Slow heating, effective Hamiltonians, prethermalization

Kapitza pendulum

Rapidly oscillating suspension point: “inverted” pendulum

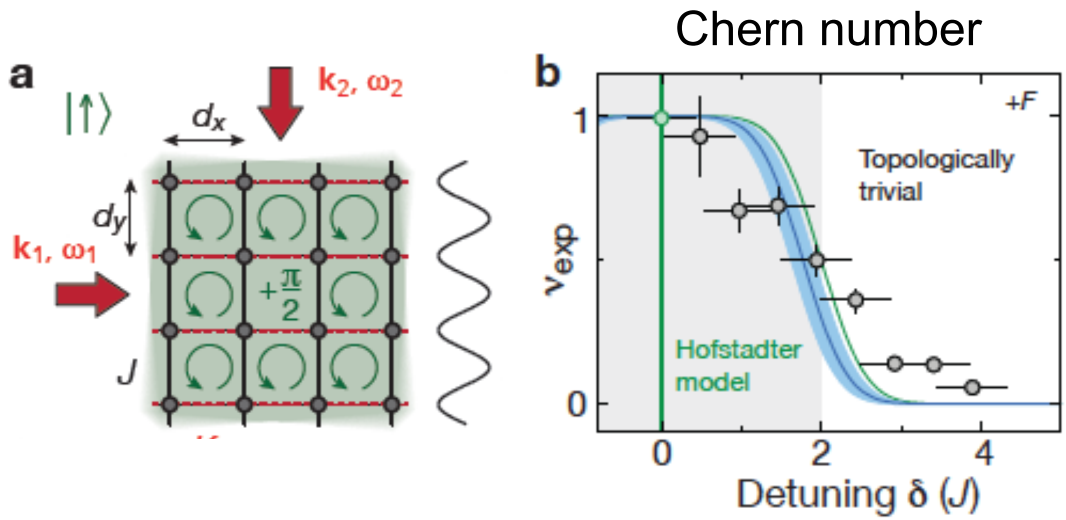
Periodic driving → new states, not possible in static systems

Realization of topological states by driving

Driving → topological Bloch bands

Bands with non-zero Chern number (recently measured)

Jotzu et al'14, Aidelsburger et al'14



Many theory works: Floquet topological insulators, fractional Chern insulators, SPTs

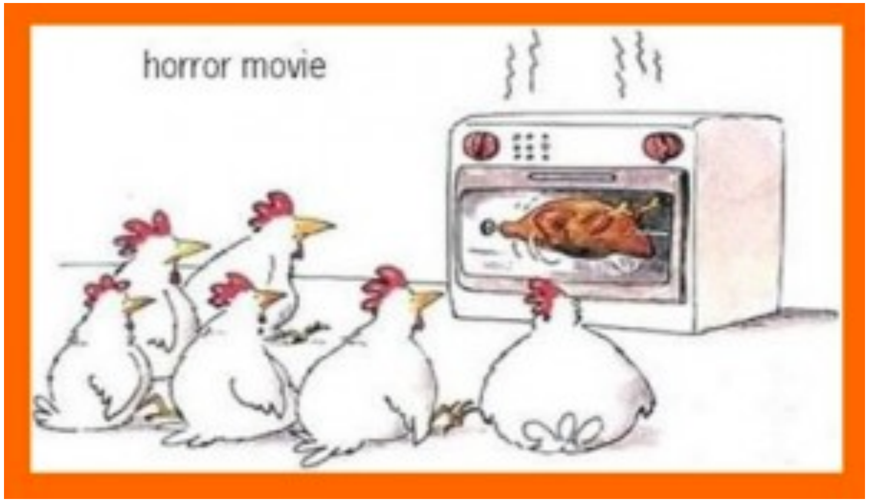
Oka, Aoki'09, Lindner, Refael, Galitski'10, Kitagawa, Rudner, Berg, Demler'10, Caysoll, Moessner'12, Rudner et al'13, Neupert, Grushin'14, many others

BUT: Theory mostly limited to single-particle physics

This talk: Driven **many-body** systems

Driven many-body systems

-Many-body systems heat up to infinite T -- a challenge for “Floquet engineering”



How rapid is heating in driven lattice ergodic systems?

-At fast driving, heating is exponentially slow (theorem for spins and fermions)

Is this the fate of all driven systems?

NO! Driven MBL systems do not heat up (at high enough driving frequency)

Conventional approach: Magnus expansion

$$H(t+T)=H(t)$$

Floquet operator $$F = T\exp - i\int_0^T H(t)\,dt$$

$$F=\exp[-iH_{eff}T]$$

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$$H_{eff} = H_{eff}^{(0)} + H_{eff}^{(1)} + H_{eff}^{(2)} + \ldots$$

$$H_{eff}^{(0)} = \frac{1}{T}\int_0^T H(t)\,dt \qquad H_{eff}^{(1)} = \frac{1}{2T}\int_0^T dt_1 \int_0^{t_1} dt_2 [H(t_1), H(t_2)], \ldots..$$

Magnus expansion → effective, time-independent Hamiltonian

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BUT: known to converge only for bounded Hamiltonians

$$\|H(t)\| T < \pi$$

PROBLEM: BREAKS DOWN IN MANY-BODY SYSTEMS

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Validity of Magnus expansion in many-body systems? Asymptotic expansion?

