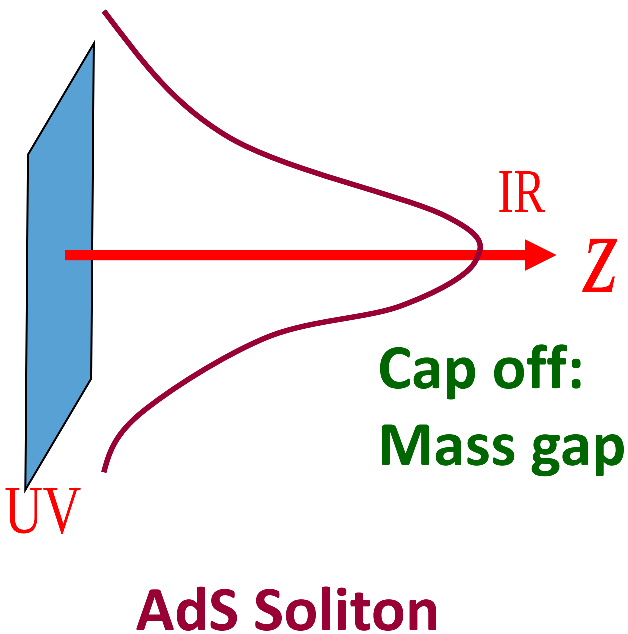
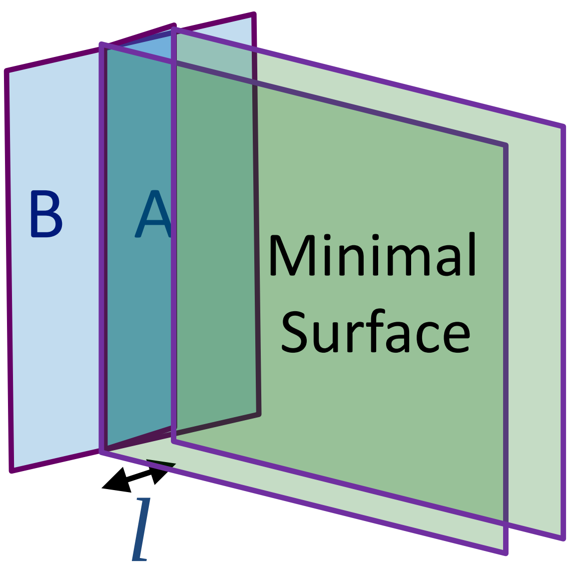
Confinement/deconfinement Phase Transitions

Here we study a confinement/deconfinement phase transition to see if the HEE can be an order parameter. One of the simplest gravity duals of confining gauge theories is the AdS soliton.

The AdS5 soliton ⇔ (2+1) dim. pure SU(N) gauge theory.



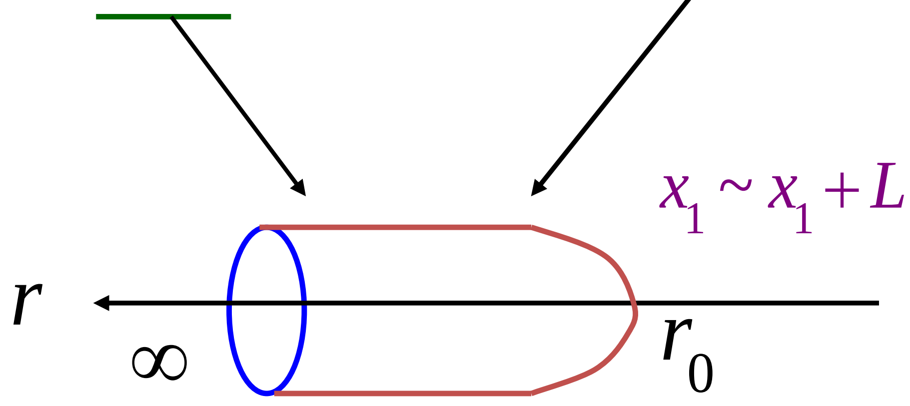


The metric of AdS soliton is given by the double Wick rotation of the AdS black hole solution.

$$ds^2_{\mathrm{AdS\,BH}} = \frac{R^2 dr^2}{r^2 f(r)} + \frac{r^2}{R^2}(-f(r)dt^2 + dx_1^2 + dx_2^2 + dx_3^2),$$

