Quantum matter and gauge-gravity duality

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Subir Sachdev

Talk online at sachdev.physics.harvard.edu

"Complex entangled" states of quantum matter, *not* adiabatically connected to independent particle states

Gapped quantum matter

Z_2 Spin liquids, quantum Hall states

Conformal quantum matter

Graphene, ultracold atoms, antiferromagnets

Compressible quantum matter

Strange metals, Bose metals

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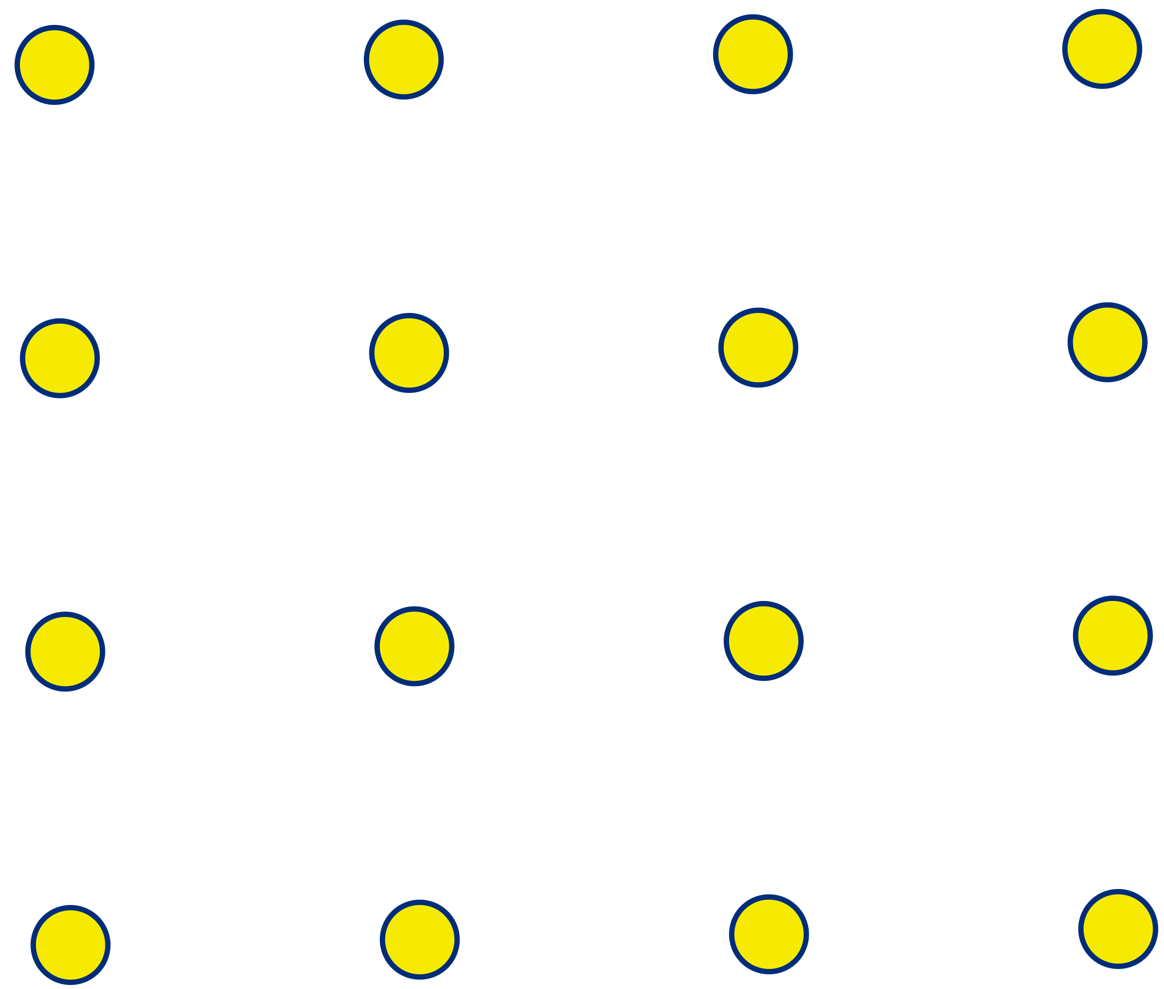
- Conformal systems are compressible in $d = 1$, but not for $d > 1$.

Compressible quantum matter

One compressible state is the **solid** (or "Wigner crystal" or "stripe").
This state breaks translational symmetry.



Compressible quantum matter

Another familiar compressible state is the **superfluid**.
This state breaks the global U(1) symmetry associated with $\mathcal{Q}$

Condensate of fermion pairs

Compressible quantum matter

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- Conjecture: *All* compressible quantum states which do not break the U(1) symmetry exhibit logarithmic violation of the area law of entanglement entropy

Compressible quantum matter

A. Fermi liquids:graphene

B. Holography: Reissner - Nördstrom solution

C. Non-Fermi liquids: Bose metals and U(1) spin liquids

D. Holography: scaling arguments for entropy and entanglement entropy

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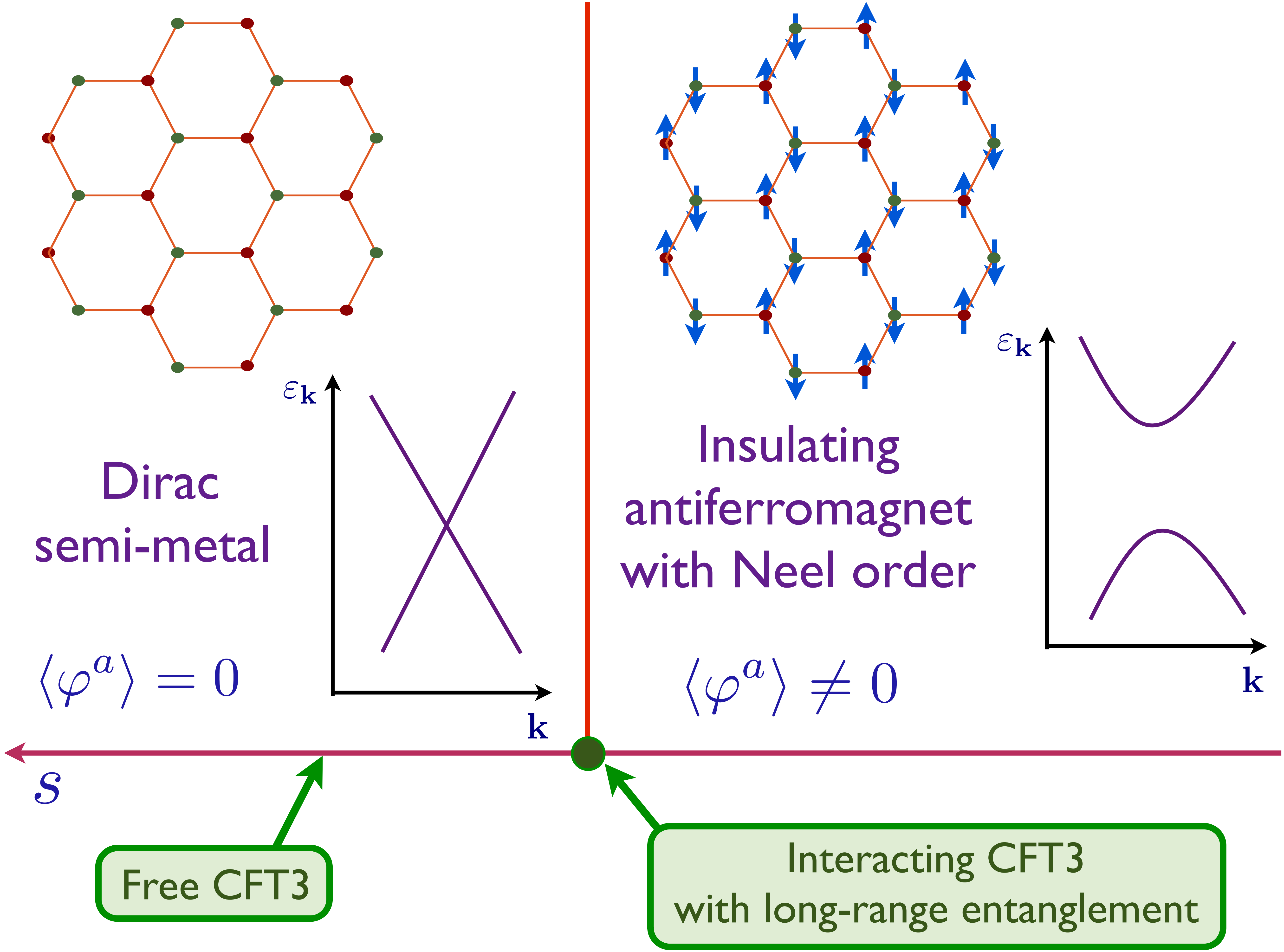
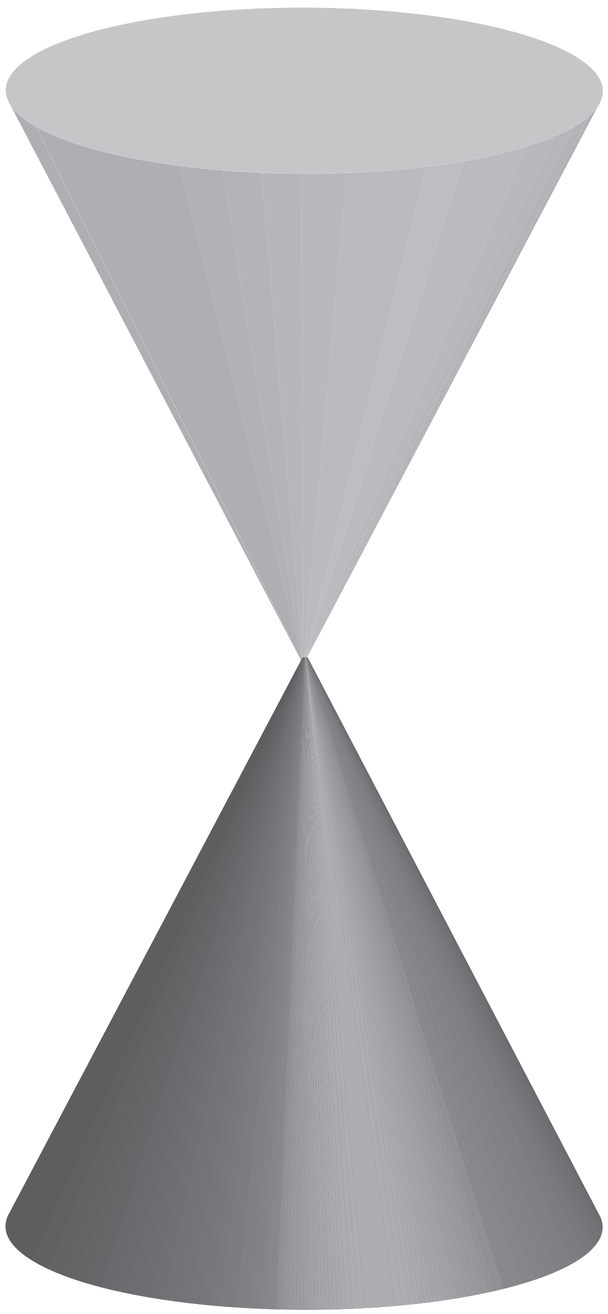


Dirac semi-metal
$\varepsilon_{\mathbf{k}}$
$\mathbf{k}$
$\langle \varphi^a \rangle = 0$
Insulating antiferromagnet with Neel order
$\varepsilon_{\mathbf{k}}$
$\mathbf{k}$
$\langle \varphi^a \rangle \neq 0$
s
Free CFT3
Interacting CFT3 with long-range entanglement

Quantum phase transition in graphene tuned by a chemical potential (gate voltage)



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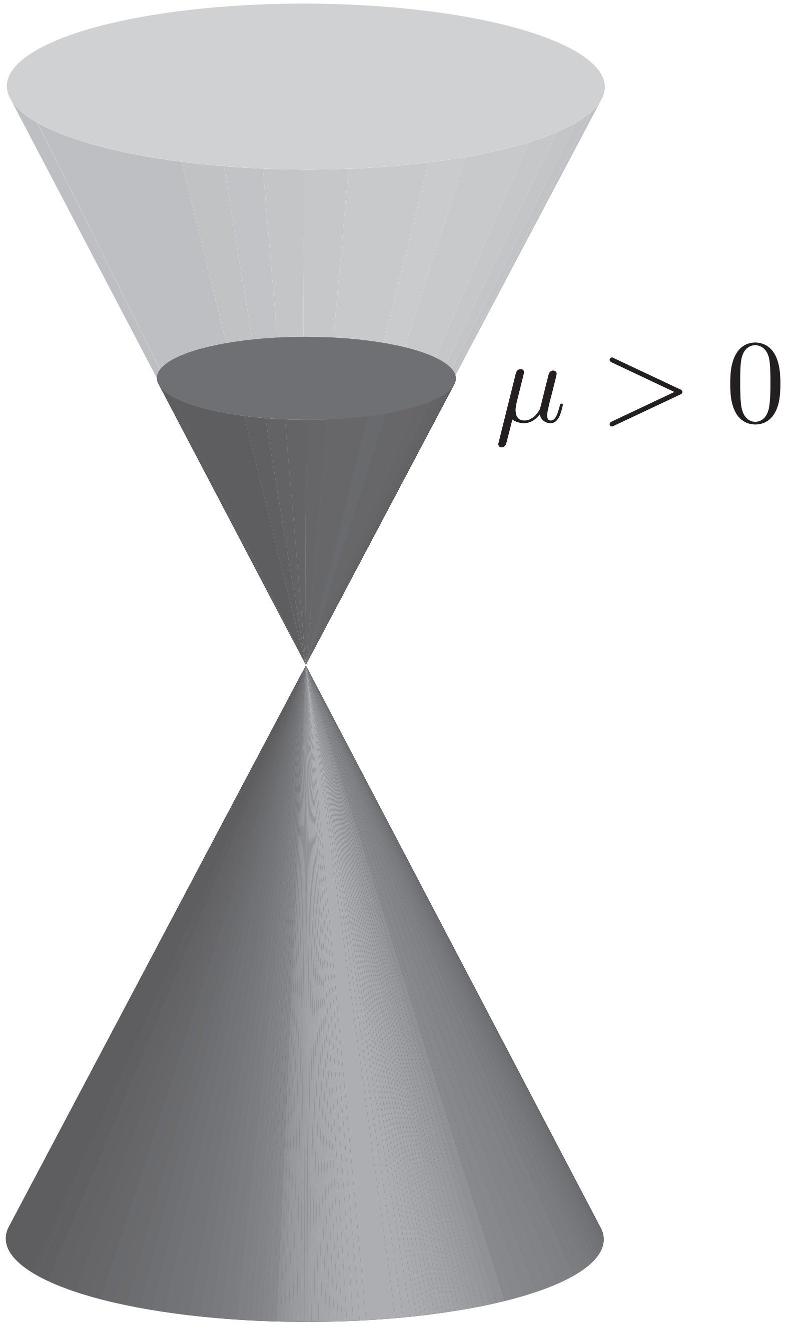
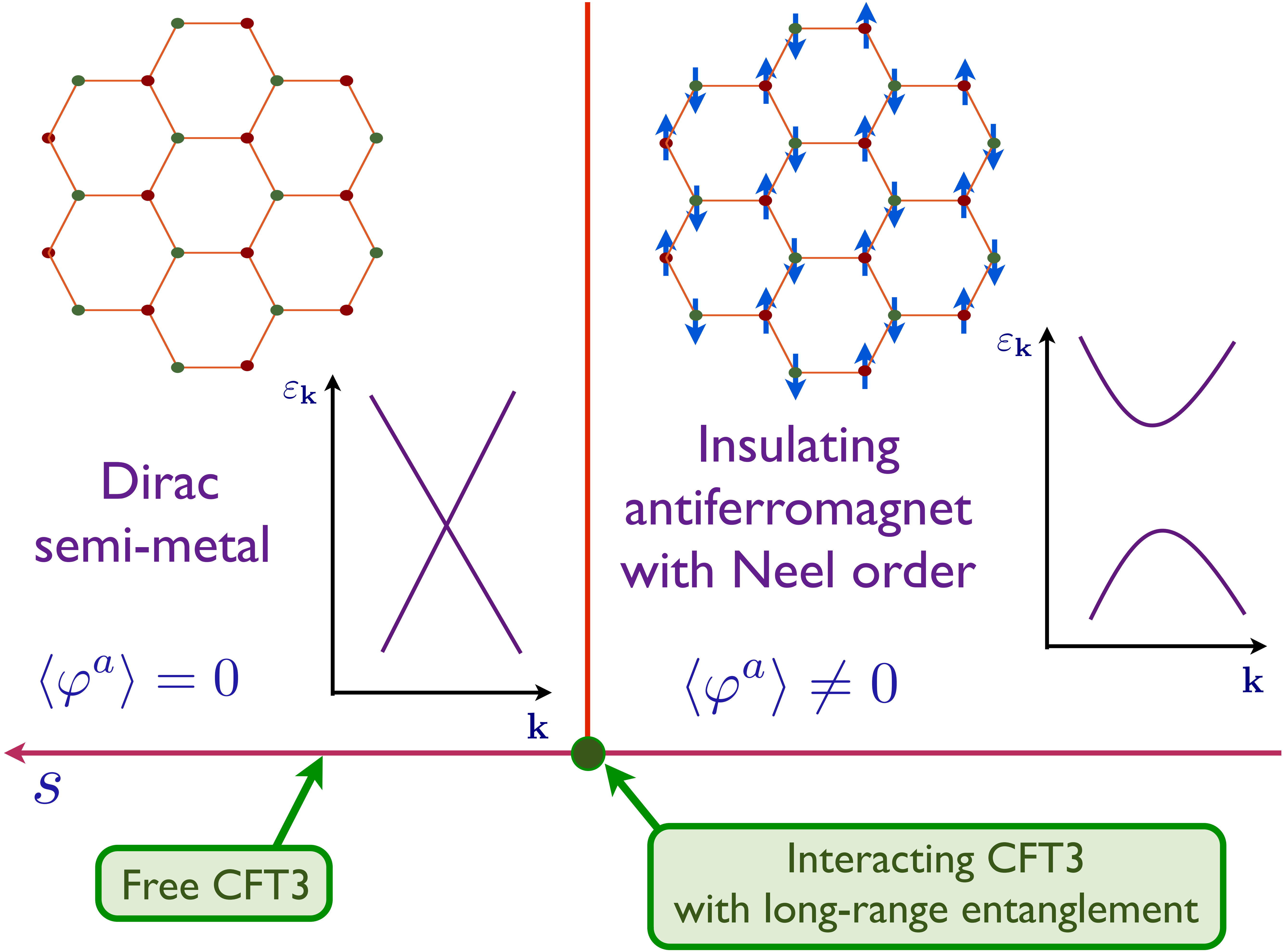


Electron
Fermi surface

Quantum phase transition in graphene tuned by a chemical potential (gate voltage)

Hole
Fermi surface

Electron
Fermi surface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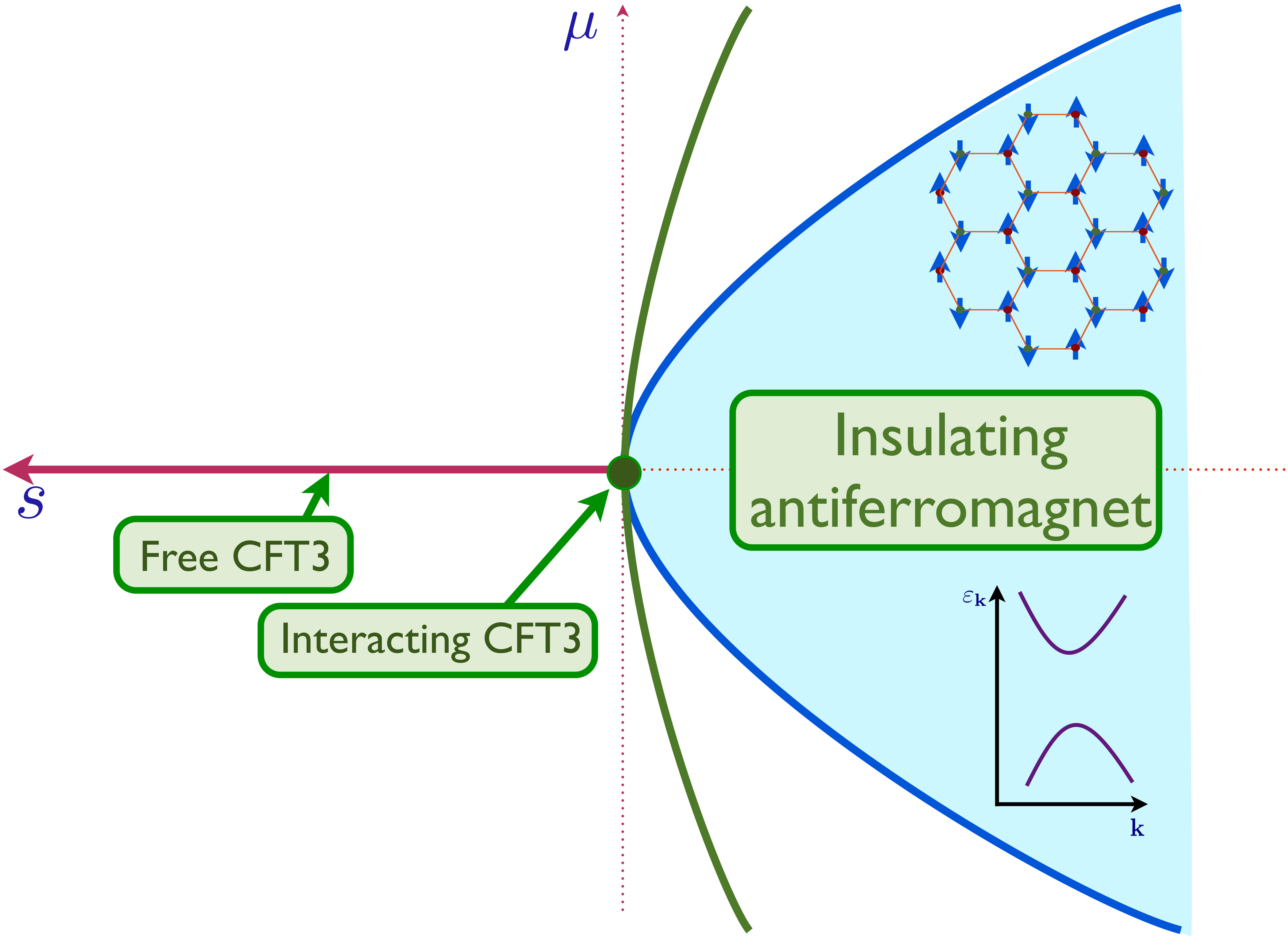

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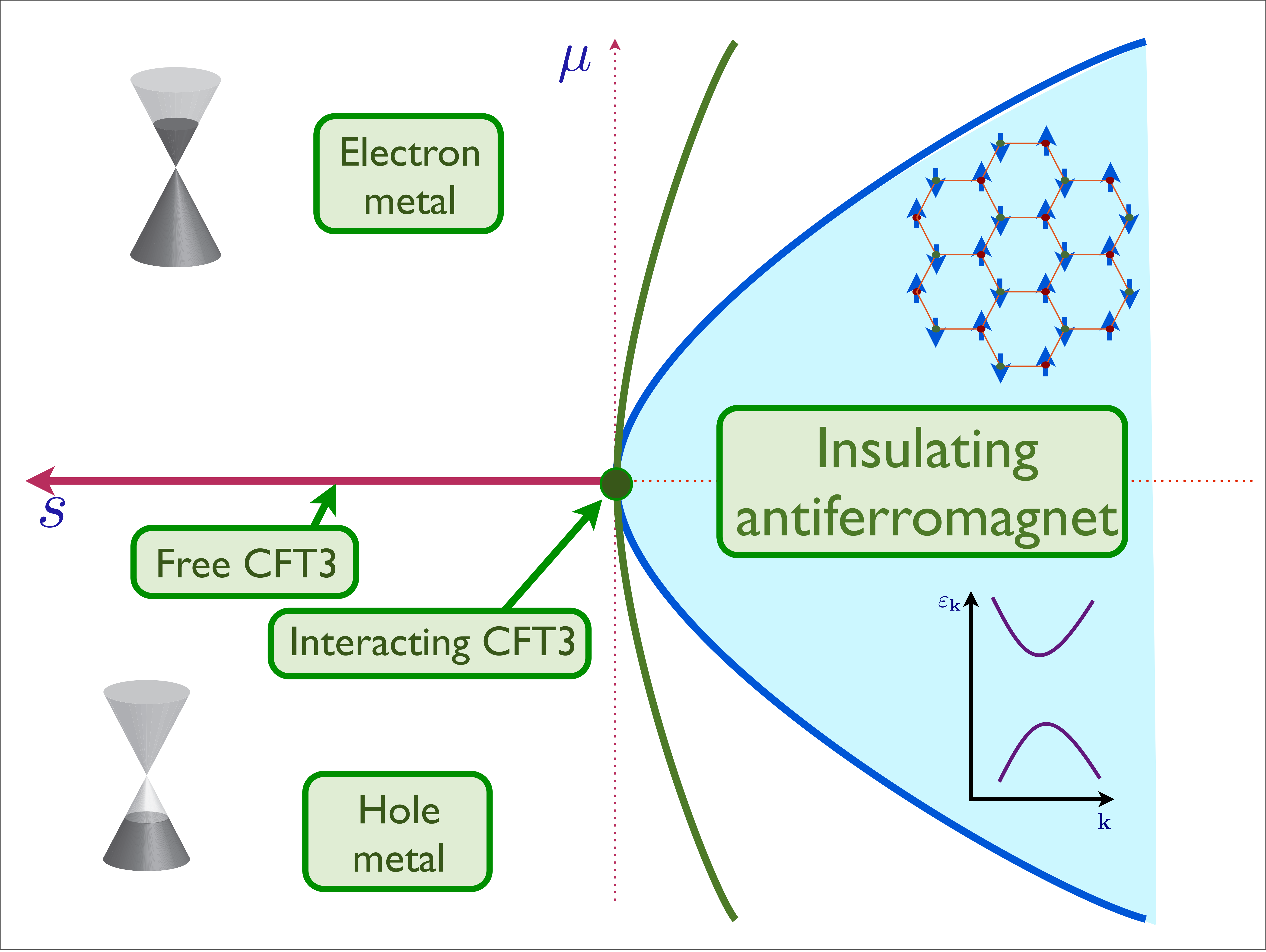
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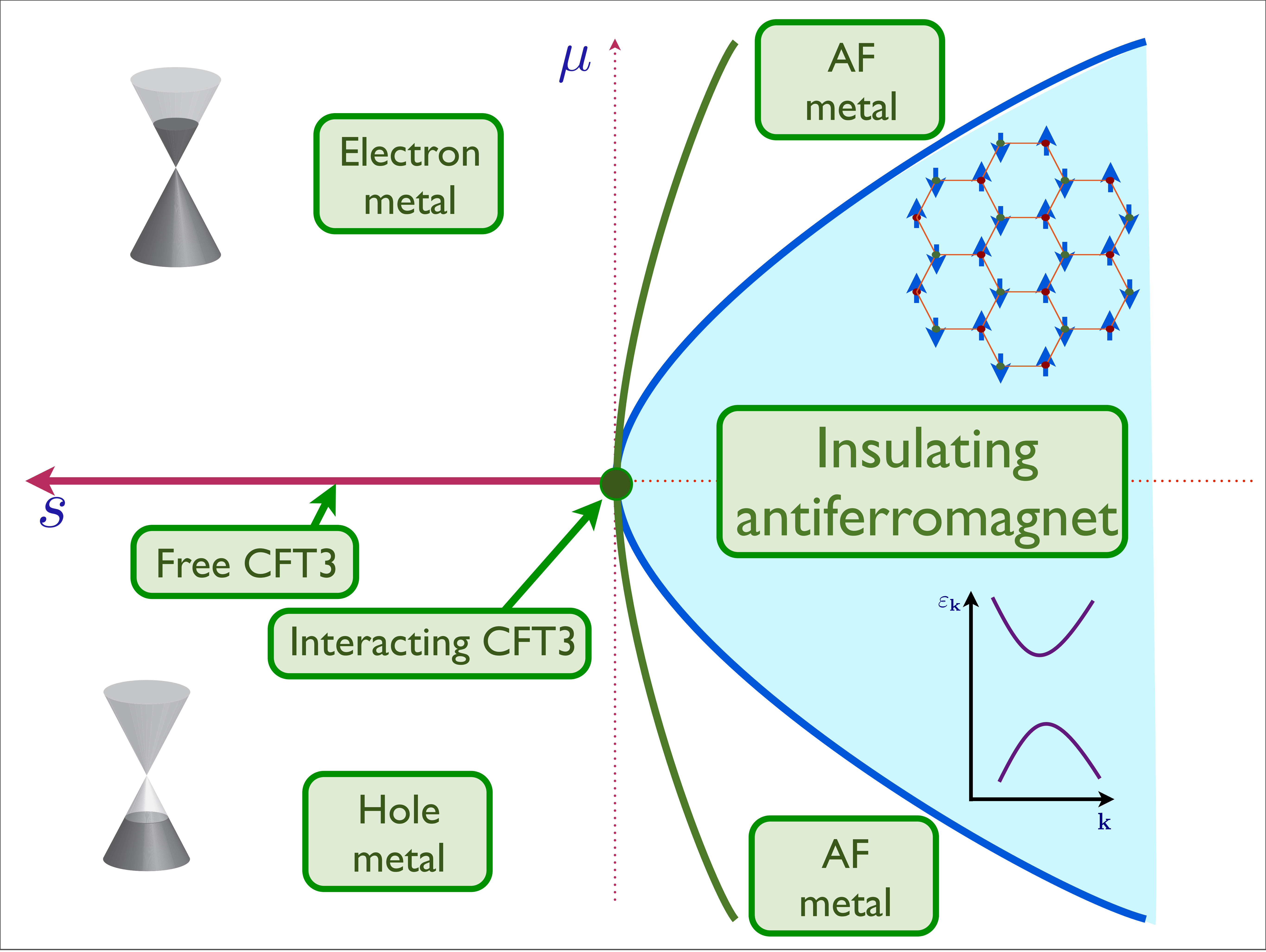