Optimal order of expansion? Rigorous results?

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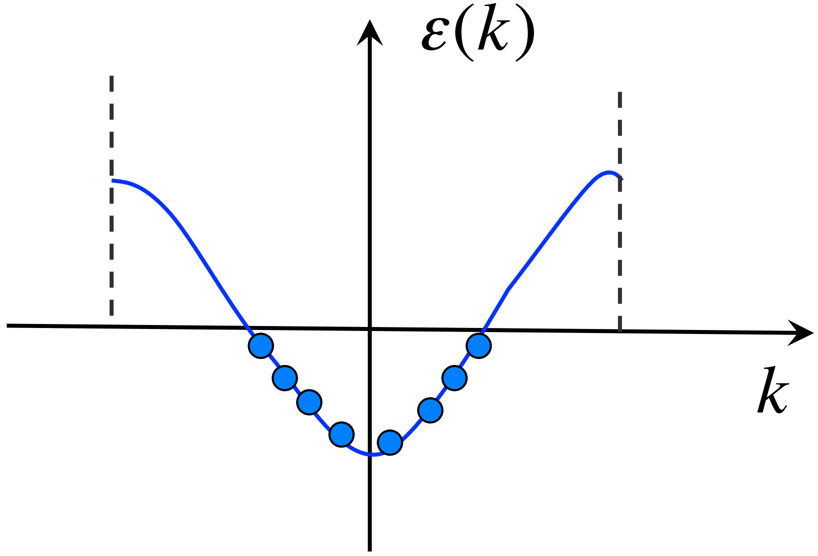
PROBLEM: BREAKS DOWN IN MANY-BODY SYSTEMS

Heating in lattice systems: qualitative considerations

Driven many-body lattice system $H(t) = H + gV\cos\omega t$

$$H = t\sum_i c_i^+ c_{i+1} + h.c. + U\sum_i n_i n_{i+1} \qquad \omega >> t, U$$



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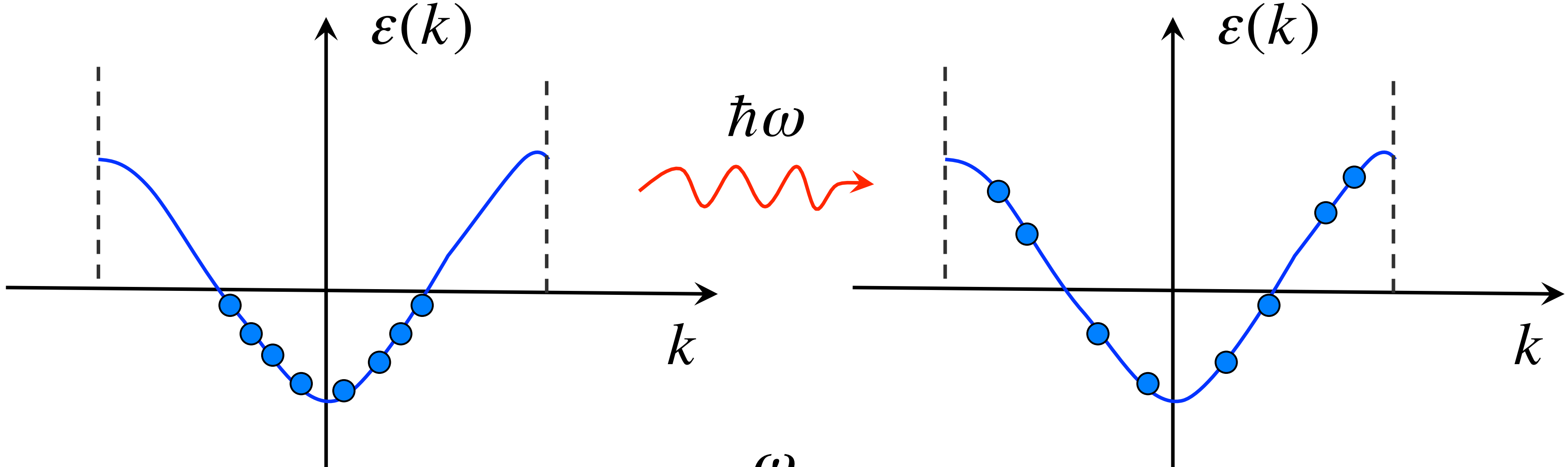