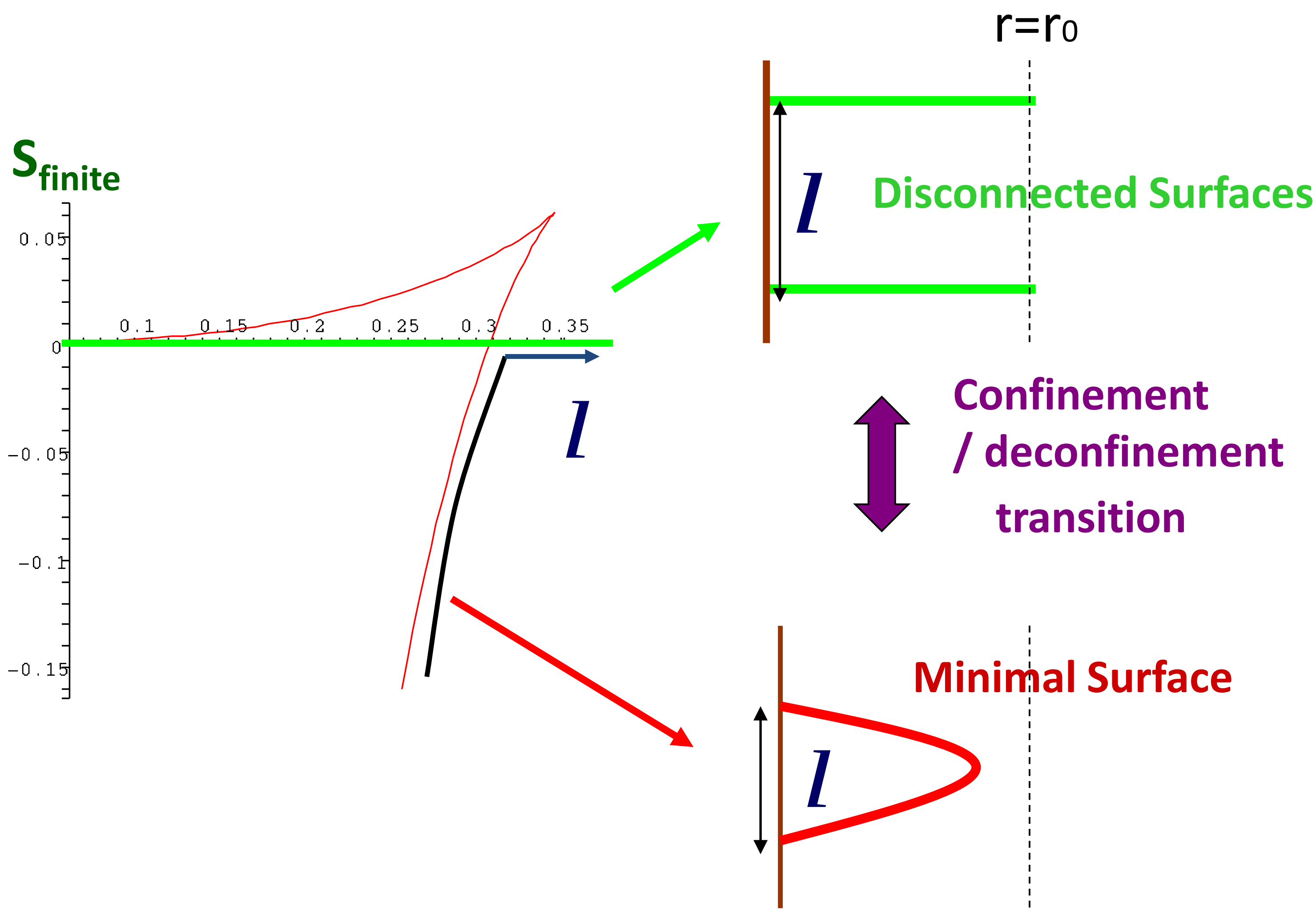
$$f(r) \equiv 1 - \frac{r_0^4}{r^4},$$

$$ds^2_{\mathrm{AdS\,Soliton}} = \frac{R^2 dr^2}{r^2 f(r)} + \frac{r^2}{R^2}(-dt^2 + f(r)dx_1^2 + dx_2^2 + dx_3^2),$$



In the holographic calculation, two different surfaces compete and this leads to the phase transition.

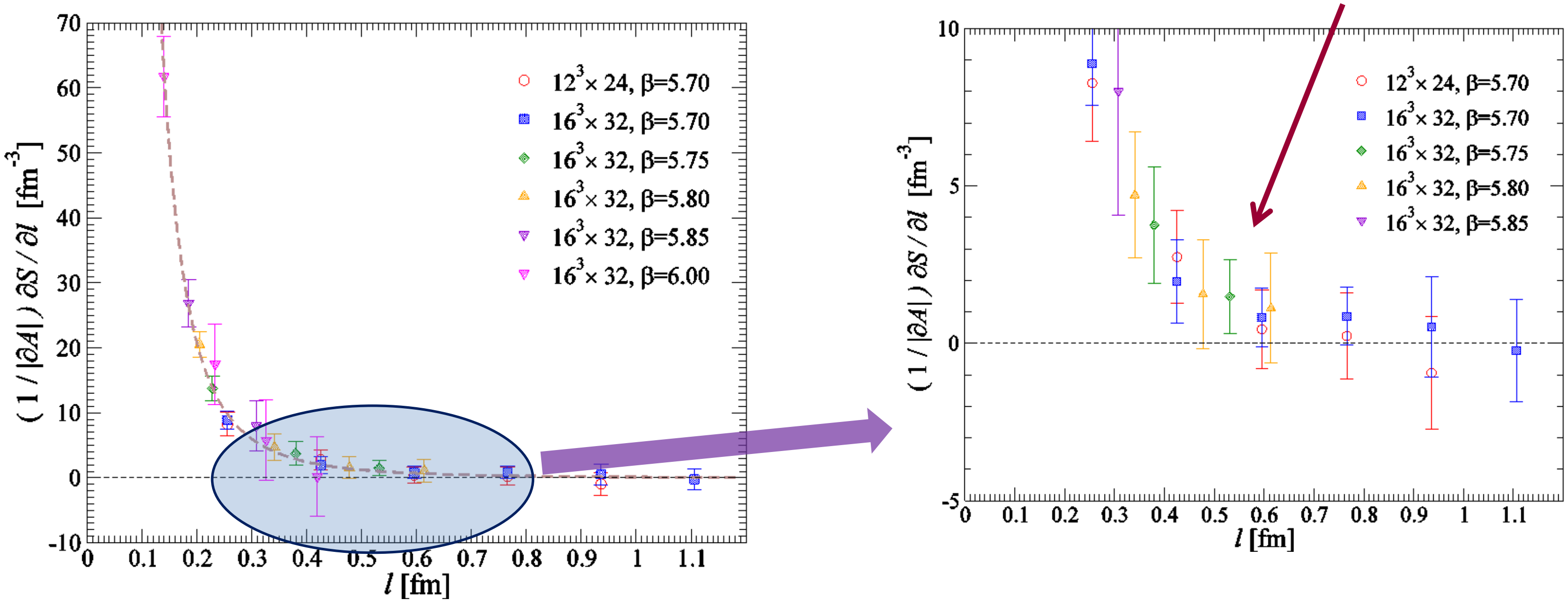
[Nishioka-TT 06', Klebanov-Kutasov-Murugan 07']



