R^2}$$

R: AdS radius

For $\varrho_H \to 0$, metric of $AdS_5 \times S^5$ is recovered.

D7 brane embedding in black hole background





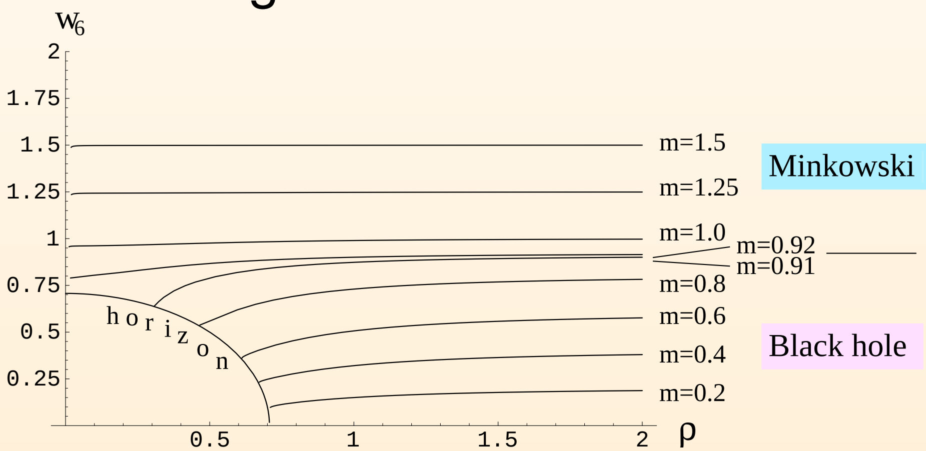
First order phase transition

Babington, J.E., Evans, Guralnik, Kirsch
Mateos, Myers, Thomson

D7 brane embedding in black hole background

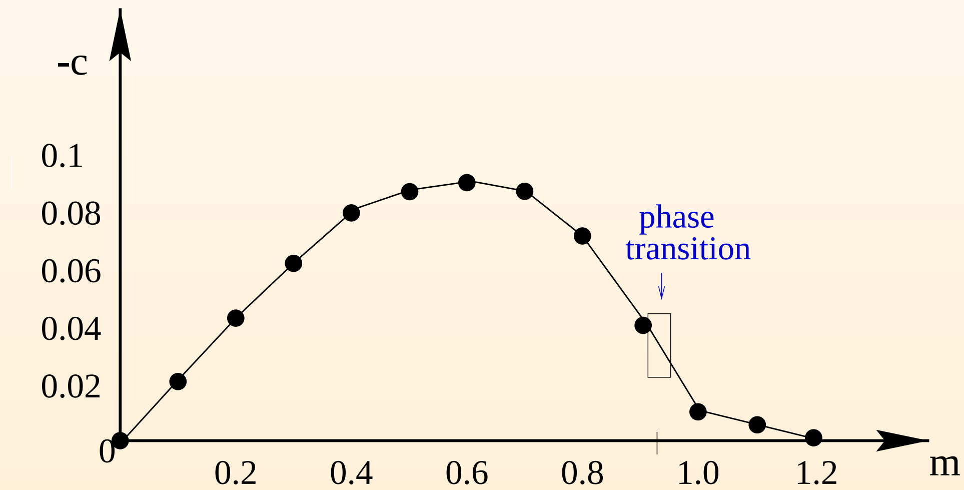
Babington, J.E., Evans, Guralnik, Kirsch 0306018

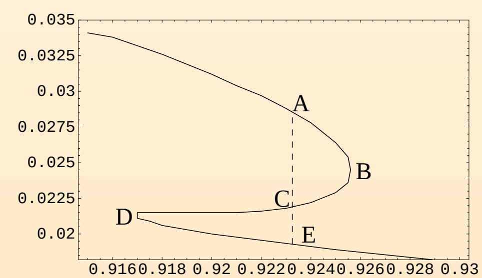
Embeddings



Phase transition at $m_c \approx 0.92$
(1st order)

Condensate $c \equiv \langle \bar{\psi}\psi \rangle$ vs. quark mass m
m in units of T





Kirsch 2004

Masses and decay widths of mesons - Spectral functions

Standard procedure in D3/D7: Mateos, Myers et al 2003

Meson masses calculated from linearized fluctuations of D7 embedding

Fluctuations: $\delta w(x,\rho) = f(\rho) e^{i(\vec{k}\cdot\vec{x} - \omega t)}$, $M^2 = -k^2$

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For black hole embeddings, ω develops negative imaginary part

$\Rightarrow$ damping $\Rightarrow$ decay width

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Make contact with hydrodynamics: Starinets, Kovtun ....