1 photon absorbed → create $N \sim \dfrac{\omega}{t} >> 1$ e-h pairs

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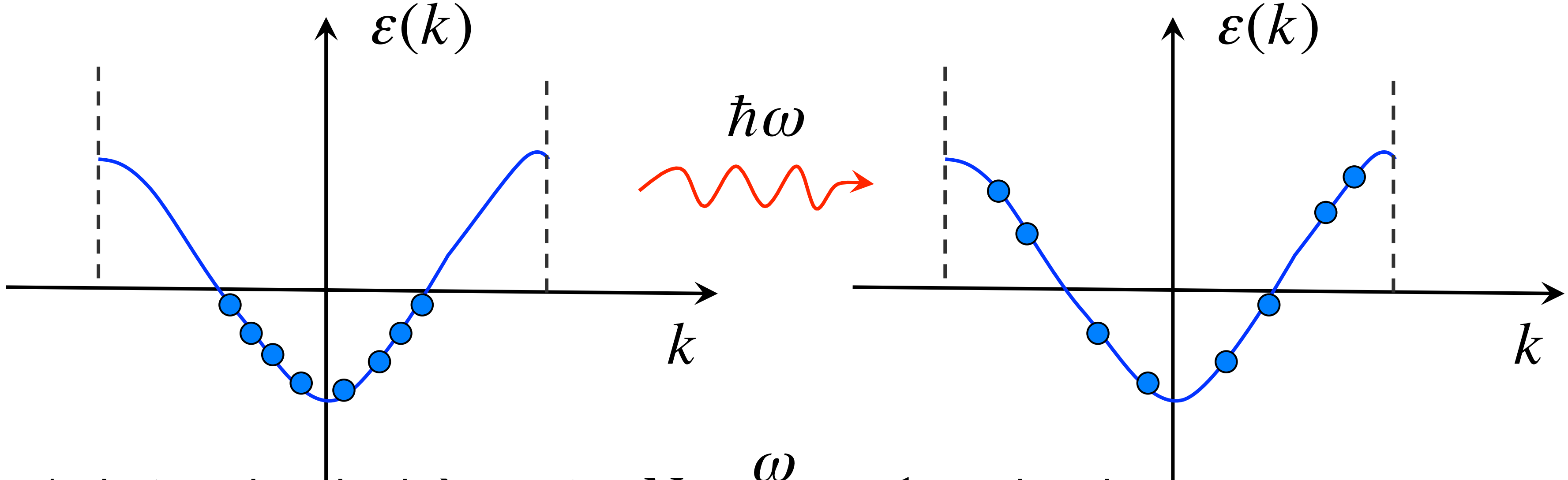

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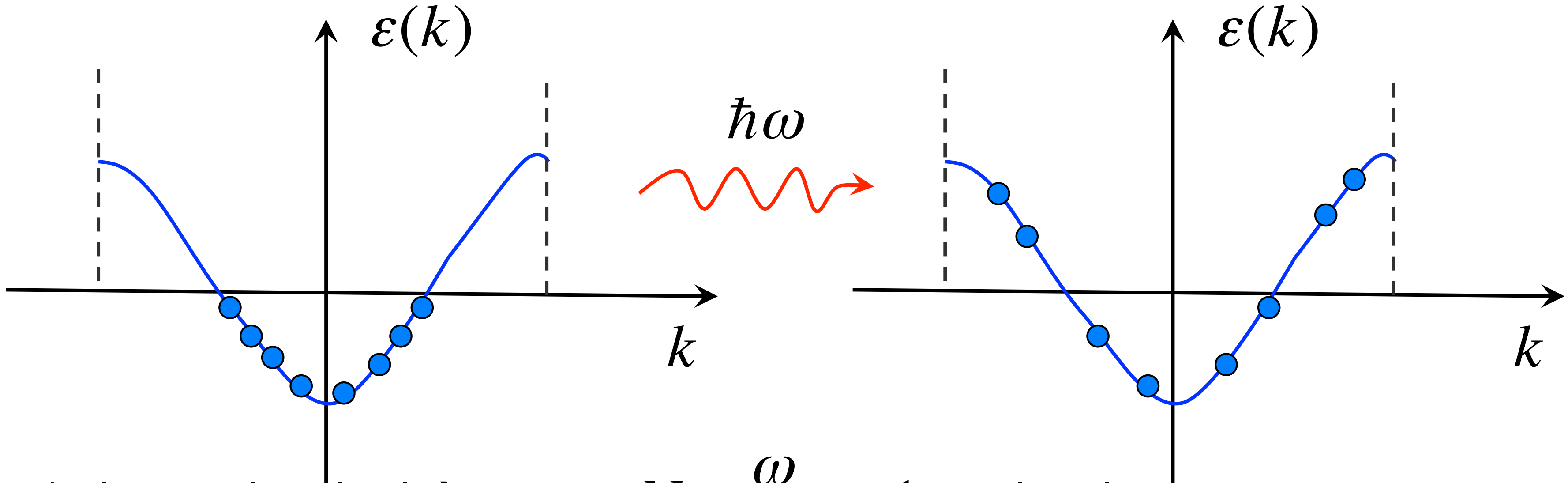

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Seems difficult, starting from ground state at weak interaction (*V* is local!)