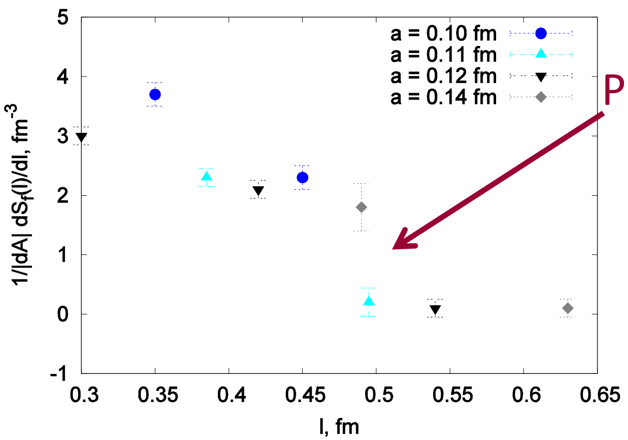
Lattice Results for 4D Pure YM

[4d SU(3): Nakagawa-Nakamura-Motoki-Zakharov 0911.2596]

Phase Transition



[4d SU(2): Buividovich-Polikarpov 0802.4247]



Phase Transition

[See for other calculations of EE in lattice gauge theory: Velytsky 0801.4111, 0809.4502; Buividovich-Polikarpov 0806.3376, 0811.3824]

⑧ AdS/MERA

(8-1) Basic Motivation

<u>Relation between the metric and wavefunction</u>

a CFT state ⇔ Information (~EE) = Minimal Areas ⇔ metric

$$|\Psi\rangle \qquad S_A \qquad \mathrm{Area}(\gamma_{\mathrm{A}}) \qquad g_{\mu\nu}$$

One candidate of such frameworks is so called the entanglement renormalization (MERA) [Vidal 05 (for a review see 0912.1651)] as pointed out by [Swingle 09].

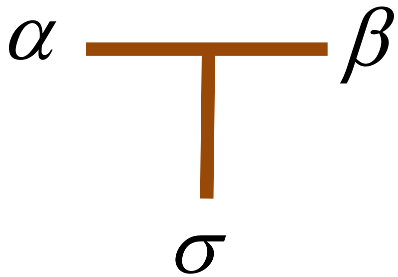
(8-2) Tensor Network (TN)

[See e.g. the review Cirac-Verstraete 09]

Recently, there have been remarkable progresses in numerical algorithms for quantum lattice models, based on so called *tensor product states*.

This leads to various nice variational ansatzs for the ground state wave functions in various quantum many-body systems.

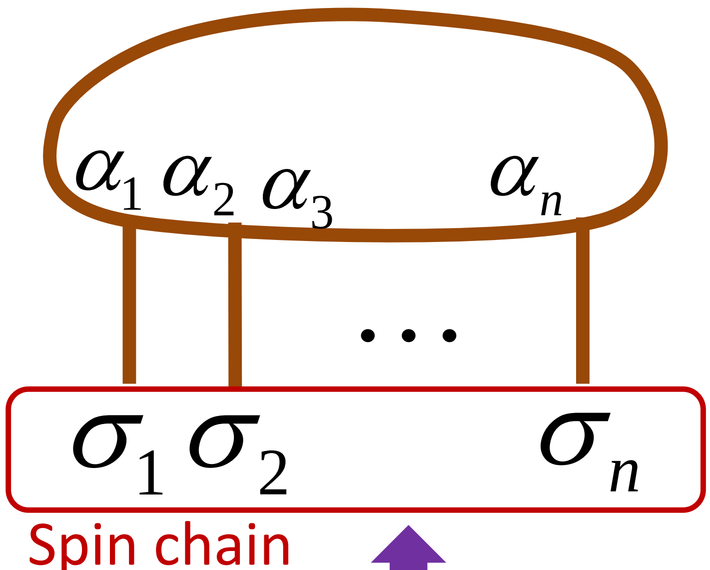
$\Rightarrow$ An ansatz is good if it respects the quantum entanglement of the true ground state.



$$\Longleftrightarrow \quad M_{\alpha\beta}(\sigma)$$

Ex. Matrix Product State (MPS)

[DMRG: White 92,...,
Rommer-Ostlund 95,..]