Spectral function determined by poles of retarded Green function

Quasinormal modes

Identify mesons with resonances in spectral function

Landsteiner, Hoyos, Montero

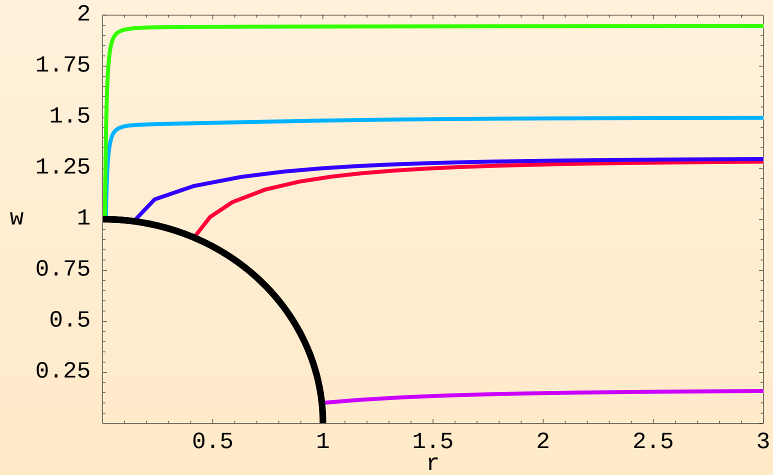
Finite $U(1)$ baryon density

Mateos, Myers, Matsuura et al

Baryon density n_B and $U(1)$ chemical potential μ
from VEV for gauge field time component:

$$\bar{A}_0(\rho) \sim \mu + \frac{\tilde{d}}{\rho^2}\,, \qquad \tilde{d} = \frac{2^{5/2}}{N_f\sqrt{\lambda}T^3}\, n_B$$

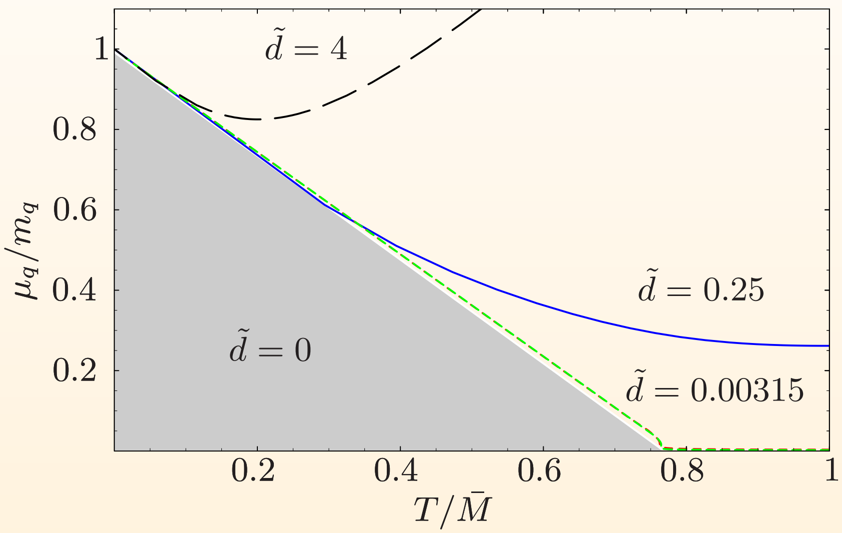
At finite baryon density, all embeddings are black hole embeddings



Phase diagram with finite $U(1)$ baryon density

Phase diagram:

grey region: $n_B = 0$
white region: $n_B \neq 0$



Sin, Yogendran et al; Mateos, Myers et al; Karch, O'Bannon; . . .

Isospin chemical potential and density

- Embed two coincident D7-branes into AdS-Schwarzschild
  gauge fields $A_\mu = A_\mu^a \sigma^a \in u(2) = u(1)_B \oplus su(2)_I$
- Finite isospin density: $A_0^3 \neq 0 \Rightarrow$ Explicit breaking to $u(1)_3$
- Dynamics of Flavour degrees is described by non-abelian DBI action

Field theory described:

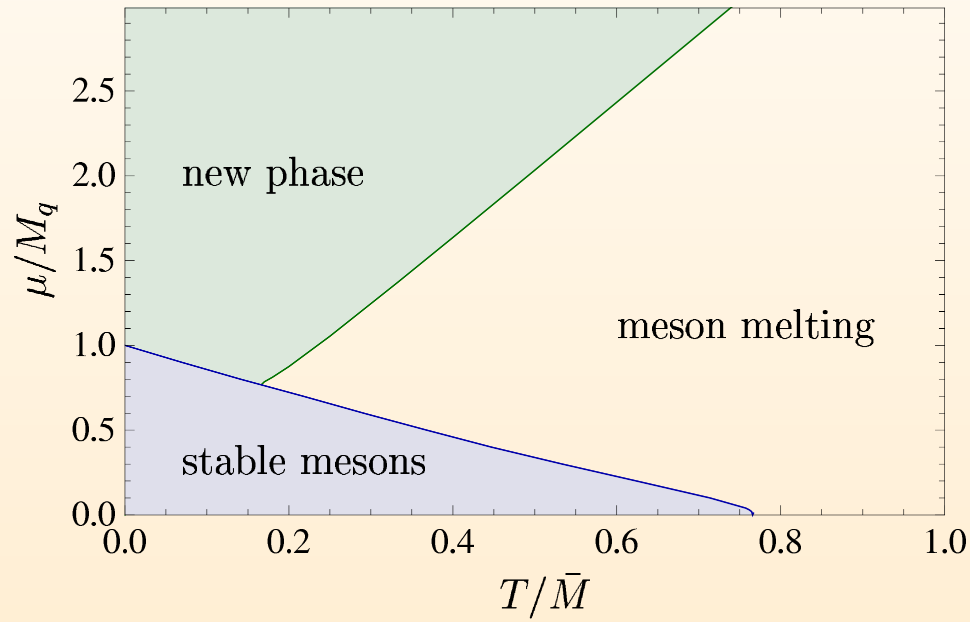
$\mathcal{N} = 4$ Super Yang-Mills plus two flavors of fundamental matter

at finite temperature and finite isospin density

ρ meson condensation

J.E., Kaminski, Kerner, Rust 0807.2663

Above a critical isospin density, a new phase forms



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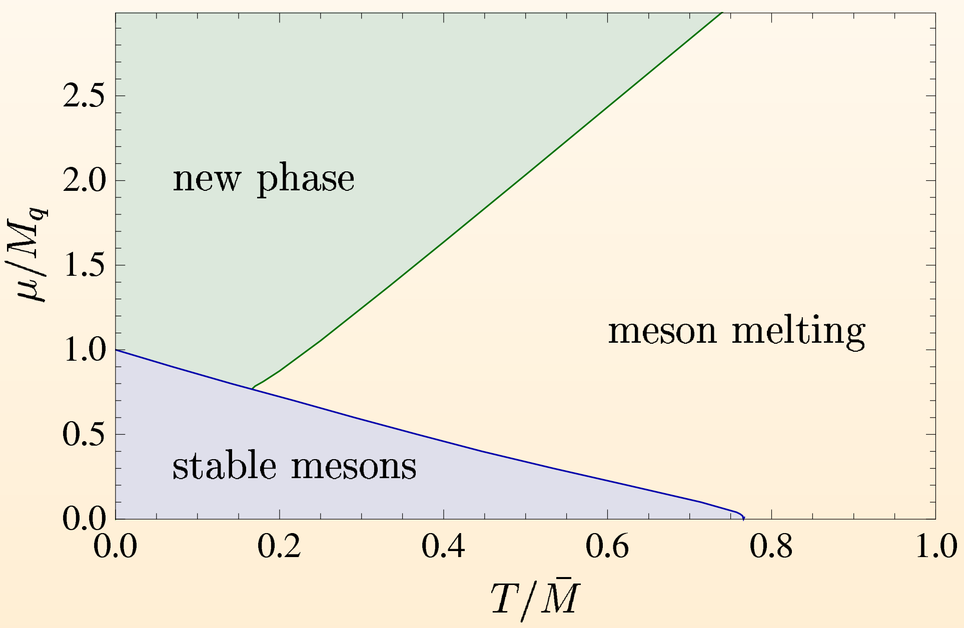
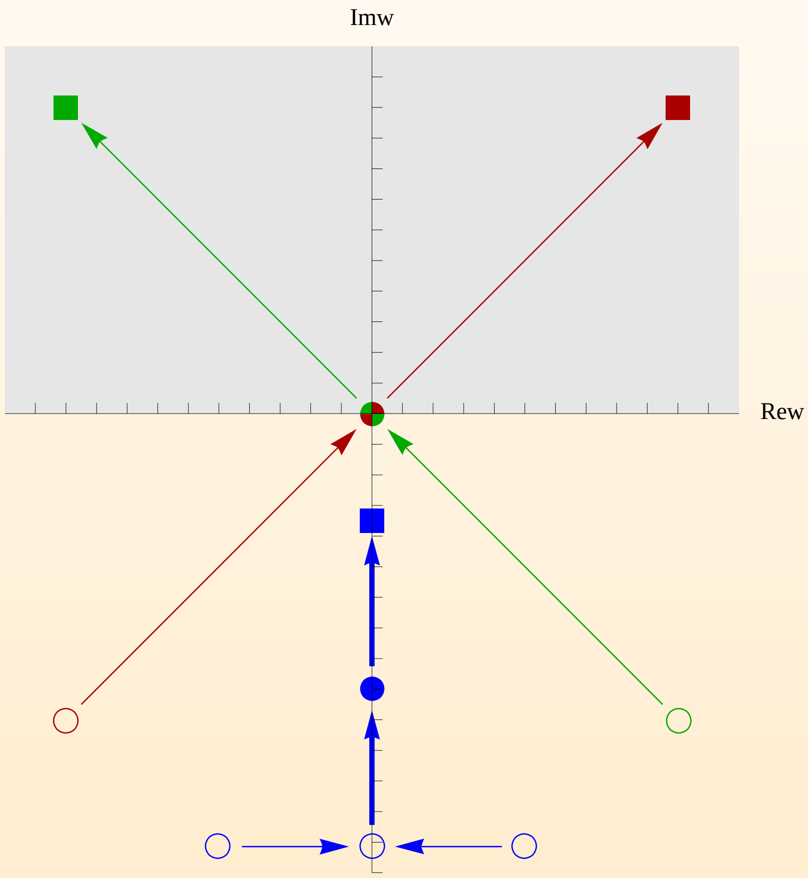