μ
AF metal
Electron metal
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S
Free CFT3
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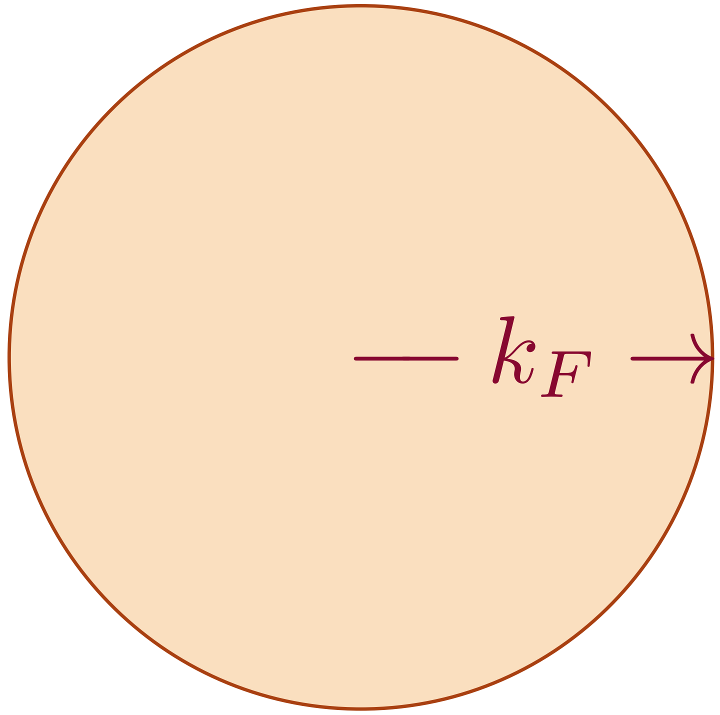
$\varepsilon_{\mathbf{k}}$
$\mathbf{k}$
Hole metal
AF metal

The Fermi liquid

$$\mathcal{L} = f^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f$$
$$+ \, 4 \text{ Fermi terms}$$



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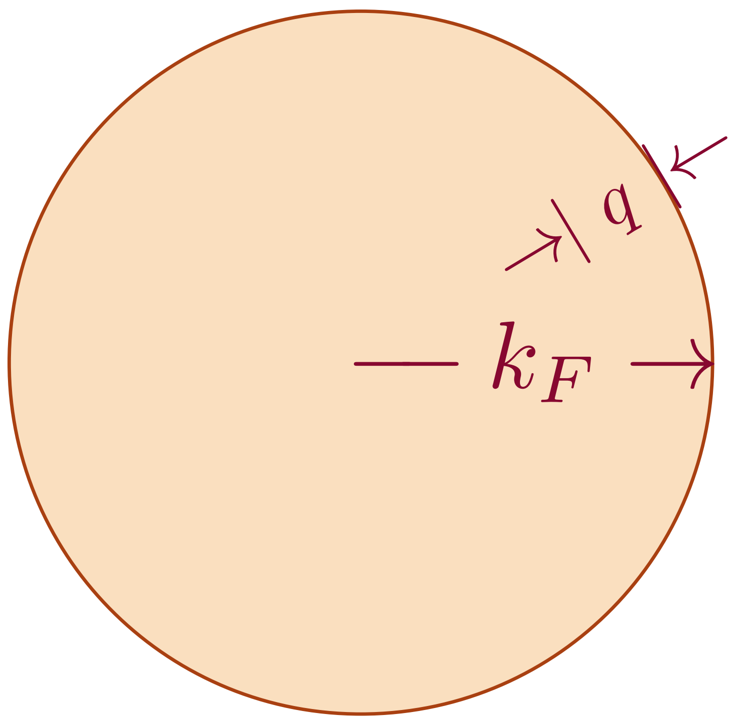

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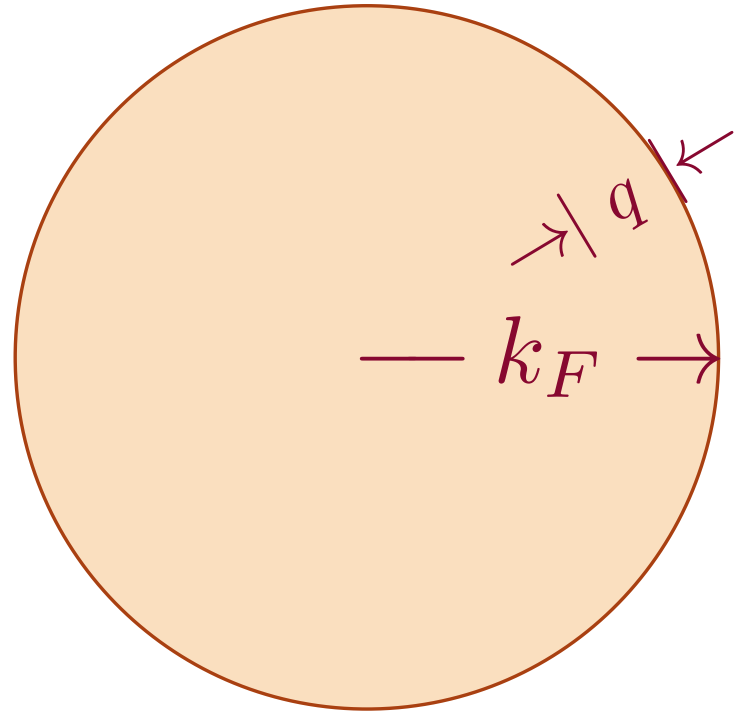
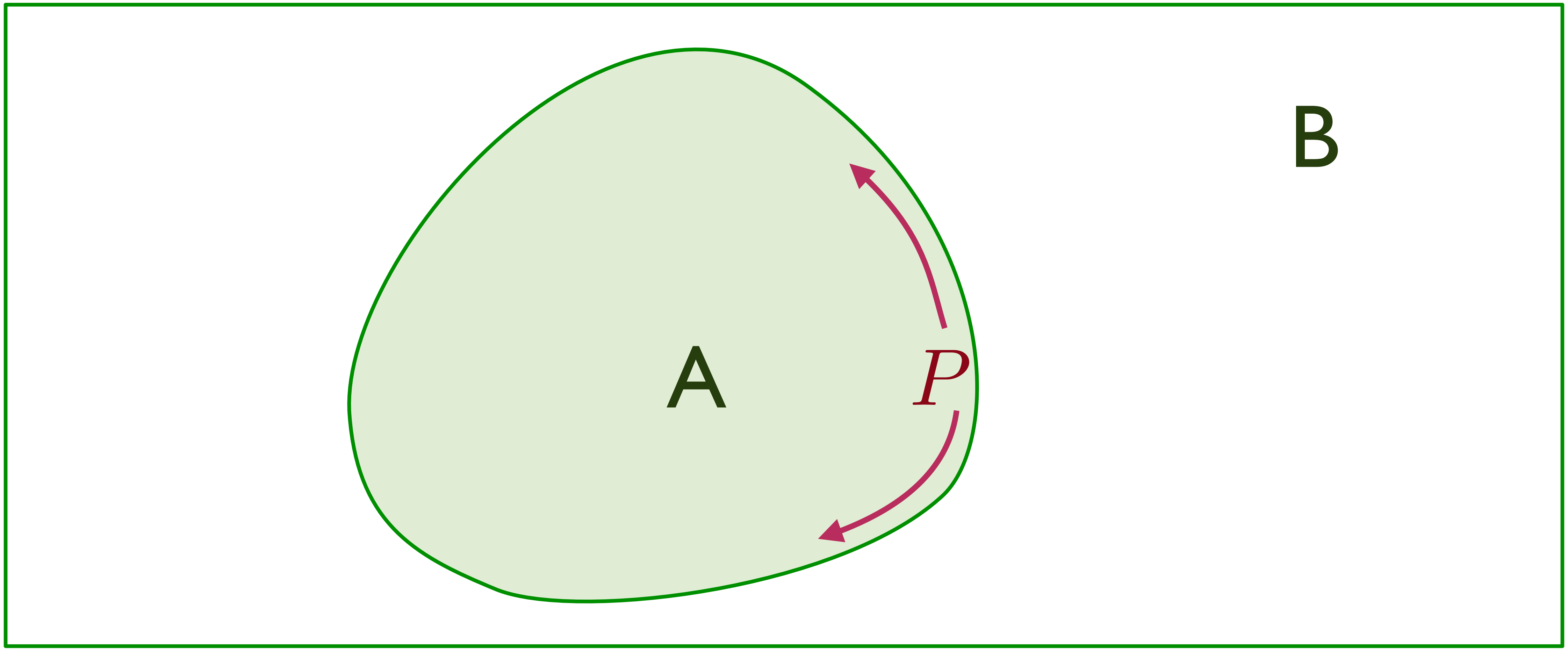


- Fermi wavevector obeys the Luttinger relation $k_F^d \sim \mathcal{Q}$, the fermion density
- Sharp particle and hole of excitations near the Fermi surface with energy $\omega \sim |q|^z$, with dynamic exponent $z = 1$.
- The phase space density of fermions is effectively one-dimensional, so the entropy density $S \sim T$. It is useful to write this is as $S \sim T^{(d-\theta)/z}$, with violation of hyperscaling exponent $\theta = d - 1$.

Entanglement entropy of the Fermi liquid



Logarithmic violation of "area law": $S_E = \frac{1}{12}(k_F P)\ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)

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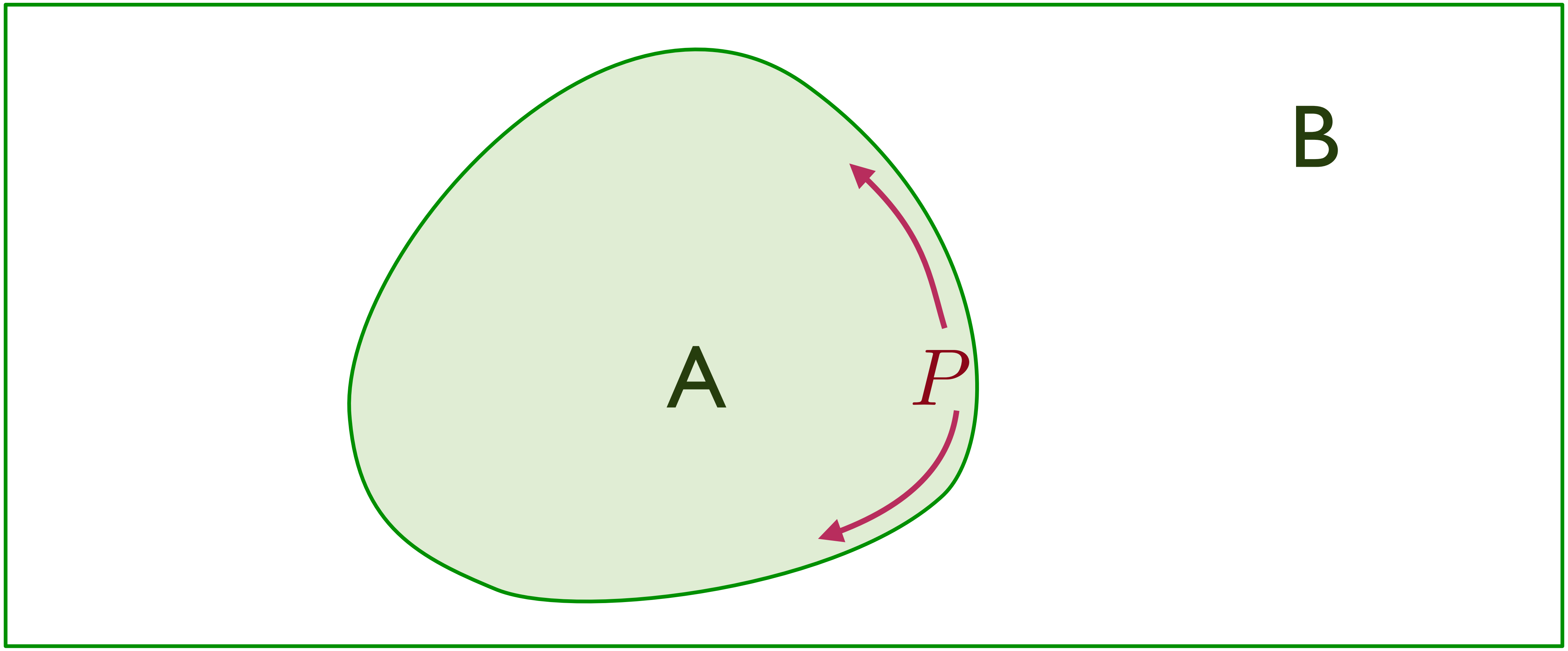

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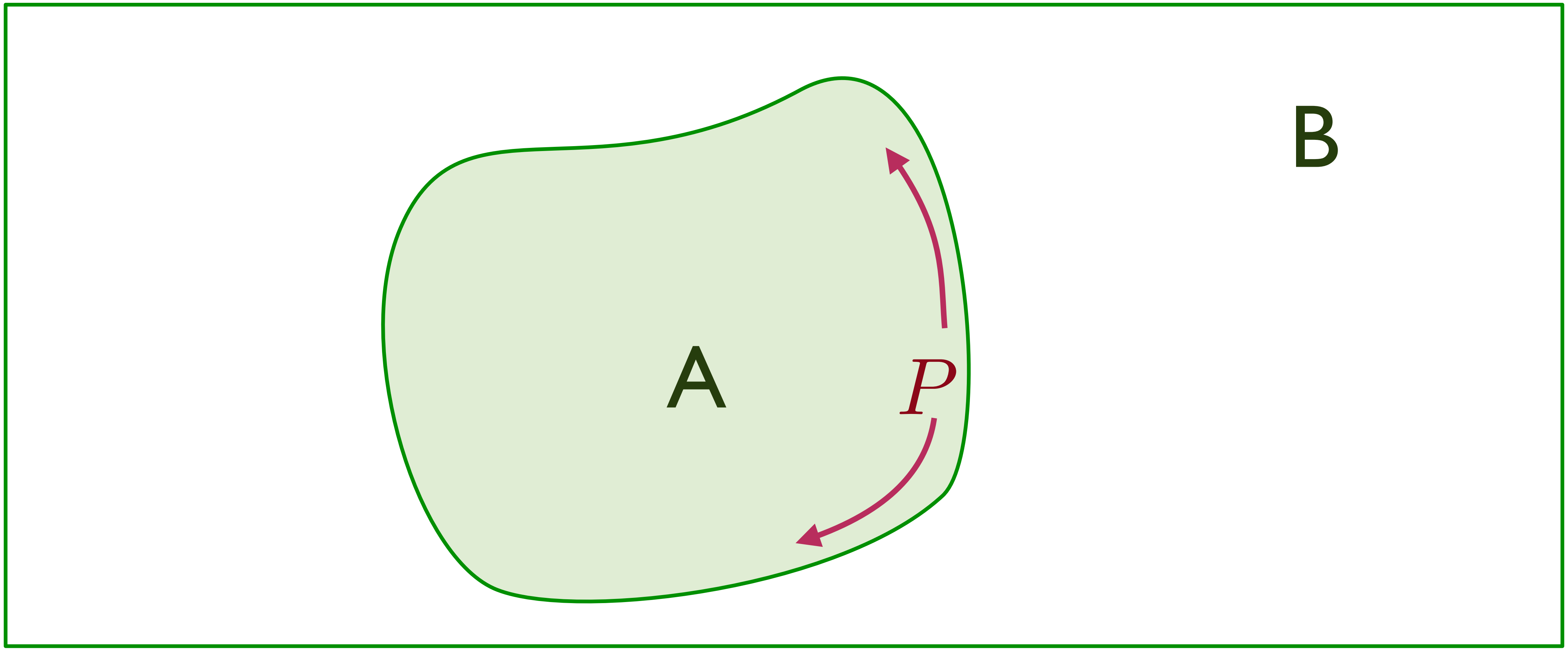

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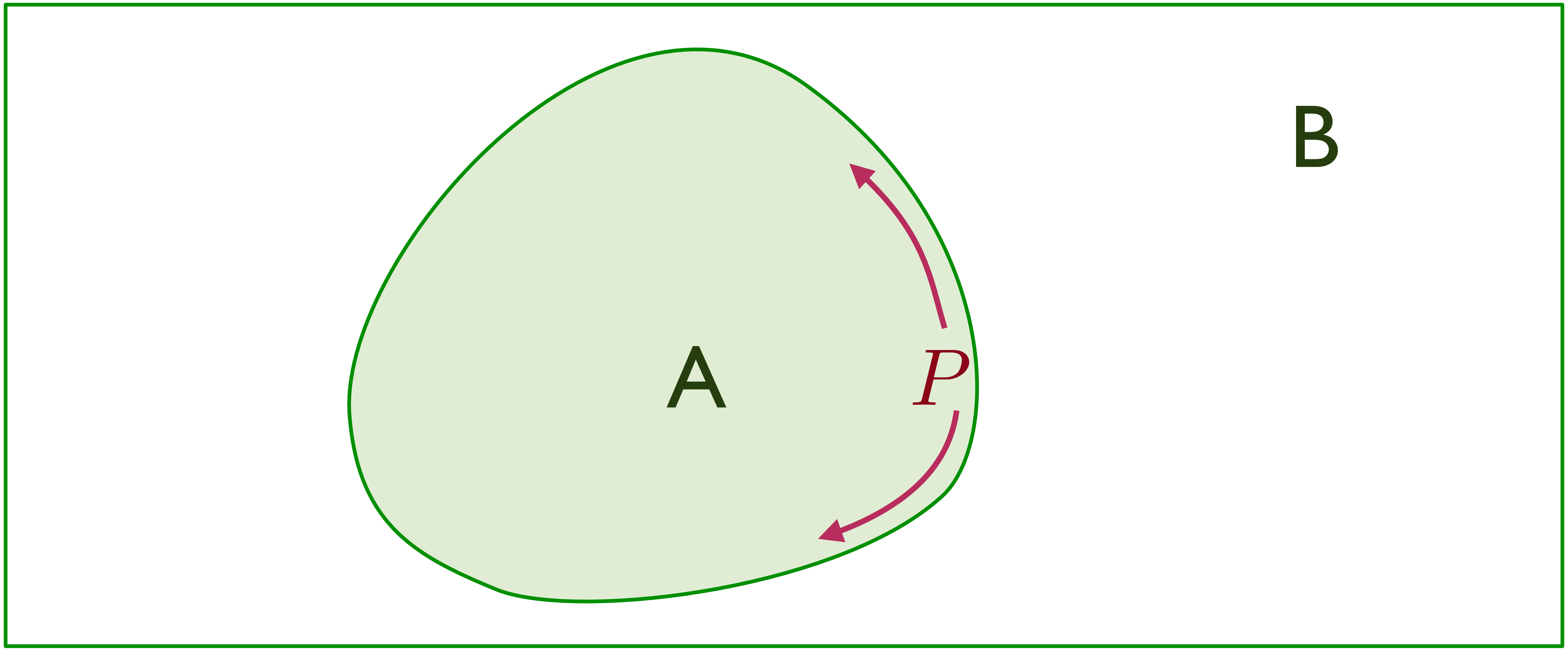

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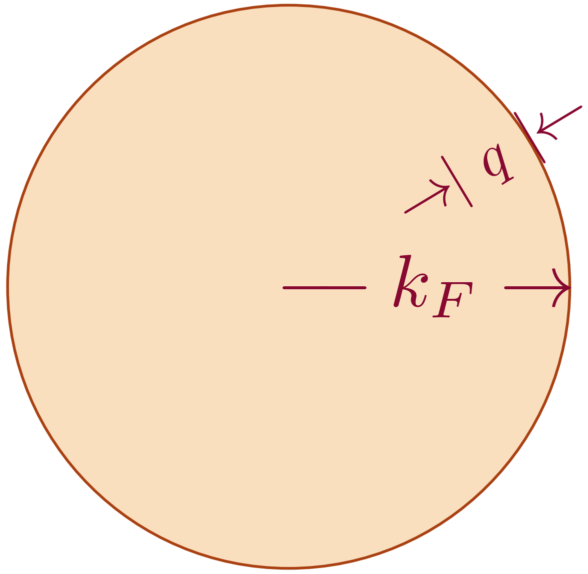
FL Fermi liquid



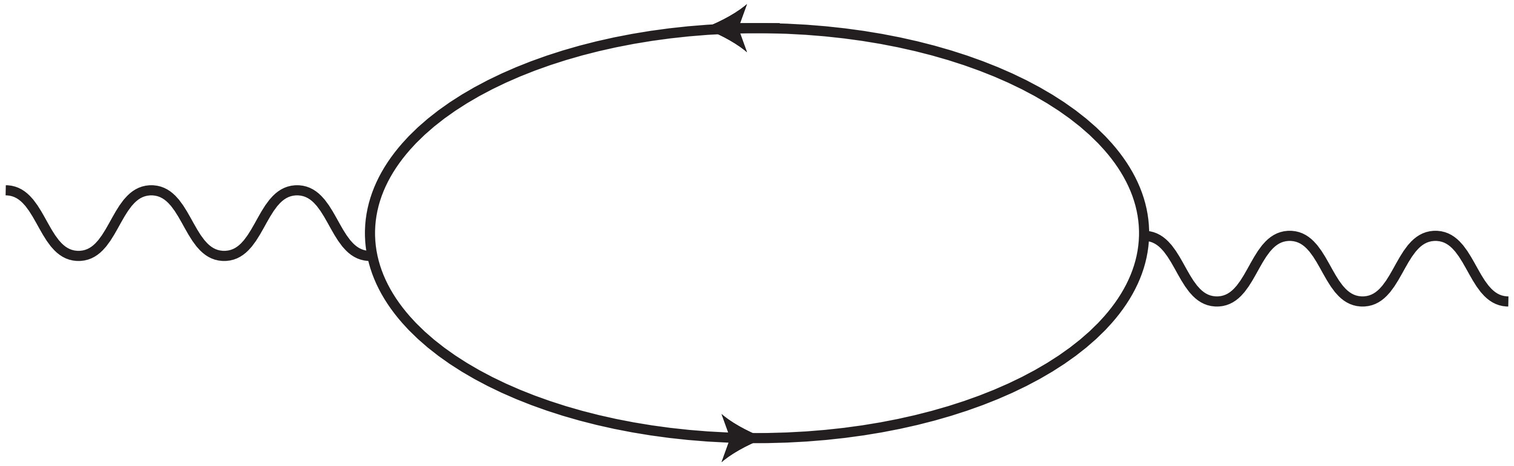
- $k_F^d \sim \mathcal{Q}$, the fermion density

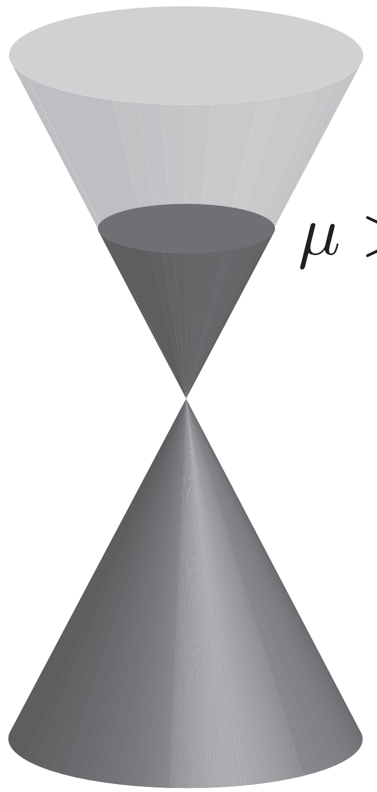
- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.

- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.

- Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.

