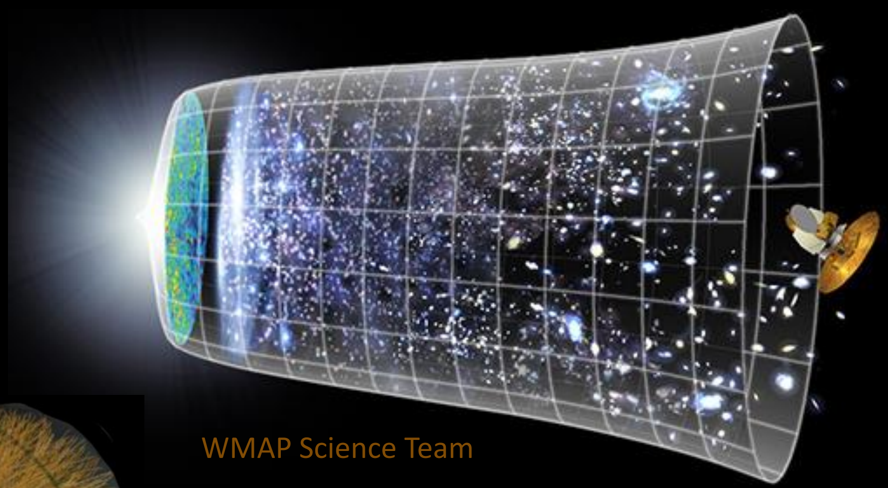
# Lecture 1

# Far from equilibrium quantum fields: From ultracold atoms to cosmology



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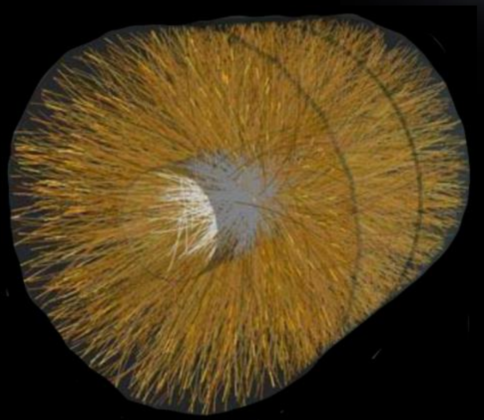
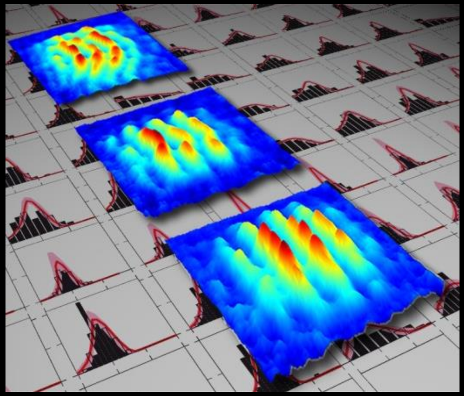


ALICE/CERN

J. Berges

Heidelberg University

ISOQUANT SFB1225



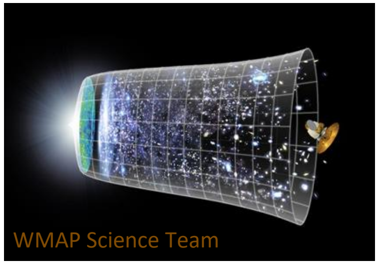
TU Vienna

Arnold Sommerfeld summer school on „Quantum many-body physics far from equilibrium", September 2016

# Isolated quantum systems in extreme conditions

**Early-universe inflaton dynamics**

*Preheating after inflation* (~$10^{16}$ GeV)




**Relativistic heavy-ion collision experiments**

*Quark-Gluon Plasma* (~100 MeV ~$10^{12}$ K)

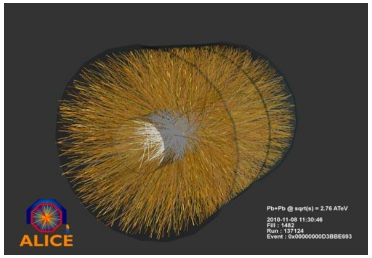
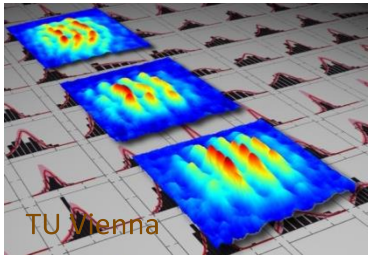


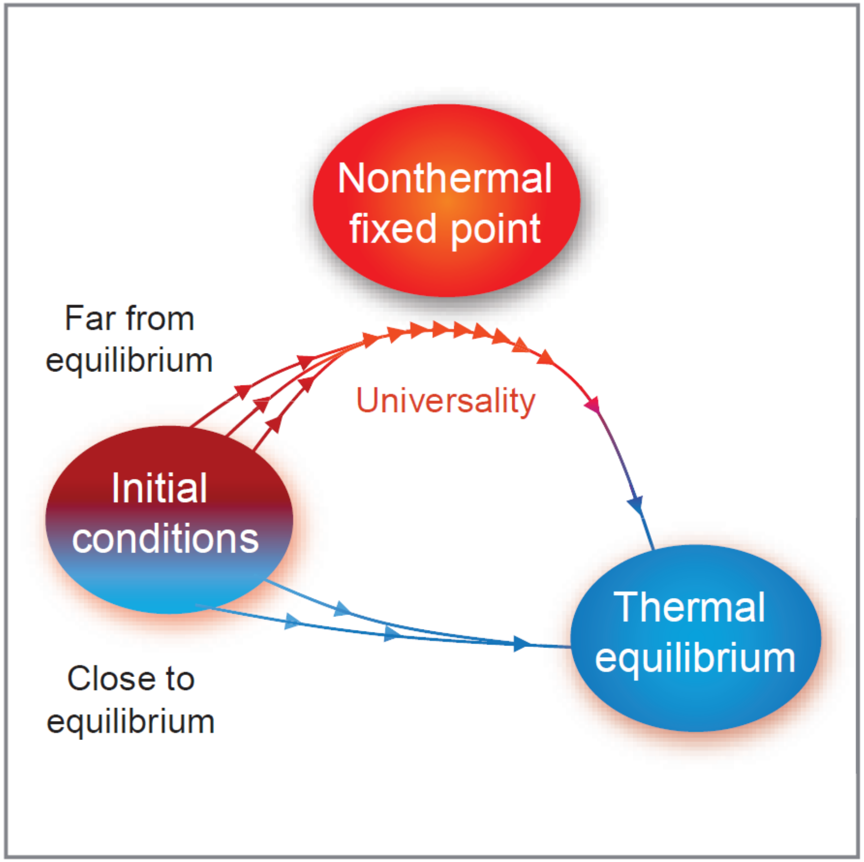

**Table-top experiments with ultracold atoms**

*Strong quenches at nanokelvins*



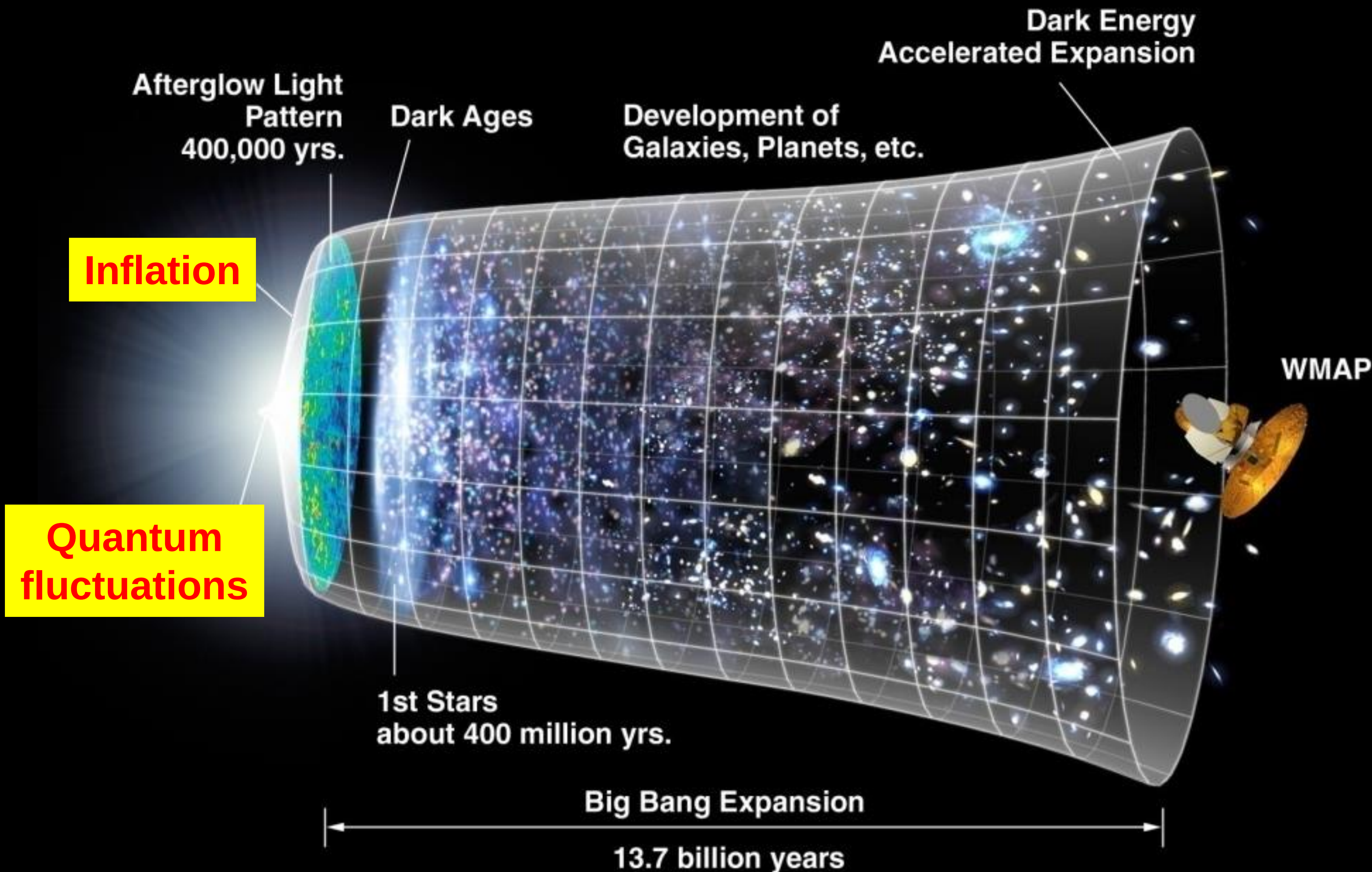
# Universality far from equilibrium

## Schematic thermalization process for isolated quantum systems:




→ **New universal regimes, where even quantitative agreements between seemingly disparate physical systems can be observed!**