New phase is unstable

Quasinormal modes

Instability:



A new ground state forms

There is a new solution to the equations of motion

with non-zero vev for $A_3^1\sigma^1$ in addition to the non-zero $A_0^3\sigma^3$

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$$A_0^3 = \mu - \frac{\tilde{d}_0^3}{2\pi\alpha'}\frac{\rho_H}{\rho^2} + \dots\,, \qquad A_3^1 = -\frac{\tilde{d}_1^3}{2\pi\alpha'}\frac{\rho_H}{\rho^2} + \dots$$

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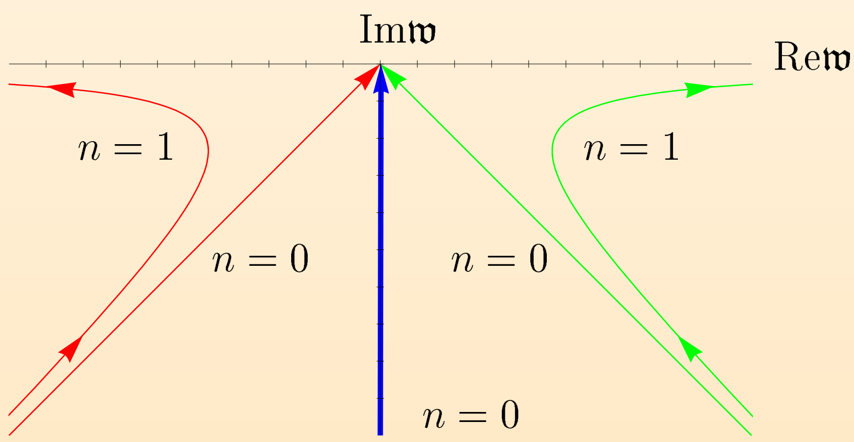
Pole structure:



Superconductivity

Ammon, J.E., Kaminski, Kerner 0810.2316, 0903.1864

The new ground state has properties known from superconductors:

- infinite DC conductivity, gap in the AC conductivity
- second order phase transition, critical exponent of 1/2 (mean field)
- a remnant of the Meissner–Ochsenfeld effect

Superfluidity and Superconductivity

Order parameter $\tilde{d}_3^1 \propto \langle \bar{\psi}_u \gamma_3 \psi_d + \bar{\psi}_d \gamma_3 \psi_u + \, bosons \, \rangle \neq 0$

Dual to $A_3^1 \sigma^1$ in gravity theory

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Spontaneous breaking of (global) $U(1)_3$