Transport in graphene at non-zero μ





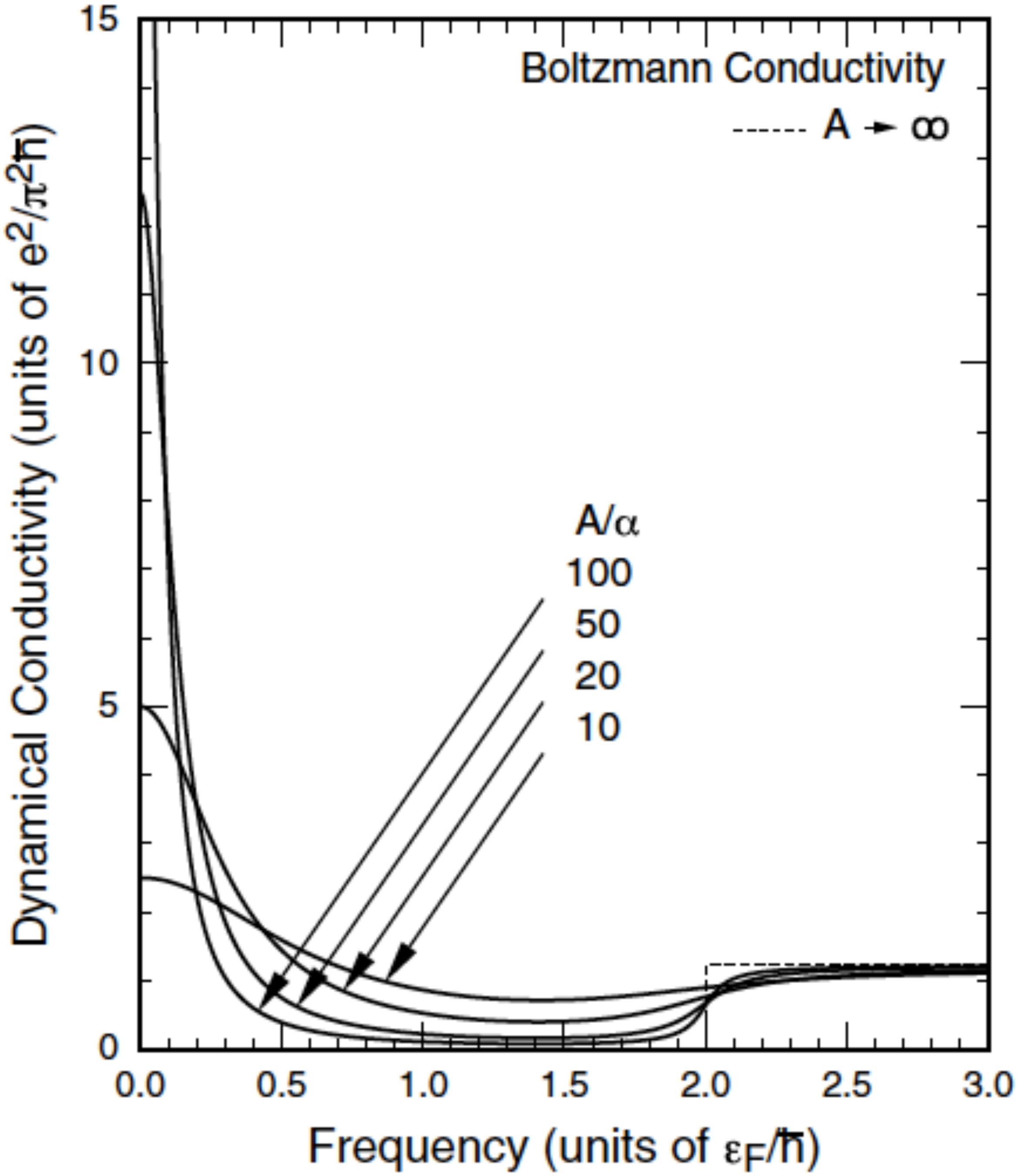
From the Kubo formula

$$\sigma(\omega) = 2\,(ev_F)^2\,\frac{\hbar}{i}\sum_{ss'}\int\frac{d^2k}{4\pi^2}\,\frac{f(\varepsilon_s(\mathbf{k})) - f(\varepsilon_{s'}(\mathbf{k}))}{(\varepsilon_s(\mathbf{k}) - \varepsilon_{s'}(\mathbf{k}))(\varepsilon_s(\mathbf{k}) - \varepsilon_{s'}(\mathbf{k}) + \hbar\omega + i\eta)}$$

where $\varepsilon_s(\mathbf{k}) = s\hbar v_F|\mathbf{k}|$ and $s, s' = \pm 1$ for the valence and conduction bands.

T. Ando, Y. Zheng and H. Suzuura, J. Phys. Soc. Jpn. **71** (2002) pp. 1318-1324

Transport in graphene at non-zero μ



A is inversely proportional to disorder. In the clean limit $A \to \infty$, at $T = 0$

$$\mathrm{Re}[\sigma(\omega)] = \frac{e^2}{\hbar}\left[\frac{\varepsilon_F}{\hbar}\delta(\omega) + \frac{1}{4}\theta(|\omega| - 2\varepsilon_F)\right]$$

Notice delta function is present even at $T = 0$ at non-zero density: this is a generic consequence of the conservation of momentum in any clean interacting Fermi liquid. Only "umklapp" scattering can broaden this delta function.

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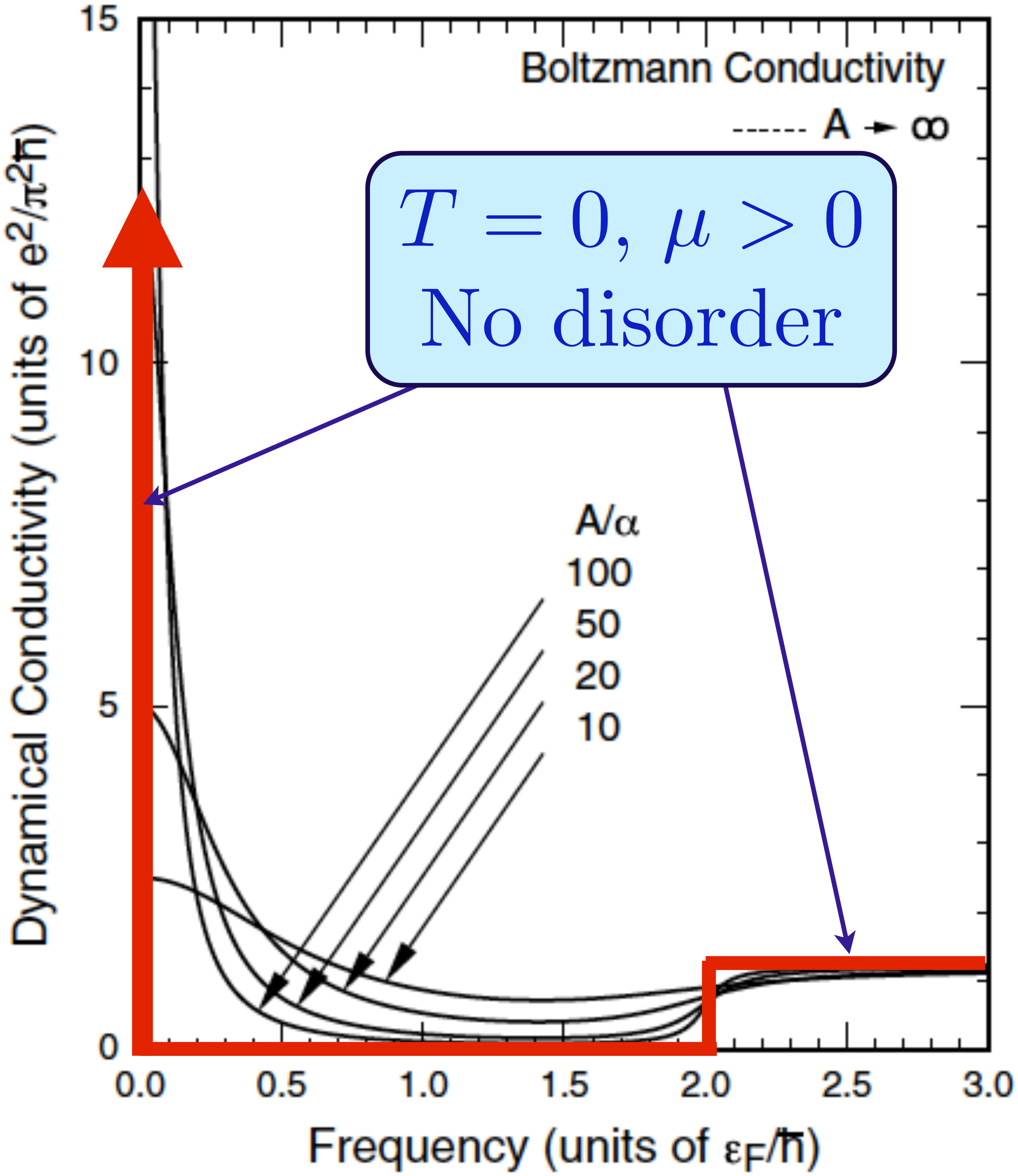

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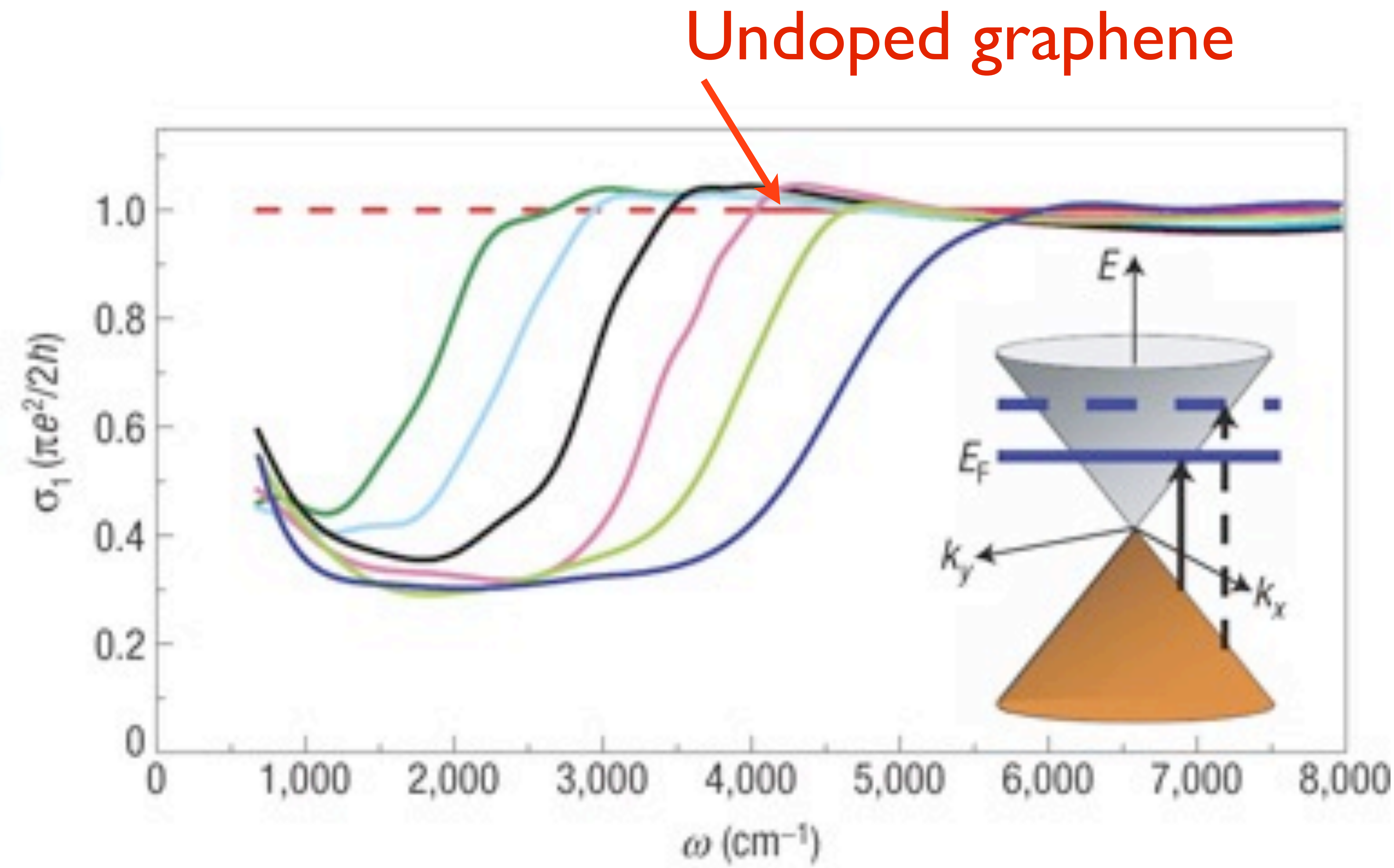
Optical conductivity of graphene



Z. Q. Li, E. A. Henriksen, Z. Jiang, Z. Hao, M. C. Martin, P. Kim, H. L. Stormer, and D. N. Basov, *Nature Physics* **4**, 532 (2008).

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B. Holography: Reissner - Nördstrom solution

C. Non-Fermi liquids: Bose metals and U(1) spin liquids

D. Holography: scaling arguments for entropy and entanglement entropy

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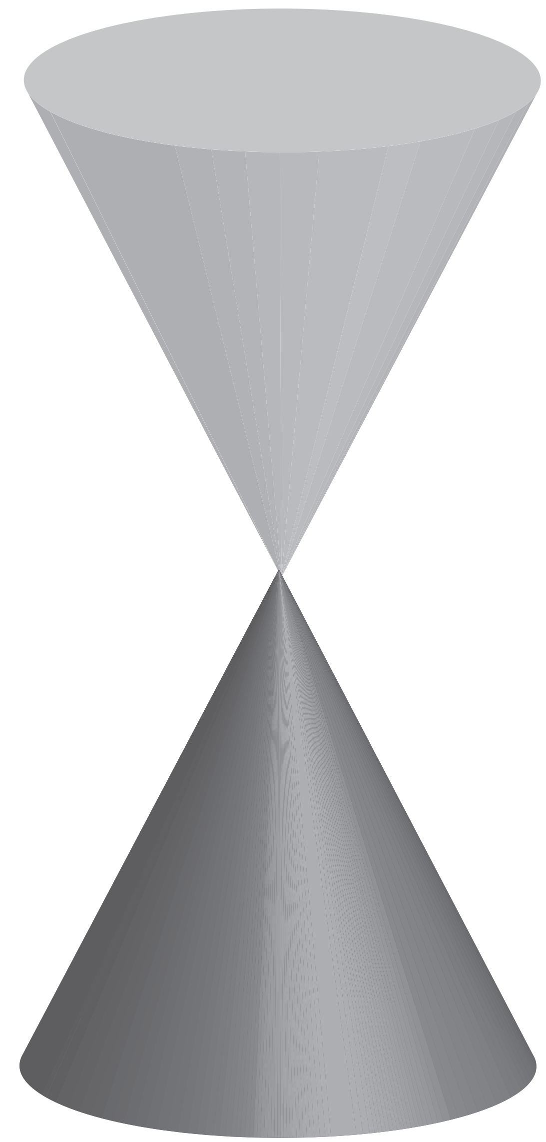
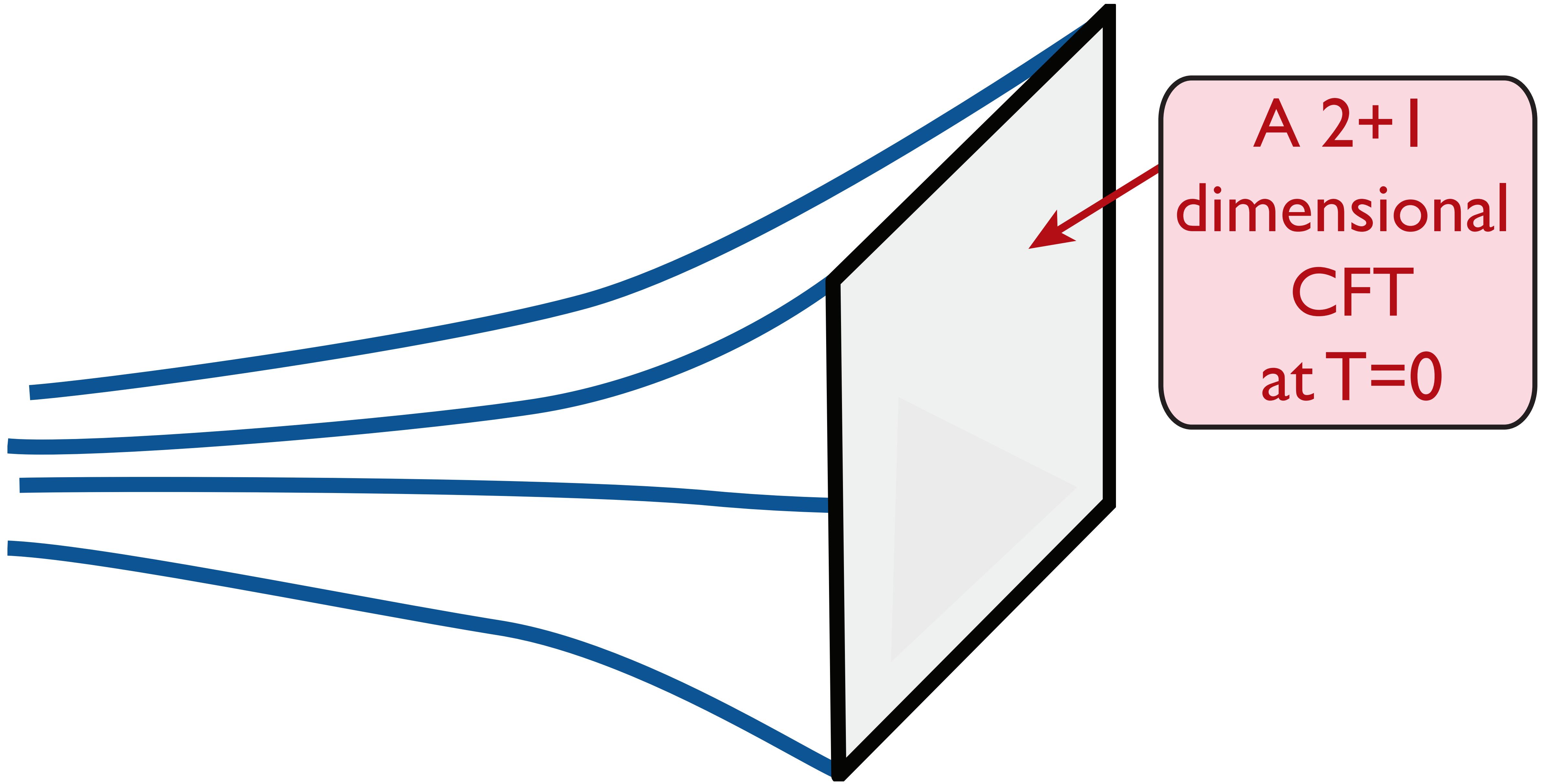
Begin with a CFT

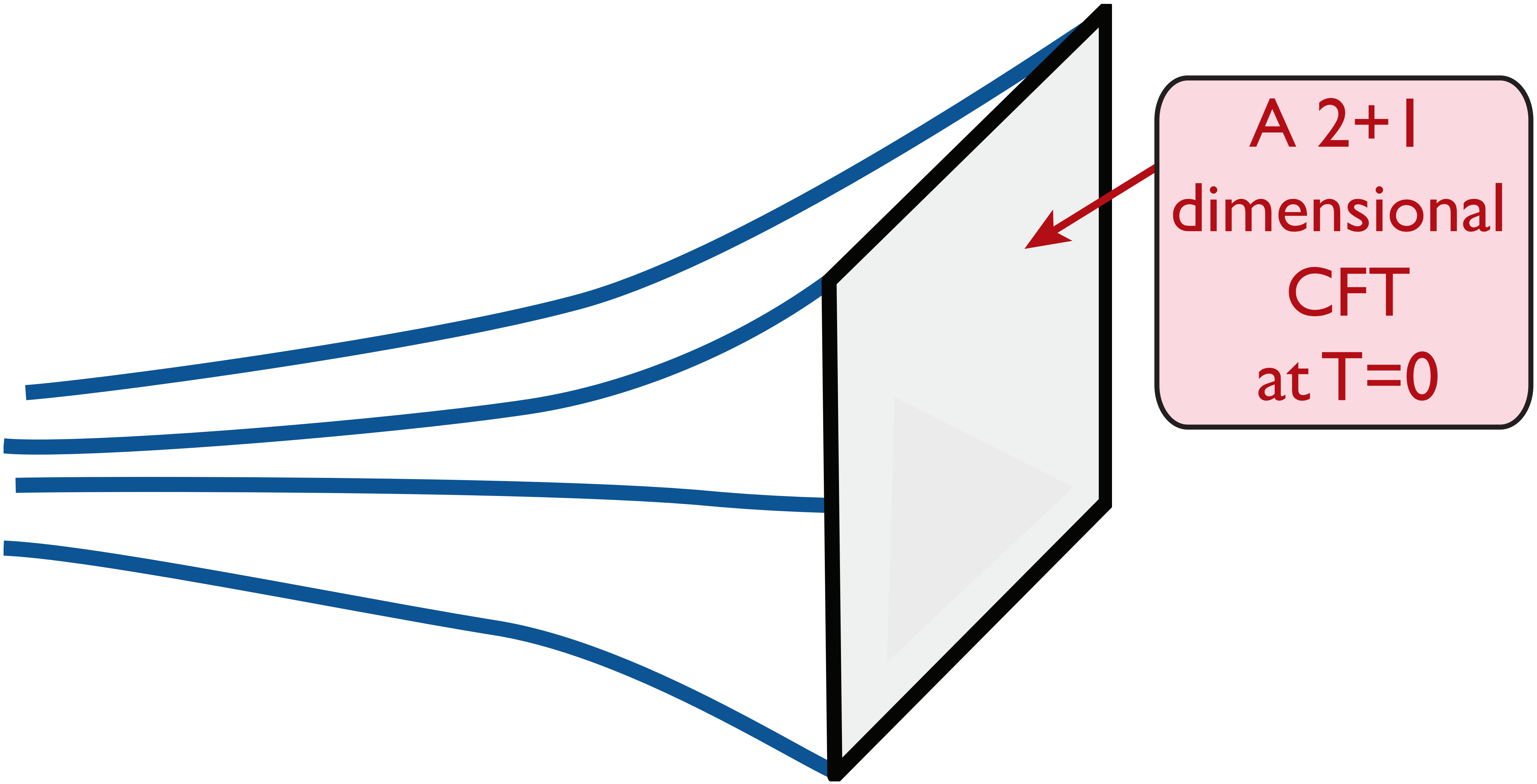


Holographic representation: AdS$_4$



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

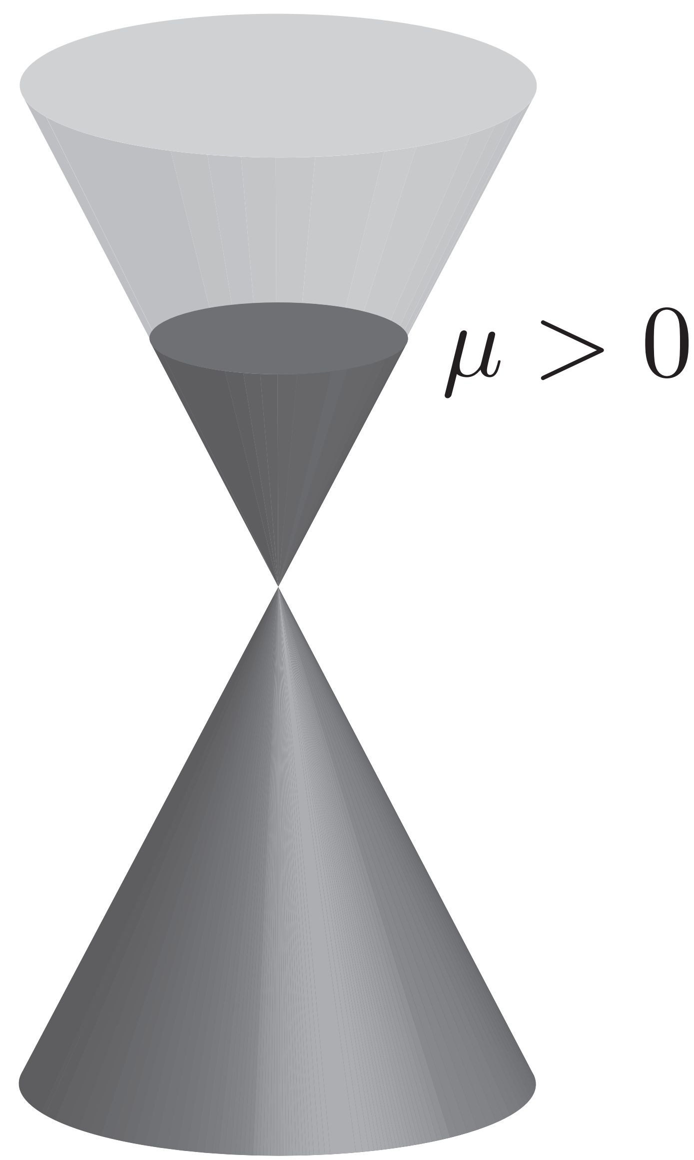
Holographic representation: AdS₄



$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2\right]$$

with $f(r) = 1$

Apply a chemical potential



AdS$_4$ theory of "nearly perfect fluids"

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS$_4$-Schwarzschild

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4g_4^2} F_{ab} F^{ab} \right] .$$

This is to be solved subject to the constraint

$$A_\mu(r \to 0, x, y, t) = \mathcal{A}_\mu(x, y, t)$$

where $\mathcal{A}_\mu$ is a source coupling to a conserved U(1) current J_μ of the CFT3

$$\mathcal{S} = \mathcal{S}_{CFT} + i \int dx dy dt \mathcal{A}_\mu J_\mu$$

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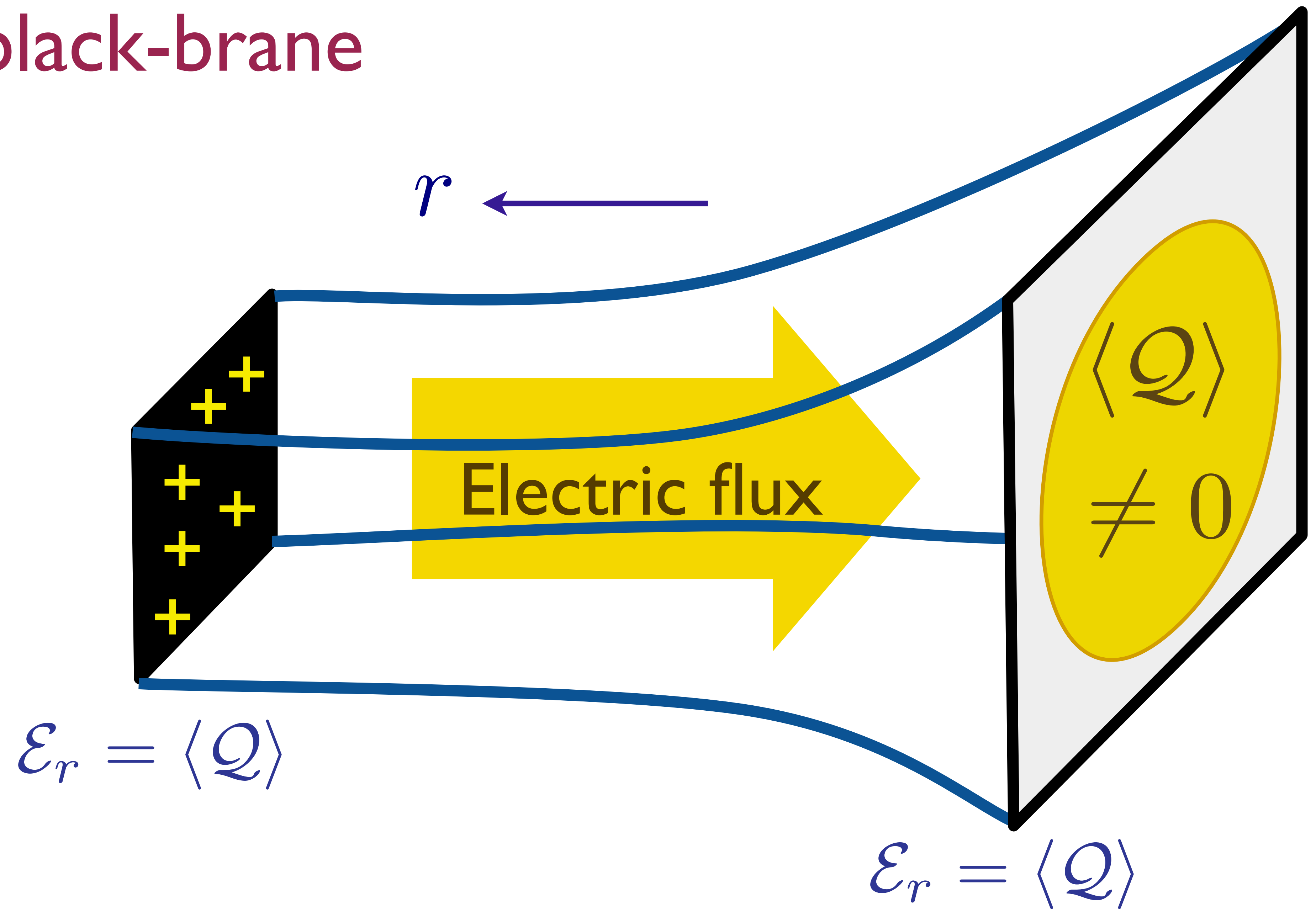
At non-zero chemical potential we simply require $\mathcal{A}_\tau = \mu$.