# Evolution of the Universe

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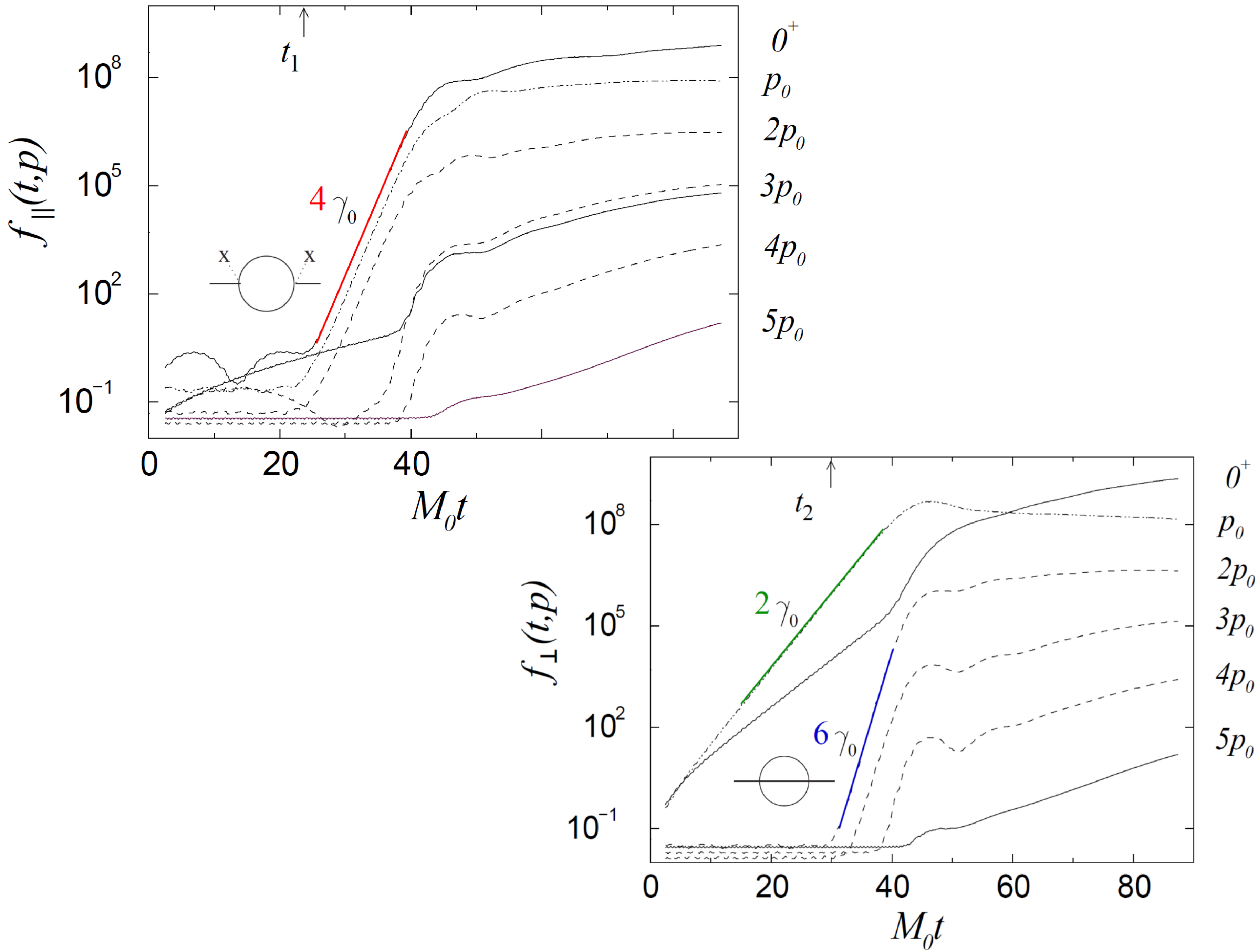
# Lecture 3



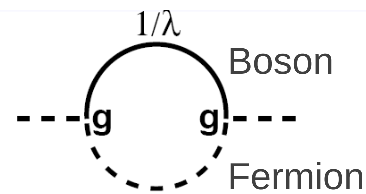
$t_1$
$0^+$
$p_0$
$2p_0$
$3p_0$
$4p_0$
$5p_0$
$4\gamma_0$
$f_{||}(t,p)$
$10^8$
$10^5$
$10^2$
$10^{-1}$
0
20
40
$M_0t$
$t_2$
$2\gamma_0$
$6\gamma_0$
$f_\perp(t,p)$
60
80

# Fermion production amplification from highly occupied bosons

Semi-classical: $$\left[i\gamma^\mu\partial_\mu - m_\psi - \frac{g}{N_f}\phi(t)\right] F_{\psi,ij}(x,y) = 0$$

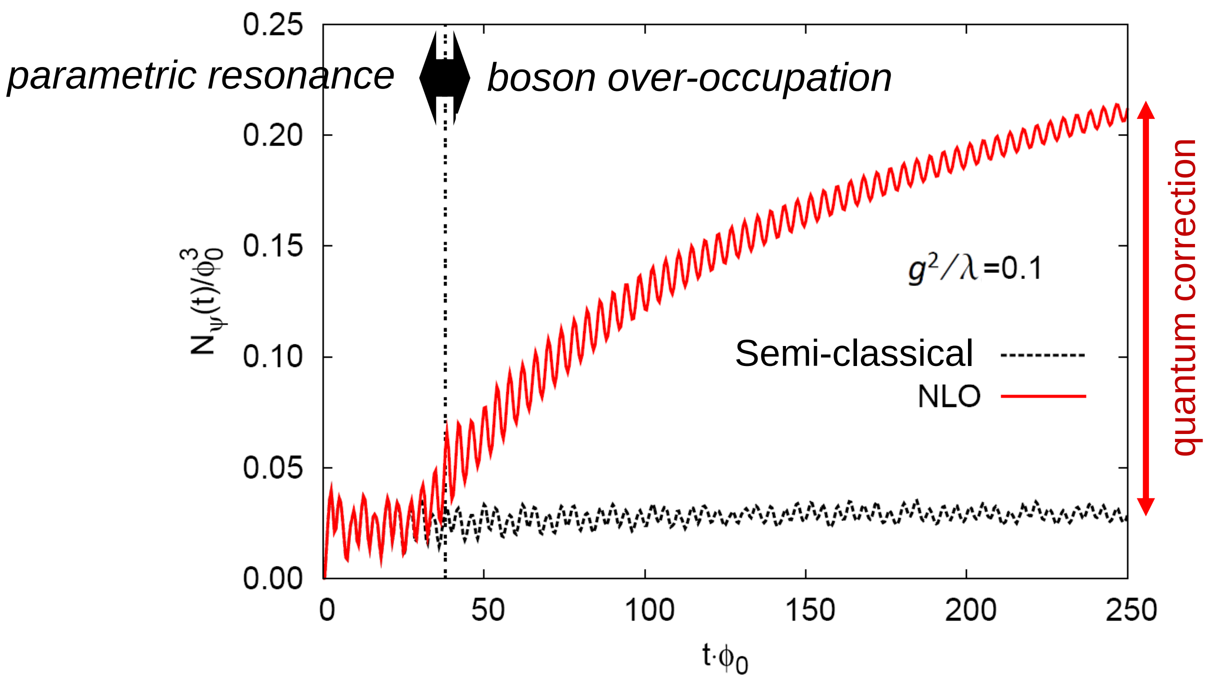
*Baacke, Heitmann, Pätzold, PRD 58 (1998) 125013; Greene, Kofman, PLB 448 (1999) 6; Giudice, Peloso, Riotto, Tkachev, JHEP 9908 (1999) 014; Garcia-Bellido, Mollerach, Roulet, JHEP 0002 (2000) 034; …*

NLO: +




$\sim \frac{g^2}{\lambda}$

*Berges, Gelfand, Pruschke, PRL 107 (2011) 061301*




- strongly enhanced fermion production due to high boson cccupancies!
- backreaction on bosons controlled by small $\sim g^2$

$\phi = \phi_0\,\sqrt{6N_s/\lambda}$ , $m_\psi = 0$

# Impact of bosons on nonequilibrium fermion spectral function

$$\rho(x,y) = i\,\langle\{\psi(x),\bar{\psi}(y)\}\rangle$$

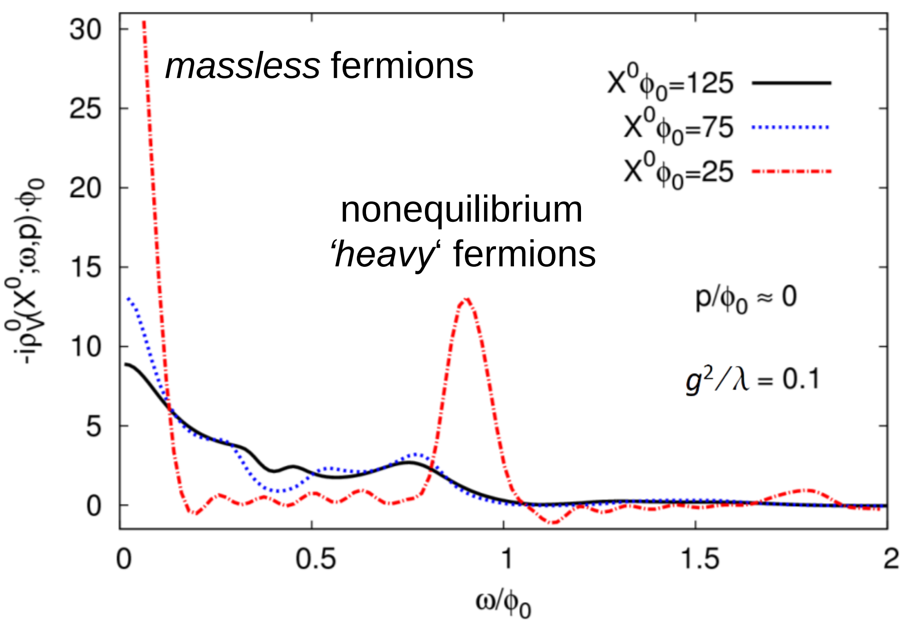
$$\rho_V^\mu = \frac{1}{4}\mathrm{tr}\,(\gamma^\mu\rho)$$ vector components

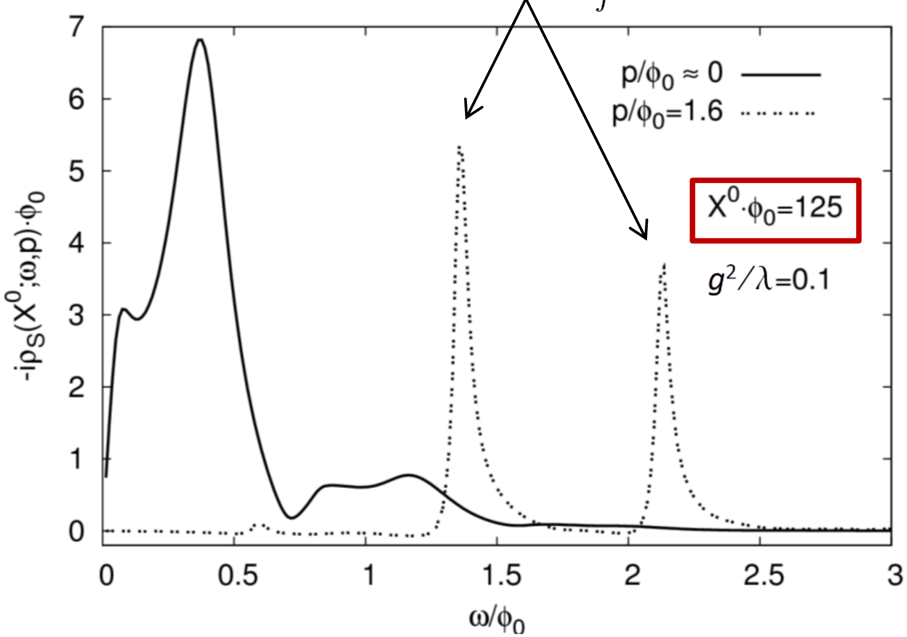
$$\rho_S = \frac{1}{4}\mathrm{tr}\,(\rho)$$ scalar component

quantum field anti-commutation relation: $-i\rho_V^0(t,t;\mathbf{p}) = 1$

**Wigner transform:** ( $X^0 = (t + t')/2$ )

$$M_\psi^{\mathrm{eff}}(t) \simeq \pm\frac{g}{N_f}|\phi(t)|$$



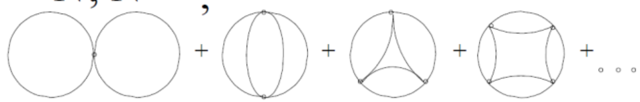



# Summary: Preheating

time ↑

**nonperturbative regime: nonthermal fixed point**

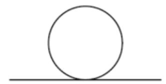
(IV) $\sim N, N^0$ ; $\sim N^0$



$\sim O(\lambda^{-1})$

$t_3 \sim \ln(\lambda^{-1}) / 2\gamma_0$

$F_\perp \sim O(N^0 \lambda^{-1})$



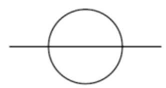
$\sim O(N^0 \lambda^0)$

**nonlinear regime: secondary amplification**

(III)

