Lecture 4 : The BH is a quantum state

1209.2686 + Many papers with Joerg Medved
+ To appear with Lasma Alberte, Andrei Khmelnitsk

- Approximating a physical, finite mass BH by a strictly classical geometry is the origin of the information paradox (QFT in curved spacetime
- The emitted radiation is weakly coherent, coherent effects build up over time ⇒ modified Page curve
- The BH is time-dependent and not eternal
  ⇒ - The energy density near the horizon is small but non-vanishing
  "Wall of fire" → "Wall of smoke"
  - Near horizon state deviates substantially from TFD in the late stage of the evaporation
  - Red shift is not infinite
  - BH's have some hair

BH as a quantum state

How "quantum" is the BH?

Use standard measure of "quantumness"

Quantum wave length of the BH $\lambda_{BH} = \frac{\hbar}{M_{BH}}$

Size of the BH : R_S

$$C_{BH} = \frac{\lambda_{BH}/2\pi}{R_S} = \frac{\hbar}{2\pi M_{BH} R_S} = \frac{\hbar G_N}{\pi R_S^2} = \frac{l_P^2}{\pi R_S^2} = \frac{1}{S_{BH}}$$

↑

dimensionless $\hbar$

QFT in curved space time

$C_{BH} = 0$, $M_{BH} \to \infty$, $G_N \to 0$

$2 M_{BH} G_N = R_S$ finite

(See also Dvali's lecture)

More formal discussion

$|\Psi_{BH,M}\rangle$, M - Matter

Need to calculate $\langle \hat{O}_M \rangle = \langle \Psi_{BH,M} | \hat{O}_M | \Psi_{BH,M} \rangle$

QFT is curved spacetime $\langle O_M \rangle_c = \langle \Psi_M(g_c) | \hat{O}_M | \Psi_M(g_c) \rangle$

So $$\langle \hat{O}_M \rangle_c = \lim_{C_{BH} \to 0} \langle \hat{O}_M \rangle$$

Claim (supported by many calculations)

$$\langle \hat{O}_M \rangle = \langle \hat{O}_M \rangle_c + \mathcal{O}(C_{BH})$$
$$\overset{?}{\mathcal{O}}\left(\frac{1}{S_{BH}}\right)$$

and <u>not</u> $e^{-S_{BH}}$.

From the point of view of QFTCST large, non-perturbative quantum corrections.

(*) States in quantum gravity expected to depend on boundary charges, but in our set up, expect approximate local observables as superposition of "off-shell" geometries.

Semiclassical gravity

Example 1: Wave function of two spherical masses at rest in a superposition

$|1\rangle =$

$|2\rangle =$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle + |2\rangle\right)$$

Such superpositions can be created using optical devices (see, for example, 1302.1924)

How would a test particle move?
Approximately on a superposition of geodesics
"Inverse of back reaction"

Back reaction: quantum fluctuations of the probe particle make the big masses fluctuate.

- What does it mean to do an interference experiment

Example 2: A semiclassical collapsing shell

$$\psi_{shell} = \frac{1}{N} e^{-\frac{(R_{shell} - \bar{R}(t))^2}{2\sigma^2}}$$

$$\langle \Psi_{shell} | \hat{O}(R_{shell}) | \Psi_{shell} \rangle = \frac{4\pi}{N} \int_0^\infty dR_{shell}\, R_{shell}^{2+1}\, e^{-\frac{(R_{shell}-\bar{R}(t))^2}{2\sigma^2}} \times \mathcal{O}(R_{s}$$

<u>Example 3</u>: Wave function of BH

$$\Psi_{BH} = \frac{1}{N} e^{-\frac{(R-R_s)^2}{2\sigma^2}}$$

"off-shell" geometries

$$ds^2 = -\left(1-\frac{R}{r}\right)dt^2 + \left(1-R/r\right)^{-1}dr^2 + r^2 d\Omega^2$$

$R \neq 2GM$, R/r is physical

Can be measured by measuring the components of $R_{\alpha\beta\gamma\delta}$

(Visser 1407.7295)

Idea of horizon fluctuations has a long history

This treatment: 1202.5273 w/ M. Hadad.

For references see, for example, gr-qc-0610066

Horizon fluctuations "destroy the horizon"

Infinite redshift $\rightarrow$ finite redshift on average.