$$\alpha_i = 1,2,\ldots,\chi,$$

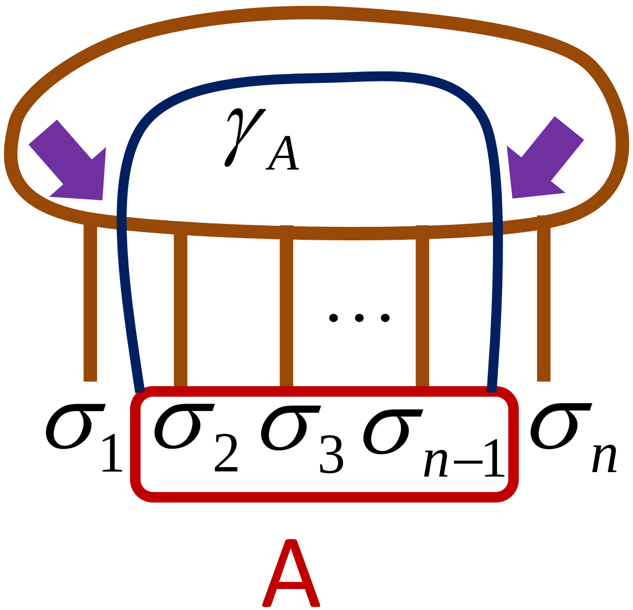
$$\sigma_i = \uparrow \text{ or } \downarrow .$$

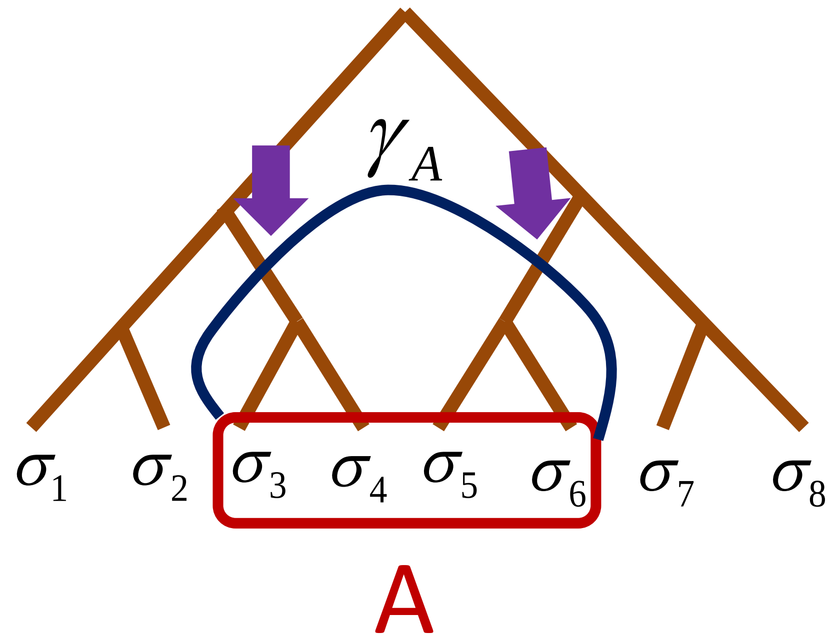
$$\Updownarrow$$

$$|\Psi\rangle = \sum_{\sigma_1,\sigma_2,\cdots,\sigma_n} \mathrm{Tr}[M(\sigma_1)M(\sigma_2)\cdots M(\sigma_n)]\,|\underbrace{\sigma_1,\sigma_2,\cdots,\sigma_n}_{n\ \mathit{Spins}}\rangle$$

MPS and TTN are not so suitable near quantum critical points (CFTs) because their entanglement entropies are too small:

$$S_A \leq 2\log\chi \quad (<< \log L \sim S_A^{CFT}).$$





In general,

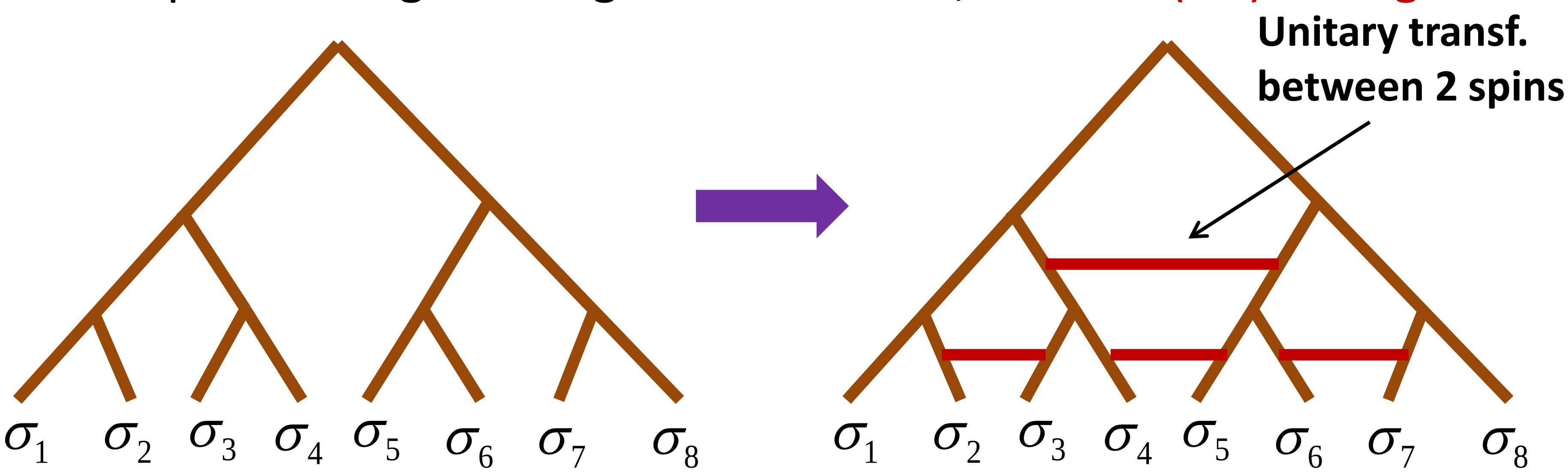
$$\boxed{\begin{aligned} &S_A \sim N_{\text{int}} \cdot \log\chi, \\ &N_{\text{int}} \equiv \min[\#\,\text{Intersections of } \gamma_A]. \end{aligned}}$$