$t_2 \sim 2\, t_3 / 3 + \ln(N) / 6\gamma_0$

$F_\perp \sim O(N^{1/3} \lambda^{-2/3})$ for $N \lesssim \lambda^{-1}$



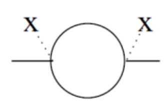
$\sim O(N^0 \lambda^0)$

rate: $6\gamma_0$ for $F_\perp (p \neq p_0)$

(II)

$t_1 \sim t_3/2$

$F_\perp \sim O(N^0 \lambda^{-1/2})$




$\sim O(N^0 \lambda^0)$

rate: $4\gamma_0$ for $F_\parallel (p \lesssim 2p_0)$

**linear regime: parametric resonance**

(I)

$F_\perp(t,t;p_0) \sim \exp(2\gamma_0 t)$

rate: $2\gamma_0$

$t = 0$ , $F_\perp \sim O(N^0 \lambda^0)$

**slow**

*Nonperturbative:* saturated occupation numbers $\sim 1/\lambda$

→ *all processes $O(1)$*

→ *universal*

**fast**

*Nonlinear – perturbative:* occupation numbers < $1/\lambda$

secondary growth rates $c\,(2\gamma_0)$ with $c = 2,3,\ldots$

*Classical/linear:* primary growth rate

# Insensitivity to initial condition details

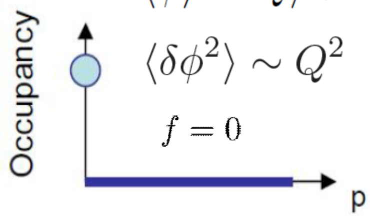
**Example:** `*Inflaton*´ $\lambda\phi^4$ theory ($\lambda \ll 1$), $\phi = \phi_0 + \delta\phi$

1. Large initial field:

$\langle\phi\rangle \sim Q/\sqrt{\lambda}$

$\langle\delta\phi^2\rangle \sim Q^2$

$f = 0$

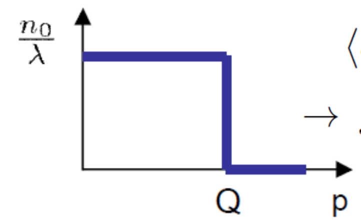



$f(Q) \sim e^{\gamma_Q \Delta t}$

$\Delta t \sim Q^{-1} \log \lambda^{-1}$

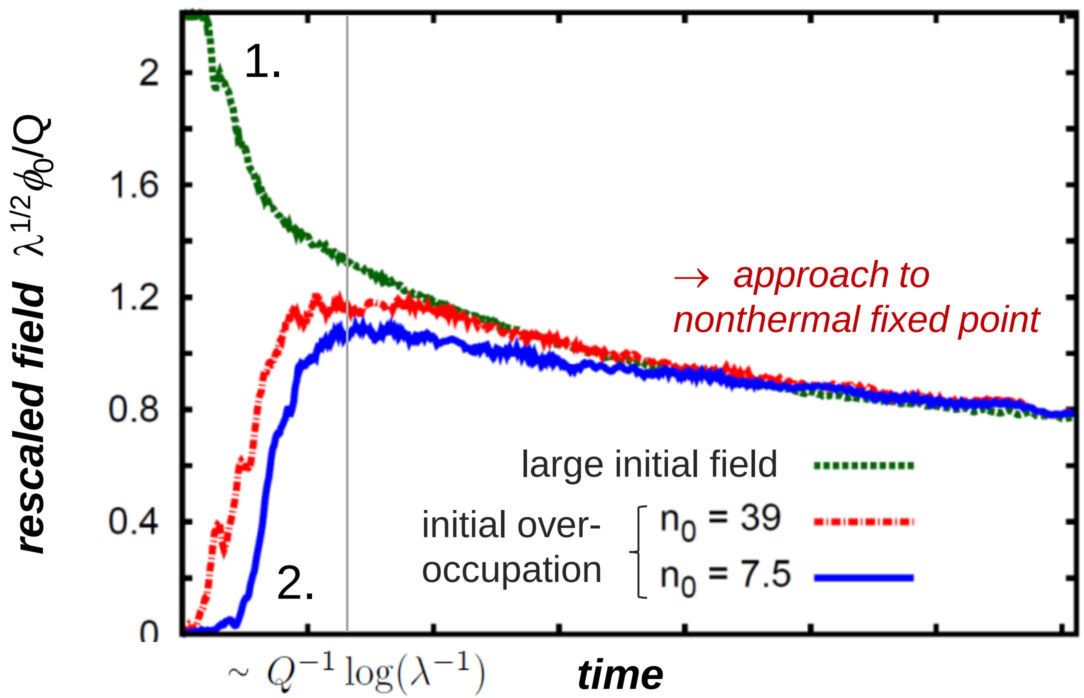
*instability*

2. High occupancy:




$\langle\delta\phi^2\rangle \sim Q^2/\lambda$

$\rightarrow f(Q) \sim 1/\lambda \gg 1$



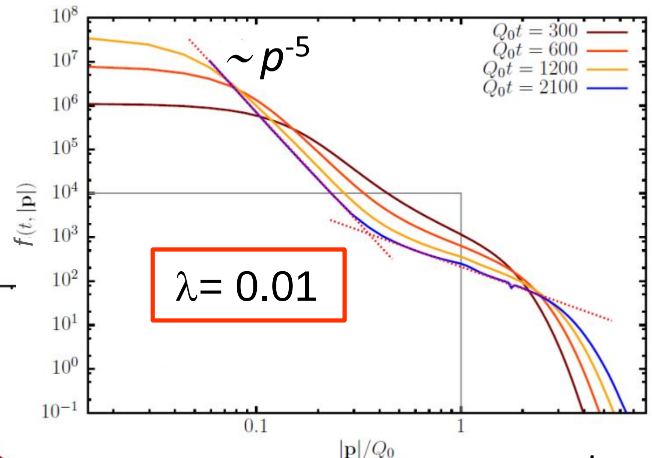

*Berges, Boguslavski, Schlichting, Venugopalan, JHEP 1405 (2014) 054*

# Insensitivity to coupling strength

**E.g. scalar *N*-component $\lambda\phi^4$ *quantum* theory** (1/*N* to NLO)**:**

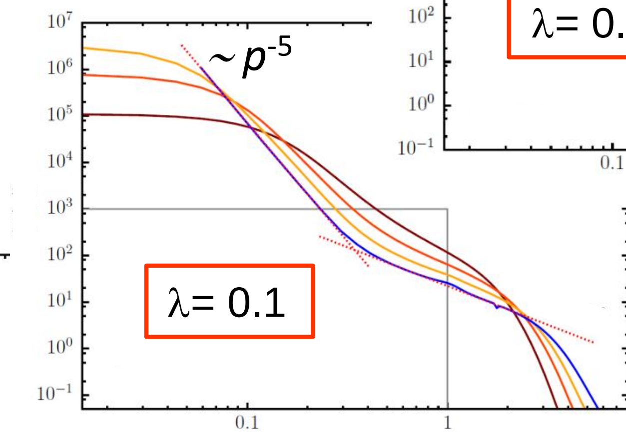
Occupation number distribution:

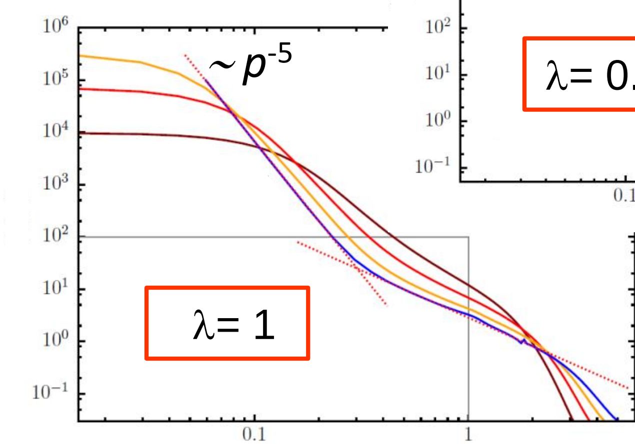
*Berges, Wallisch, arXiv:1607.02160*




momentum

*N* = 4



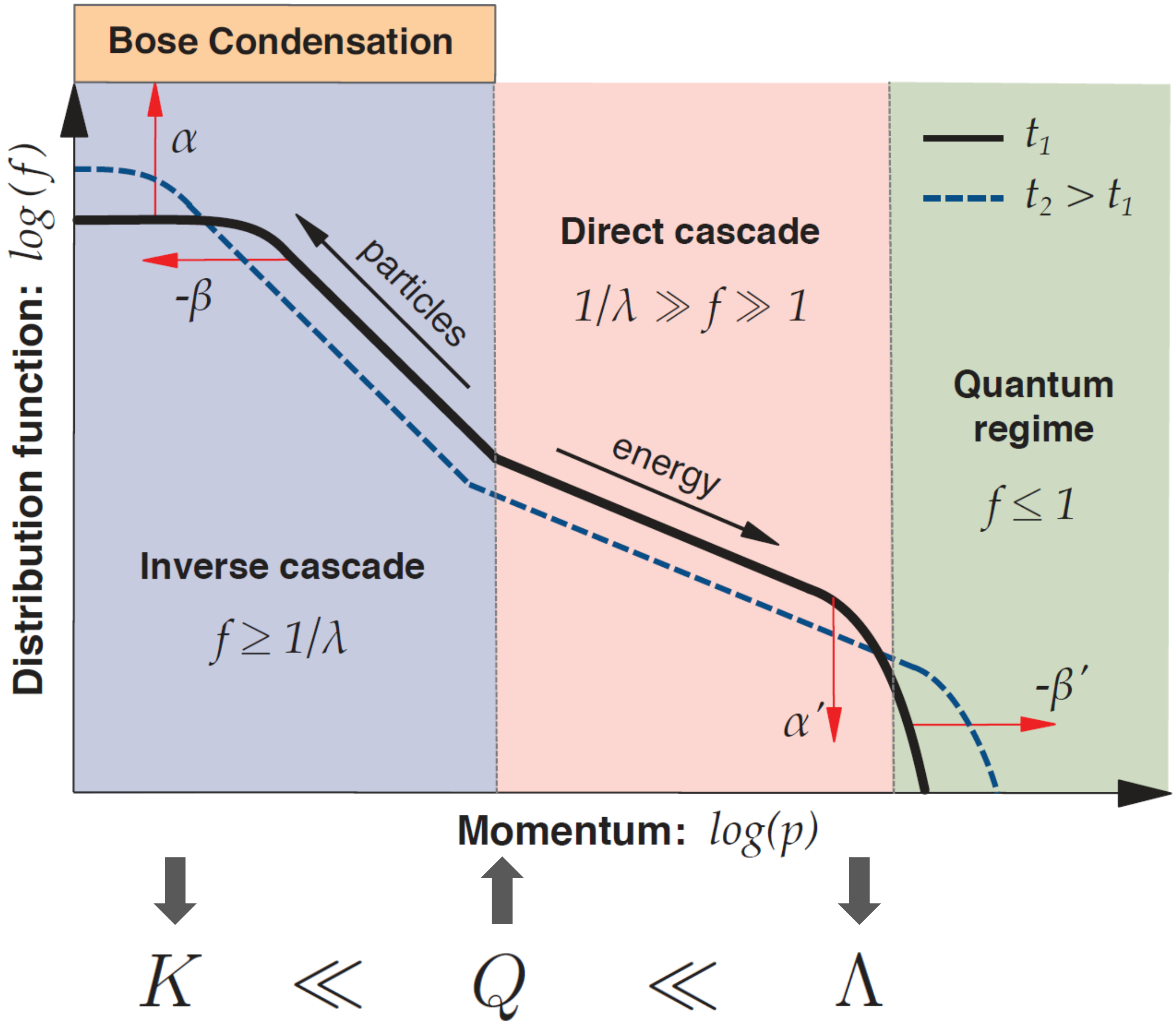




(similar results for strong-field initial conditions)

***Universal scaling behavior for wide range of couplings!***

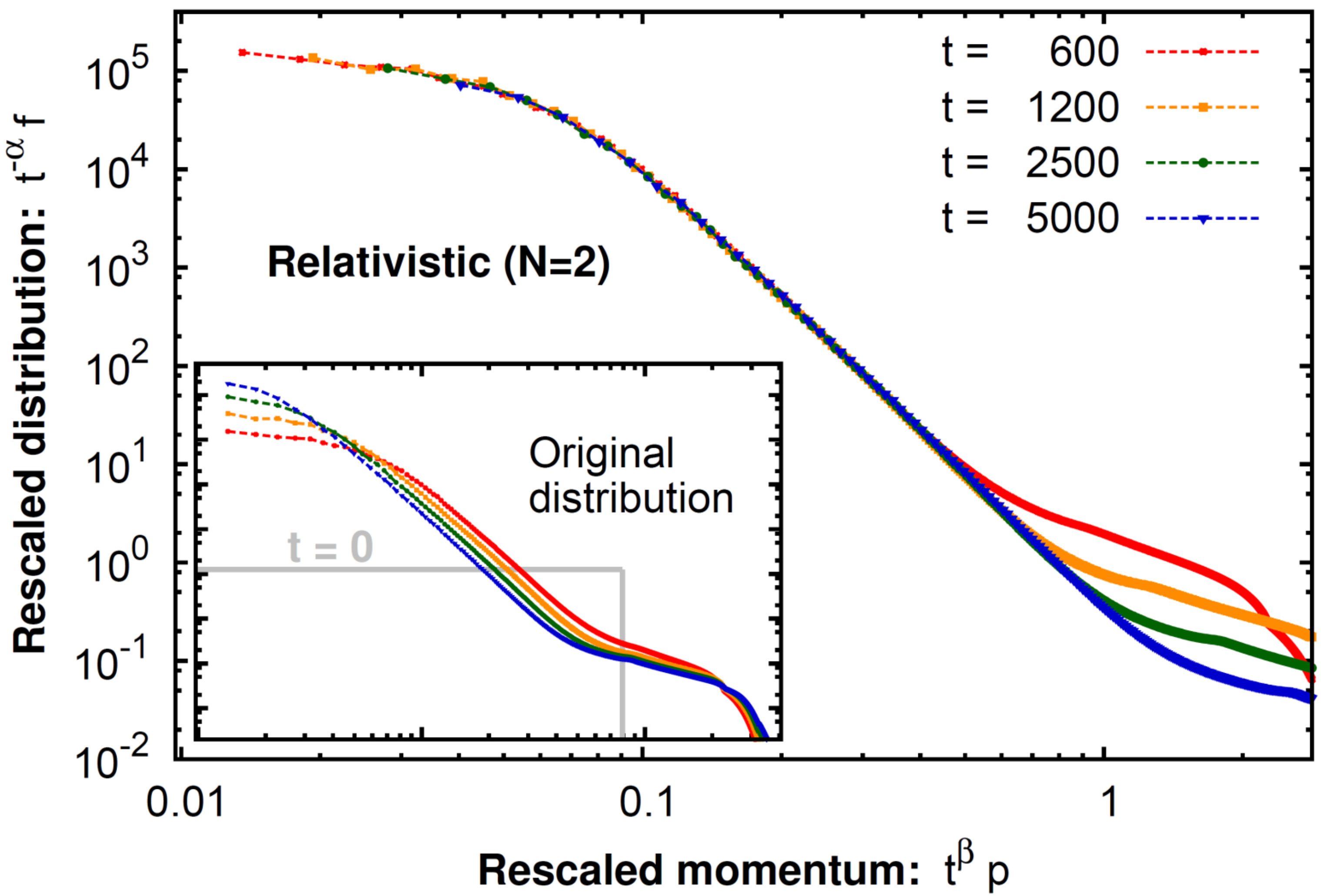
# Lecture 4

# Schematic behavior near nonthermal fixed point: dual cascade



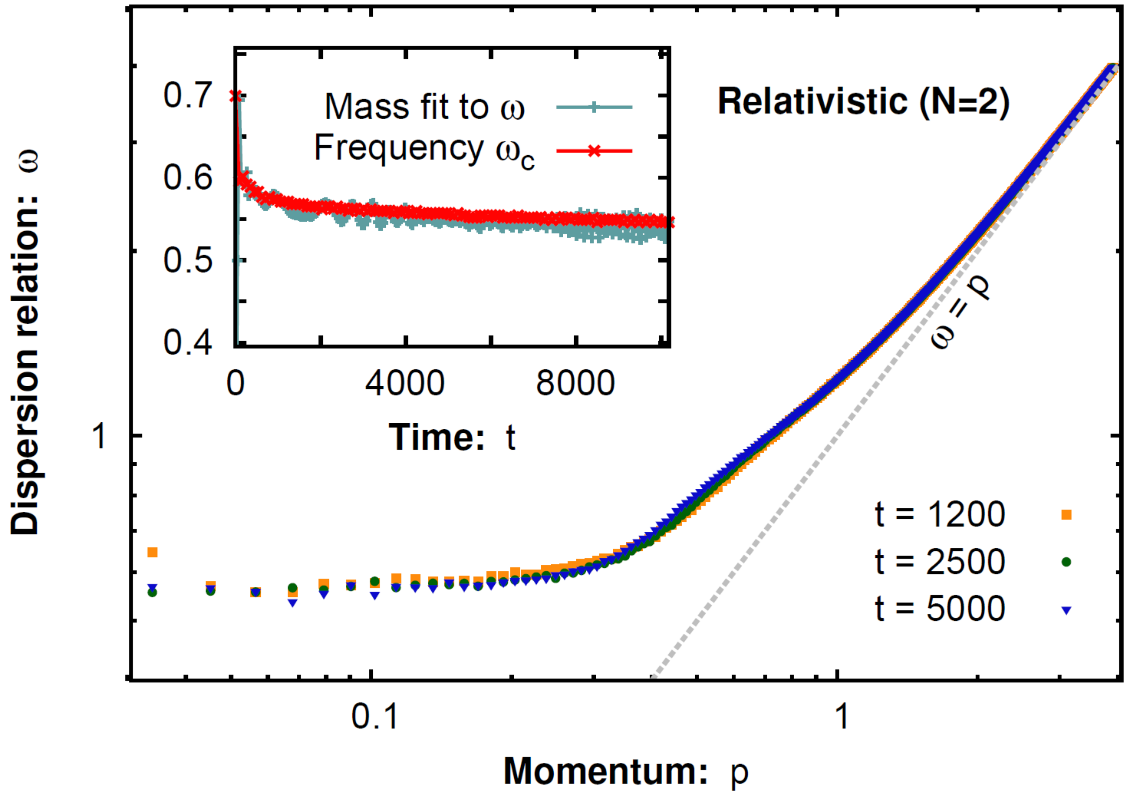
# Self-similar dynamics: infrared scaling

$$f(t,\mathbf{p}) = t^{\alpha}\, f_S(t^{\beta}\mathbf{p})\ , \quad \alpha = 1.51 \pm 0.13\,, \qquad \beta = 0.51 \pm 0.04$$




*Piñeiro Orioli, Boguslavski, Berges, PRD 92 (2015) 025041*

# Mass scale separating non-relativistic infrared regime

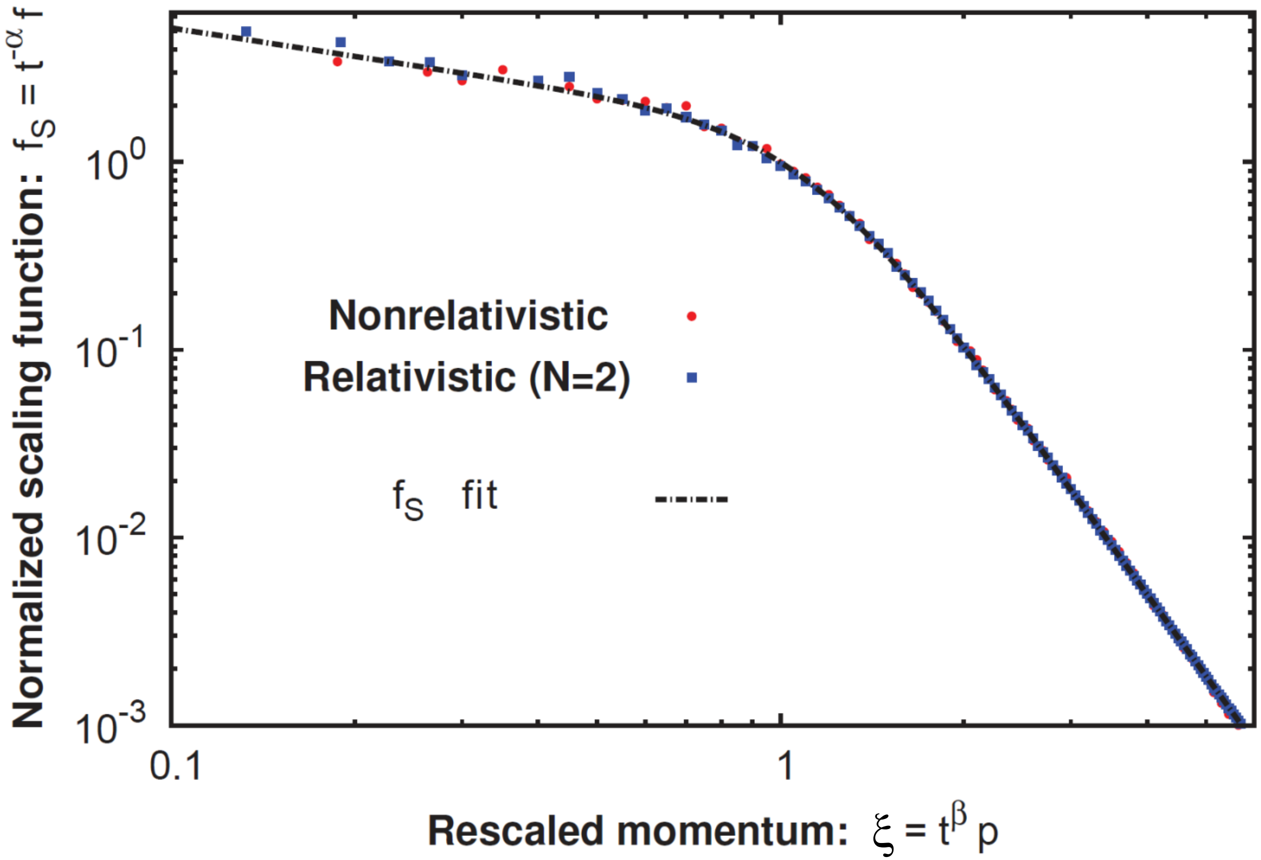



- ***non-relativistic infrared dynamics*** expected because of the generation of a ***mass gap*** (condensate + medium)