Flavor superfluid

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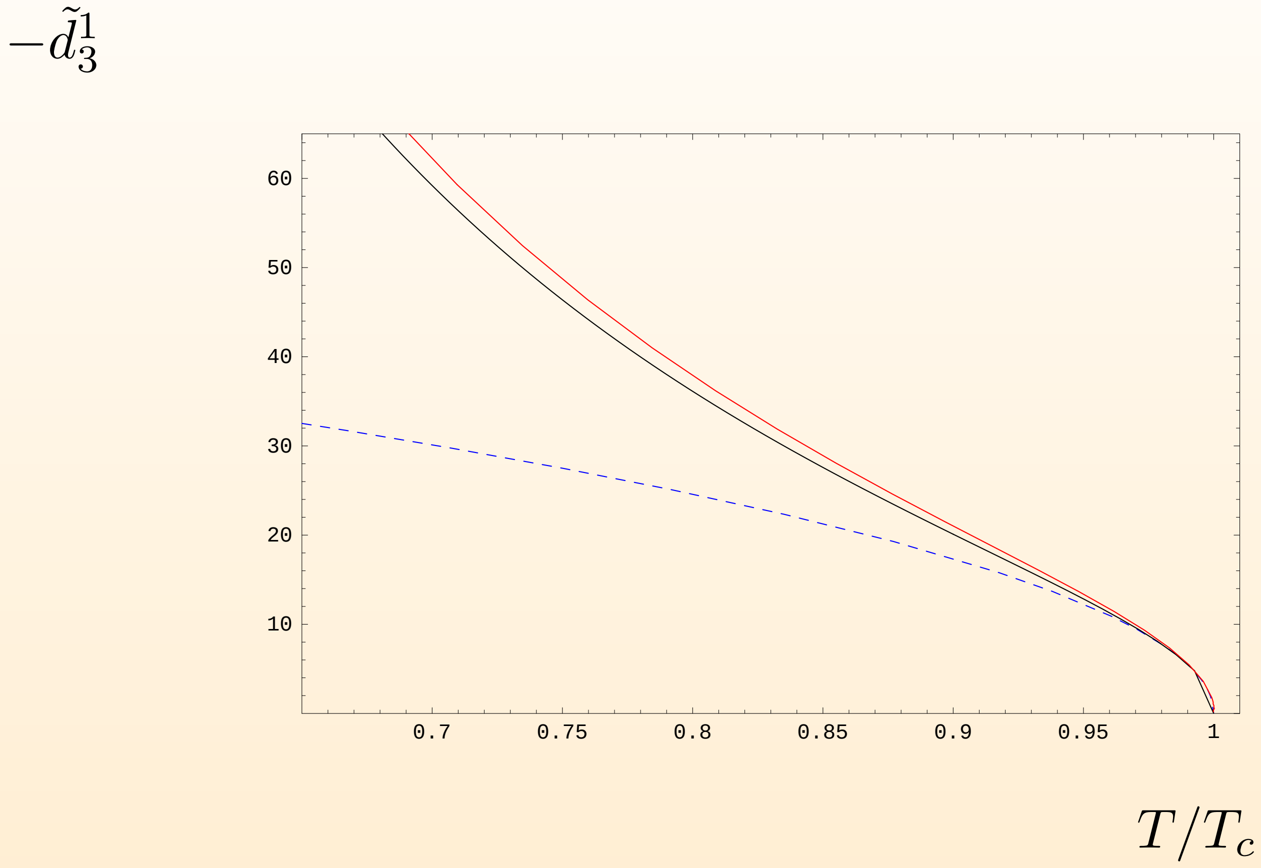
Flavor superfluid

FFLO condensate corresponds to ρ meson superfluid

discussed in QCD literature Son, Stephanov; Splittorff; Sannino ...