Radiation emitted from a semiclassical collapsing shell

1305.3139
1310.5861

$\rho_{ij} = \langle 0_- | b_i^\dagger b_j | 0_- \rangle$ 2-point function

$$\rho_H(\omega, \tilde{\omega}) = \int_0^\infty d\omega' \, \beta^*_{\omega'\omega} \beta_{\omega'\tilde{\omega}} \propto \delta(\omega - \tilde{\omega})$$

$$= \int_{-\infty}^{\infty} dv \int_0^\infty d\omega' \int_0^\infty d\omega'' \, \beta^*_{\omega'\omega} \beta_{\omega''\tilde{\omega}} \frac{e^{iv(\omega'-\omega'')}}{2\pi}$$

$$\rho_{sc}(\omega, \tilde{\omega}) = \int_{-\infty}^{\infty} dv \int_0^\infty d\omega' \int_0^\infty d\omega'' \, \langle \psi_{shell/BH}(v_{shell}) | \beta^*_{sc}(\omega', \omega; v_{shell}) \beta_{sc}(\omega'', \tilde{\omega}; v_{shell}) | \psi_{shell}(v_{shell}) \rangle \times \frac{1}{2\pi} e^{iv(\omega'-\omega'')}$$

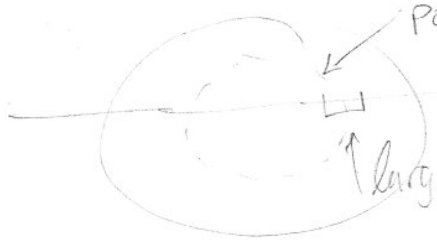
$$(\beta_{classical})_{\omega'\omega} \propto \frac{1}{2\pi} \int_{-\infty}^{v_0} dv \, e^{i\omega' v} e^{-\frac{2i\omega R_s}{4iM\omega} \ln(v_0 - v)}$$

$$(\beta_{sc})_{\omega'\omega}(v_{shell}) \propto \frac{1}{2\pi} \int_{-\infty}^{v_{shell}} dv \, e^{i\omega' v} e^{-2i\omega R_s \ln(v_{shell} - v)}$$

The total phase shift is a superposition of phase-shifts from each shell configuration.

$$(\beta_{sc})_{\omega'\omega}(v_{shell}) = \Gamma(1 - 2i\omega R_s) (i\omega')^{-1 + 2i\omega R_s} \frac{1}{2\pi} e^{i v_{shell} \omega'}$$

position of the shell fluctuates as the mode crosses.



large phase shift

added phase shift w.r.t to classical case.

Final result for $\rho_{sc}(\omega,\tilde{\omega};v_{shell}) \propto I_{sc}(\omega'-\omega'';v_{shell})$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} dv\, e^{i(v-v_{shell})(\omega'-\omega'')}$$

If v_{shell} was fixed - v_0 $\quad I_{sc} \propto \delta(\omega'-\omega'') \Rightarrow \rho \propto \delta(\omega-\tilde{\omega})$

But it is not!

Using that the rest of the integrals are symmetric under $\omega' \leftrightarrow \omega''$

$$I_{sc} = \frac{1}{\pi}\int_0^{\infty} dv \cos\left[(v-v_{shell})(\omega'-\omega'')\right]$$

$$= \frac{1}{2\pi}\int_0^{v_{shell}} dv \cos\left[v(\omega'-\omega'')\right] + \delta(\omega'-\omega'')$$

semiclassical correction. $\qquad$ classical contribution

$$\langle \Psi_{shell}(v_{shell}) | I_{sc}(v_{shell}) | \Psi_{shell}(v_{shell}) \rangle$$

$\Rightarrow \rho_{sc}(\omega,\tilde{\omega})$ contains off diagonal terms

$\Delta\rho_{sc}(\omega,\tilde{\omega}) \sim \sqrt{C_{BH}} \times$ Uniform magnitude matrix in the thermal window

A heuristic explanation: fluctuations in v_{shell} induce dispersion in phase shift of modes crossing the shell

Magnitude correct: generic corrections to physical quantities $\sim (\sqrt{C_{BH}})^2 \sim \hbar$

Perturbative parameter $N_{RAD} \cdot C_{BH}$ (not C_{BH})

$$P(\rho_{ij}) = Tr(\rho^2) = \underbrace{\frac{1}{N_{RAD}}}_{\text{Diagonal}} + \underbrace{\frac{1}{N_{RAD}} \cdot N_{RAD} C_{BH}}_{\text{off diagonal}} \quad \text{(with "tracking")}$$