→ relativistic & non-relativstic field theories have same infrared scaling

*Piñeiro Orioli, Boguslavski, Berges, PRD 92 (2015) 025041*

# Universal scaling form of the distribution function




Piñeiro Orioli, Boguslavski, Berges, PRD 92 (2015) 025041

$$f_S(\xi) \simeq \frac{A(\kappa_> - \kappa_<)}{(\kappa_> - 2)(\xi/B)^{\kappa_<} + (2 - \kappa_<)(\xi/B)^{\kappa_>}} \quad , \ \kappa_< \simeq 0.5 \ , \ \kappa_> \simeq 4.5$$

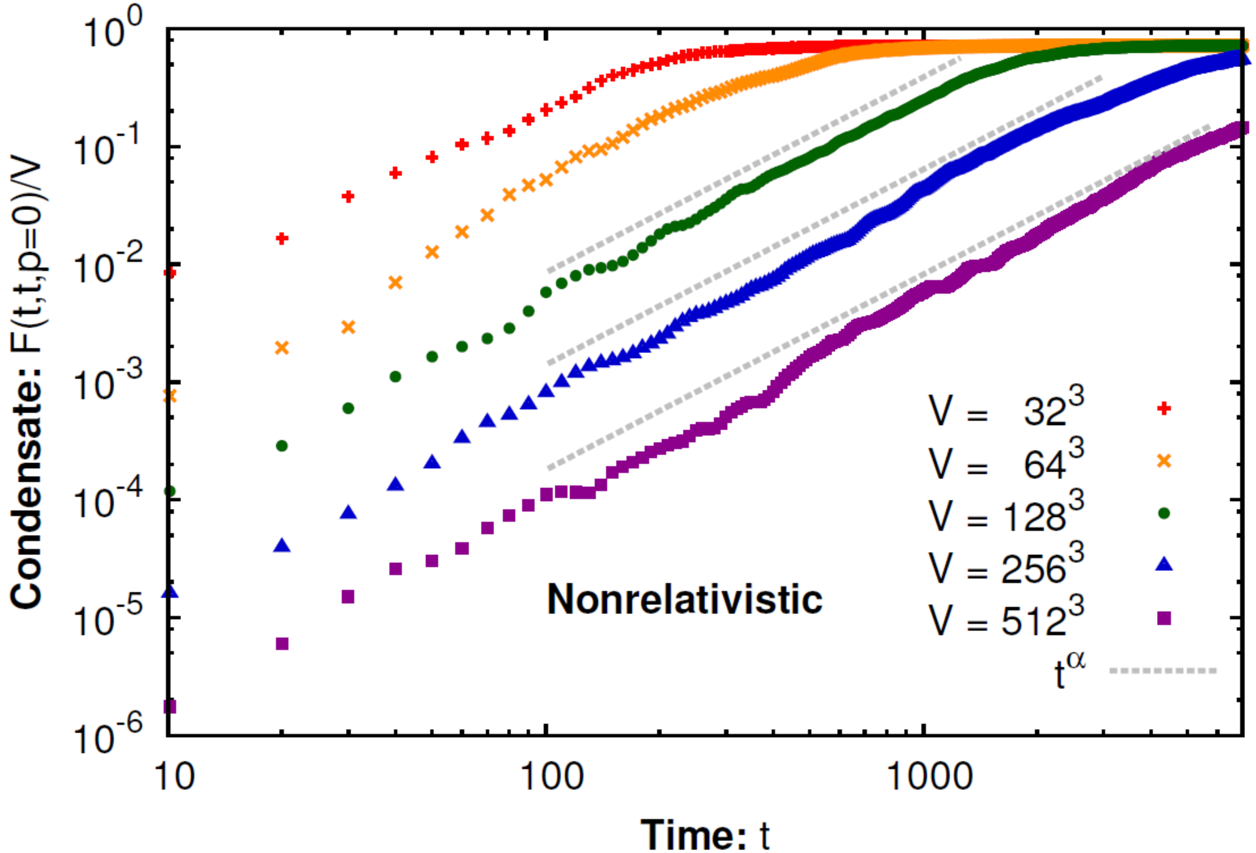
$$f_S(\xi = B) = A \quad , \quad df_S(\xi = B)/d\xi = -2A/B$$

# Condensation far from equilibrium

$$F(t,t',\mathbf{x}-\mathbf{x}') = \frac{1}{2}\langle \psi(t,\mathbf{x})\psi^*(t',\mathbf{x}') + \psi(t',\mathbf{x}')\psi^*(t,\mathbf{x})\rangle$$

$$f(t,\mathbf{p}) + (2\pi)^3\delta^{(3)}(\mathbf{p})|\psi_0|^2(t) \equiv \int d^3x\, e^{-i\mathbf{p}\mathbf{x}}\, F(t,t,\mathbf{x})$$

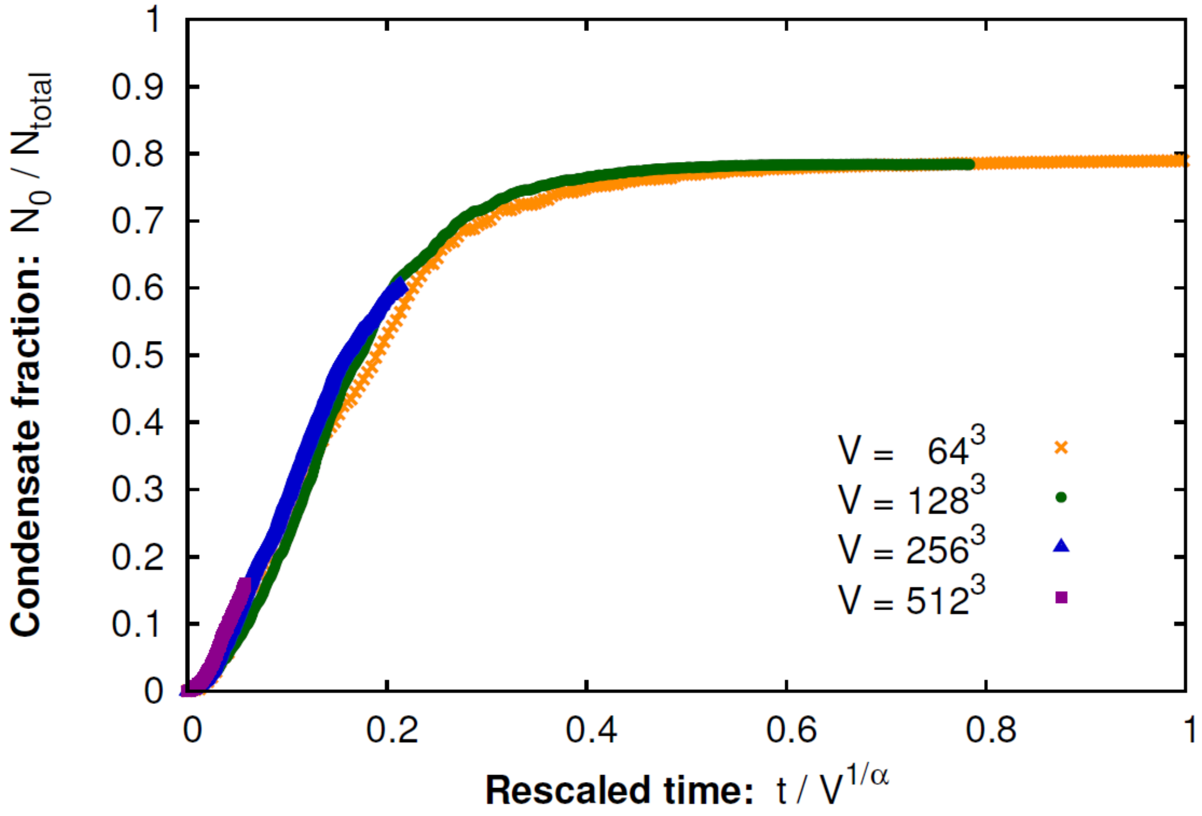
*volume:* $(2\pi)^3\delta(\mathbf{0}) \to V$



# Condensation time

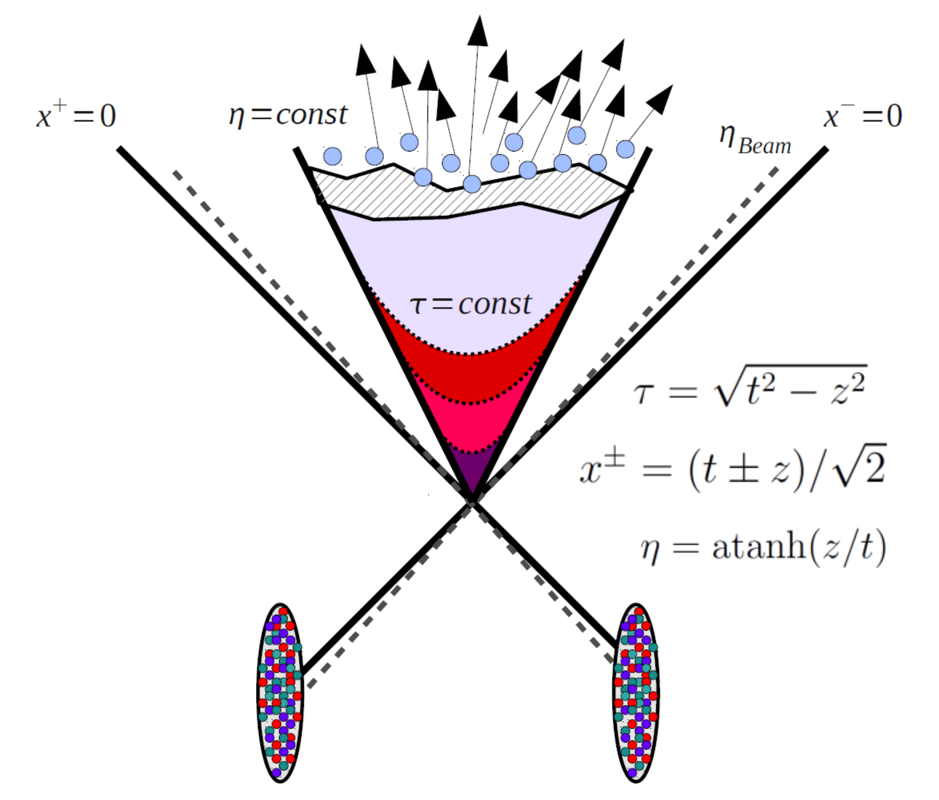
$$\frac{N_0(t)}{N_{\rm total}} = \frac{|\psi_0|^2(t)}{\int d^3p/(2\pi)^3 f(t,\mathbf{p}) + |\psi_0|^2(t)} \quad , \quad V^{-1}F(t,t,\mathbf{p}=0) \sim t^{\alpha}$$

$$\Rightarrow \qquad t_f \simeq t_0 \left( \frac{|\psi_0|^2(t_f)}{f(t_0,0)} \right)^{1/\alpha} V^{1/\alpha}$$