E.g.: Doublon decay in large-U Hubbard model is slow (exp+perturbation th.)

Strohmaier et al'10; Sensarma et al'10

General bound for the heating rate

Driven many-body lattice system $H(t) = H + gV\cos\omega t$

Locality: $H = \sum_i h_i \qquad V = \sum_i V_i$

<u>Heating rate (golden rule)</u>

$$\Gamma_\beta(\omega) \propto g^2 \sum_{\eta,\eta'} e^{-\beta E_\eta} \left|\langle \eta' | V | \eta \rangle\right|^2 \delta(\omega - (E_{\eta'} - E_\eta))$$

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EXPONENTIALLY SLOW HEATING AT FAST DRIVING

General: holds for any initial state and for strong interactions

The idea of the proof

1) Consider the case of a local operator *V*. Rewrite

$$A(\omega) \sim \sum_{\eta,\mu} \left|\langle \mu | V | \eta \rangle\right|^2 \delta(\omega - (E_\mu - E_\eta)) = \sum_{\eta,\mu} \frac{\left|\langle \mu | [V,H] | \eta \rangle\right|^2}{\omega^2} \delta(\omega - (E_\mu - E_\eta)) =$$

$$= \sum_{\eta,\mu} \frac{\left|\langle \mu | \overbrace{[V,H,H,\ldots,H]}^{k \text{ commutators}} | \eta \rangle\right|^2}{\omega^{2k}} \delta(\omega - (E_\mu - E_\eta))$$

2) Use locality of *H to* bound the commutators

$$\left\| [V,H,H,\ldots,H] \right\| < C^k k!$$

3) Choose optimal $k_* \approx \frac{\omega}{Ce}$

4) Bound: $A(\omega) < e^{-\kappa\omega} \qquad \kappa = \frac{2}{ce}$

5) For global driving, use Lieb-Robinson bounds to estimate “cross-terms” involving V_i, V_j with large $|i-j|$

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The

Result can be made fully non-perturbative
(in driving strength and not only in interaction!)
→ Controlled version of high-frequency expansion

7.01474

Fundamental reason: locality of quantum dynamics

EXPONENTIALLY SLOW HEATING AT FAST DRIVING

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Driven many-body systems: non-perturbative results

Very long heating times

$$\tau_* \sim e^{C\omega}$$

What governs dynamics at intermediate times? $t \leq \tau_*$

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Dynamics is described by a quasi-conserved effective Hamiltonian

$$H_* = H_0 + \frac{1}{\omega} H_1 + \frac{1}{\omega^2} H_2 + \ldots + \frac{1}{\omega^n} H_n \qquad n \sim C\omega >> 1$$

Conservation breaks down only <u>at exponentially long times</u> $t \sim \tau_*$

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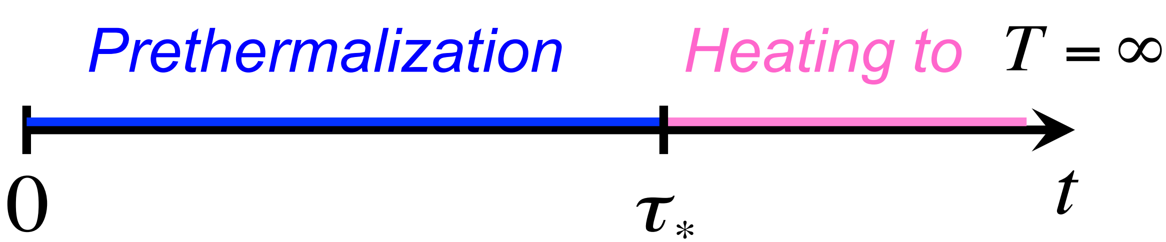

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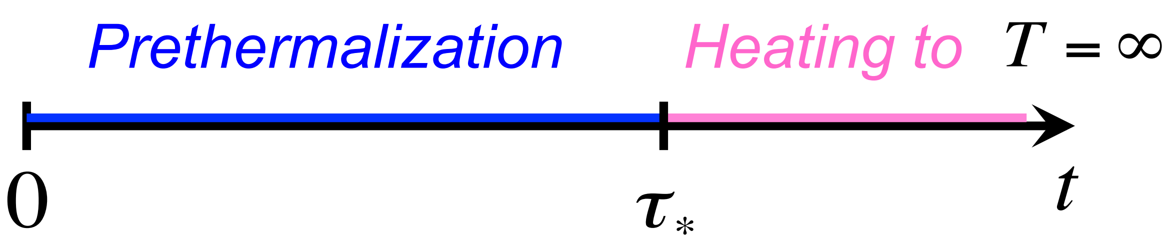