(8-3) AdS/CFT and (c)MERA

<u>MERA</u> (Multiscale Entanglement Renormalization Ansatz):

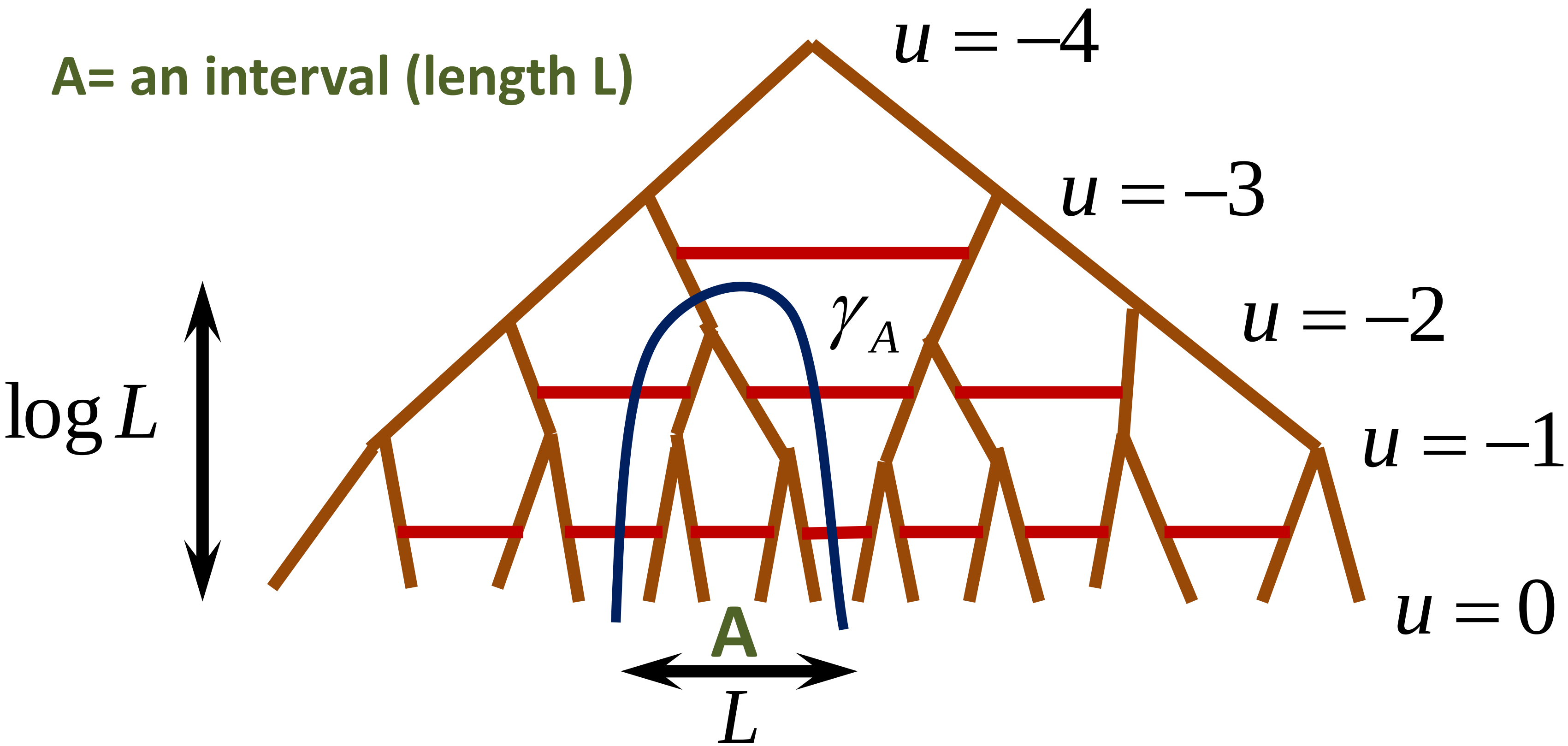
An efficient variational ansatz to find CFT ground states have been developed recently. [Vidal 05 (for a review see 0912.1651)].

To respect its large entanglement in a CFT, we add (dis)entanglers.