*Analytic estimates agree well with simulations!*

# Thermalization dynamics in relativistic heavy-ion collisions



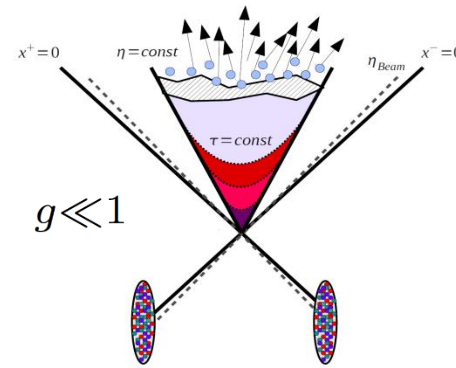
# Heavy-ion collisions in the high-energy limit

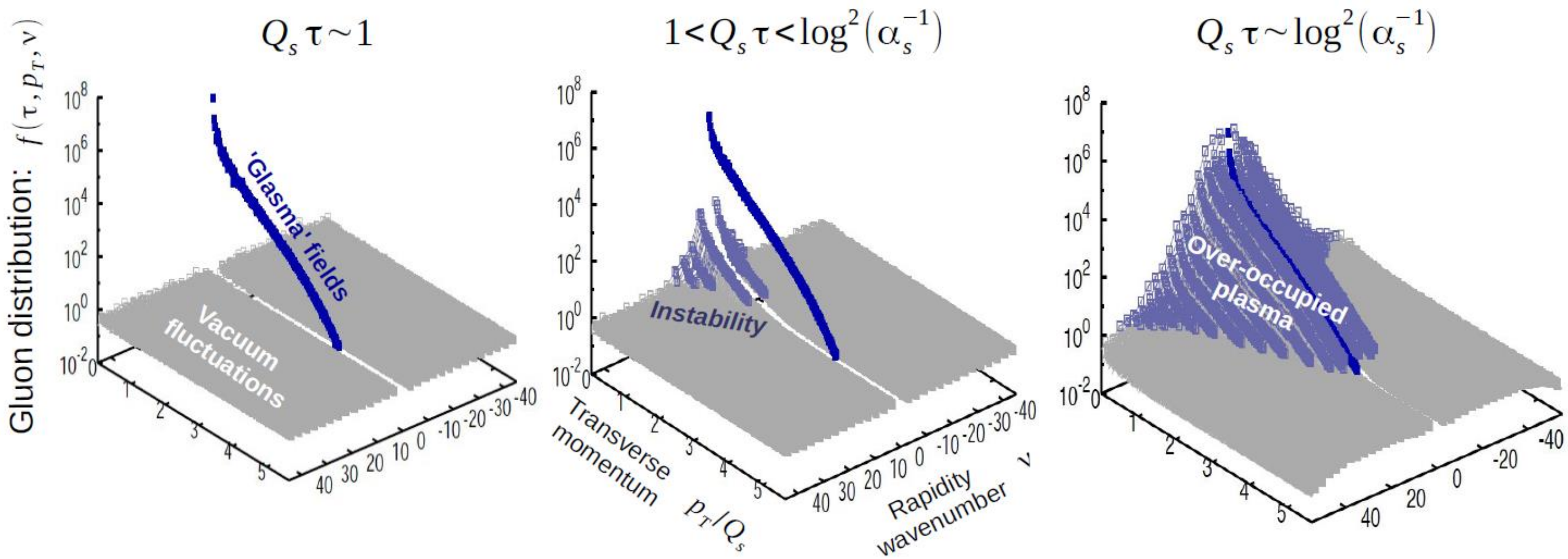
**Large** initial gauge fields: $\langle A \rangle \sim Q_s/g$

*CGC: Lappi, McLerran, Dusling, Gelis, Venugopalan, Epelbaum…*

**Small** initial (vacuum) fluctuations: $\langle \delta A^2 \rangle \sim Q_s^2$

$\rightarrow$ ***plasma instabilities***

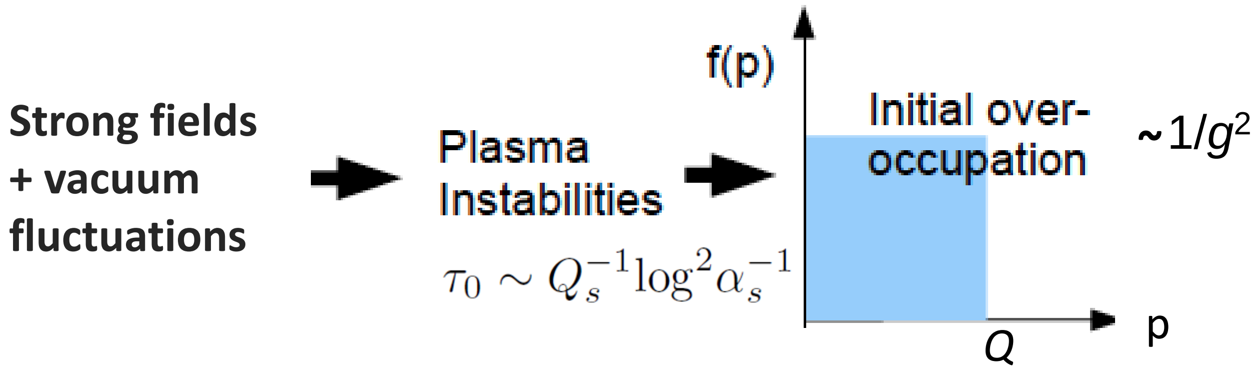



*JB, Schenke, Schlichting, Venugopalan,* *NPA931 (2014) 348* for initial spectrum from *Epelbaum, Gelis, PRD88 (2013) 085015*. ***Plasma instabilities from wide range of initial conditions:***

*Mrowczynski; Rebhan, Romatschke, Strickland; Arnold, Moore, Yaffe; Bödecker; Attems, … Romatschke, Venugopalan; J.B., Scheffler, Schlichting, Sexty; Fukushima, Gelis …*

# Overoccupied non-Abelian plasma




- To discuss attractor: Initial overoccupation described by family of distributions at $\tau_0$ (read-out in Coulomb gauge)

occupancy parameter

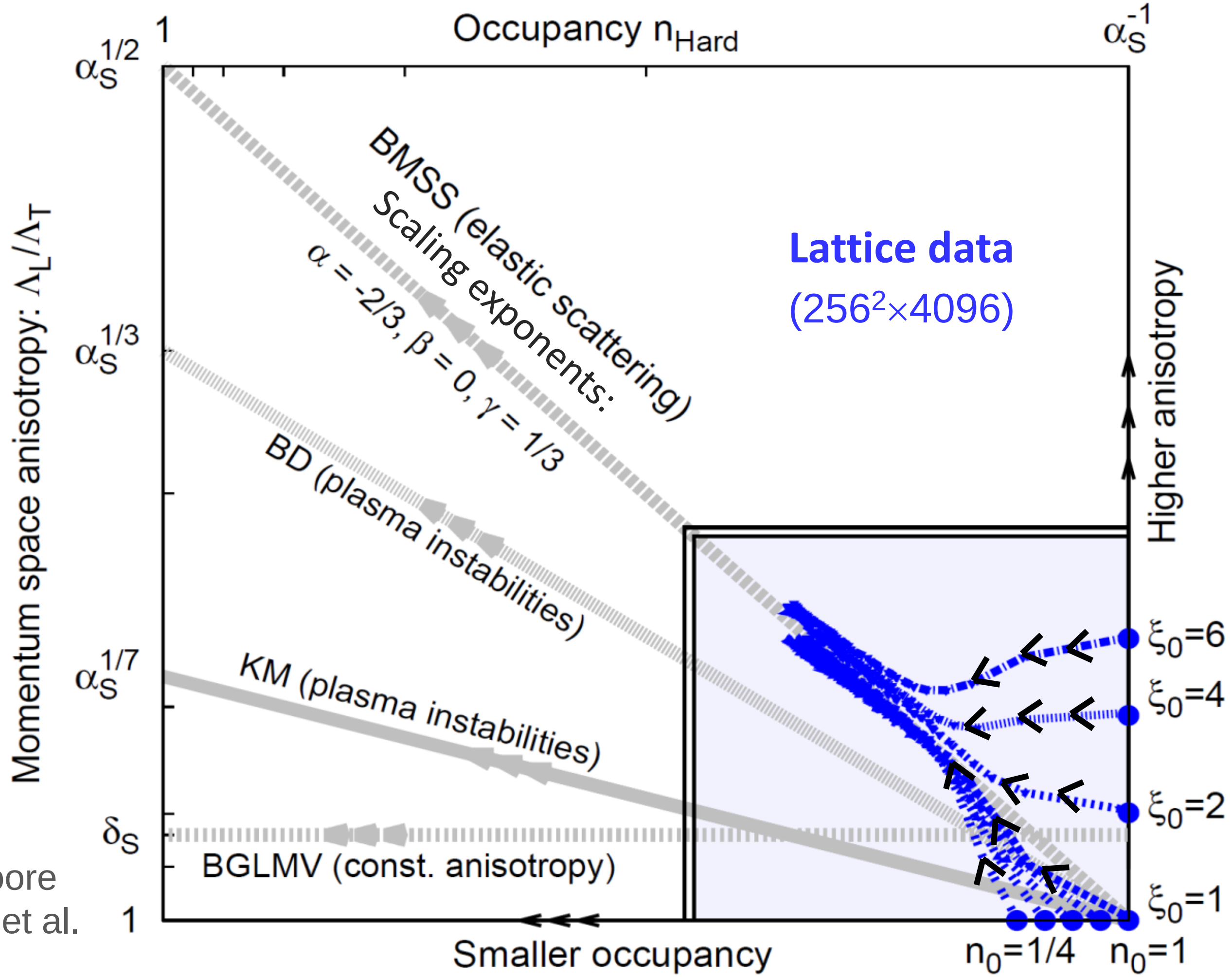
$$f(\mathrm{p_T}, \mathrm{p_z}, \tau_0) = \frac{n_0}{2g^2} \Theta\left(Q_s - \sqrt{\mathrm{p_T^2} + (\xi_0 \mathrm{p_z})^2}\right)$$

anisotropy parameter
(controls “prolateness” or “oblateness”
of initial momentum distribution)

*J.B., Boguslavski, Schlichting, Venugopalan, PRD89 (2014) 074011; 114007; PRL114 (2015) 061601*