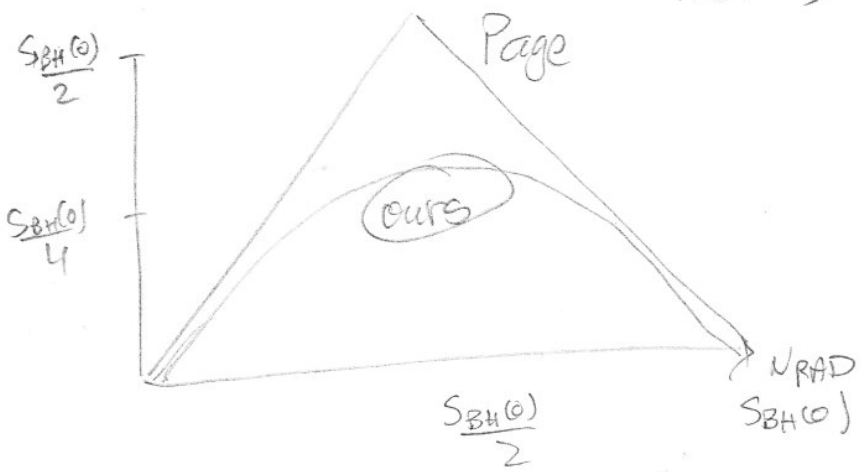
$$PR = 1/\left(\frac{1}{N_T} + C_{BH}\right) = \frac{N_{RAD}}{1 + N_{RAD} C_{BH}} = \frac{N_{RAD}}{1 + \frac{N_{RAD}}{(S_{BH}(0) - N_{RAD})}} =$$

$$C_{BH} = \frac{1}{S_{BH}} = \frac{1}{S_{BH}(0) - N_T}$$

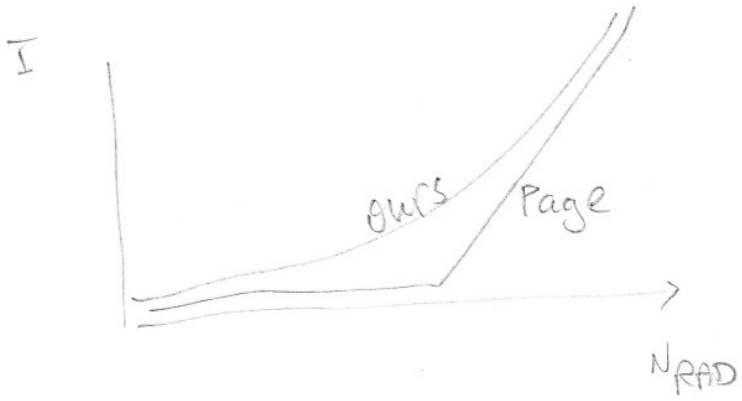
$$= \frac{N_{RAD}(S_{BH}(0) - N_{RAD})}{S_{BH}(0)}$$

$$H_2(\hat{\rho}) = Tr \ln(1 + 2\rho) \Longleftrightarrow PR(\rho_{ij}) \Leftrightarrow H_2(\hat{\rho})$$

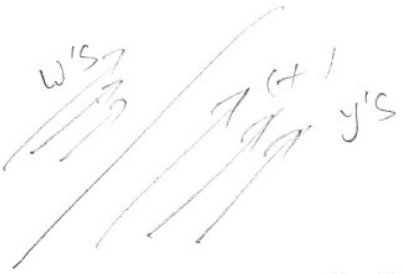
Albeite, RB, Khmelnitsky, Medved



1407.4914



The near horizon state



In the Hawking calculation $C_{BH} = 0$

The modes "graze the horizon" and stay there formally for an finite time.

To find a finite energy density and decide whether it is small or large we need a scale, M_{BH}, or C_{BH}.

Two ways to estimate the number of near horizon modes:

1.