Applies to: driven systems of spins and fermions with <u>local interactions</u>

Rapidly driven many-body systems show a long prethermalization regime

Driven many-body systems: idea of the approach

1)"Gauge transformation": $\left|\psi(t)\right\rangle = \hat{Q}(t)\left|\varphi(t)\right\rangle$

$$\hat{Q}^{+}\hat{Q} = I$$

$$\hat{Q}(t+T) = \hat{Q}(t)$$

Stroboscopic evolution not affected, but Hamiltonian changed:

$$H'(t) = \hat{Q}^{+} H(t)\hat{Q} - i\hat{Q}^{+}\frac{d\hat{Q}}{dt} \qquad (1)$$

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2) Goal: choose $\hat{Q}$ to minimize the driving term

Result:

Using locality, can find $\hat{Q} = e^{T\Omega_1 + T^2\Omega_2 + .. + T^n\Omega_n}$ for $n \sim C\omega$

that decreases driving term exponentially, by $e^{-C\omega}$

3) Obtain the quasi-conserved quantity H_* using (1)

Numerics on finite-size systems

Kick protocol:

$$U_F = e^{-iH_0 t} e^{-iH_1 t}$$

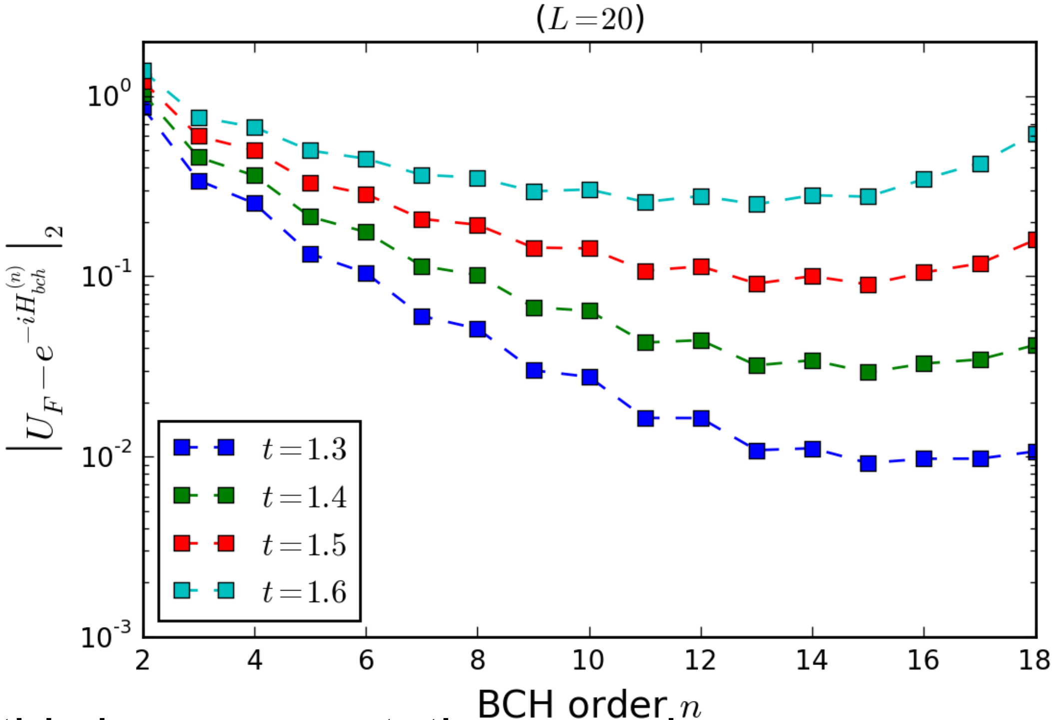
t -driving period

$$H_0 = \sum_i h s_i^z + J s_i^z s_{i+1}^z$$

$$H_1 = \sum_i J s_i^x s_{i+1}^x$$

Prosen model '97

Error, expansion order n



-Confirmed that this is an asymptotic expansion

$$n_* = C\omega$$

The bound is saturated

Implications

1) Broad prethermalization regime in driven many-body systems

2) Optimal order of the Magnus expansion $n_* \sim C\omega$

3) "Floquet topological insulators" can be very long-lived

4) Suggests ways of experimental preparation of "Floquet fractional Chern insulators" and other correlated states in driven systems

OTHER APPLICATIONS:

-Bounds of heating in bosonic lattice systems

-Some rigorous results about quasi-adiabatic behavior in Floquet systems

Part II: MBL in periodically driven systems

Kicked rotor: localization in a periodically driven system

Chaos, Quantum Recurrences, and Anderson Localization

Shmuel Fishman, D. R. Grempel, and R. E. Prange
Department of Physics and Center for Theoretical Physics, University of Maryland, College Park, Maryland 20742
(Received 6 April 1982)