The Maxwell-Einstein theory of the applied chemical potential yields a AdS4-Reissner-Nordtröm black-brane



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) - \frac{1}{4g_4^2} F_{ab} F^{ab} \right]$$

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, Physical Review B **76**, 144502 (2007)

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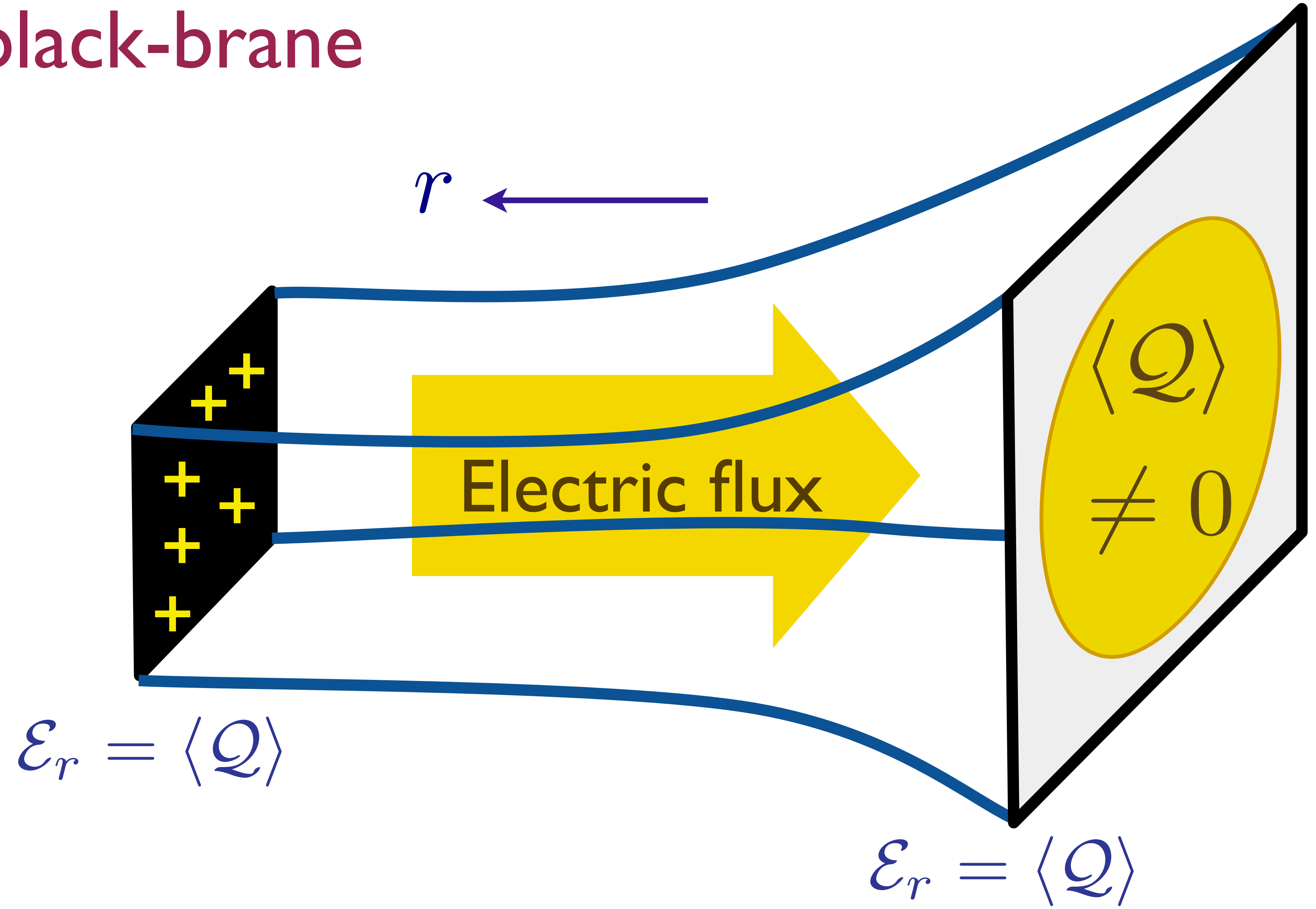

$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2\right]$$

with $f(r) = \left(1 - \frac{r}{R}\right)^2 \left(1 + \frac{2r}{R} + \frac{3r^2}{R^2}\right)$ and $R = \frac{\sqrt{6}Lg_4}{\kappa\mu}$, and $A_\tau = \mu\left(1 - \frac{r}{R}\right)$

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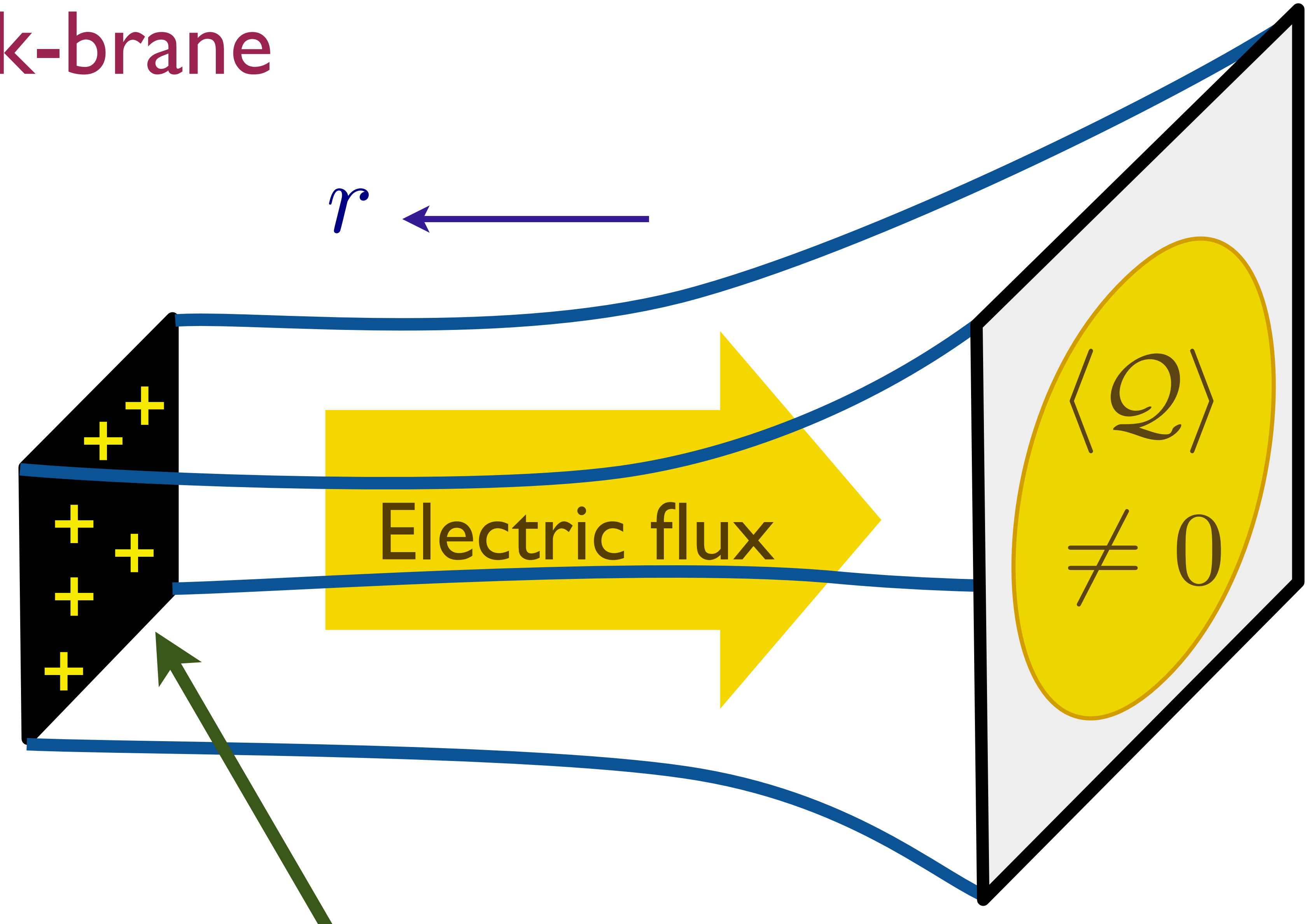
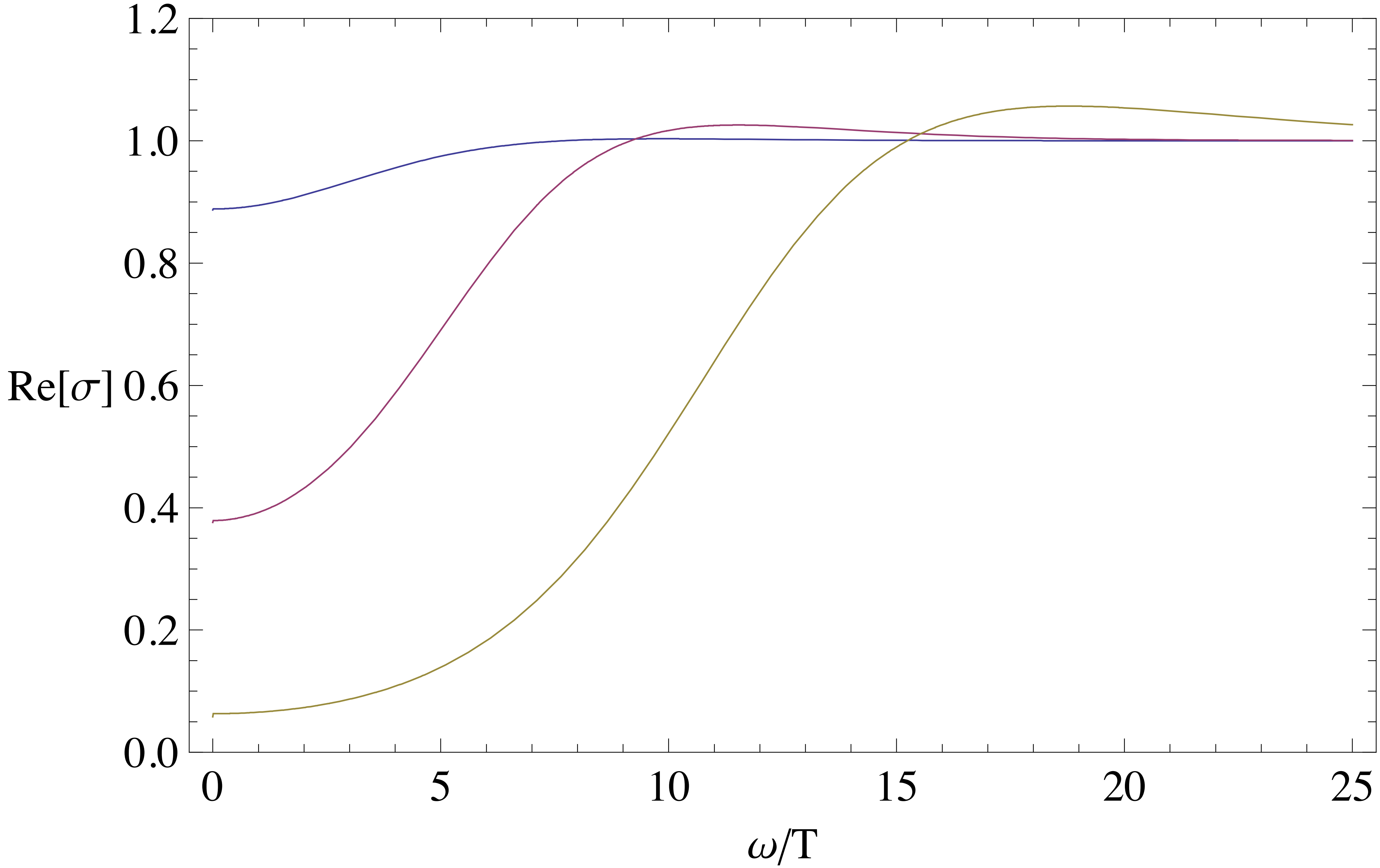


At $T = 0$, we obtain an extremal black-brane, with a near-horizon (IR) metric of $\mathrm{AdS}_2 \times R^2$

$$ds^2 = \frac{L^2}{6}\left(\frac{-dt^2 + dr^2}{r^2}\right) + dx^2 + dy^2$$

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

Compute conductivity using response to a time-dependent vector potential as a function of ω/T and μ/T



S.A. Hartnoll, arXiv:0903.3246

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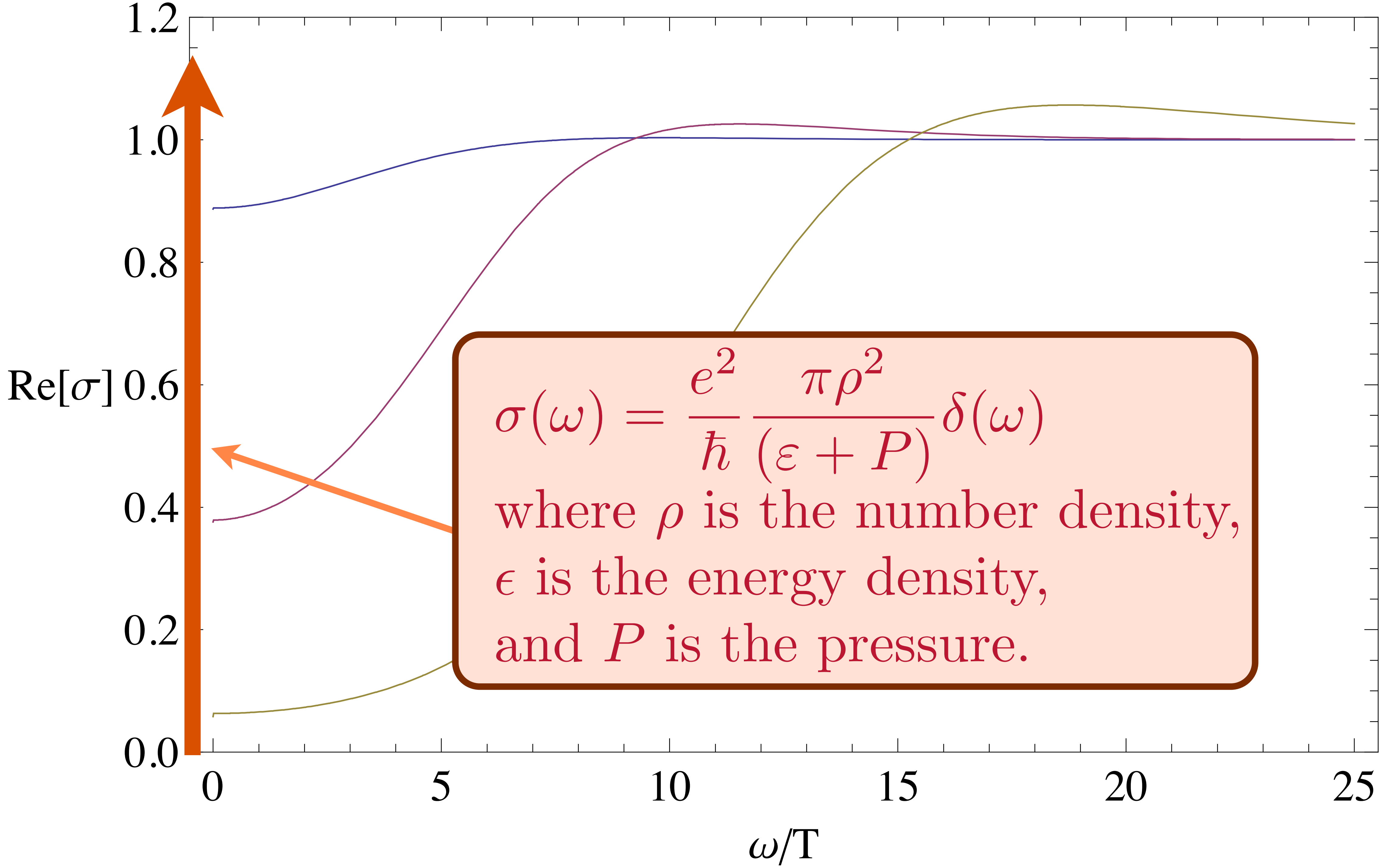

$$\sigma(\omega) = \frac{e^2}{\hbar} \frac{\pi \rho^2}{(\varepsilon + P)} \delta(\omega)$$

where ρ is the number density,
ϵ is the energy density,
and P is the pressure.

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, Physical Review B **76**, 144502 (2007)

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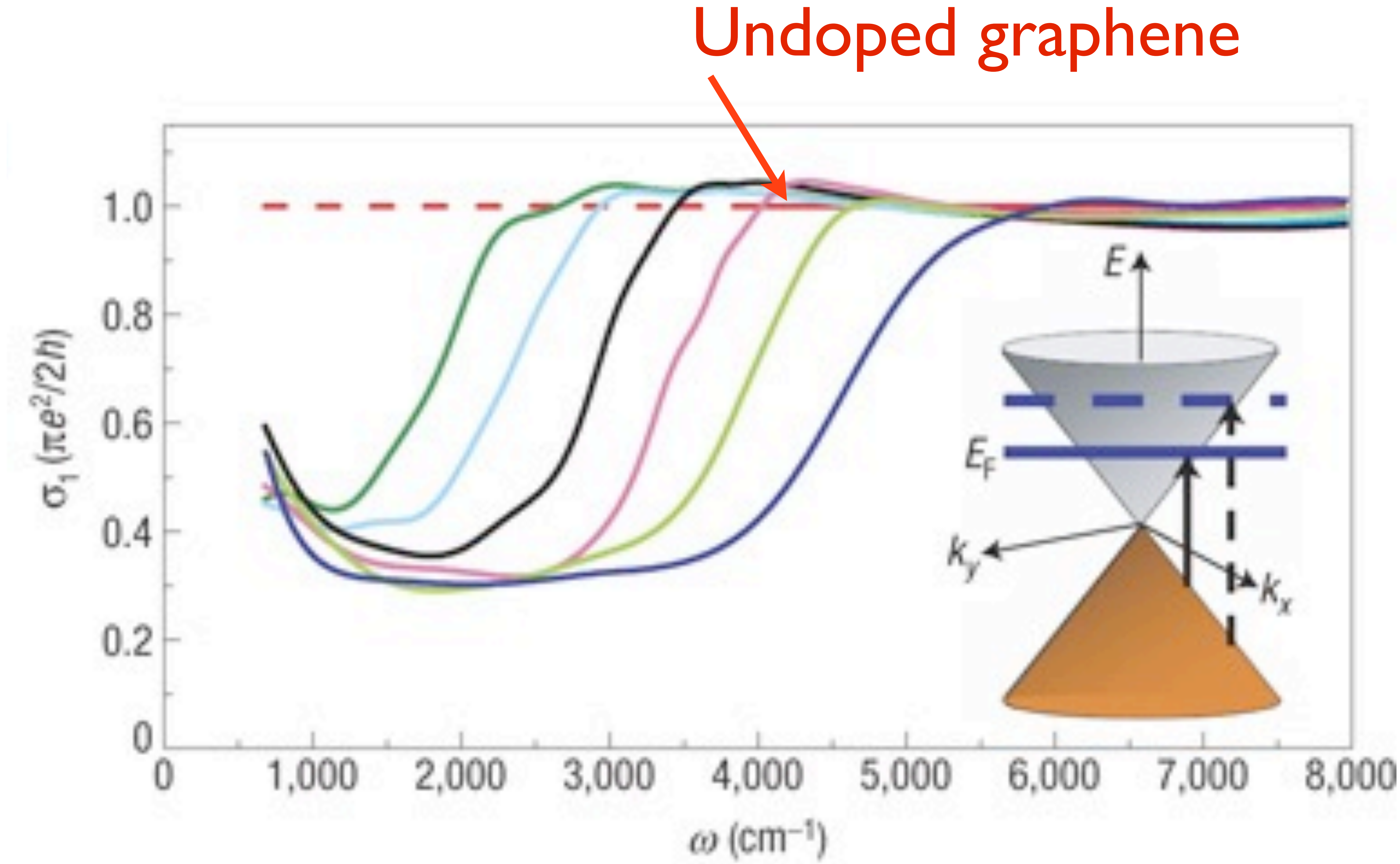

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Features of $AdS_2 \times R^2$

- Has non-zero entropy density at $T = 0$, and "volume" law for entanglement entropy.

- Green's function of a probe fermion (a *mesino*) can have a Fermi surface, but self energies are momentum independent, and the singular behavior is the same on and off the Fermi surface

- Deficit of order $\sim N^2$ in the volume enclosed by the mesino Fermi surfaces: presumably associated with "hidden Fermi surfaces" of gauge-charged particles (the *quarks*).

S.-S. Lee, *Phys. Rev. D* **79**, 086006 (2009);
M. Cubrovic, J. Zaanen, and K. Schalm, *Science* **325**, 439 (2009);
T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
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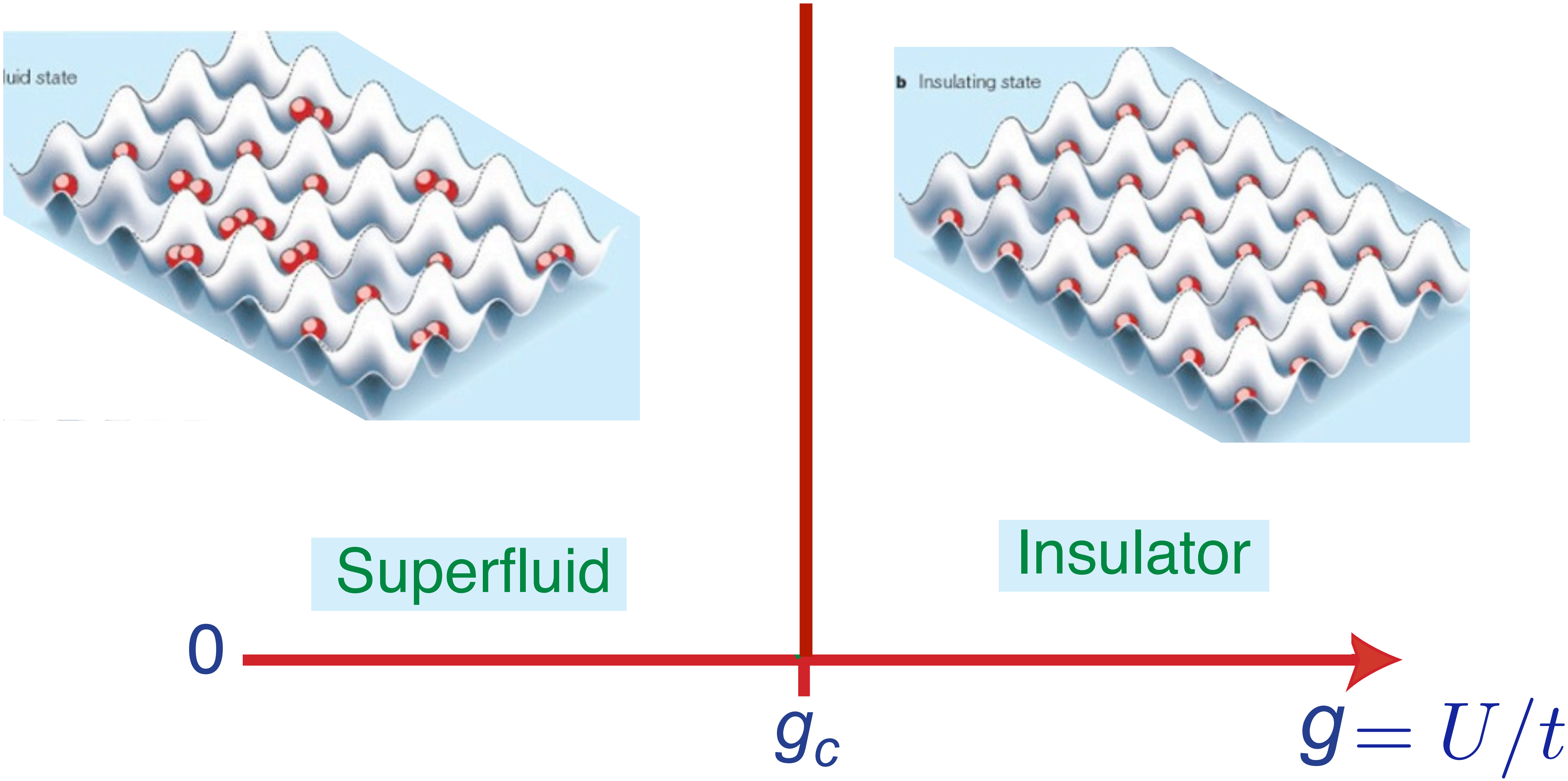
D. Holography: scaling arguments for entropy and entanglement entropy

Bose-Hubbard model at integer filling

$$H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i (n_i - 1) \quad ; \quad n_i \equiv b_i^\dagger b_i$$



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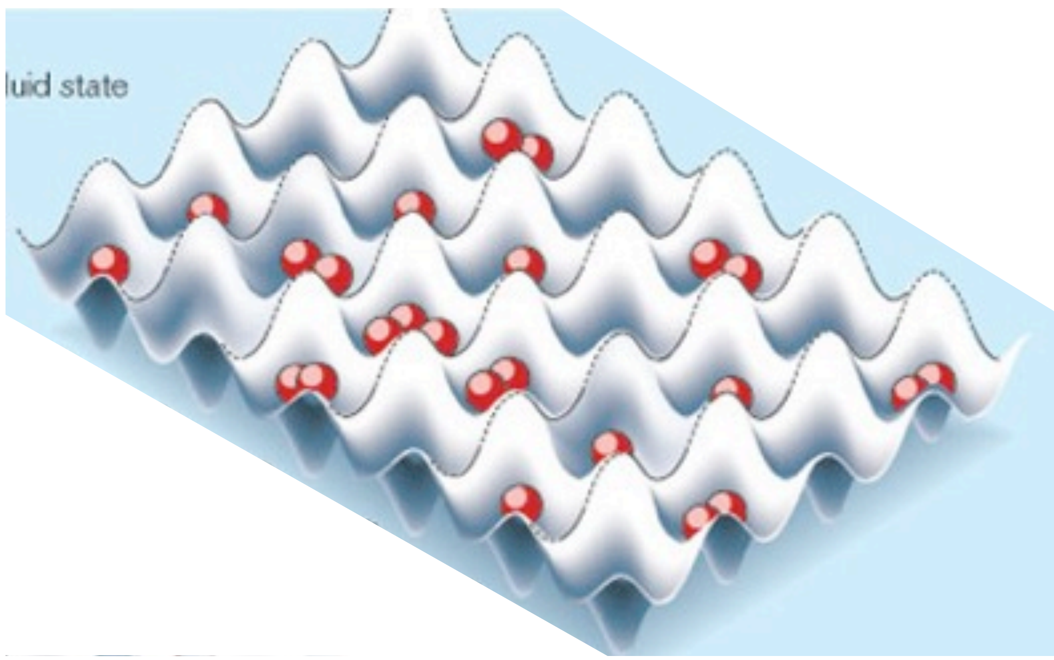
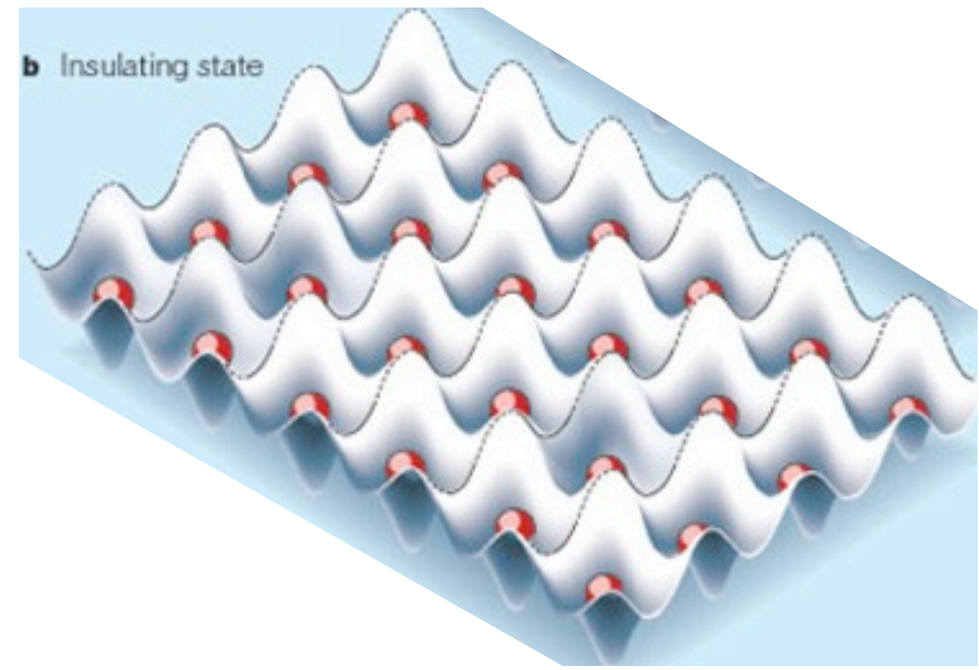