ρ condensation in Sakai-Sugimoto model: Aharony, Peeters, Sonnenschein, Zamaklar

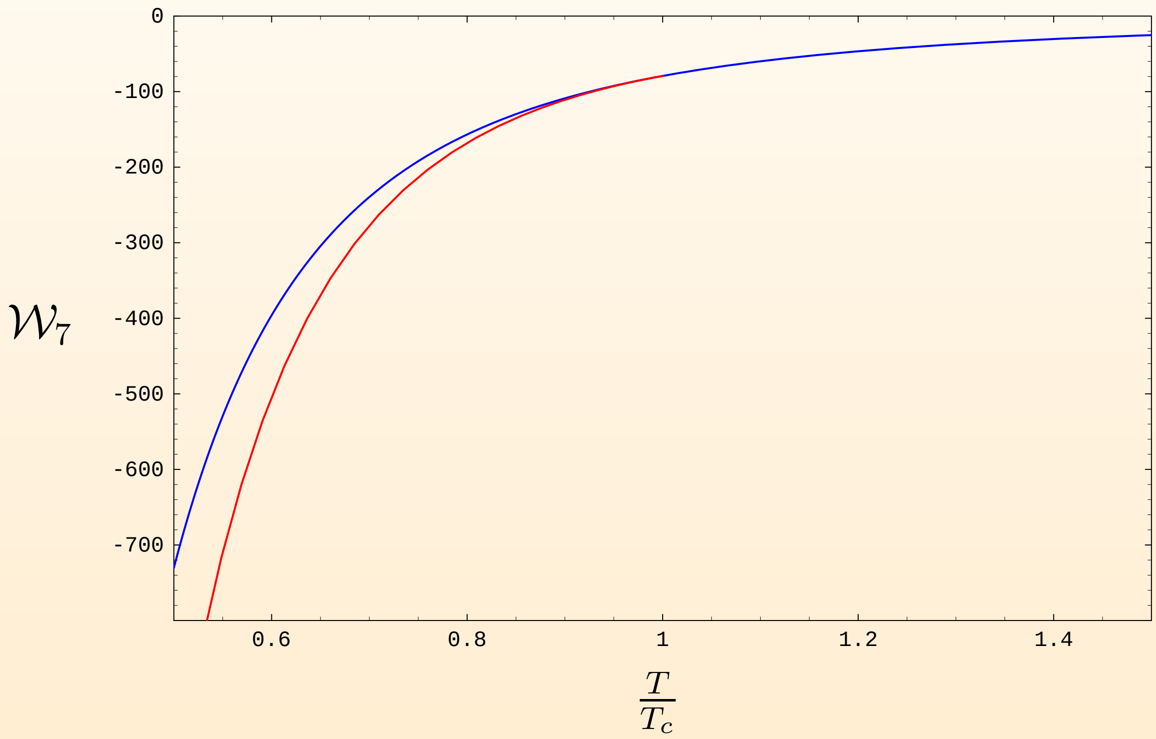
Order parameter: p wave condensate



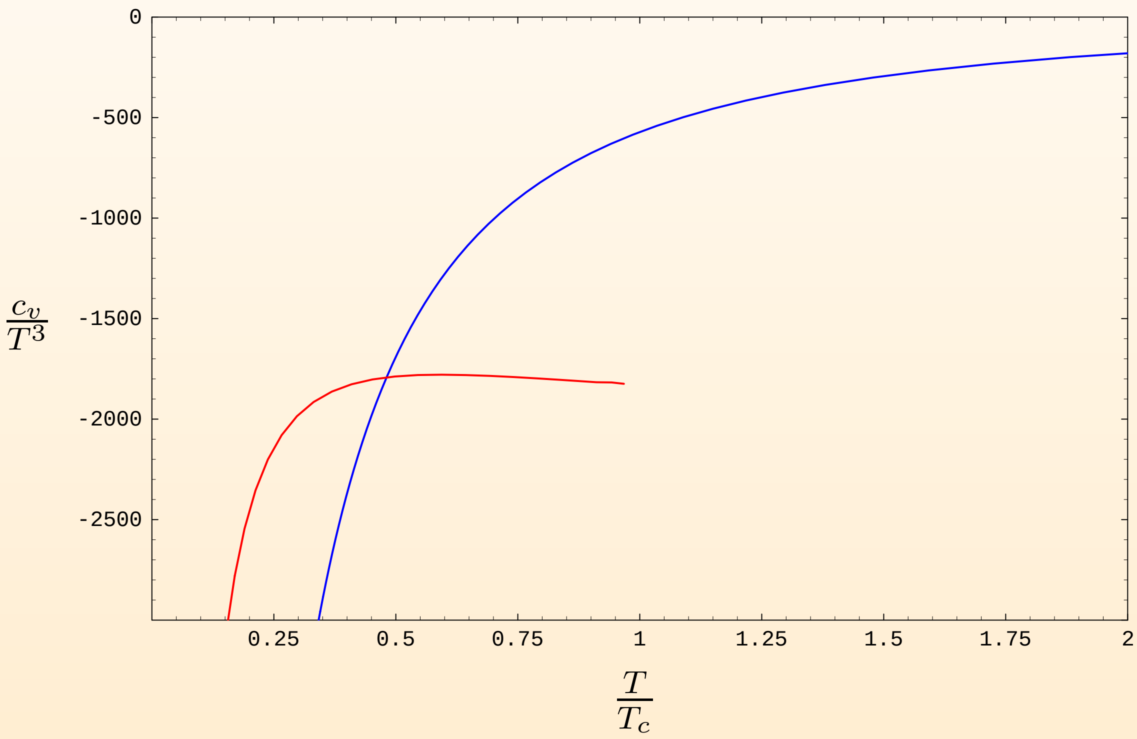
Red: Vanishing quark mass; Black: Finite quark mass, $\mu/M_q = 3$