Calculations of EE in 1+1 dim. MERA

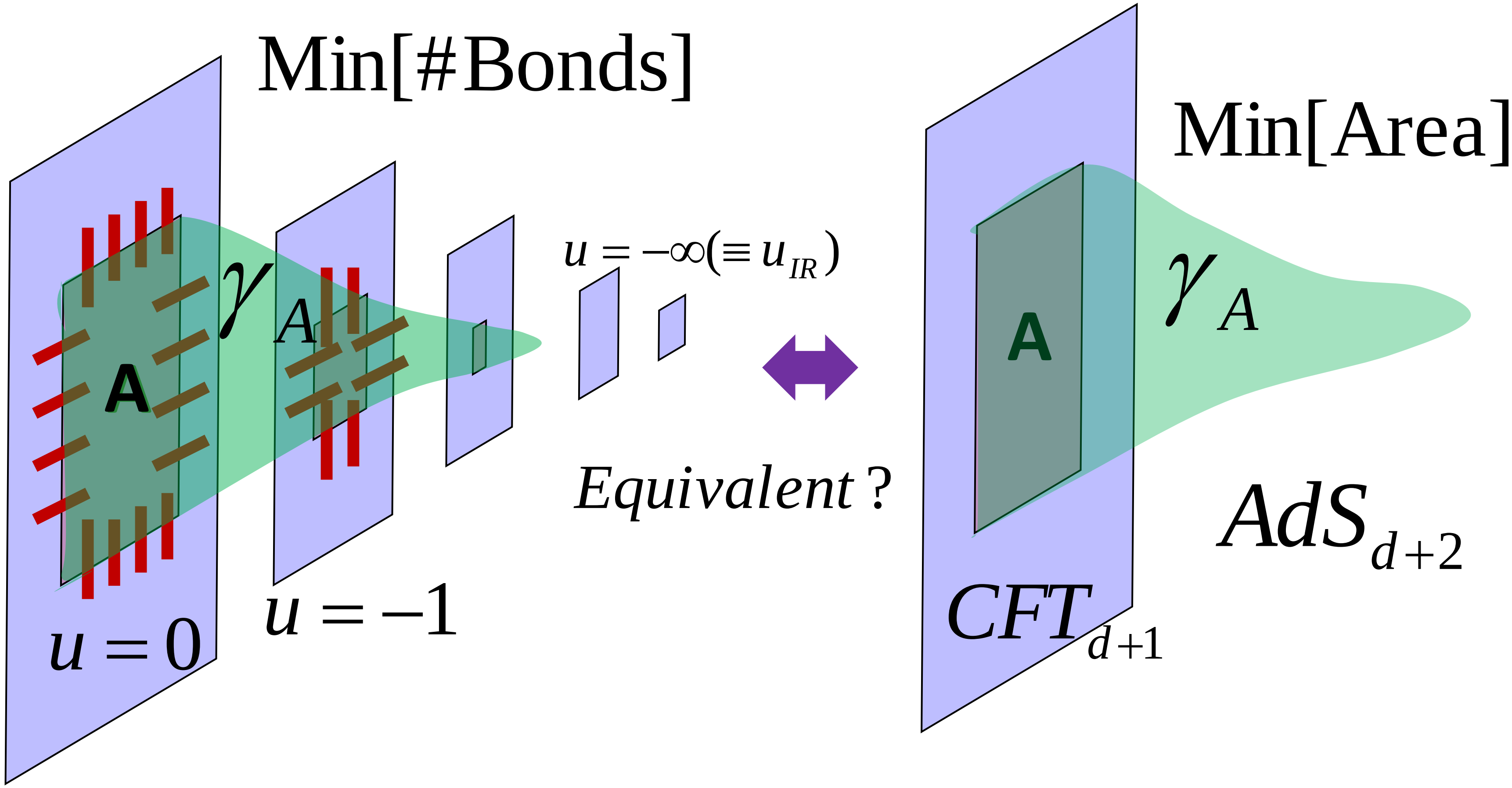
A= an interval (length L)



$$S_A \propto \mathrm{Min}[\#\mathrm{Bonds}] \propto \log L$$

$\Rightarrow$ agrees with 2d CFTs.

A conjectued relation to AdS/CFT [Swingle 09]



$$\text{Metric} = ds^2 + \frac{e^{2u}}{\varepsilon^2}(-dt^2 + d\vec{x}^2) = \frac{dz^2 - dt^2 + d\vec{x}^2}{z^2},$$

$$\textit{where} \quad z = \varepsilon \cdot e^{-u} \quad .$$

Now, to make the connection to AdS/CFT clearer, we would like to consider the MERA for quantum field theories.

Continuous MERA (cMERA)

[Haegeman-Osborne-Verschelde-Verstraete 11]

$$\underbrace{|\Psi(u)\rangle}_{\substack{\text{True ground state} \\ \text{(highly entangled)}}} = P \cdot \exp\left(-i\int_{u_{IR}}^{u} ds[K(s)+L]\right) \cdot \underbrace{|\Omega\rangle}_{\substack{\text{IR state} \\ \text{(no entanglement)}}} ,$$

$$\Rightarrow \text{Real space renormalization flow : length scale} \sim \varepsilon \cdot e^{-u}.$$

K(u) : disentangler, L: scale transformation

Conjecture

$$d+1 \ \text{dim. cMERA} = \text{gravity on AdS}_{\text{d+2}} \qquad z = \varepsilon \cdot e^{-u}.$$

(8-4) Emergent Metric from cMERA [Nozaki-Ryu-TT 12]

We focus on gravity duals of translational invariant static states, which are not conformal in general.

We conjecture that the metric in the extra direction is given by using the Bures metric (or Fisher information metric):

$$\boxed{g_{uu}du^2 = N\cdot\left(1-\left|\langle\Psi(u)|\,e^{iLdu}\,|\Psi(u+du)\rangle\right|^2\right).}$$

$$N^{-1}\equiv\int dx^d\cdot\int_0^{\Lambda e^u}dk^d =$$ The total volume of phase space at energy scale u.