Superfluid

Insulator

0

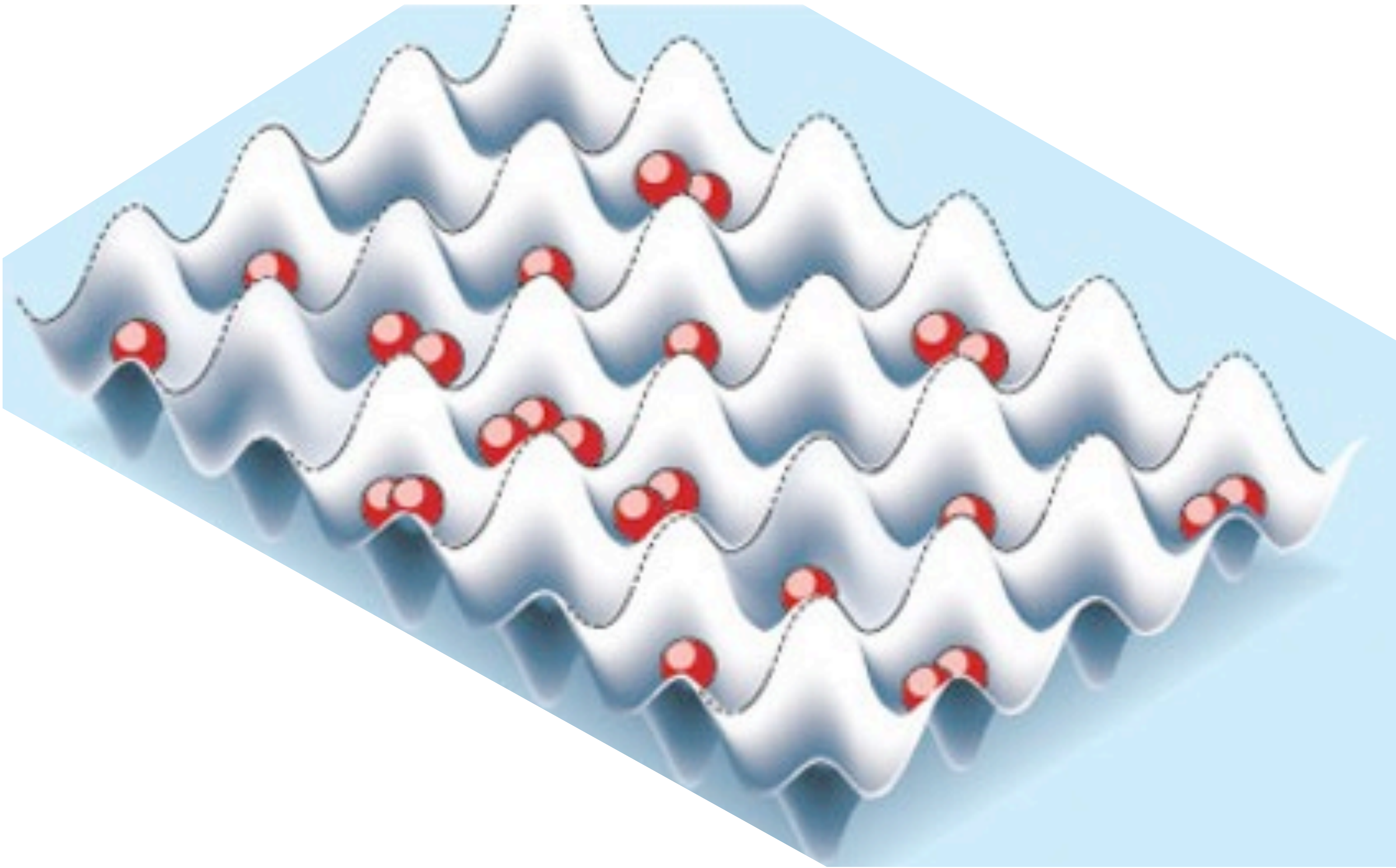
g_c

$g = U/t$

CFT3 of the XY model:
$$\mathcal{L} = |\partial \psi|^2 + s|\psi|^2 + u|\psi|^4$$

Bosons with correlated hopping

$$H = -t\sum_{\langle ij\rangle} b_i^\dagger b_j + \frac{U}{2}\sum_i n_i(n_i - 1) + w \sum_{ijk\ell\in\Box} b_i^\dagger b_k^\dagger b_j b_\ell$$



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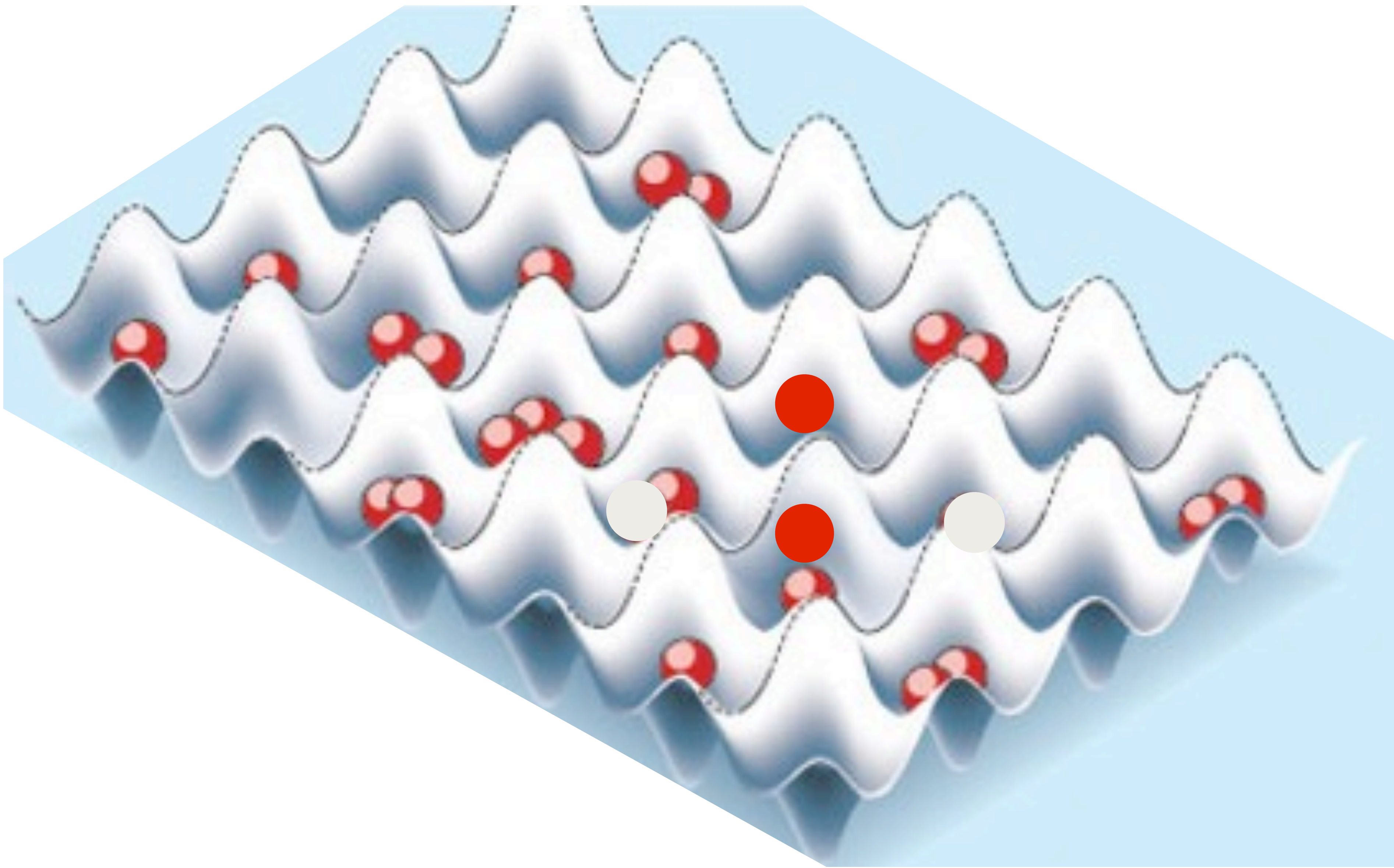

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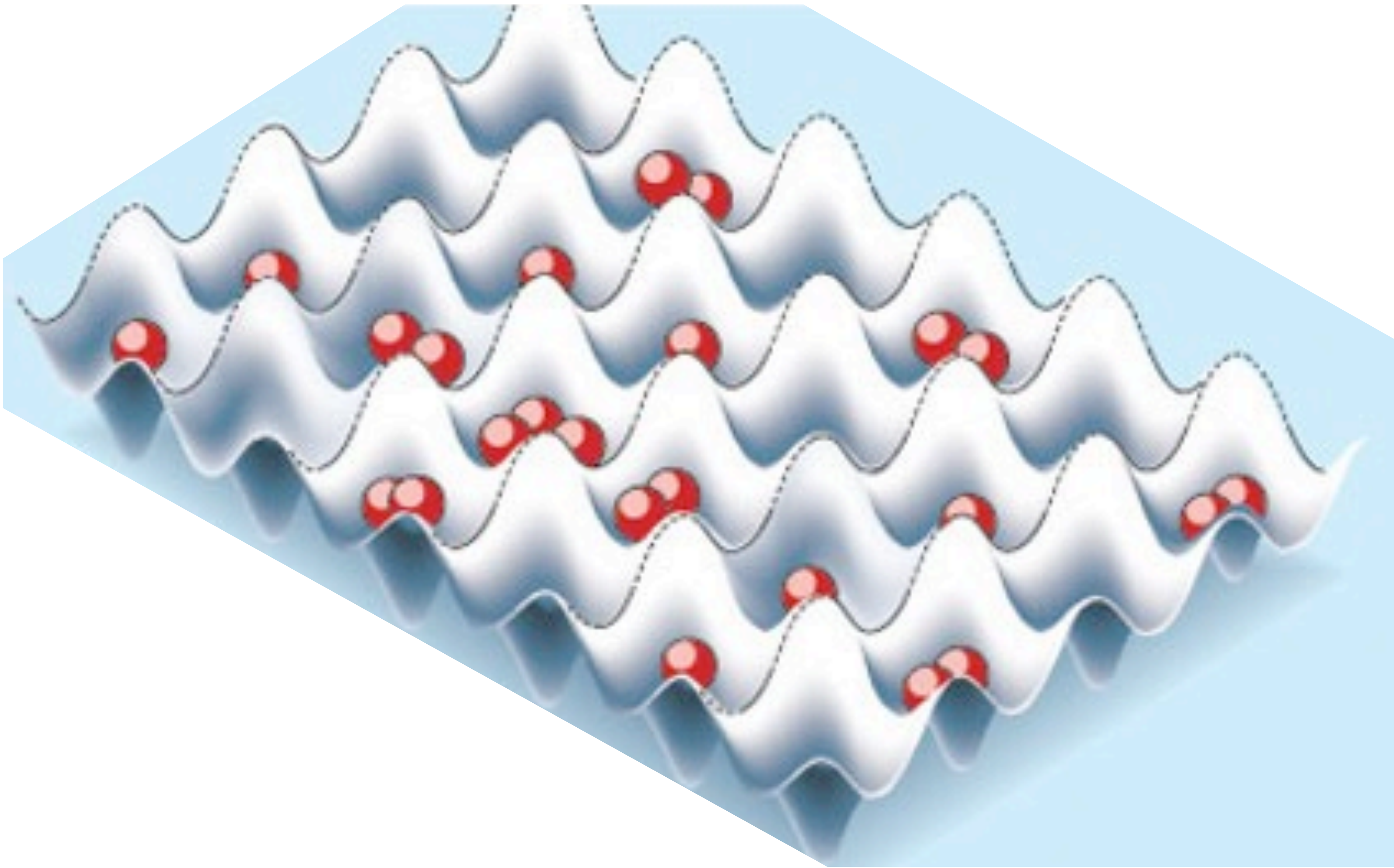
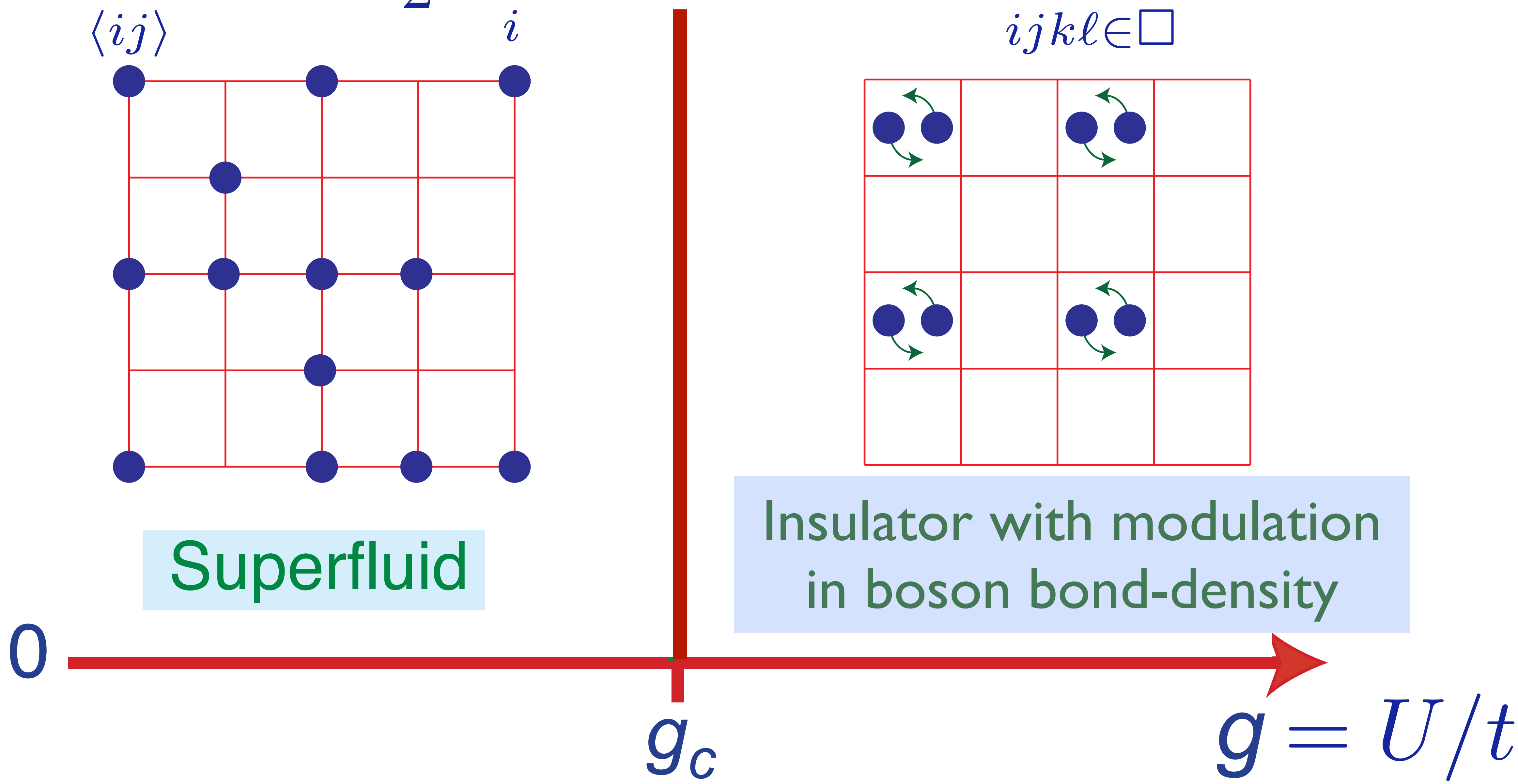


Bosons with correlated hopping at half-filling

$$H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i(n_i - 1) + w \sum_{ijk\ell \in \Box} b_i^\dagger b_k^\dagger b_j b_\ell$$

Bosons with correlated hopping at half-filling

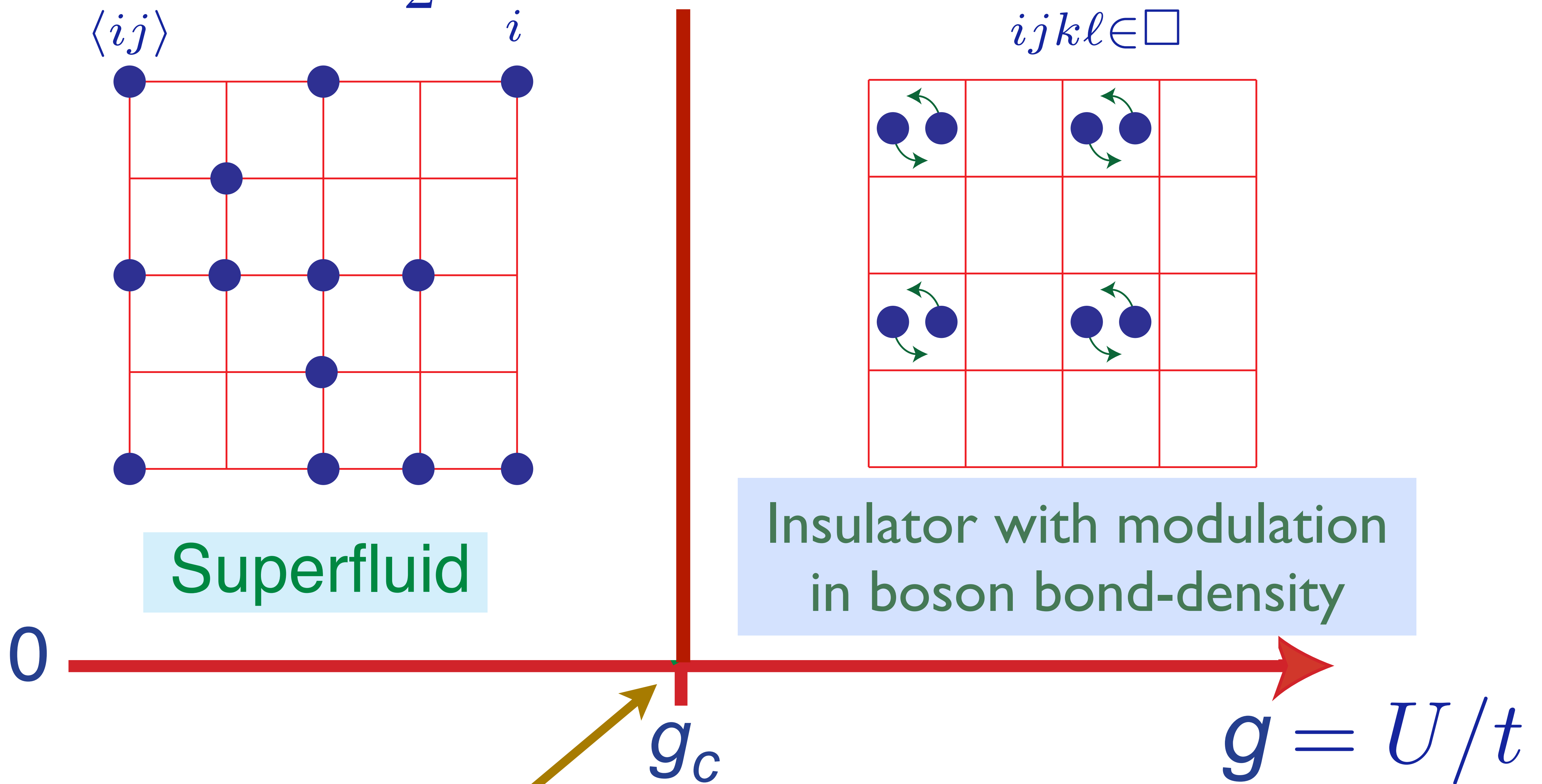
$$H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i (n_i - 1) + w \sum_{ijk\ell \in \square} b_i^\dagger b_k^\dagger b_j b_\ell$$



N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).

Bosons with correlated hopping at half-filling

$$H = -t\sum_{\langle ij\rangle} b_i^\dagger b_j + \frac{U}{2}\sum_i n_i(n_i-1) + w \sum_{ijk\ell\in\Box} b_i^\dagger b_k^\dagger b_j b_\ell$$



'Deconfined' critical point: boson fractionalizes $b \sim z_1 z_2$, and the fractionalized bosons are coupled to an emergent U(1) gauge field

$$\mathcal{L} = |(\partial_\mu - iA_\mu)z_1|^2 + |(\partial_\mu + iA_\mu)z_2|^2 + s(|z_1|^2+|z_2|^2) + u(|z_1|^2+|z_2|^2)^2 - v|z_1|^2|z_2|^2$$

O.I. Motrunich and A. Vishwanath, *Phys. Rev. B* **70**, 075104 (2004).
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

Bosons with correlated hopping close to half-filling

$$H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i(n_i - 1) + w \sum_{ijk\ell \in \Box} b_i^\dagger b_k^\dagger b_j b_\ell$$

Bosons with correlated hopping close to half-filling

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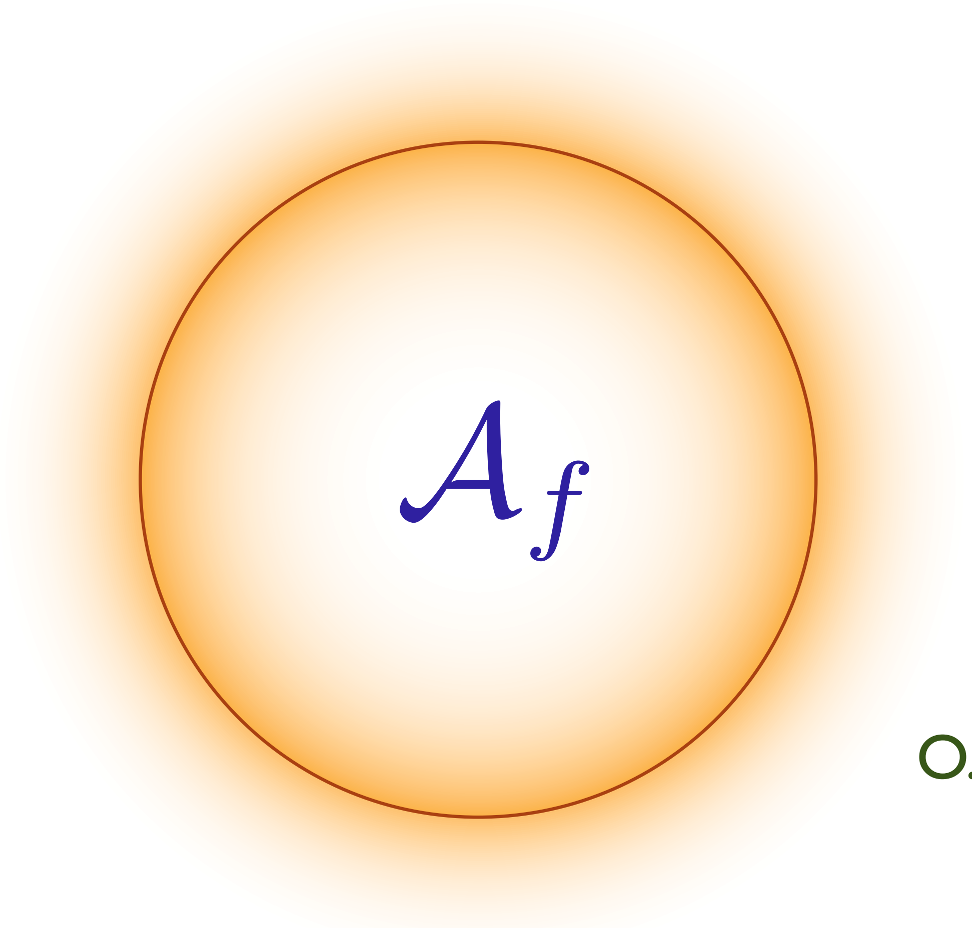
- **NFL**, the non-Fermi liquid *Bose metal*. The z_1, z_2 quanta fermionize into f_1, f_2, each of which forms a Fermi surface. Both fermions are gauge-charged, and so the Fermi surfaces are partially "hidden".



$$\mathcal{Q} = b^\dagger b$$

$$\mathcal{A}_f = \langle \mathcal{Q} \rangle$$

O. I. Motrunich and M. P. A. Fisher, *Phys. Rev.* B **75**, 235116 (2007)
L. Huijse and S. Sachdev, *Phys. Rev.* D **84**, 026001 (2011)
S. Sachdev, arXiv:1209.1637

Non-Fermi liquid Bose Metal

For suitable interactions, we can have the boson, b, *fractionalize* into two fermions $f_{1,2}$:

$$b \to f_1 f_2$$

This implies the effective theory for $f_{1,2}$ is invariant under the U(1) gauge transformation

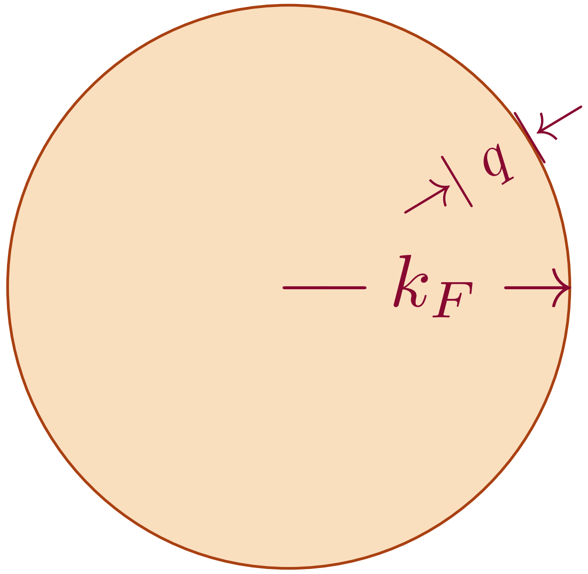
$$f_1 \to f_1 e^{i\theta(\boldsymbol{x},\tau)} \quad , \quad f_2 \to f_2 e^{-i\theta(\boldsymbol{x},\tau)}$$

Consequently, the effective theory of the Bose metal has an emergent gauge field A_μ and has the structure

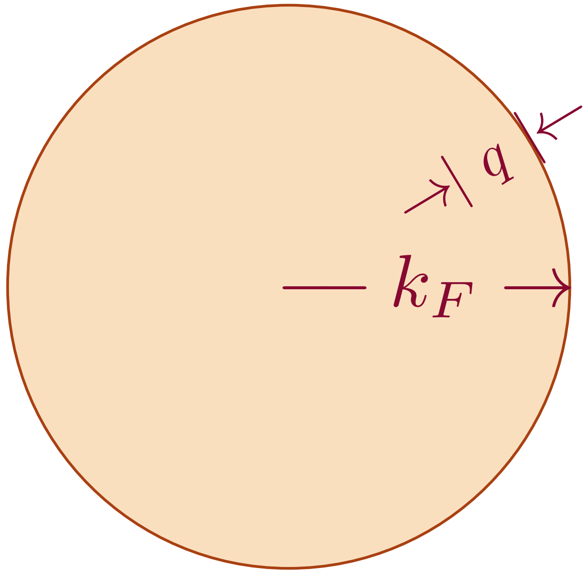
$$\mathcal{L} = f_1^\dagger \left(\partial_\tau - iA_\tau - \frac{(\boldsymbol{\nabla} - i\boldsymbol{A})^2}{2m} - \mu \right) f_1 + f_2^\dagger \left(\partial_\tau + iA_\tau - \frac{(\boldsymbol{\nabla} + i\boldsymbol{A})^2}{2m} - \mu \right) f_2$$

The gauge-dependent $f_{1,2}$ Green's functions have Fermi surfaces obeying $\mathcal{A}_f = \langle \mathcal{Q} \rangle$. However, these Fermi surfaces are not directly observable because it is gauge-dependent. Nevertheless, gauge-independent operators, such as b or $b^\dagger b$, will exhibit *Friedel oscillations* associated with fermions scattering across these <u>hidden</u> Fermi surfaces.