Floquet operator $U = e^{-iH_0T_0}e^{-2iV}$

$$H_0 = \frac{1}{2}\frac{\partial^2}{\partial\vartheta^2}$$

rotor

$$V = K\cos\vartheta$$

kick

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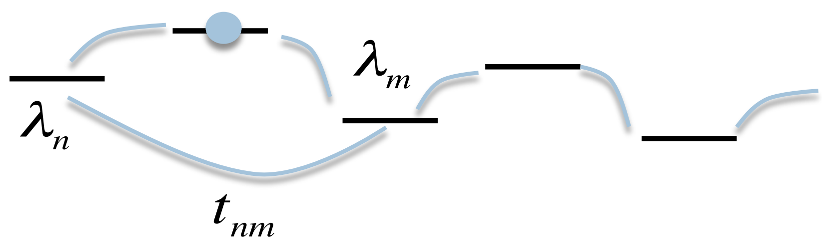
Mapping onto an effective Anderson model



On-site energy $\tan\left(\frac{E_nT_0 - \vartheta}{2}\right)$

Hopping operator $t_{nm} = \left\langle n\right|\tan\left(\hat{V}\right)\left|m\right\rangle$

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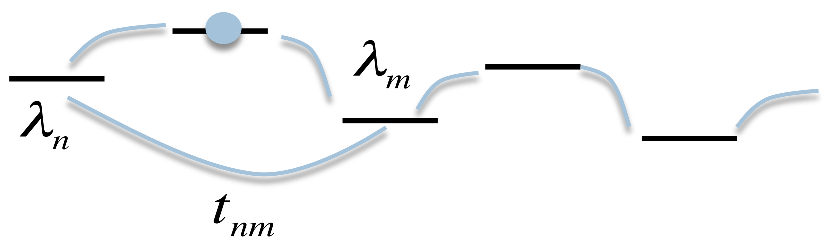

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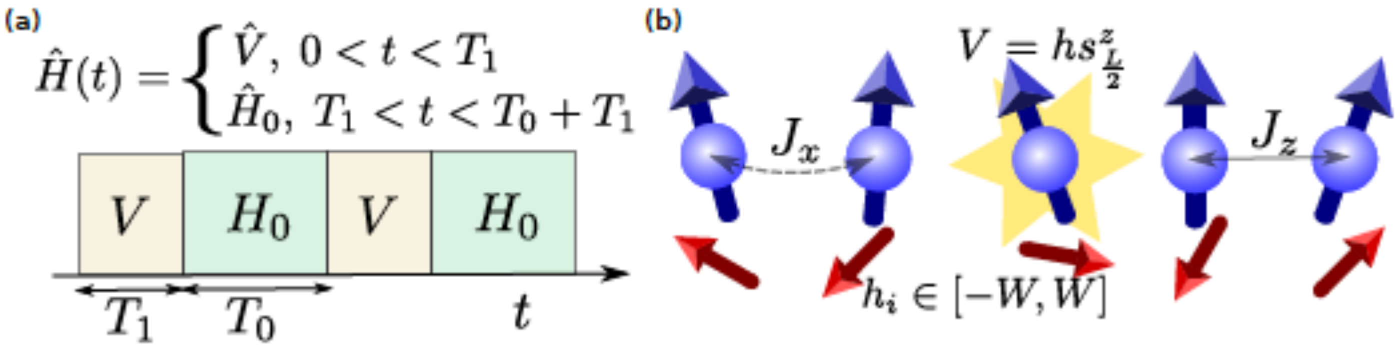
Quasi-random → dynamical localization!

Hopping decays with $|n-m|$

Driven many-body systems: ergodic vs MBL



L spins +local driving $\hat{V}$ $\qquad U = e^{-iH_0 T_0} e^{-2iV}$

Mapping onto an effective hopping problem; sites=eigenstates of H_0

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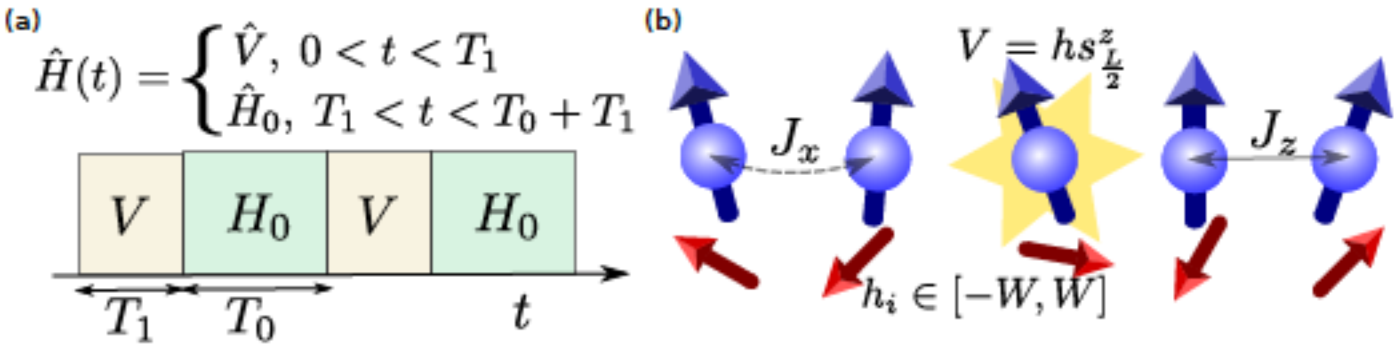