Bures Metric

The **Bures distance** between two states is defined by

$$D(\psi_1,\psi_2) = 1 - \left|\langle \psi_1 | \psi_2 \rangle\right|^2 .$$

More generally, for two mixed states ρ1 and ρ2,

$$D(\rho_1,\rho_2) = 1 - \mathrm{Tr}\sqrt{\sqrt{\rho_1}\,\rho_2\sqrt{\rho_1}} .$$

When the state depends on the parameters {ξi},

the **Bures metric (Fisher information metric)** is defined as

$$D[\psi(\xi),\psi(\xi+d\xi)] = g_{ij}\,d\xi^i d\xi^j .$$

⇒ Reparameterization invariant (in our case: coordinate u)

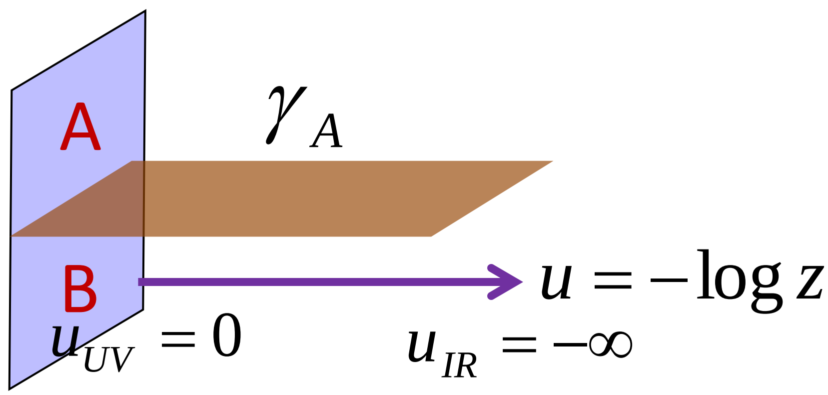
The operation e^{iLdu} removes the coarse-graining procedure to extract the strength of unitary transformations (disentanglers).

⇒ Our metric = the density of disentanglers

= the metric guu in the gravity dual

Understandable from the HEE:

$$S_A \sim \int_{u_{IR}}^{0} du \sqrt{g_{uu}} \cdot e^{(d-1)u}$$



(8-5) Emergent Metric in a (d+1) dim. Free Scalar Theory

Hamiltonian: $H = \frac{1}{2}\int dk^d [\pi(k)\pi(-k) + (k^2 + m^2)\phi(k)\phi(-k)].$

Ground state $|\Psi\rangle$: $a_k|\Psi\rangle = 0.$

Moreover, we introduce the `IR state' $|\Omega\rangle$ which has no real space entanglement.

$$a_x|\Omega\rangle = 0, \qquad a_x = \sqrt{M}\phi(x) + \frac{i}{\sqrt{M}}\pi(x),$$

$$\text{i.e.} \quad |\Omega\rangle = \prod_x |0\rangle_x \qquad a_x^+ = \sqrt{M}\phi(x) - \frac{i}{\sqrt{M}}\pi(x).$$

$$\Rightarrow \quad S_A = 0.$$

For a free scalar theory, the ground state corresponds to

$$\hat{K}(u) = \frac{i}{2}\int dk^d \left[\chi(u)\Gamma\left(ke^{-u}/M\right)a_k^+ a_{-k}^+ + (h.c.)\right],$$

where $\Gamma(\mathrm{x})$ is a cut off function: $\Gamma(\mathrm{x}) = \theta(1\text{-}|\,\mathrm{x}\,|)$.

$$\chi(s) = \frac{1}{2}\cdot\frac{e^{2u}}{e^{2u} + m^2/M^2}, \quad (\text{for } m = 0,\ \chi(\mathrm{u}) = 1/2.)$$

For the excited states, $\chi(s)$ becomes time-dependent.

One might be tempting to guess

Density of *bonds*

$$\boxed{ds^2_{Gravity} = g_{uu}du^2 + \frac{e^{2u}}{\varepsilon^2}\cdot d\vec{x}^2 - g_{tt}dt^2} \Rightarrow \sqrt{g_{uu}} \propto |\,\chi(u)\,| \quad ?$$

Indeed, the previous proposal for guu lead to $g_{uu} = \chi(u)^2$.

Explicit metric

$$\boxed{ds^2_{Gravity} = g_{uu}du^2 + \frac{e^{2u}}{\varepsilon^2}\cdot d\vec{x}^2 - g_{tt}dt^2}$$

(i) Massless scalar (E=k)

$$g_{uu} = \frac{1}{4}, \qquad g_{tt} = g_{xx} \qquad \Rightarrow \text{ the pure } AdS$$

(ii) Lifshitz scalar (E=k$^{\nu}$)

$$g_{uu} = \frac{\nu^2}{4} \qquad \Rightarrow \text{ the Lifshitz geometry}$$

(iii) Massive scalar

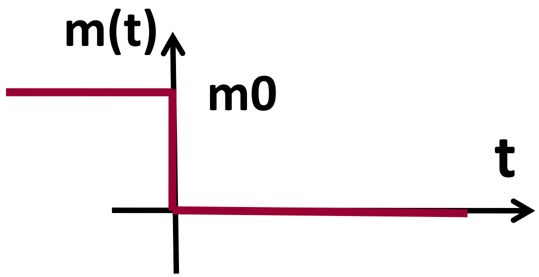
$$g_{uu} = \frac{e^{4u}}{4(e^{2u} + m^2/\Lambda^2)^2}, \qquad g_{tt} = g_{xx},$$

$$\Rightarrow ds^2 = \frac{dz^2}{z^2} + \left(\frac{1}{\Lambda^2 z^2} - \frac{m^2}{\Lambda^2}\right)(d\vec{x}^2 - dt^2).$$

Capped off in the IR z<1/m

(8-6) Excited states after quantum quenches

$$(A_k a_k + B_k a_{-k}^{+})\left|\Psi\right\rangle = 0, \qquad (|A_k|^2 - |B_k|^2 = 1).$$



$$A_k = \frac{1}{2}\left(\left(\frac{k^2+m0^2}{k^2}\right)^{1/4} + \left(\frac{k^2}{k^2+m0^2}\right)^{1/4}\right)\cdot e^{ikt},$$

$$B_k = \frac{1}{2}\left(\left(\frac{k^2+m0^2}{k^2}\right)^{1/4} - \left(\frac{k^2}{k^2+m0^2}\right)^{1/4}\right)\cdot e^{-ikt}.$$

To realized these states, we need to extend the ansatz such that

$$\hat{K}(u) = \frac{i}{2}\int dk^d\,\Gamma\left(ke^{-u}/M\right)\left[g(u)a_k^{+}a_{-k}^{+} + g^{*}(u)a_k a_{-k}\right],$$

$\Rightarrow$ SU(1,1) Bogoliubov transf. Mk(u)

$$(A_k(u), B_k(u)) = (\alpha_k, \beta_k)\cdot M_k(u).$$

For a given UV state $|\Psi\rangle$ or equally $M_k(0)$,
the intermediate state $|\Psi(u)\rangle$ or $M_k(u)$ is determined
up to an ambiguity.