FL Fermi liquid



- $k_F^d \sim \mathcal{Q}$, the fermion density
- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.
- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.
- Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.



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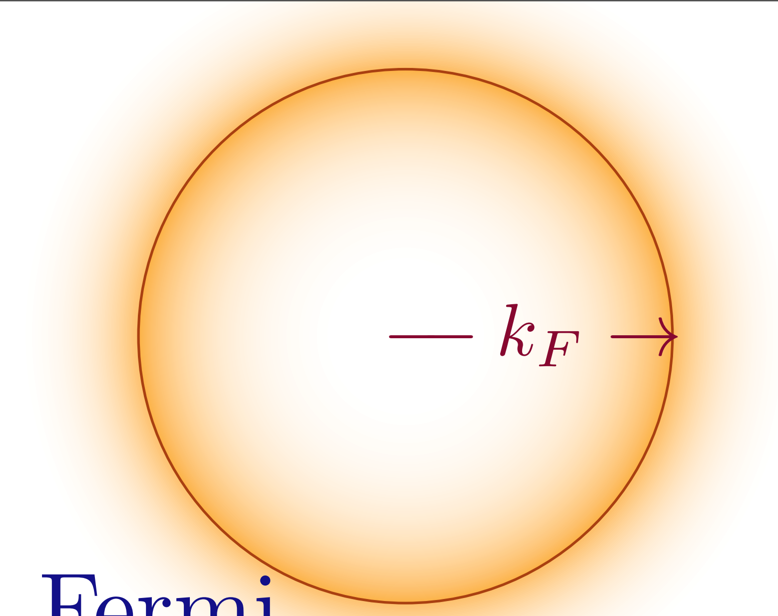
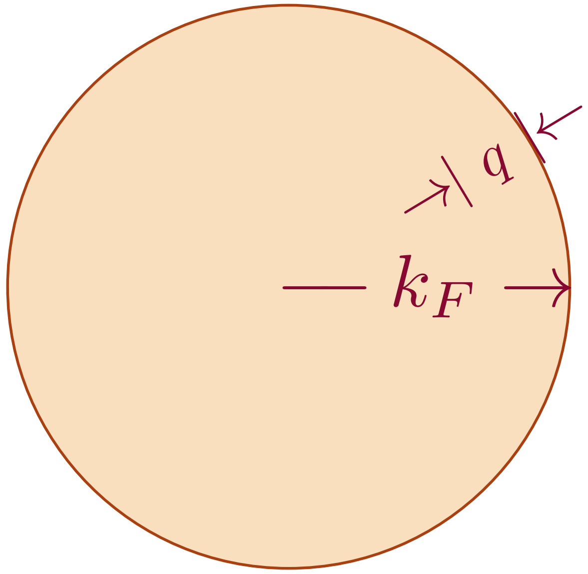


NFL
Bose metal

- Hidden Fermi surface with $k_F^d \sim \mathcal{Q}$.

FL Fermi liquid



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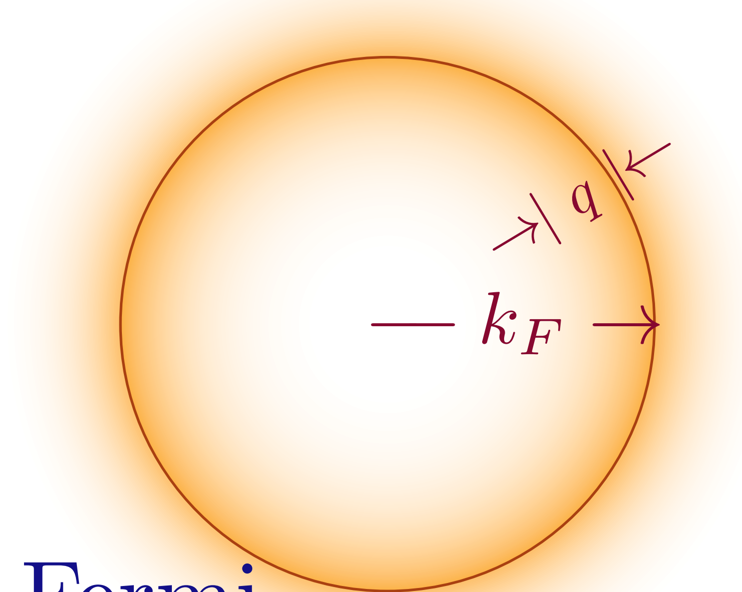
NFL Bose metal

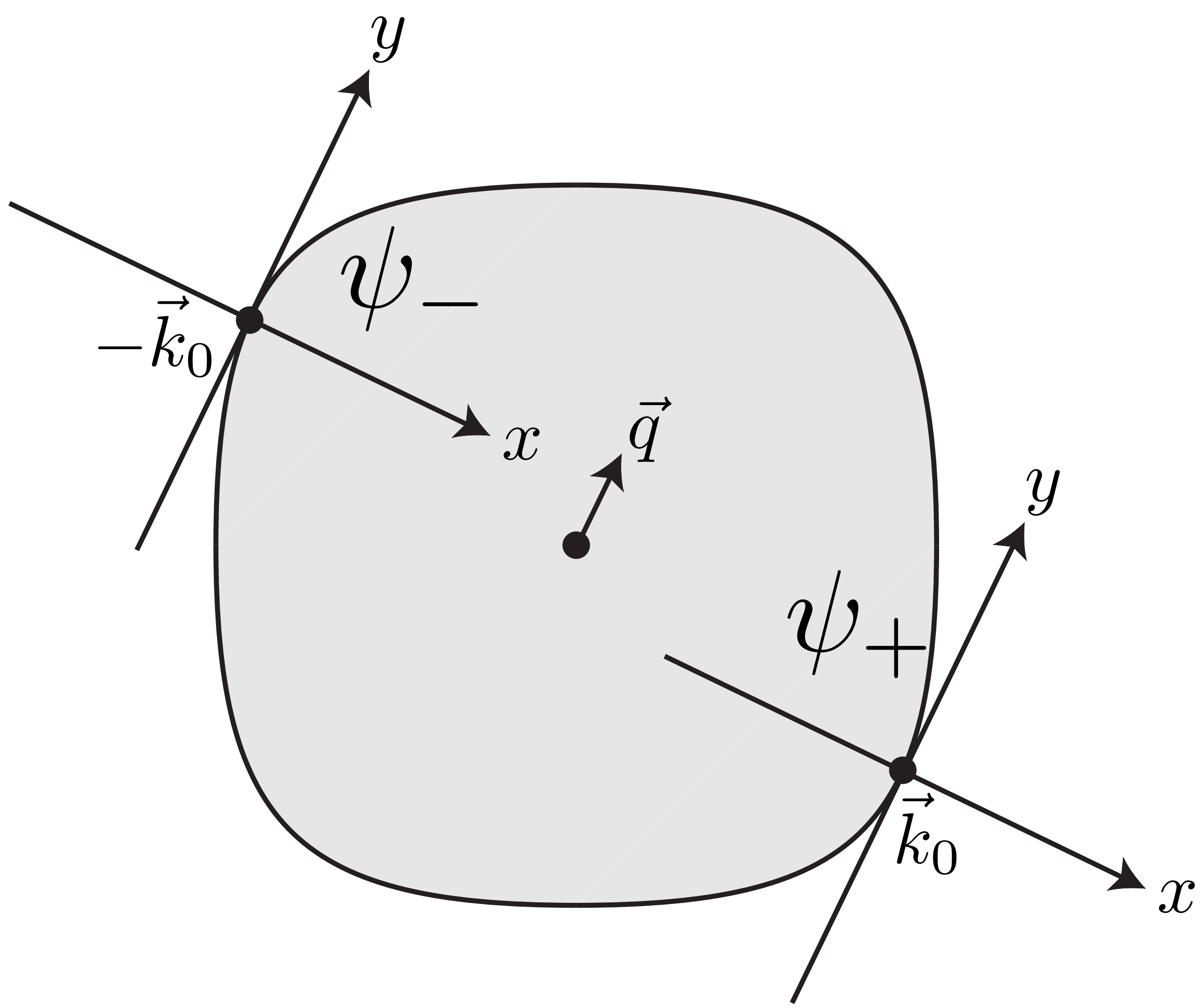


- Hidden Fermi surface with $k_F^d \sim \mathcal{Q}$.

- Diffuse fermionic excitations with $z = 3/2$ to three loops.

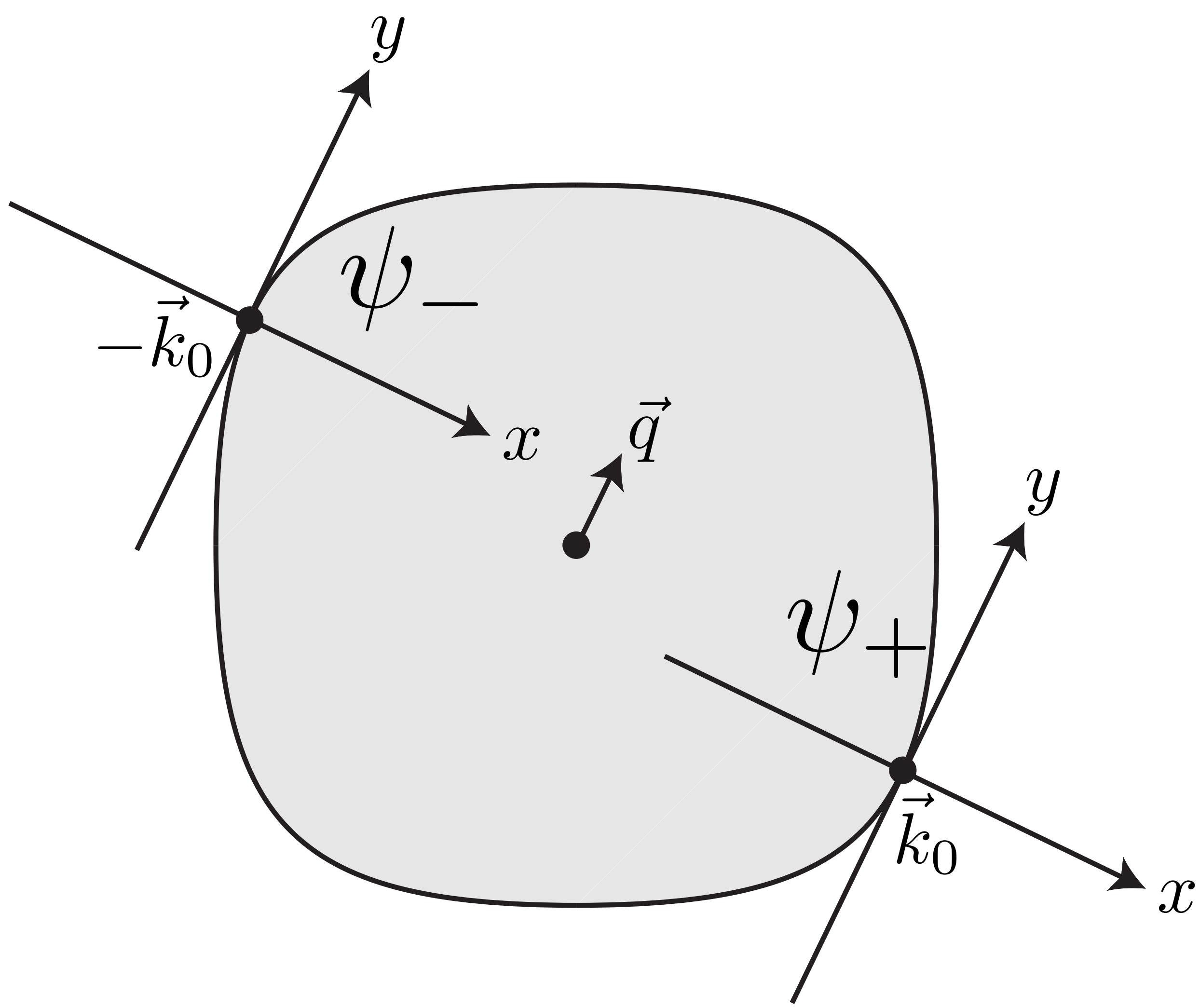
P. A. Lee, Phys. Rev. Lett. **63**, 680 (1989)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Field theory of non-Fermi liquid



- $\vec{A}$ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.

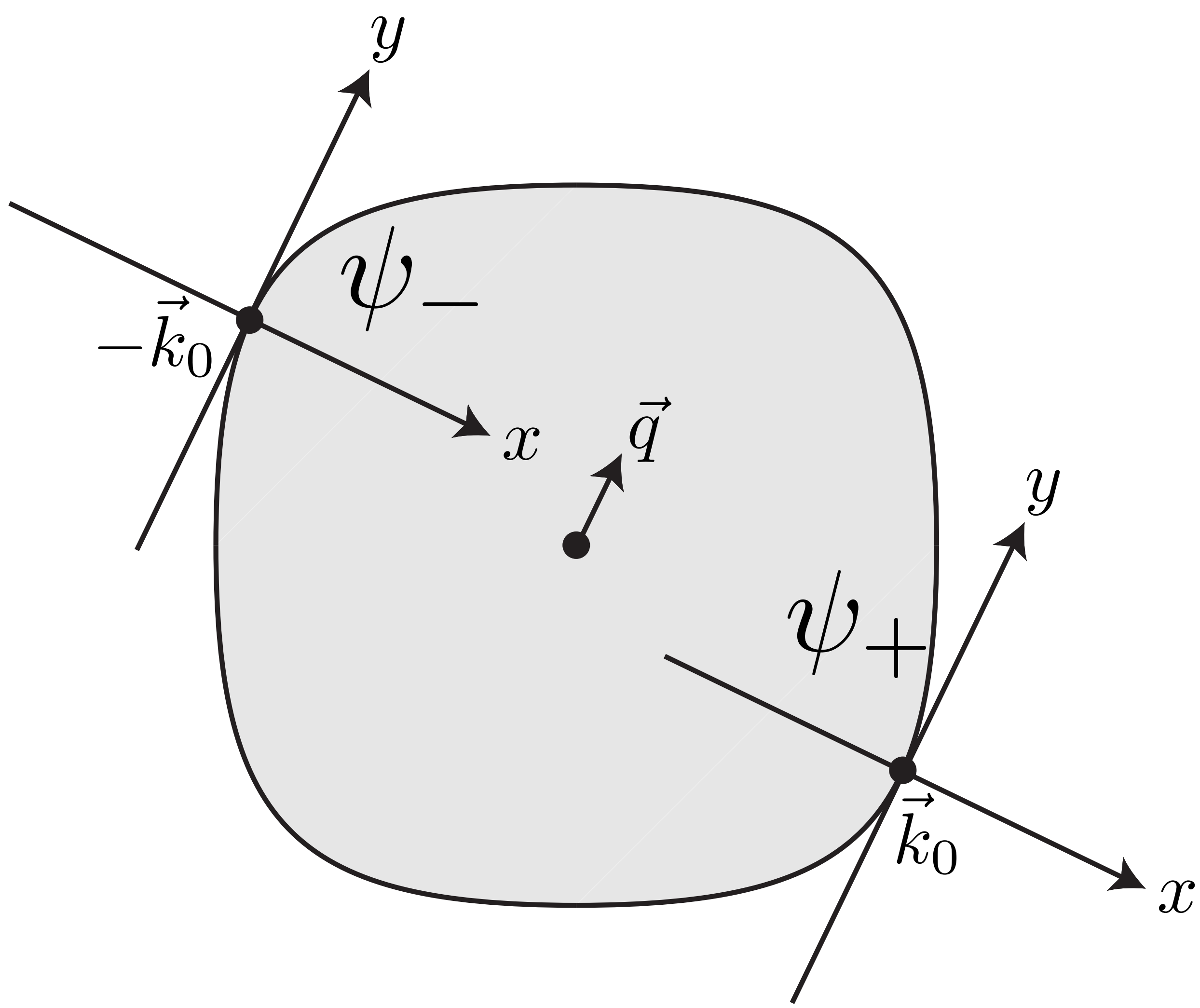
Field theory of non-Fermi liquid



- $\vec{A}$ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.

- Expand fermion kinetic energy at wavevectors about $\pm\vec{k}_0$. In Landau gauge $\vec{A} = (a, 0)$.

Field theory of non-Fermi liquid



$$\mathcal{L}[\psi_\pm, a] =$$

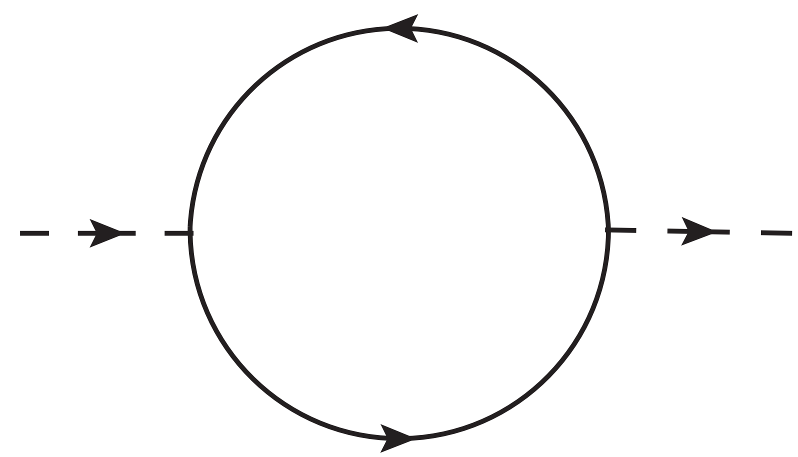
$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_-$$

$$-a\left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2}\left(\partial_y a\right)^2$$

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Field theory of non-Fermi liquid

$$\begin{aligned}\mathcal{L} &= \psi_+^\dagger\left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger\left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- \\ &\quad - a\left(\psi_+^\dagger\psi_+ - \psi_-^\dagger\psi_-\right) + \frac{1}{2g^2}\left(\partial_y a\right)^2\end{aligned}$$



One loop a self-energy with N_f fermion flavors:

$$\begin{aligned}D(\vec{q},\omega) &= N_f\int\frac{d^2k}{4\pi^2}\frac{d\Omega}{2\pi}\frac{1}{\left[-i(\Omega+\omega)+k_x+q_x+(k_y+q_y)^2\right]\left[-i\Omega-k_x+k_y^2\right]} \\ &= \frac{N_f}{4\pi}\frac{|\omega|}{|q_y|}\end{aligned}$$

Landau-damping

We first explicitly evaluate $\Pi(q, \omega_n)$. We will only be interested in terms that are singular in q and ω_n, and will drop regular contributions from regions of high momentum and frequency. In this case, it is permissible to reverse the conventional order of integrating over frequency first in (17), and to first integrate over k_x. It is a simple matter to perform the integration over k_x in using the method of residues to yield

$$\begin{aligned}\Pi(q, \omega_n) &= \frac{1}{2v_F} \int \frac{d^{d-1}k_y}{(2\pi)^{d-1}} \int \frac{d\epsilon_n}{2\pi} \frac{\mathrm{sgn}(\epsilon_n + \omega_n) - \mathrm{sgn}(\epsilon_n)}{\left(\zeta\omega_n + iv_F q_x + i\kappa q_y^2/2 + i\kappa \vec{q}_y \cdot \vec{k}_y\right)} \\ &= \frac{|\omega_n|}{2\pi v_F} \int \frac{d^{d-1}k_y}{(2\pi)^{d-1}} \frac{1}{\left(\zeta\omega_n + iv_F q_x + i\kappa q_y^2/2 + i\kappa \vec{q}_y \cdot \vec{k}_y\right)}. \end{aligned} \quad (19)$$

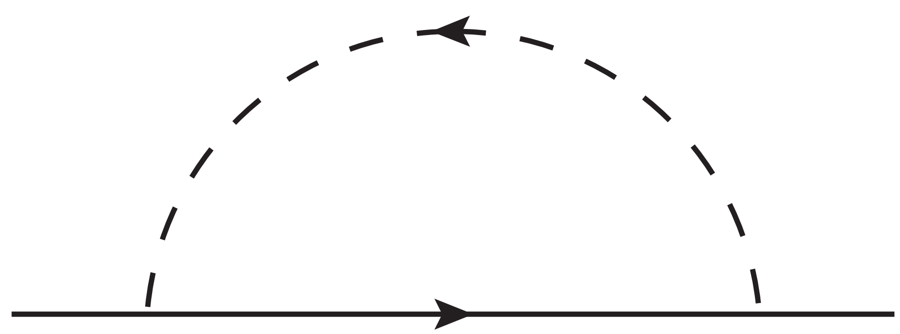
We now integrate along the component of $\vec{k}_y$ parallel to the direction of $\vec{q}_y$ to obtain

$$\begin{aligned}\Pi(q, \omega_n) &= \frac{|\omega_n|}{2\pi v_F \kappa |q_y|} \int \frac{d^{d-2}k_y}{(2\pi)^{d-2}} \\ &= \frac{|\omega_n|}{2\pi v_F \kappa |q_y|} \Lambda^{d-2} \end{aligned} \quad (20)$$

Note that in $d = 2$ the last non-universal factor is not present, and the result for Π is universal with $\Lambda^{d-2} = 1$. Note also that ζ has dropped out of the result Π: this will be important in our subsequent treatment of quantum critical points.

Field theory of non-Fermi liquid

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- - a\left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2}\left(\partial_y a\right)^2$$



Electron self-energy at order $1/N_f$:

$$\Sigma(\vec{k},\Omega) = -\frac{1}{N_f}\int \frac{d^2q}{4\pi^2}\frac{d\omega}{2\pi}\frac{1}{\left[-i(\omega+\Omega)+k_x+q_x+(k_y+q_y)^2\right]\left[\frac{q_y^2}{g^2}+\frac{|\omega|}{|q_y|}\right]}$$

$$= -i\frac{2}{\sqrt{3}N_f}\left(\frac{g^2}{4\pi}\right)^{2/3}\mathrm{sgn}(\Omega)|\Omega|^{2/3}$$

$$\Sigma(k,\omega_n) = \lambda^2 \int \frac{d^d q}{(2\pi)^d} \int \frac{d\epsilon_n}{2\pi} \frac{1}{q_y^2 + \gamma|\epsilon_n|/|q_y|} G_0(k+q, \epsilon_n + \omega_n) \tag{45}$$

This can be evaluated by the same methods used for (18). Integrating over q_x we find the analog of (21)

$$\begin{aligned} \Sigma(k,\omega_n) &= i\,\frac{\lambda^2}{v_F} \int \frac{d^{d-1}q_y}{(2\pi)^{d-1}} \int \frac{d\epsilon_n}{2\pi} \frac{\text{sgn}(\epsilon_n+\omega_n)|q_y|}{|q_y|^3 + \gamma|\epsilon_n|} \\ &= i\,\frac{\lambda^2}{\pi v_F \gamma} \text{sgn}(\omega_n) \int \frac{d^{d-1}q_y}{(2\pi)^{d-1}} |q_y| \ln\left(\frac{|q_y|^3 + \gamma|\omega_n|}{|q_y|^3}\right). \end{aligned} \tag{46}$$

Evaluation of the q_y integral yields a result which agrees with (42) and (43) in $d=2$, and with the expected logarithmic corrections in $d=3$. In the physically important case of $d=2$, the q_y integral evaluates to

$$\Sigma(k,\omega_n) = \frac{\lambda^2}{\pi v_F \gamma^{1/3}\sqrt{3}} \text{sgn}(\omega_n)|\omega_n|^{2/3} \quad , \quad d=2, \tag{47}$$

in agreement with (43).

Field theory of non-Fermi liquid

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- - a\left(\psi_+^\dagger\psi_+ - \psi_-^\dagger\psi_-\right) + \frac{1}{2g^2}\left(\partial_y a\right)^2$$

Schematic form of a and fermion Green's functions

$$D(\vec{q},\omega) = \frac{1/N_f}{q_y^2 + \dfrac{|\omega|}{|q_y|}} \quad , \quad G_f(\vec{q},\omega) = \frac{1}{q_x + q_y^2 - i\,\mathrm{sgn}(\omega)|\omega|^{2/3}/N_f}$$

In *both* cases $q_x \sim q_y^2 \sim \omega^{1/z}$, with $z = 3/2$. Note that the bare term $\sim \omega$ in G_f^{-1} is irrelevant.

Strongly-coupled theory without quasiparticles.

Field theory of non-Fermi liquid

$$\begin{aligned}\mathcal{L} &= \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_- \\ &\quad - a\left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2}\left(\partial_y a\right)^2\end{aligned}$$

Simple scaling argument for $z = 3/2$.

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Field theory of non-Fermi liquid

$$\mathcal{L} = \psi_+^\dagger \left(\cancel{\partial_\tau} - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\cancel{\partial_\tau} + i\partial_x - \partial_y^2\right) \psi_- - a\left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2}\left(\partial_y a\right)^2$$

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M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Field theory of non-Fermi liquid

$$
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& - a\left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2}\left(\partial_y a\right)^2
\end{aligned}
$$

Simple scaling argument for $z = 3/2$.

Under the rescaling $x \to x/s$, $y \to y/s^{1/2}$, and $\tau \to \tau/s^z$, we find invariance provided

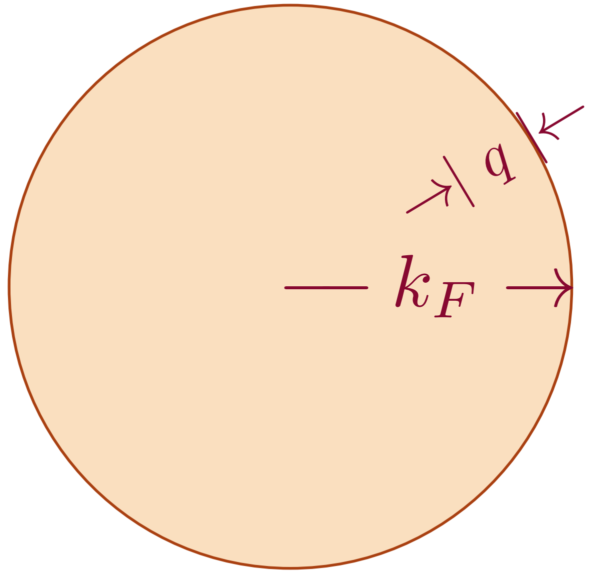
$$
\begin{aligned}
a &\to a\, s \\
\psi &\to \psi\, s^{(2z+1)/4} \\
g &\to g\, s^{(3-2z)/4}
\end{aligned}
$$

So the action is invariant provided $z = 3/2$.

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

FL

Fermi liquid



- $k_F^d \sim \mathcal{Q}$, the fermion density

- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.

- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.