Blue: Fit displaying critical exponent 1/2

Flavor contribution to Grand potential vs. temperature



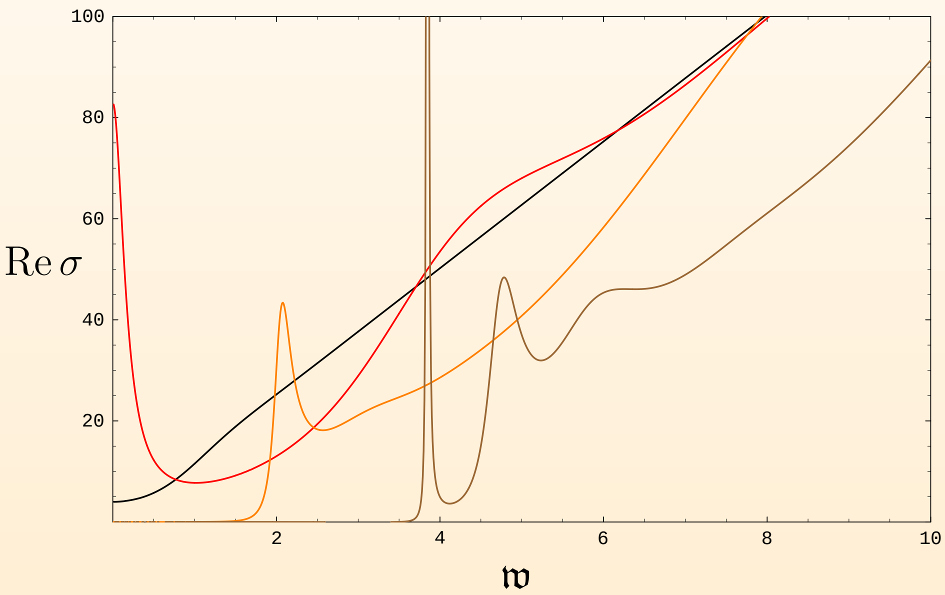
Flavor contribution to heat capacity



Conductivity

Frequency-dependent conductivity $\sigma(\omega) = \frac{i}{\omega} G^R(\omega)$

G^R retarded Green function for fluctuation a_2^3

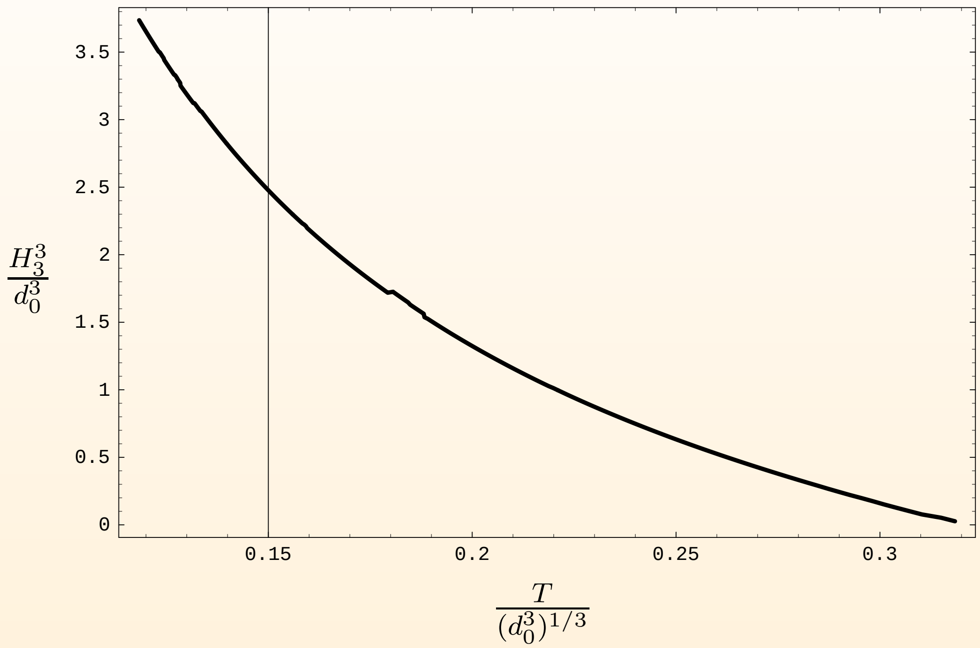


$\mathfrak{w} = \omega/(2\pi T)$

T/T_c: Black: ∞, Red: 1, Orange: 0.5, Brown: 0.28.

(Vanishing quark mass)

Meissner effect



Lower phase: magnetic field and condensate coexist

Upper phase: condensate vanishes

Non-abelian DBI action

Evaluation non-trivial in presence of both σ^0, σ^1

Two evaluation methods:

1) Expansion to fourth order

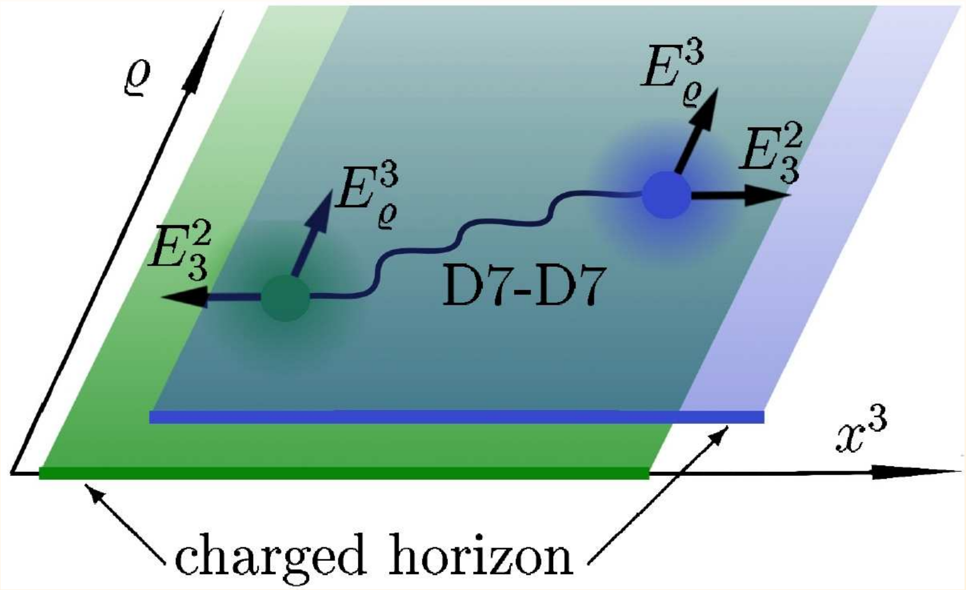
2) Simplification: Omitting commutators of Pauli matrices
Modified prescription for symmetrized trace