This stems from the phase factor ambiguity of wave function:

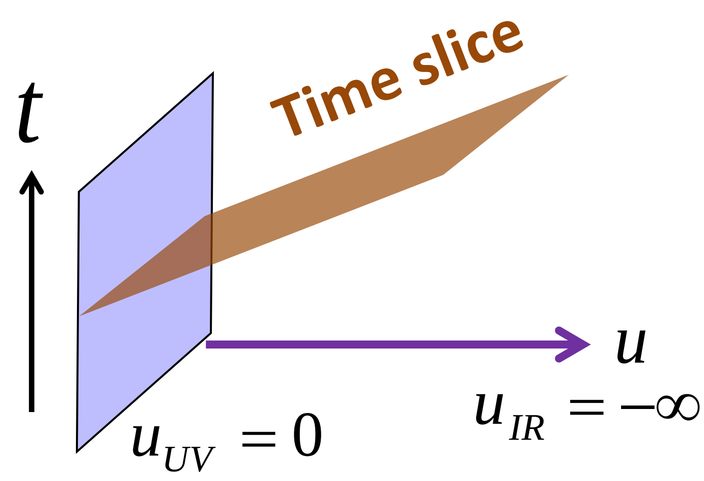
$$(A_k a_k + B_k a_{-k}^{+})|\Psi\rangle = 0 \quad \Rightarrow \quad e^{i\theta_k(t)} \cdot (A_k a_k + B_k a_{-k}^{+})|\Psi\rangle = 0 \ .$$

Our conjecture:

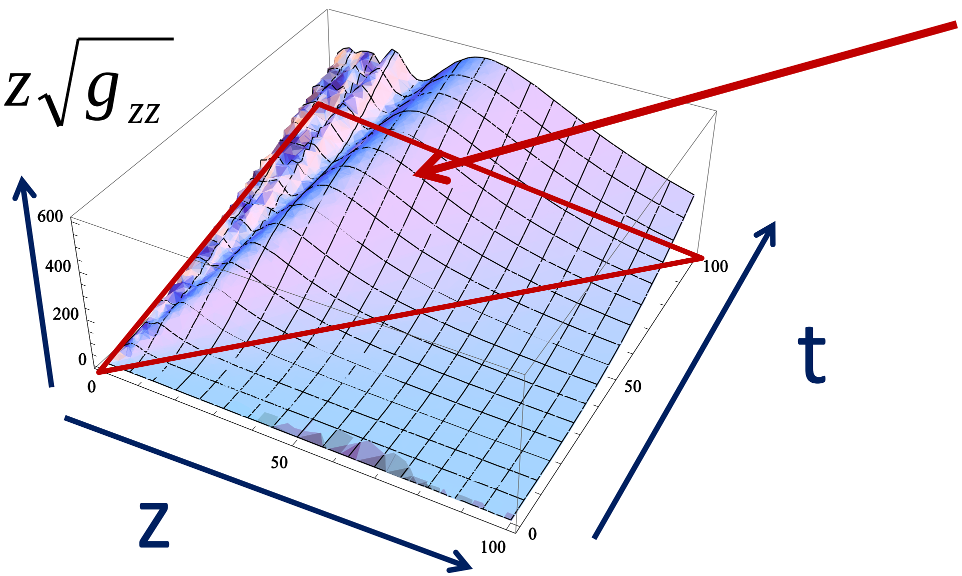
the phase ambiguity $\theta_k(t)$

⇔ the choice of the time slice

$$F(t,u) = const.$$



Time dependent metric from the Quantum Quench



looks like a propagation of gravitational wave.

We can also (analytically) confirm the linear growth: $S_A \propto t$.

This is consistent with the known CFT (2d) [Calabrese-Cardy 05] and recent holographic results in 2 or higher dimensions

[Arrastia-Aparicio-Lopez 10 , Albash-Johnson 10, Balasubramanian-Bernamonti-de Boer-Copland-Craps- Keski-Vakkuri-Müller-Schäfer-Shigemori-Staessens 10, 11, Hartman-Maldacena 13, Liu-Suh 13]

(8-7) Towards Holographic Dual of Flat Space

If we consider the (almost) flat metric

$$ds^2 = e^{2u} du^2 + e^{2u} dx^2 \Rightarrow g_{uu} = e^{2u},$$

the corresponding dispersion relation reads

$$\chi(u) = \frac{1}{2} \cdot \left(\frac{k \partial_k E_k}{E_k} \right) \Bigg|_{k=\Lambda e^u} = e^u \qquad \Rightarrow E_k = e^k.$$

This leads to the **highly non-local** Hamiltonian:

$$H = \int dx^d \phi(x) e^{\sqrt{-\partial^2}} \phi(x).$$

[cf. Li-TT 10]