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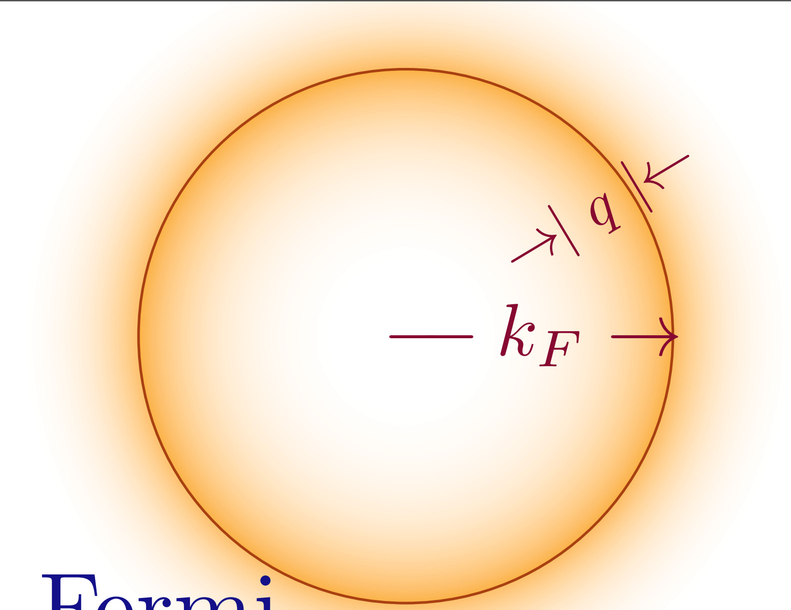
NFL

Bose metal



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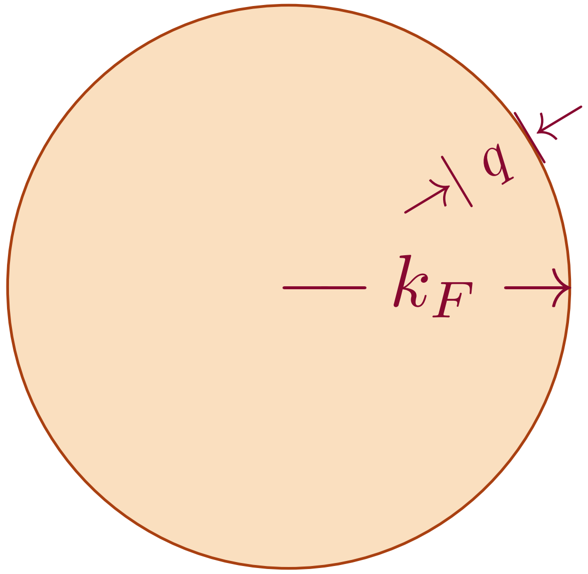
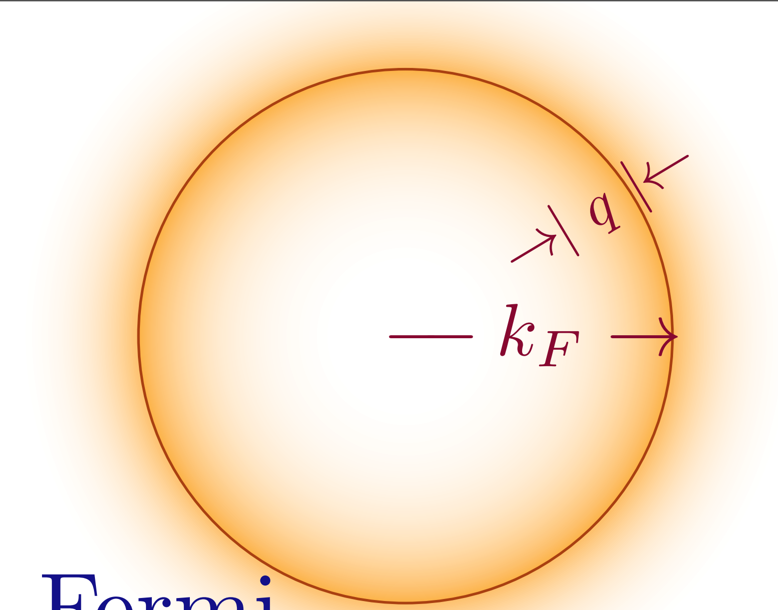
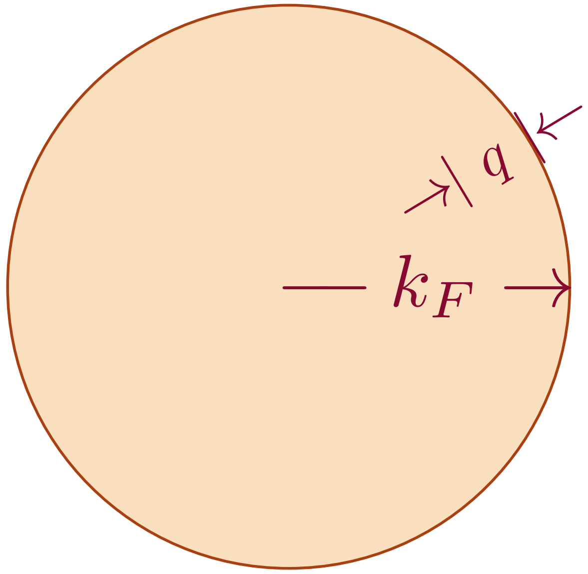
FL Fermi liquid	NFL Bose metal
	
• $k_F^d \sim \mathcal{Q}$, the fermion density	• Hidden Fermi surface with $k_F^d \sim \mathcal{Q}$.
• Sharp fermionic excitations near Fermi surface with $\omega \sim \lvert q \rvert^z$, and $z = 1$.	• Diffuse fermionic excitations with $z = 3/2$ to three loops.
• Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.	• $S \sim T^{(d-\theta)/z}$ with $\theta = d - 1$.
• Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.	

FL Fermi liquid



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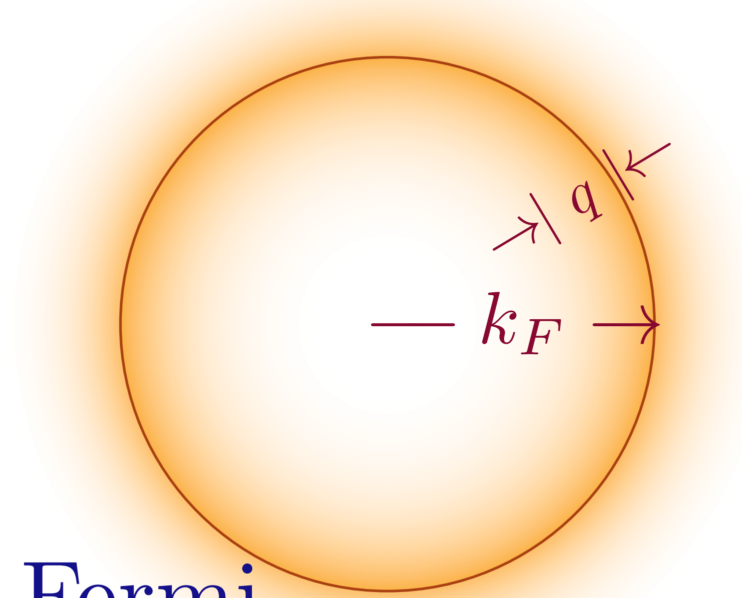

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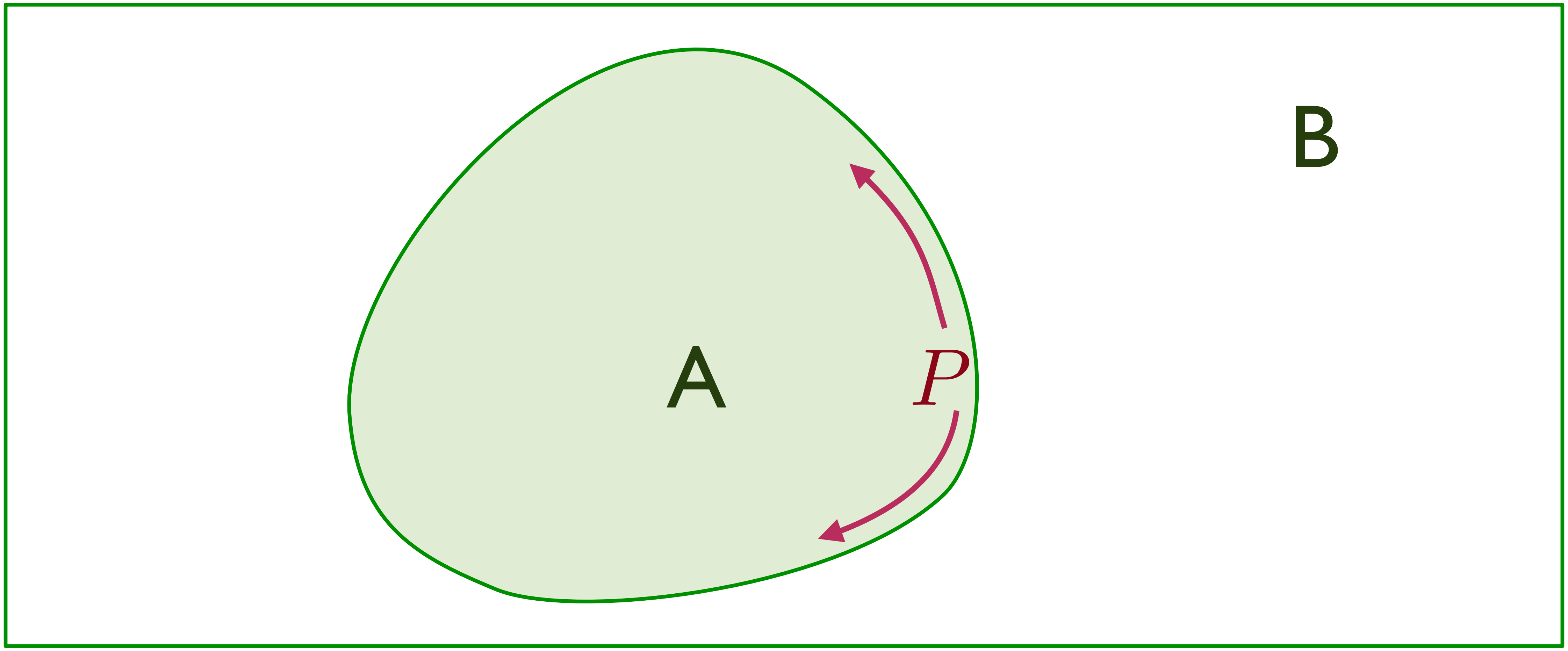
- $S_E \sim k_F^{d-1} P \ln P$.

Entanglement entropy of the non-Fermi liquid



Logarithmic violation of "area law": $S_E = \mathcal{C}_E \, k_F P \ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.
The prefactor $\mathcal{C}_E$ is expected to be universal but $\neq 1/12$:
independent of the shape of the entangling region, and dependent
only on IR features of the theory.

B. Swingle, *Physical Review Letters* **105**, 050502 (2010)
Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

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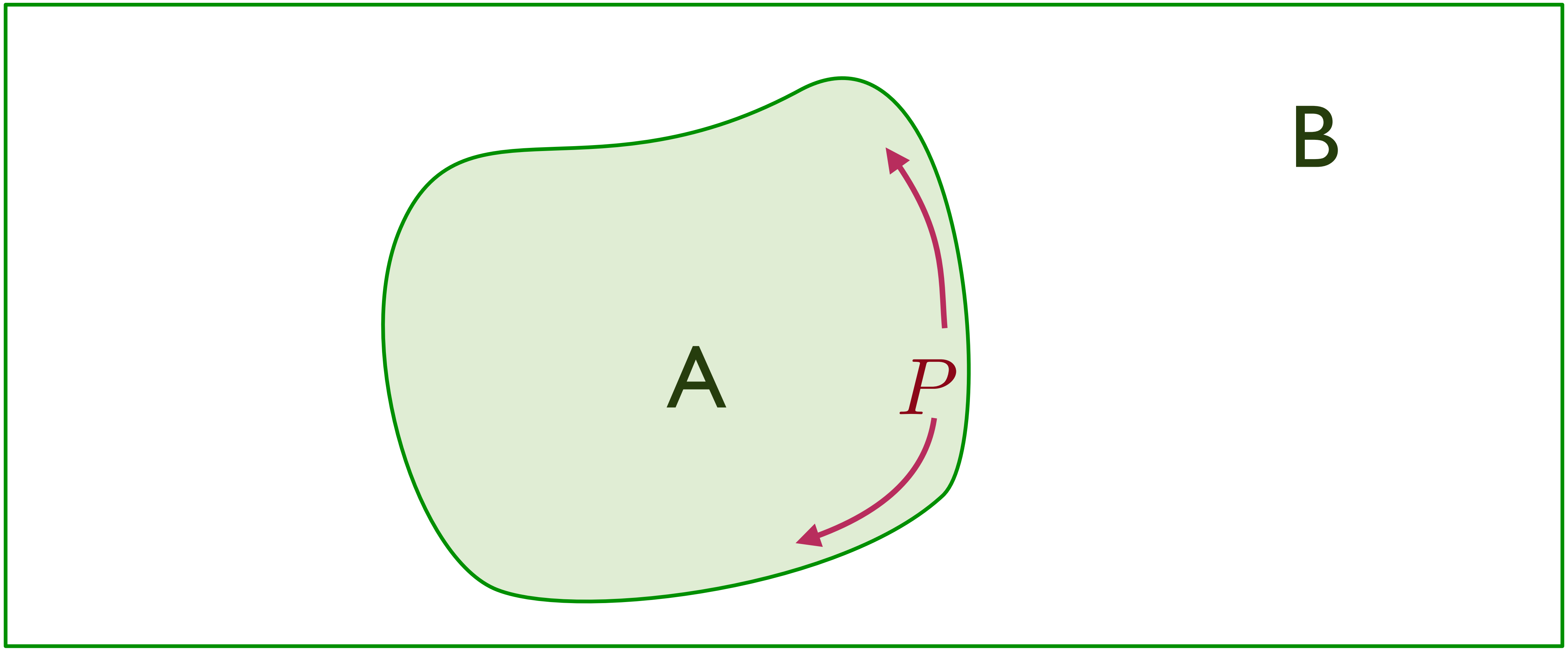

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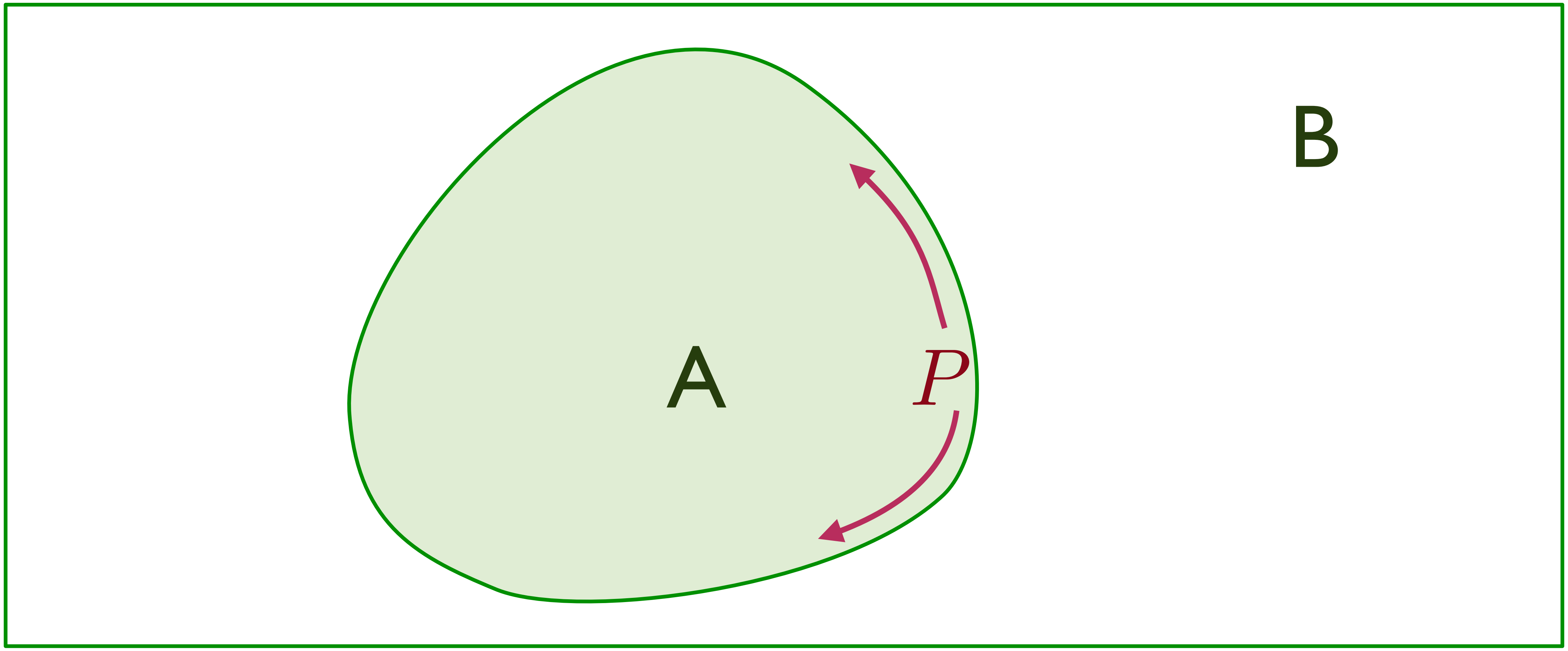

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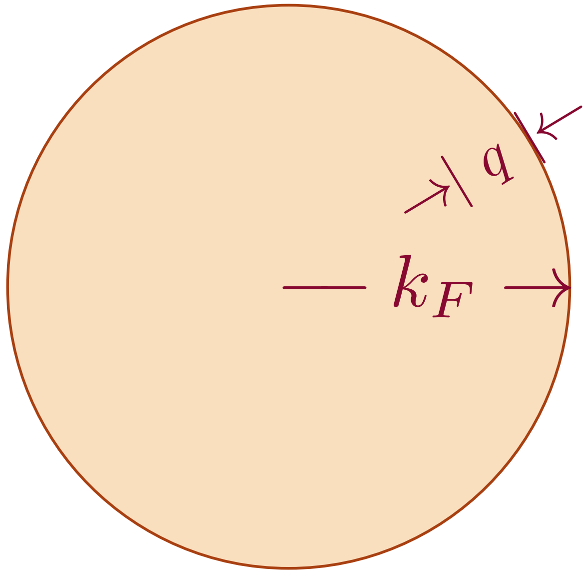
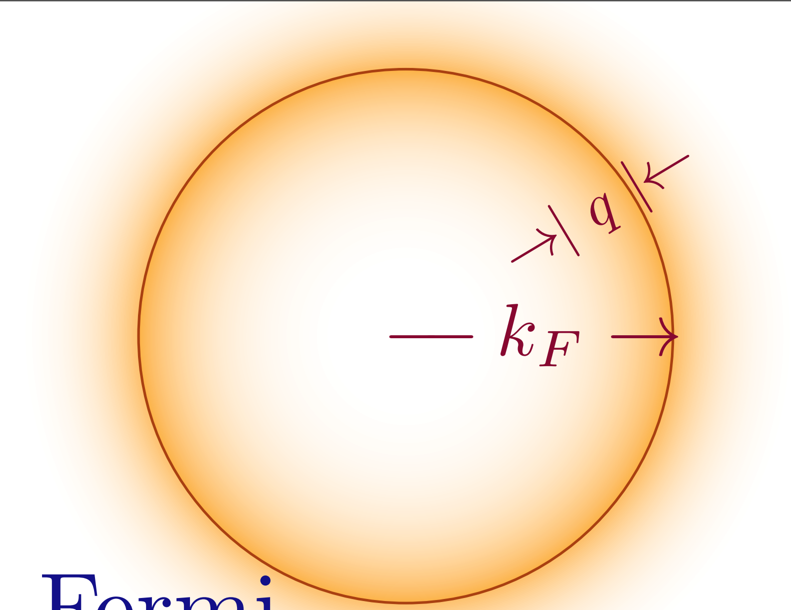
FL Fermi liquid	NFL Bose metal
	
• $k_F^d \sim \mathcal{Q}$, the fermion density	• Hidden Fermi surface with $k_F^d \sim \mathcal{Q}$.
• Sharp fermionic excitations near Fermi surface with $\omega \sim \lvert q \rvert^z$, and $z = 1$.	• Diffuse fermionic excitations with $z = 3/2$ to three loops.
• Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.	• $S \sim T^{(d-\theta)/z}$ with $\theta = d - 1$.
• Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.	• $S_E \sim k_F^{d-1} P \ln P$.

Compressible quantum matter

A. Fermi liquids:graphene

B. Holography: Reissner - Nördstrom solution

C. Non-Fermi liquids: Bose metals and U(1) spin liquids

D. Holography: scaling arguments for entropy and entanglement entropy

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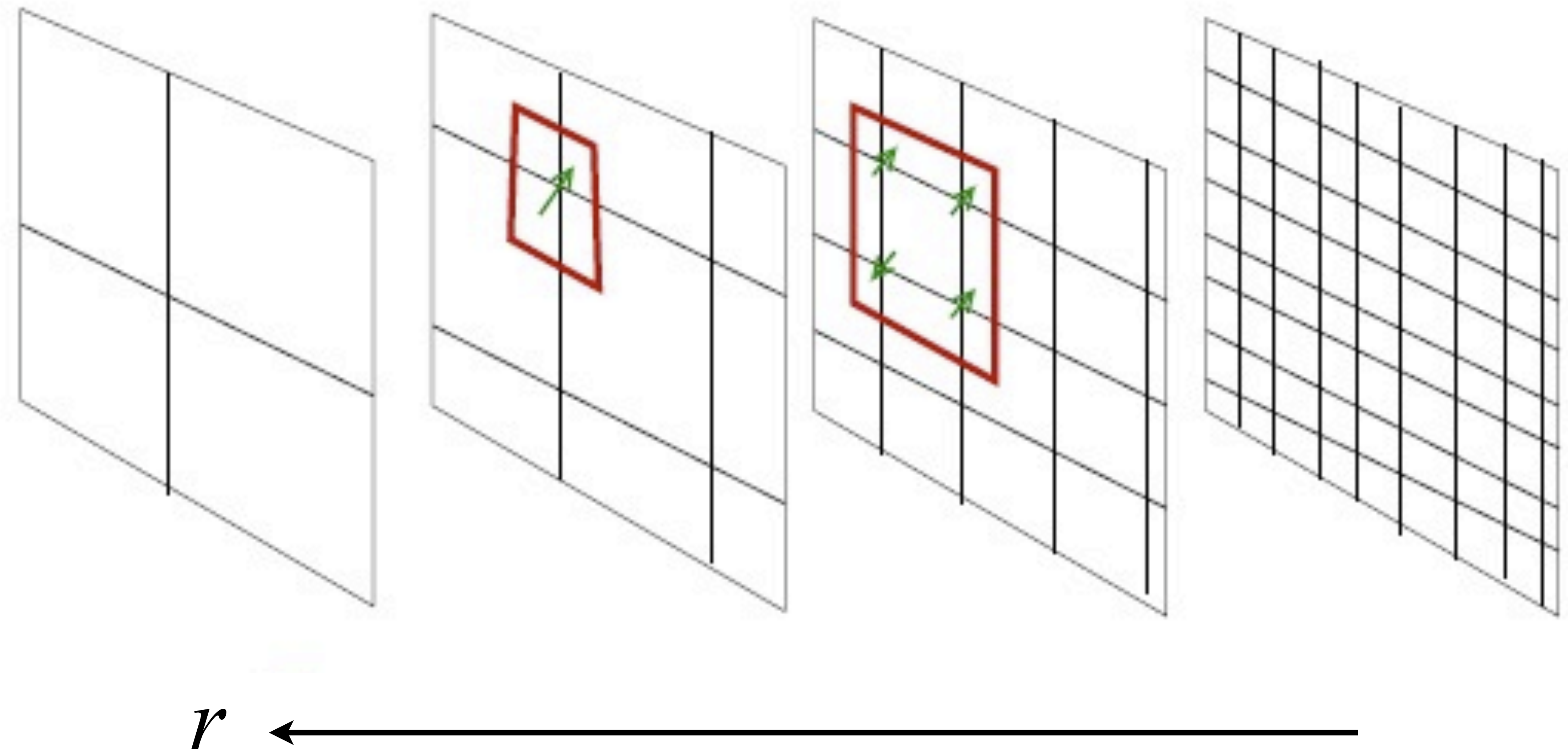
Holography



Consider the metric which transforms under rescaling as

$$\begin{aligned} x_i &\rightarrow \zeta\, x_i \\ t &\rightarrow \zeta^z\, t \\ ds &\rightarrow \zeta^{\theta/d}\, ds. \end{aligned}$$

This identifies z as the dynamic critical exponent ($z = 1$ for "relativistic" quantum critical points).

θ is the violation of hyperscaling exponent.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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This identifies z as the dynamic critical exponent ($z = 1$ for "relativistic" quantum critical points).

θ is the violation of hyperscaling exponent.
The most general choice of such a metric is

$$
\boxed{ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)}
$$

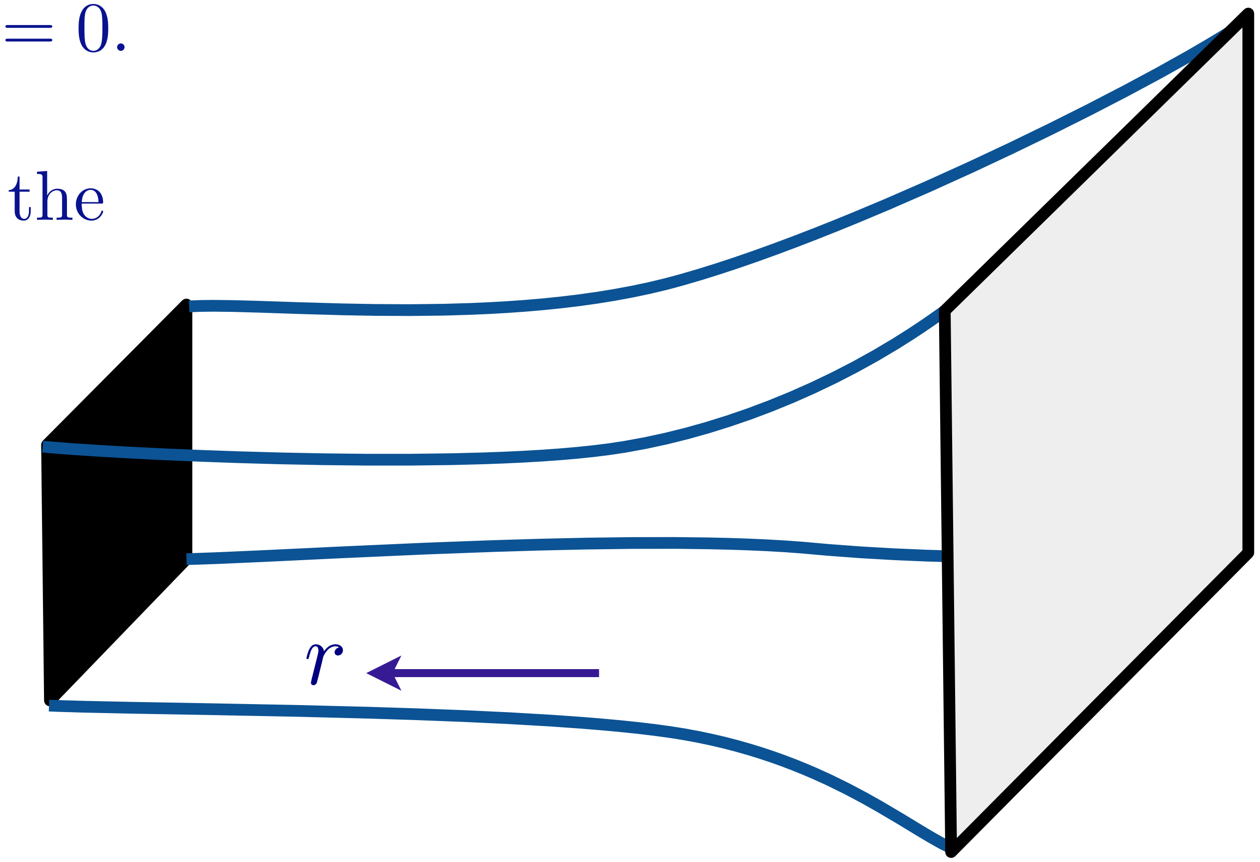
We have used reparametrization invariance in r to choose so that it scales as $r \rightarrow \zeta^{(d-\theta)/d} r$.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

At $T > 0$, there is a "black-brane" at $r = r_h$.