# Nonthermal fixed point

## Evolution in the `anisotropy-occupancy plane´



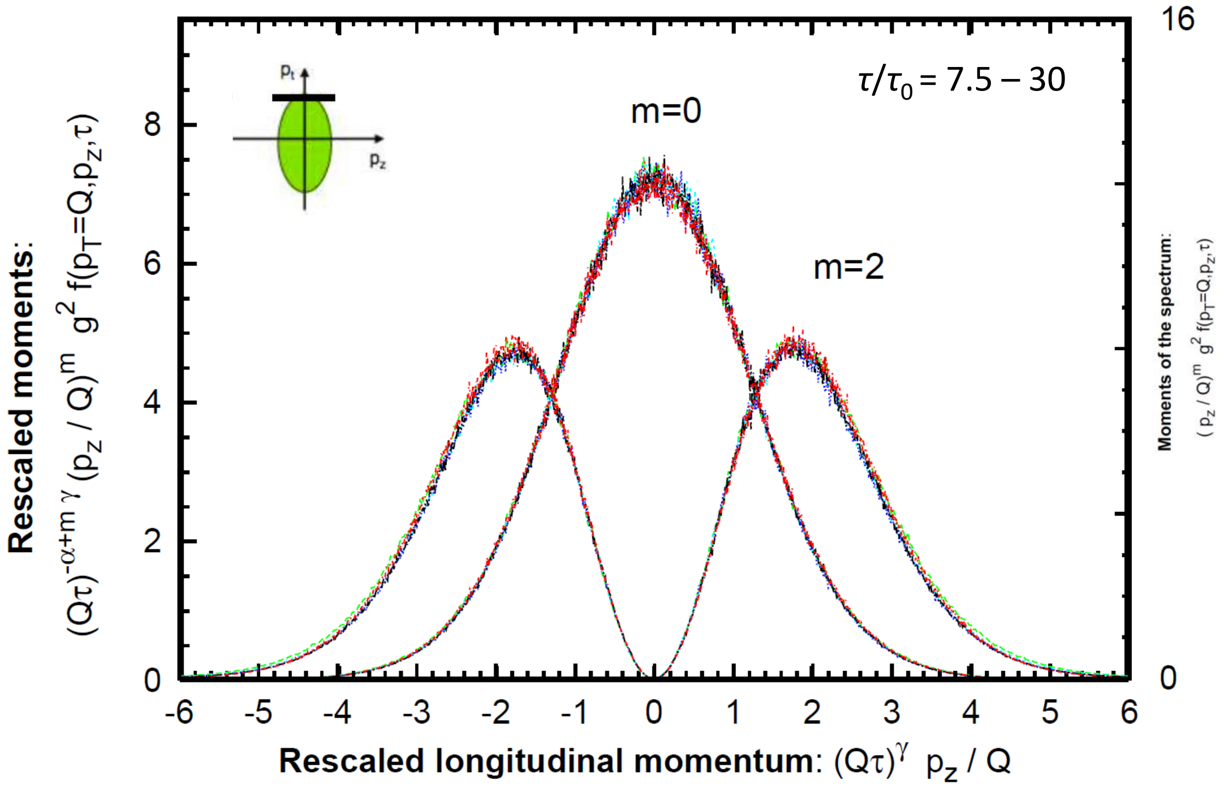

BD: Bodecker
KM: Kurkela, Moore
BGLMV: Blaizot et al.

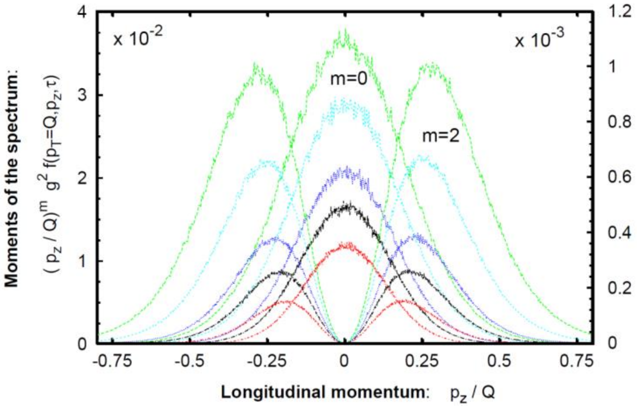
*J.B., Boguslavski, Schlichting, Venugopalan, PRD 89 (2014) 074011; ibid. 114007*

*Early stage of `bottom-up´* scaling emerges as a consequence of the attractor*

**Baier et al (BMSS), PLB 502 (2001) 51*

# Self-similar evolution




Scaling exponents: $\alpha = -2/3\,,\qquad \beta = 0\,,\qquad \gamma = 1/3$

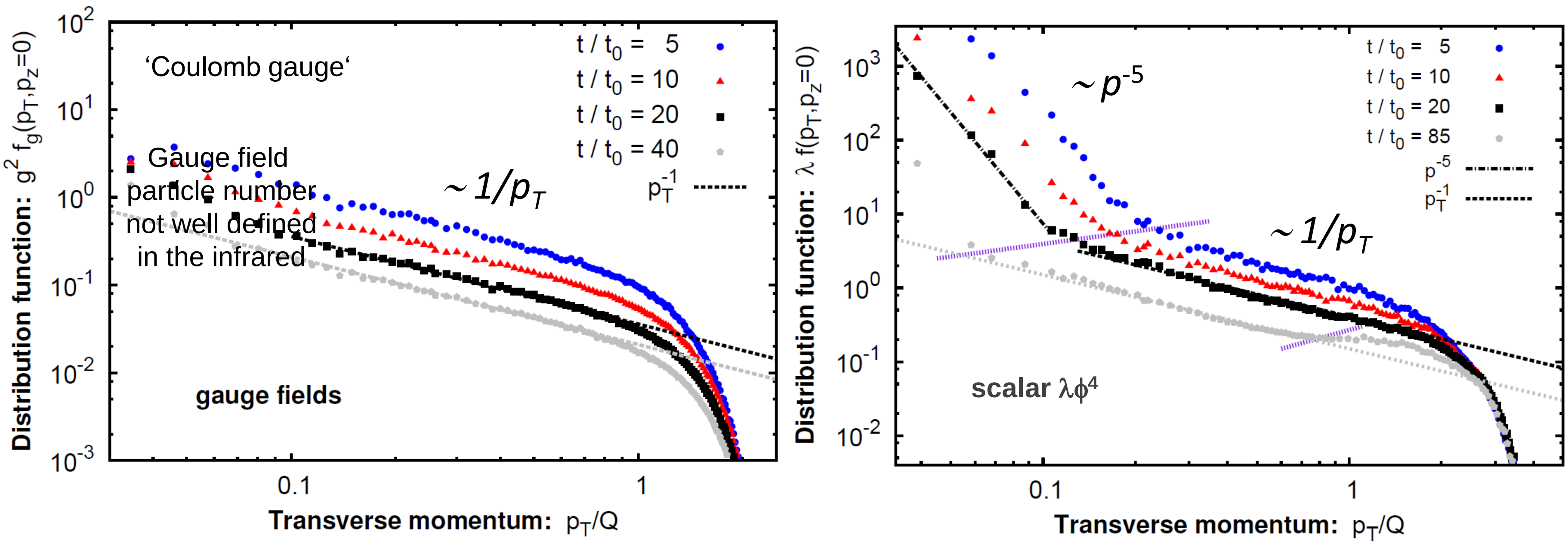
and scaling distribution function $f_S$:

$$f(\mathrm{p_T}, \mathrm{p_z}, \tau) = (Q\tau)^{\alpha} f_S\Big((Q\tau)^{\beta}\mathrm{p_T}, (Q\tau)^{\gamma}\mathrm{p_z}\Big)$$

*nonthermal fixed-point distribution*

# Comparing gauge and scalar field theories

with longitudinal expansion



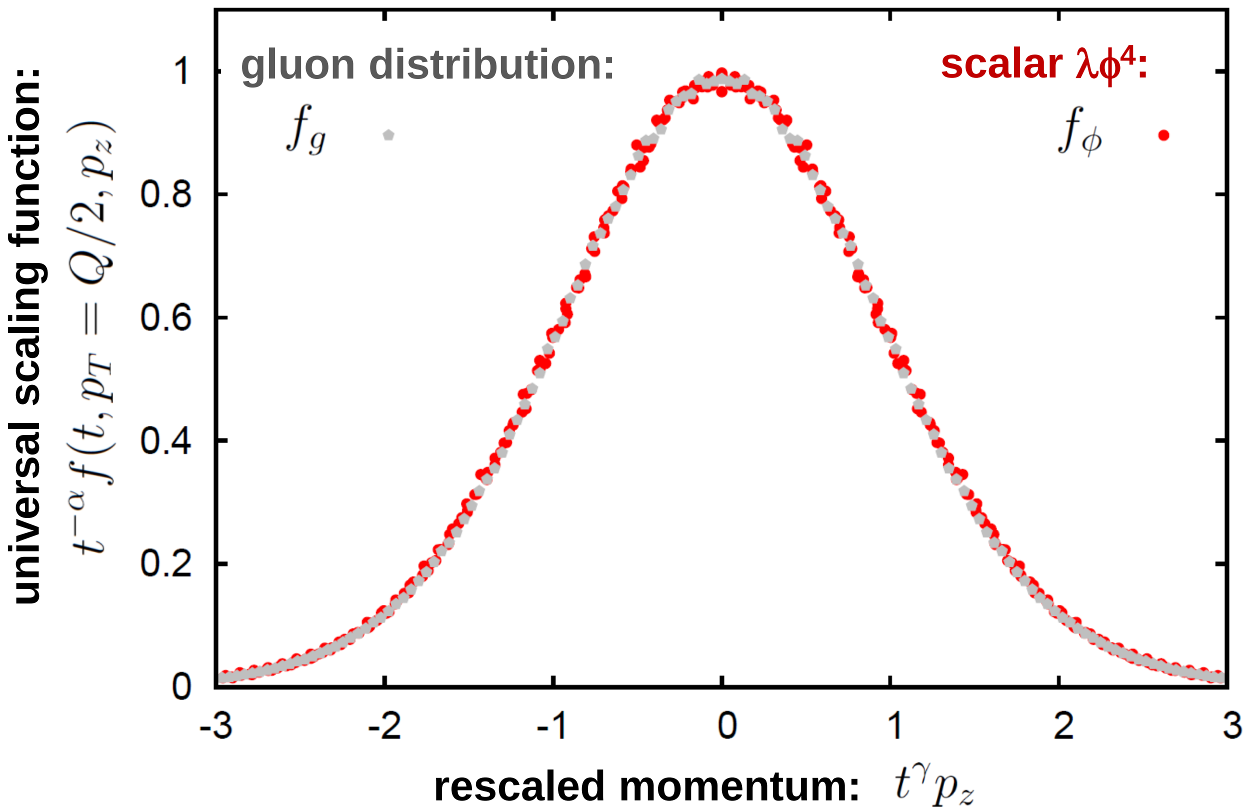

- For gauge & scalar fields: range of ***thermal-like transverse spectrum ~1/$p_T$*** even as longitudinal distribution is being `squeezed´
- Strongly enhanced infrared regime for scalars: ***inverse particle cascade leading to Bose condensation***, $\sim 1/p^5$ as in ***isotropic superfluid turbulence***
- At latest available times for scalars a flat distribution for $p_T \gtrsim Q$ emerges

*J.B., Boguslavski, Schlichting, Venugopalan, Phys. Rev. Lett. 114 (2015) 6, 061601*