Allows for all-order calculation of the non-abelian DBI

Error of order $1/N_f$

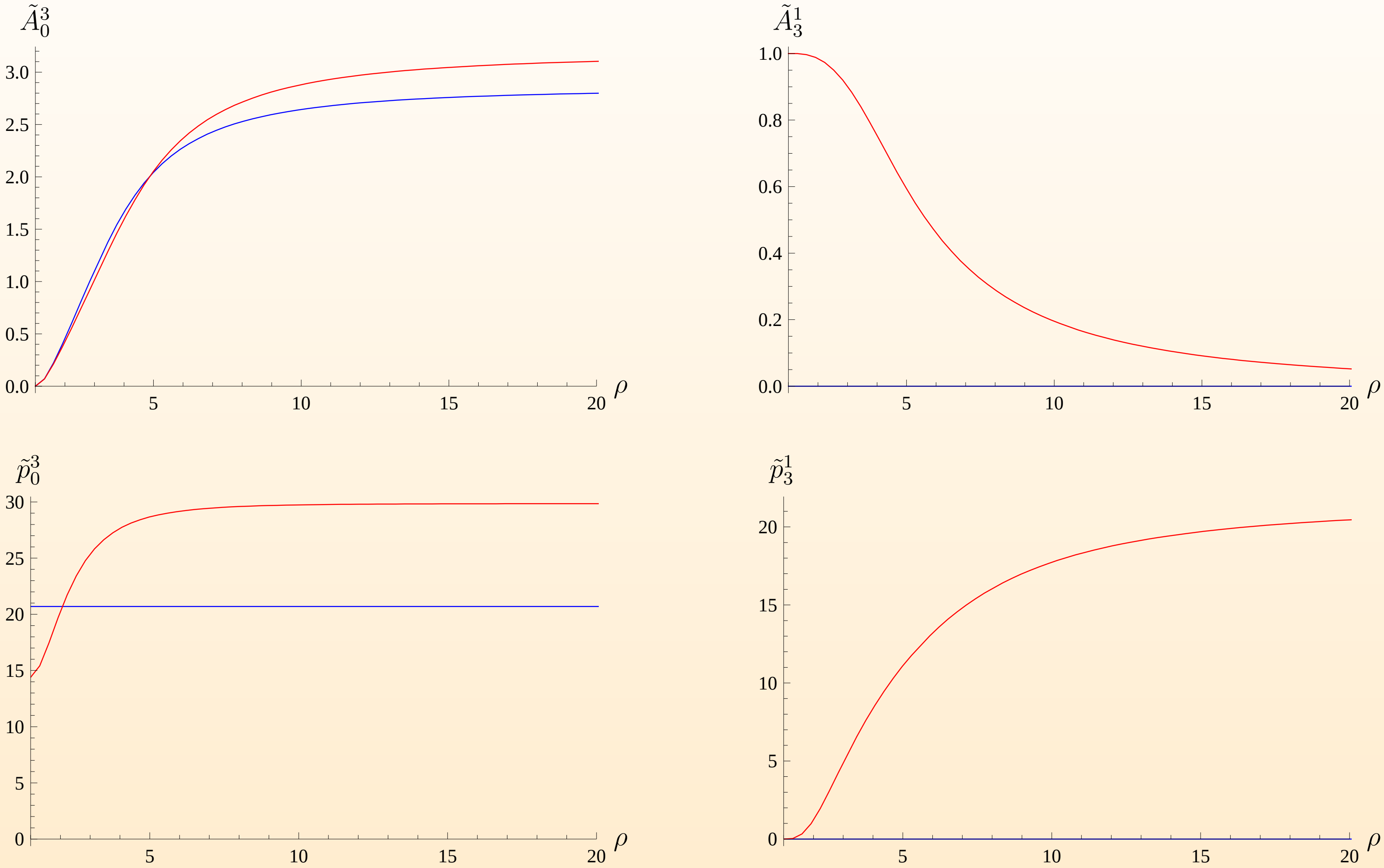
cf. Myers, Constable, Tafjord 1999

String picture



- Strings stretched between D7 branes and horizon induce a charge near the horizon
- System unstable above a critical charge density
- Horizon strings recombine to $D7-D7$ strings
- $D7-D7$ strings propagate into the bulk, balancing flavorelectric and gravitational forces
- $D7-D7$ strings distribute isospin charge into the bulk $\rightarrow$ condensate

Charge distributions



Conclusion and Outlook

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- Outlook: Fermions
- Outlook: Space-time dependent solutions