The horizon is shrinking due to the black body emission

when it shrunk by σ the negative energy modes have been absorbed

$N_{modes} = N_{coh} = \sqrt{R_S^2/\ell_p^2} \gg 1$ but $\sqrt{\frac{R_S^2}{\ell_p^2}} \ll S_{BH} = \frac{R_S^2}{\ell_p^2}$

$\mathcal{E}_{pair} = \frac{1}{R_S^3}\frac{1}{\ell_p}$ amount of discontinuity

$\mathcal{E} = \mathcal{E}_{pair} \times N_{pairs} = \frac{1}{R_S^3}\frac{1}{\ell_p} \cdot \frac{R_S}{\ell_p} = \frac{1}{R_S^2 \ell_p^2}$

near horizon energy density

if $N_{pairs} = S_{BH}$

$\frac{1}{R_S^2 \ell_p^2} \cdot \frac{R_S}{\ell_p}$ Firewall

Another way

deformation / tidal forces $\propto N_{pairs} \times \frac{l_p l}{R_s^2}$

$N_{pair} \sim S_{BH}$

$N_{pairs} \times \frac{l_p l}{R_s^2} = l/l_p \gg 1$

$N_{pairs} \sim \sqrt{S_{BH}}$

$N_{pairs} \times \frac{l_p l}{R_s^2} = l/R_s \ll 1$

Another way to estimate the number of pairs

For Hawking $V_\perp = 0$ $c_{BH} =$

For $c_{BH} \neq 0$ $V_\perp \sim c_{BH}$

$\Delta X = l_p \Rightarrow t = \frac{l_p}{c_{BH}} = R_s^2/l_p$

$\Delta N = R_s/l_p$

O.K. with EP

Redshift effectively finite

Fluctuating horizon → horizon disappears

$$ds^2 = -\left(1 - \frac{R}{r}\right) dt^2 + \frac{1}{\left(1 - R/r\right)} dr^2 + r^2 d\Omega^2$$

$$\langle \psi_{BH} | \left(1 - R/r\right) | \psi_{BH} \rangle \neq 0 \propto G_{BH}$$

$$\langle \psi_{BH} | \frac{1}{1 - R/r} | \psi_{BH} \rangle \neq \infty \propto \frac{1}{G_{BH}}$$

So there's always some finite probability to escape from inside the BH.

BH's have hair

Classical no-hair thm.

$$ds^2 = -F(r)dt^2 + \frac{1}{F(r)}dr^2 + r^2 d\Omega_2^2$$

$$F(R_S) = 0, \quad F'(R_S) \neq 0$$

$$\frac{1}{\sqrt{-g}}\partial_\mu \sqrt{-g}\, g^{\mu\nu}\partial_\nu \phi = 0, \quad \text{BC} \begin{cases} \phi(\infty) = 0, \\ \phi'(\infty) = 0 \end{cases}$$

$$0 = \int_{R_S}^{\infty} dr\, \phi\, \partial_r\left(\sqrt{-g}\, g^{rr}\partial_r\phi\right) = \int_{R_S}^{\infty} dr\, \phi\, \partial_r\left[r^2 F(r)\partial_r\phi\right]$$

Integrate by parts: $0 = -\int_{R_S}^{\infty} dr\, (\partial_r\phi)^2 r^2 F(r) + \int_{R_S}^{\infty} \partial_r\left[\phi\, r^2 F(r)\partial_r\phi\right]$

$$0 = F(r)\, r^2\, \phi\phi'\Big|_{R_S}^{\infty} = \int_{R_S}^{\infty} dr\, (\partial_r\phi)^2 r^2 F(r)$$

$\phi(\infty) = 0$

$F(R_S) = 0 \quad \Rightarrow \quad \partial_r\phi = 0 \Rightarrow \phi(r) = \text{const}$

$\phi = 0 \quad (\phi(\infty) = 0)$

Hair with a fluctuating horizon:

$$\langle \Psi_{BH} | F(r)\, r^2 \phi'\phi | \Psi_{BH}\rangle \propto C_{BH} \neq 0$$

$$\langle \Psi_{BH} | \left(F(r)\, r^2 \phi'\phi\right)^2 | \Psi_{BH}\rangle \propto C_{BH} \neq 0$$

$\Rightarrow \phi$ not constant

redshift effectively finite $\Rightarrow$ Hair!

Summary:

Treating the BH as strictly classical

Equivalence principle ✓ $\Rightarrow$ Unitarity ✗

Unitarity ✓ $\Rightarrow$ Equivalence principle ✗

Message: treat BH as a quantum state

$$C_{BH} \doteq \frac{1}{S_{BH}} = \frac{\hbar G_N}{R_S^2} \quad \text{finite (small)}$$

Issues look much more relaxed.

Detailed discussion in papers.