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	H_0 ergodic	H_0 MBL
Effective level spacing	$\Delta \sim \frac{1}{D} \sim \frac{1}{2^L}$	$\Delta \sim \frac{1}{2^L}$
Hopping	$t_{nm} \sim \frac{1}{\sqrt{D}} \sim \frac{1}{2^{L/2}}$ (from ETH)	$t_{nm} \sim \frac{e^{-L/\xi}}{2^L}$
	$t_{nm} >> \Delta$ Nonlocal hopping delocalization	$t_{nm} << \Delta$ localization

Periodically driven many-body localized systems

$$H(t) = H_{\text{MBL}} + gV(t)$$

H_{MBL} many-body localized

ω driving frequency

1) Construct Floquet Hamiltonian iteratively (different from Magnus!)
Show convergence and MBL at high driving frequency

2) Argue delocalization at low driving frequency

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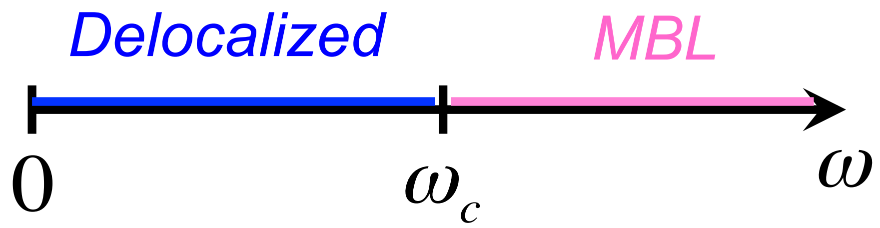

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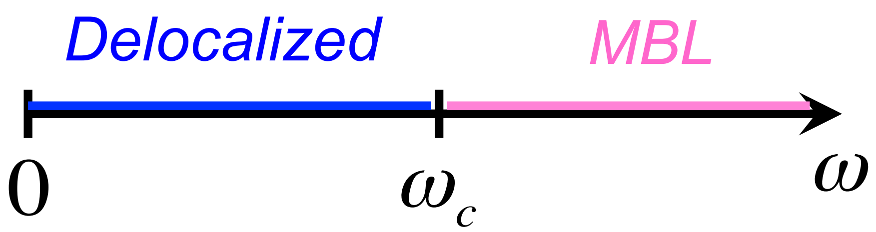


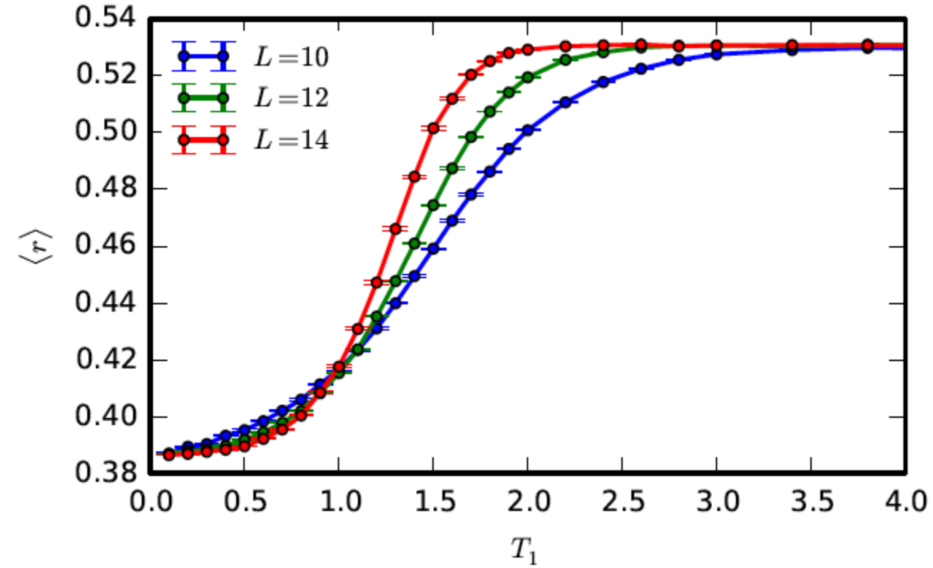
4) Driving: probe MBL (little/no heating), access MBL-ergodic transition