# Universality far from equilibrium

- Same *universal exponents and scaling function* in common $1/p_T$ range

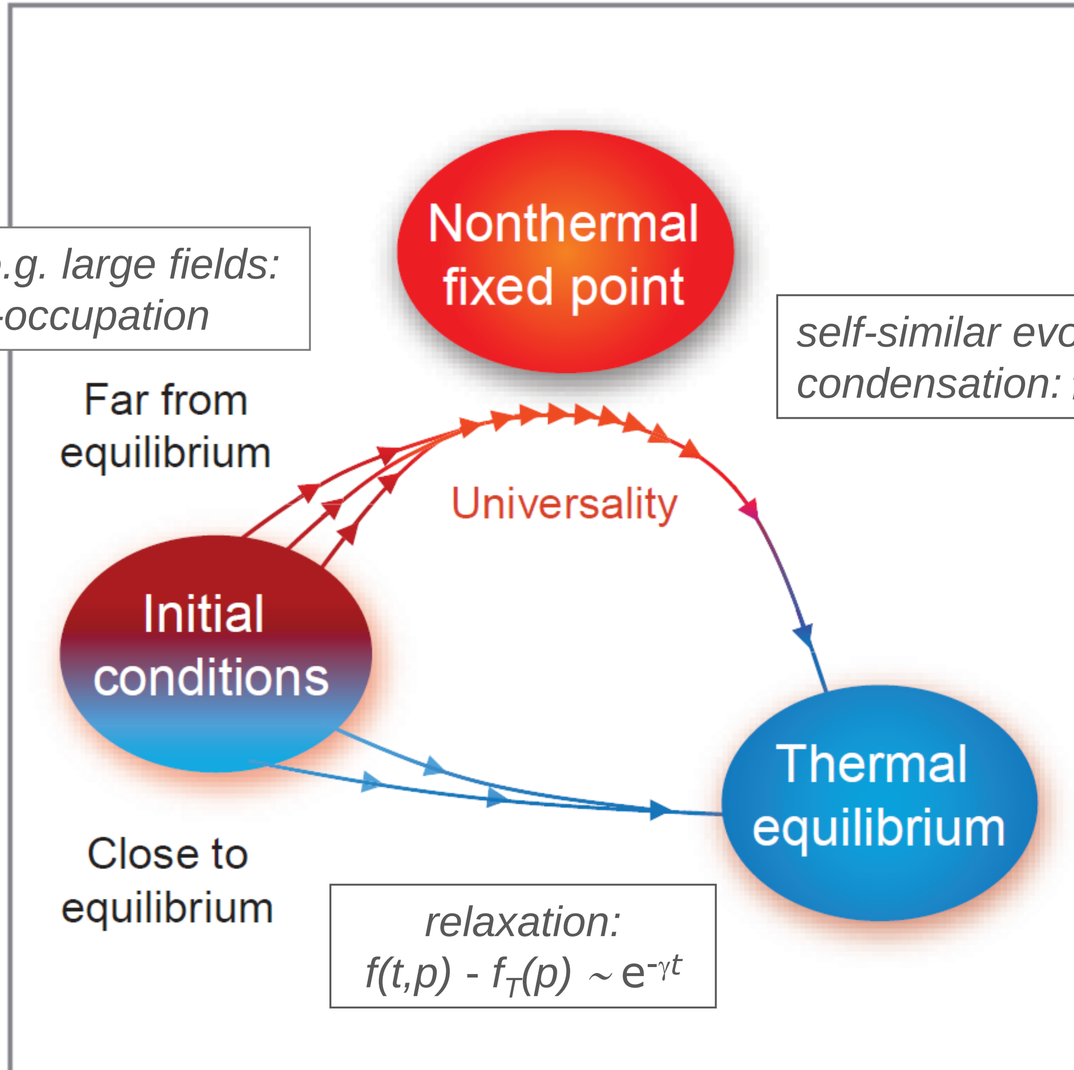
$$\alpha = -2/3 \quad , \quad \beta = 0 \quad , \quad \gamma = 1/3$$




$\rightarrow$ Remarkably large universality class far from equilibrium!

# Conclusions

*extreme conditions, e.g. large fields:*
$f(t,p) \sim e^{\lambda t} \rightarrow$ *over-occupation*




*self-similar evolution/*
*condensation:* $f(t,p) \sim t^{\alpha} f_S(t^{\beta} p)$

*relaxation:*
$f(t,p) - f_T(p) \sim e^{-\gamma t}$

# Real-time lattice simulations with fermions

$$\mathcal{L} = \frac{1}{2}\partial\Phi^*\partial\Phi - V(\Phi) + \sum_k^{N_f}\left[i\bar{\Psi}_k\gamma^\mu\partial_\mu\Psi_k - \bar{\Psi}_k(MP_L + M^*P_R)\Psi_k\right]$$

$M = m - g\Phi(x)$, $P_L = \frac{1}{2}(1-\gamma^5)$, $P_R = \frac{1}{2}(1+\gamma^5)$

Highly-occupied bosons at weak coupling: employ classical-statistical simulations

Fermion quantum fluctuations:

$$\int\prod_k D\Psi_k^+ D\Psi_k e^{i\int\mathcal{L}(\Phi,\Psi^+,\Psi)} \quad\rightarrow\quad \boxed{\partial_x^2\Phi(x) + V'(\Phi(x)) + N_f J(x) = 0}$$

$$J(x) = J^{\mathrm{S}}(x) + J^{\mathrm{PS}}(x)$$

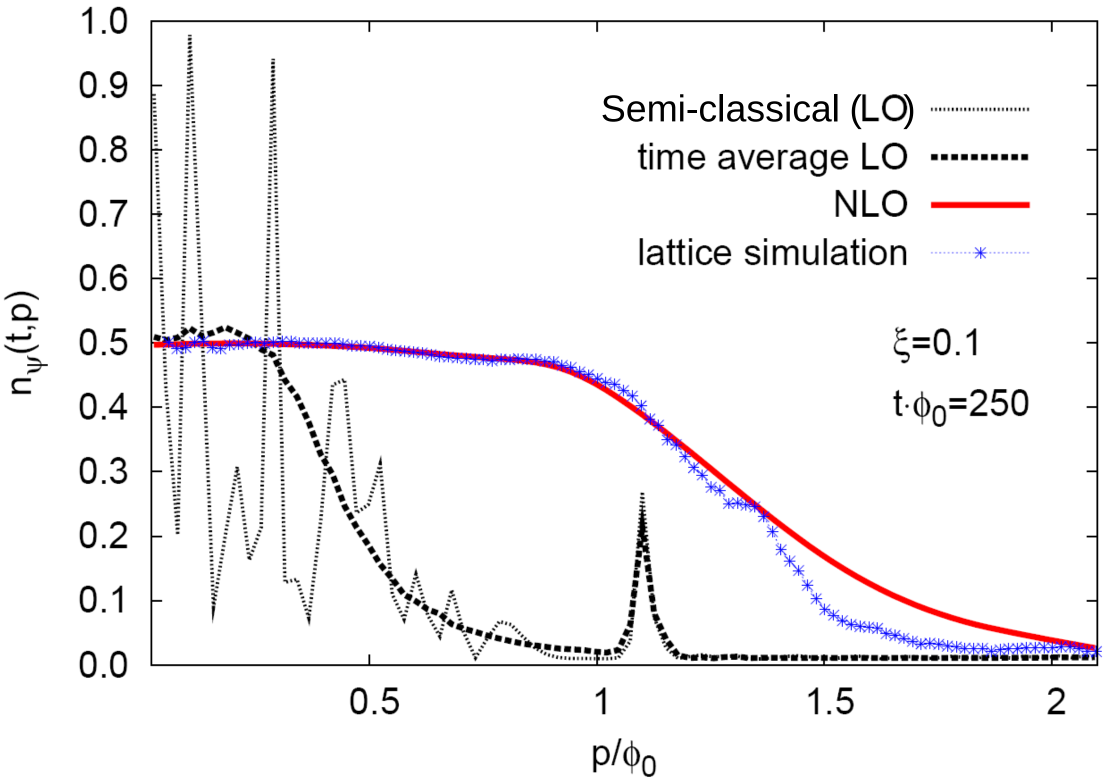
$$\begin{aligned} J^{\mathrm{S}}(x) &= -g\left\langle\bar{\Psi}(x)\Psi(x)\right\rangle = g\mathrm{Tr}\,D(x,x)\,, \\ J^{\mathrm{PS}}(x) &= -g\left\langle\bar{\Psi}(x)\gamma^5\Psi(x)\right\rangle = g\mathrm{Tr}\,D(x,x)\gamma^5 \end{aligned}$$

For classical $\Phi(x)$ the exact equation for the fermion $D(x,y)$ reads:

$$\boxed{(i\gamma^\mu\partial_{x,\mu} - m + g\mathrm{Re}\,\Phi(x) - ig\mathrm{Im}\,\Phi(x)\gamma^5)D(x,y) = 0}$$

*Aarts, Smit; Borsanyi, Hindmarsh; Berges, Gelfand, Pruschke; Saffin, Tranberg; Kasper, Hebenstreit; Mace, Mueller, Schlichting, Sharma, Tanji,…*

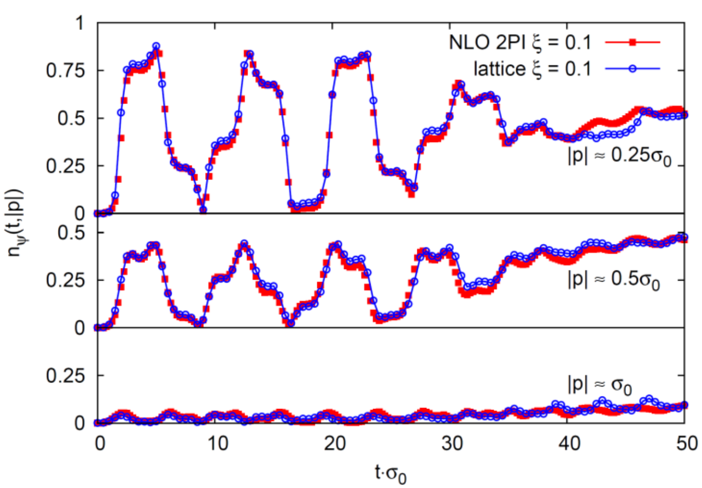
# Comparing lattice simulations to NLO quantum results

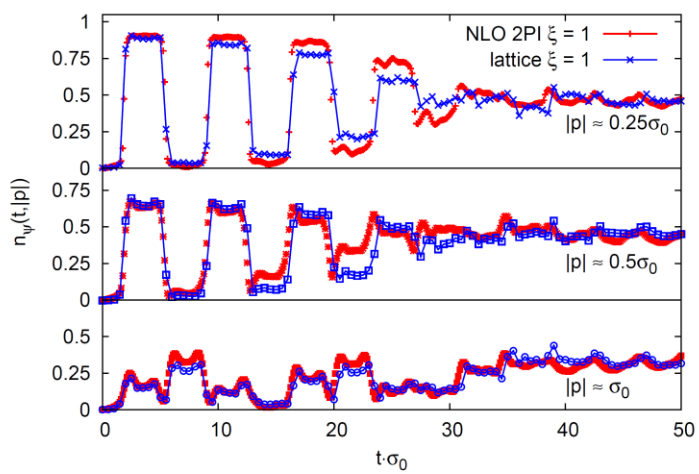



• Wilson fermions on a $64^3$ lattice

$$\xi = g^2/\lambda$$

*Berges, Gelfand, Pruschke PRL 107 (2011) 061301*




*good agreement even for ξ = 1*

*Berges, Gelfand, Sexty PRD 89 (2014) 025001*