Driven MBL systems: numerical results

Kicked spin chain $\hat{F} = e^{-iH_0T_0}e^{-iH_1T_1}$

$$H_0 = \sum_i h_i\sigma_i^z + J_z\sigma_i^z\sigma_{i+1}^z$$

$$H_1 = J_x\sum_i \sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y$$

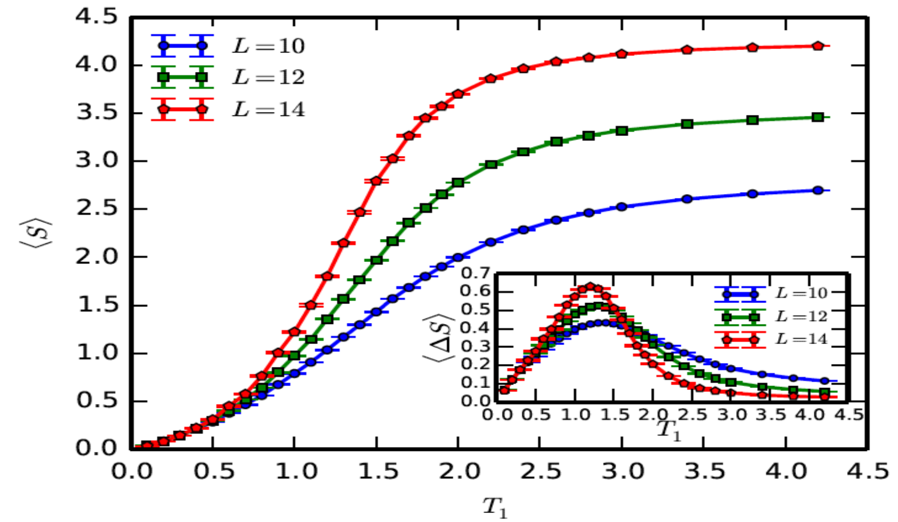


1) Spectral statistics: transition Between Poisson and Wigner-Dyson

2) Area-law for eigenstates in MBL phase

3) Log-growth of entanglement

4) Constructed a set of local integrals of motion



Confirms analytical theory

Iterative procedure and its convergence

Evolution operator: $$i\frac{d}{dt}U(t) = H(t)U(t)$$

Decompose $U(t) = P(t)e^{-iH_{eff}t}$ $\quad$ $P(t+T) = P(t)$

$$P^{+}(t)\left(H(t) - i\frac{d}{dt}\right)P(t) = H_{eff}$$

Solve for $P(t)$ iteratively, gradually eliminating time-dependent terms

Perturbation theory in

$$\frac{g}{\nu} << 1 \qquad \frac{g^2}{\nu W} << 1$$

Convergence criteria

Higher orders: combinatorics equivalent to time-independent MBL problem

Quasi-local Floquet Hamiltonian, which itself is MBL

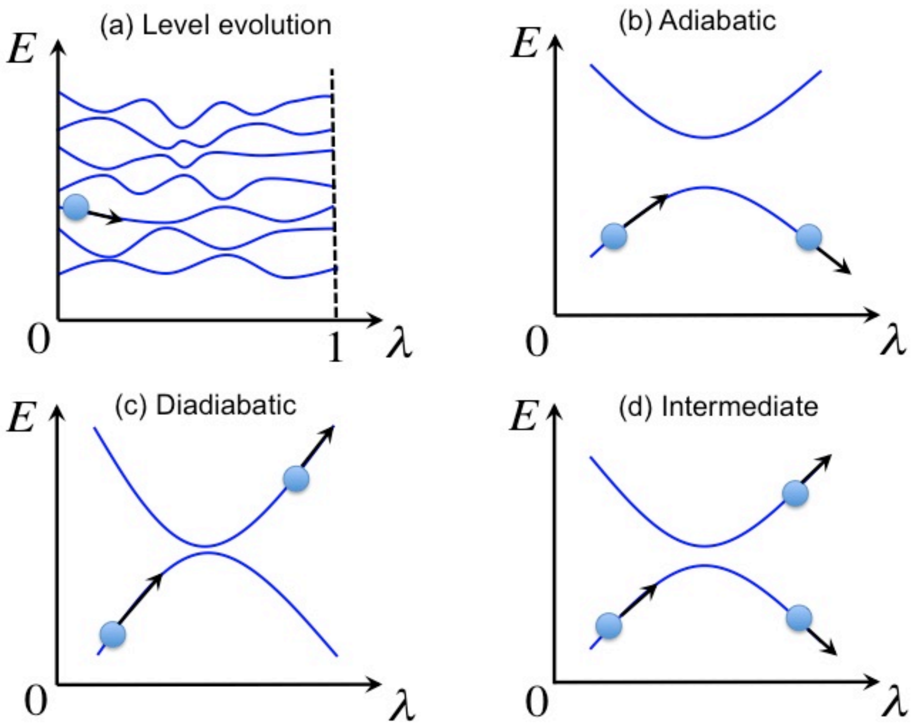
Low frequency: delocalization via Landau-Zener transitions

Consider instantaneous eigenstates of

$$H(\lambda) = H_{\text{MBL}} + gV(\lambda)$$

Change $\lambda = \frac{t}{T}$

Multi-level Landau-Zener problem



Diabatic crossing: not dangerous

"Intermediate" crossings:
the state get mixed, delocalization

At low frequency, many intermediate/adiabatic crossings → delocalization

DA, De Roeck, Huveneers, arXiv:1412.4752

See also: Khemani, Nandkishore, Sondhi'14 (local ramp)