The Beckenstein-Hawking entropy of the black-brane is the thermal entropy of the quantum system $r = 0$.

The entropy density, S, is proportional to the "area" of the horizon, and so $S \sim r_h^{-d}$



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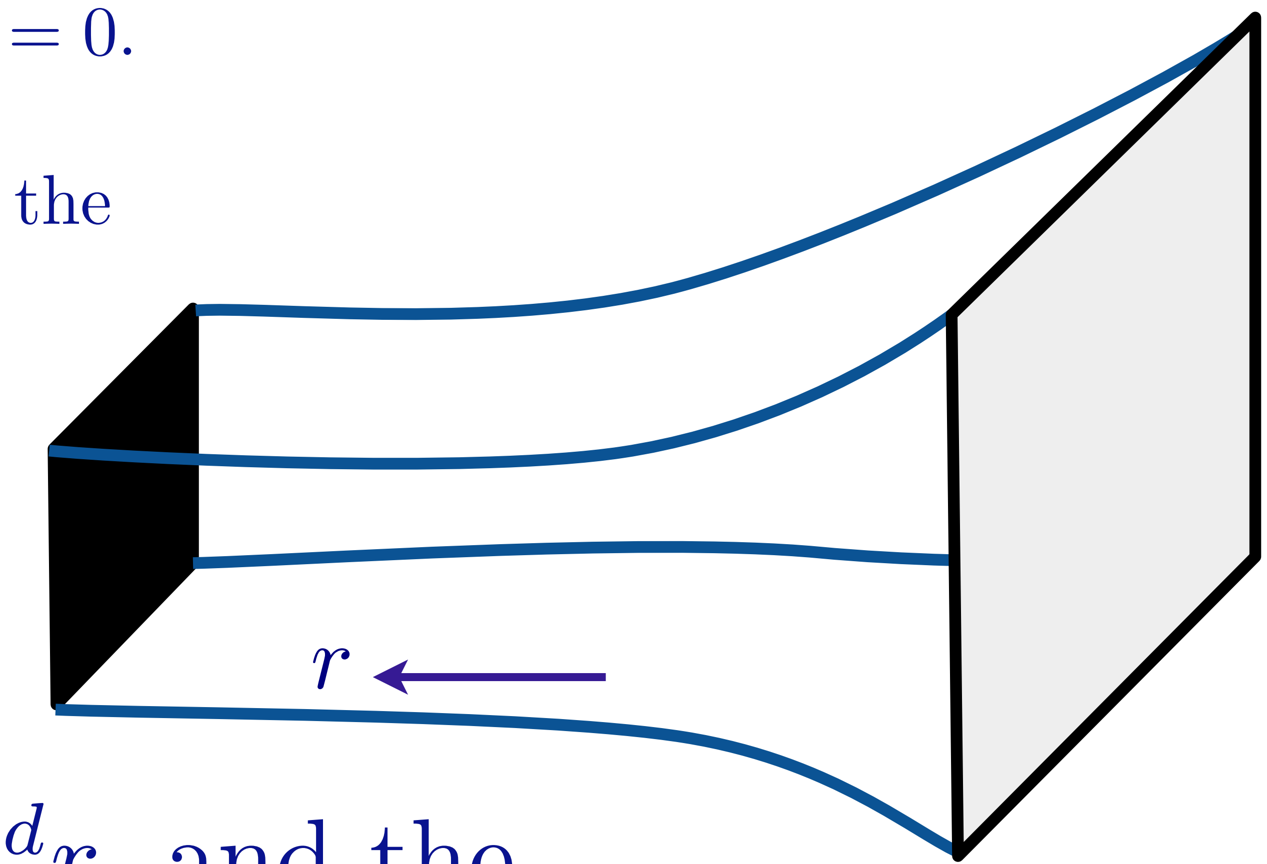

Under rescaling $r \to \zeta^{(d-\theta)/d} r$, and the temperature $T \sim t^{-1}$, and so

$$S \sim T^{(d-\theta)/z} = T^{d_{\rm eff}/z}$$

where $\theta = d - d_{\rm eff}$ measures "dimension deficit" in the phase space of low energy degrees of a freedom.

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

At $T > 0$, there is a *horizon*, and computation of its Bekenstein-Hawking entropy shows

$$S \sim T^{(d-\theta)/z}.$$

So θ is indeed the violation of hyperscaling exponent as claimed. For a compressible quantum state we should therefore *choose* $\theta = d - 1$.
No additional choices will be made, and all subsequent results are consequences of the assumption of the existence of a holographic dual.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holography of a non-Fermi liquid

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

$$\theta = d - 1$$

The null energy condition (stability condition for gravity) yields a new inequality

$$z \geq 1 + \frac{\theta}{d}$$

In $d = 2$, this implies $z \geq 3/2$. So the lower bound is precisely the value obtained from the field theory.

N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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$$\theta = d - 1$$

Application of the Ryu-Takayanagi minimal area formula to a dual Einstein-Maxwell-dilaton theory yields

$$S_E \sim P \ln P$$

with a co-efficient *independent* of the shape of the entangling region. These properties are just as expected for a circular Fermi surface.

N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).
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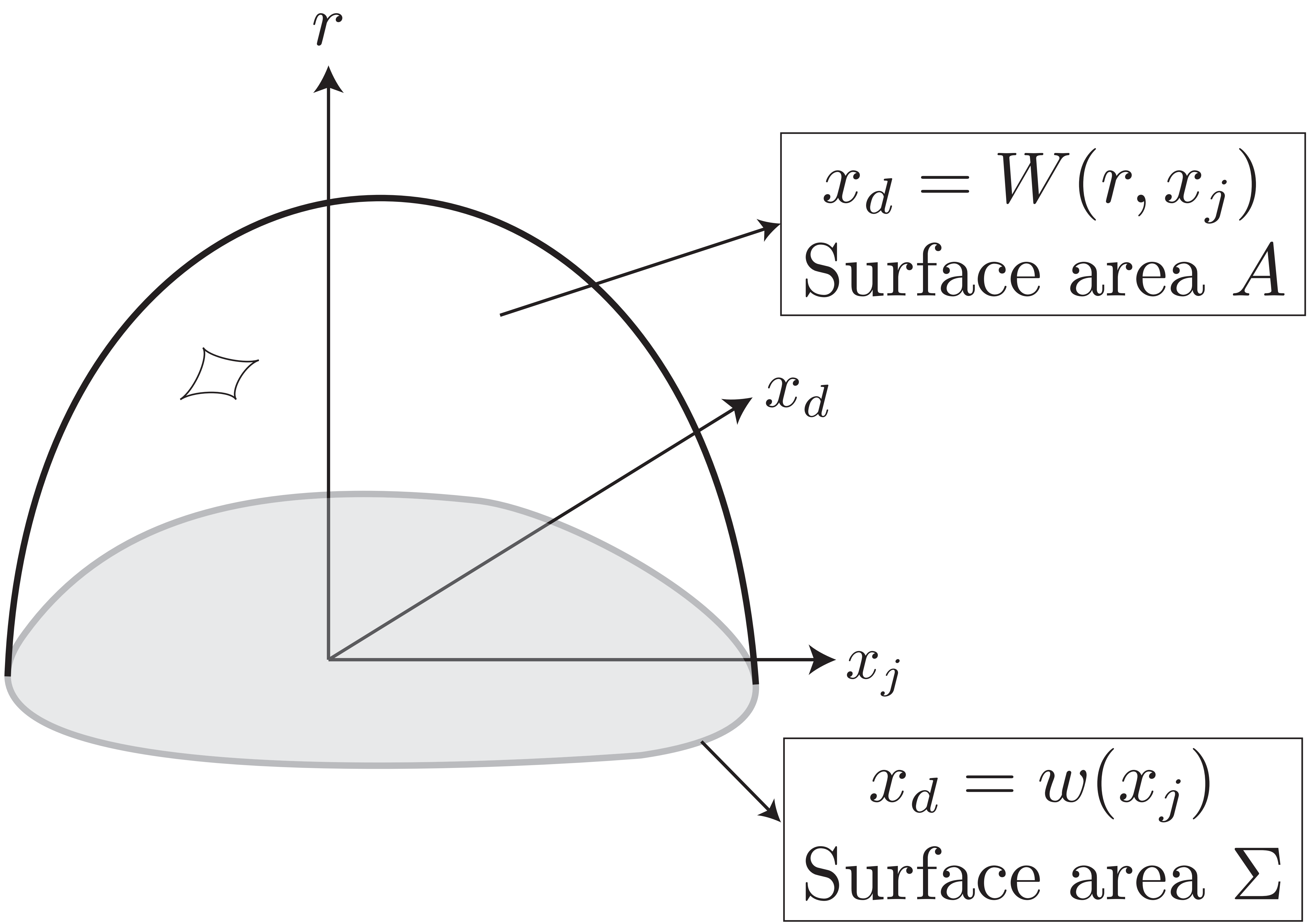

r
$x_d = W(r, x_j)$
Surface area A
x_d
x_j
$x_d = w(x_j)$
Surface area Σ

Let us parameterize the extended surface by (see Fig. 1)

$$x_d = W(r, x_j). \tag{3.4}$$

Then we have to find the optimum function $W(r, x_j)$ subject to the constraint

$$W(0, x_j) = w(x_j). \tag{3.5}$$

Let us compute the area of the general holographic surface in (3.4). The induced metric on this surface is

$$d\sigma^2 = \frac{L^2}{r^2}\left[\left(\hat{g}_0 r^{2\theta/(d-\theta)} + \left(\frac{\partial W}{\partial r}\right)^2\right) dr^2 + 2\frac{\partial W}{\partial r}\frac{\partial W}{\partial x_j} dr dx_j + \left(\delta_{jj'} + \frac{\partial W}{\partial x_j}\frac{\partial W}{\partial x_{j'}}\right) dx_j dx_{j'}\right] \tag{3.6}$$

The area element on the surface is determined by the square-root of the determinant of the induced metric, which is

$$dA = L^d\, \hat{g}_0^{1/2}\, \frac{dr}{r^{d-\theta/(d-\theta)}}\, d^{d-1}x_j \left[1 + \left(\frac{\partial W}{\partial x_j}\right)^2 + \frac{r^{-2\theta/(d-\theta)}}{\hat{g}_0}\left(\frac{\partial W}{\partial r}\right)^2\right]^{1/2} \tag{3.7}$$

We now observe that for $d - \theta/(d-\theta) \geq 1$, which is equivalent to (1.8), the r integral is divergent as $r \to 0$: then the leading term to the integral over dA is an ultraviolet contribution proportional to Σ (see Fig. 1) which yields the 'area law' of entanglement entropy. Thus we expect that the inequality (1.8) applies to holographic duals of all generic local quantum field theories which do not have large accidental degeneracies in their low energy spectrum. Also, as we noted earlier, relativistic conformal field theories have $\theta = 0$.

The remainder of this section limits consideration to the case $\theta = d - 1$ of interest in this paper, where we have a logarithmic violation of the area law. Let us study the nature of the $r \to 0$ limit more carefully. Let us expand W in this limit as

$$W(r, x_j) = w(x_j) + r^n \sigma(x_j) + \dots \quad , \quad r \to 0, \tag{3.8}$$

where it remains to determine the exponent, n, of the leading correction, and σ is an arbitrary function of the $d - 1$ co-ordinates. Inserting this in (3.7) we have

$$dA = L^d \, \hat{g}_0^{1/2} \, \frac{dr}{r} \, d^{d-1} x_j \left[1 + \left(\frac{\partial w}{\partial x_j} \right)^2 + 2 r^n \frac{\partial w}{\partial x_j} \frac{\partial \sigma}{\partial x_j} + \frac{r^{2(n-d)}}{\hat{g}_0} n^2 \sigma^2 + \dots \right]^{1/2} \tag{3.9}$$

The variational derivative of the integral of this expression with respect to $\sigma(x_j)$ must vanish. A non-trivial solution is only possible if the two leading terms in powers of r can cancel against each other. So we must have $n = 2(n - d)$ or

$$n = 2d. \tag{3.10}$$

So the r- and σ-dependent terms inside the square-root in are indeed subdominant, and to leading logarithmic accuracy we can write

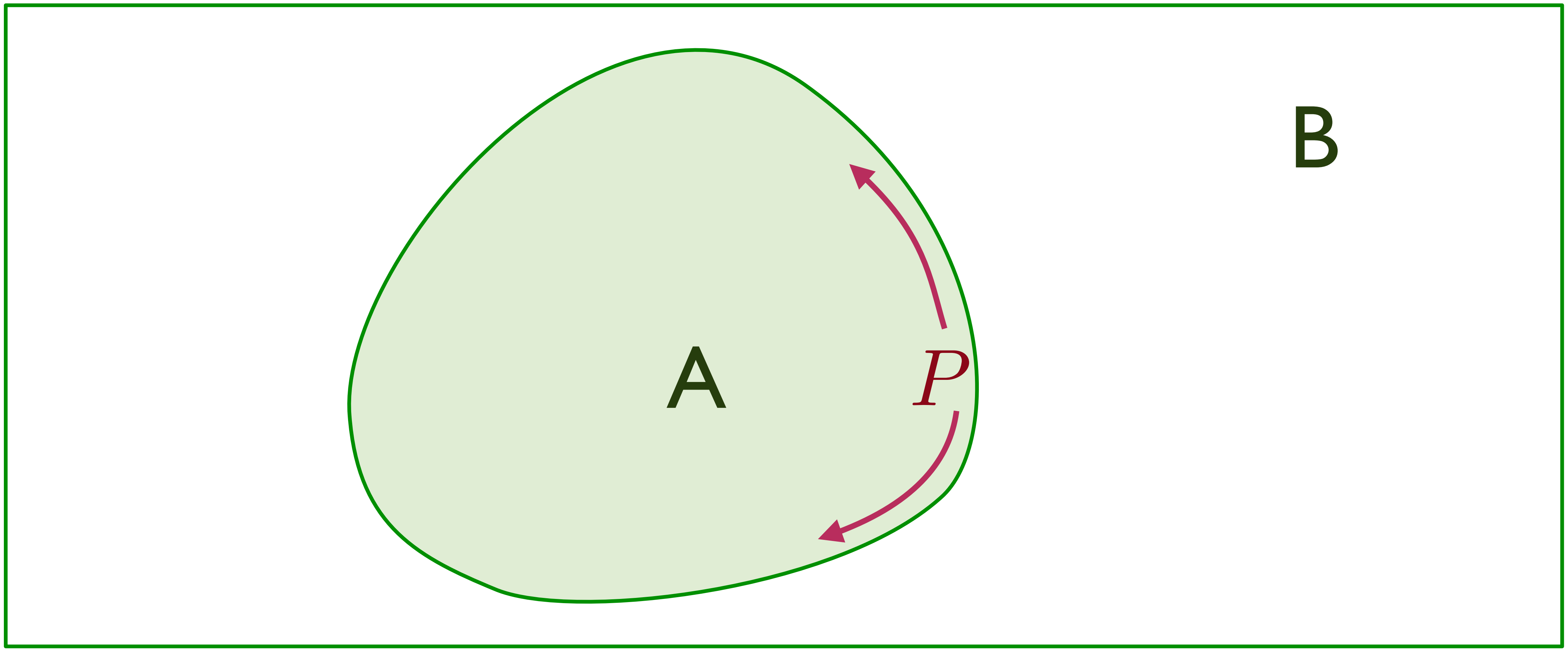
$$S_E = \frac{2\pi}{\kappa^2} \int dA = \frac{2\pi L^d}{\kappa^2} \, \hat{g}_0^{1/2} \, \Sigma \int_{r_{\rm min}}^{r_{\rm max}} \frac{dr}{r} \tag{3.11}$$

where

$$\Sigma = \int d^{d-1}x_j \left[1 + \left(\frac{\partial w}{\partial x_j} \right)^2 \right]^{1/2} . \tag{3.12}$$

The quantity Σ depends only on the entangling region on the boundary, and indeed it is just its surface area. So we conclude that the log-divergent entanglement entropy is proportional to the surface area of the entangling region, and is otherwise independent of its shape. This is precisely the property of the entanglement entropy of a spherical Fermi surface [43, 58]: our holographic analysis is for spatially isotropic systems, so a spherical Fermi surface is expected. Also note from (3.2) that the prefactor of (3.11) is of order $\mathcal{Q}^{(d-1)/d}$, and so the complete $\mathcal{Q}$-dependence of the entanglement entropy is that displayed in (1.7).

Entanglement entropy of a non-Fermi liquid in holography



$$\boxed{\theta = d - 1}$$

Logarithmic violation of "area law": $S_E = \mathcal{C}_E \, k_F P \ln(k_F P)$

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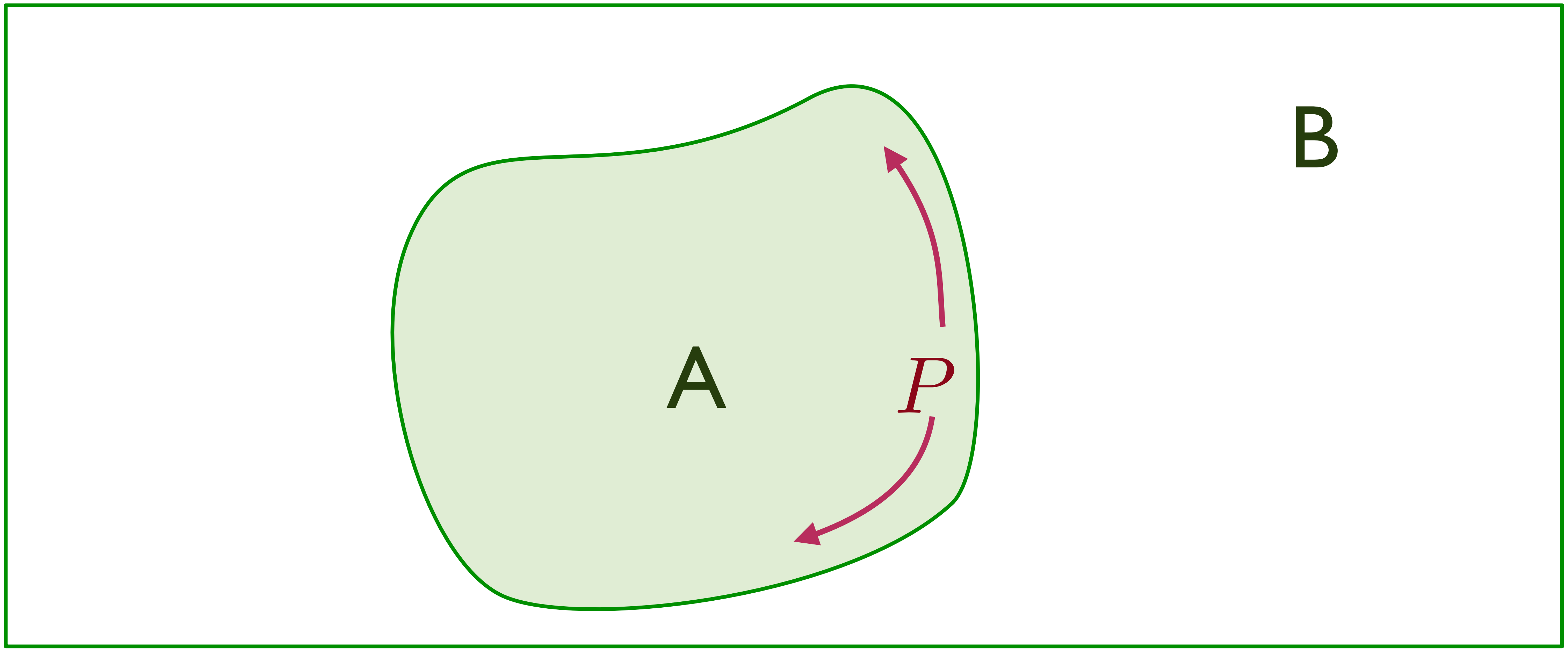

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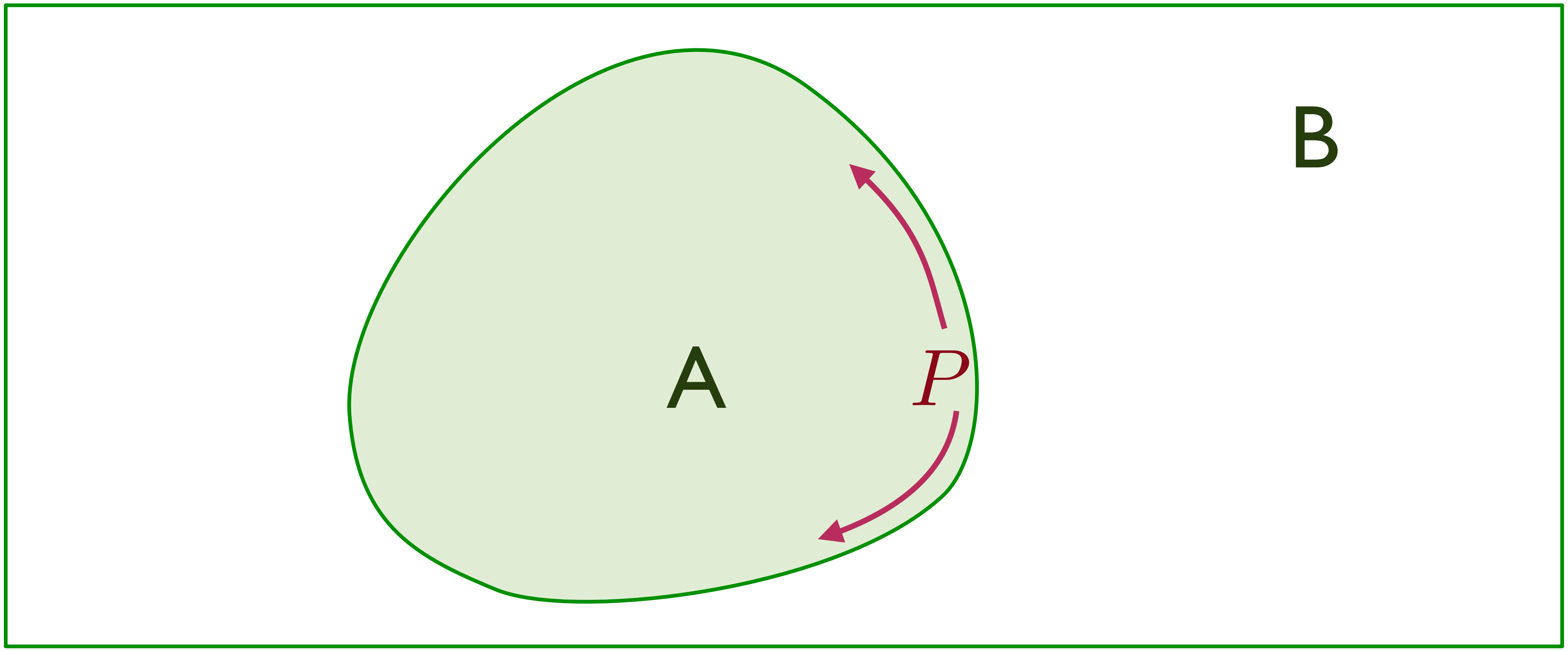

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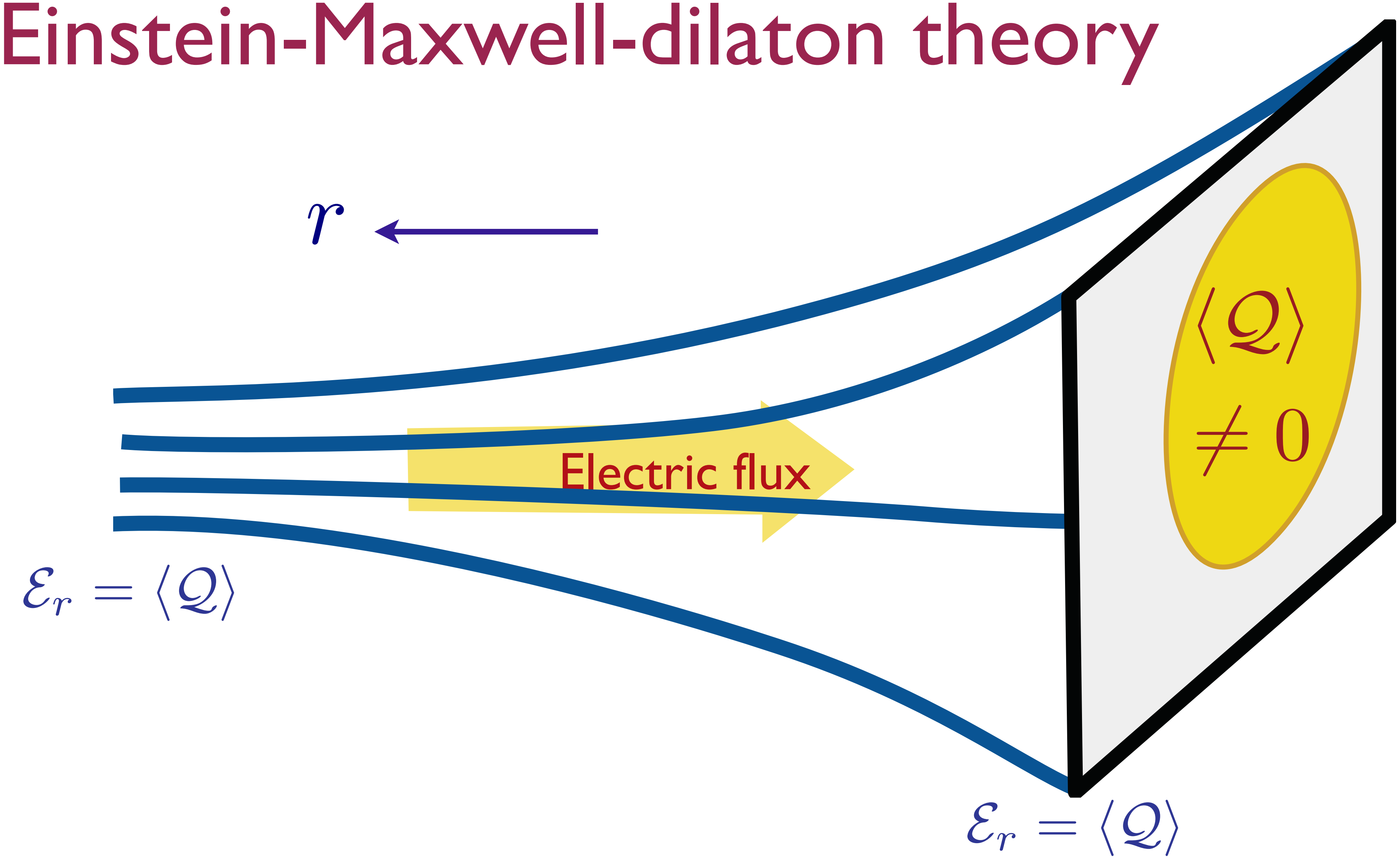
Logarithmic violation of "area law": $S_E = \mathcal{C}_E \, k_F P \ln(k_F P)$

Holography of a non-Fermi liquid

Einstein-Maxwell-dilaton theory

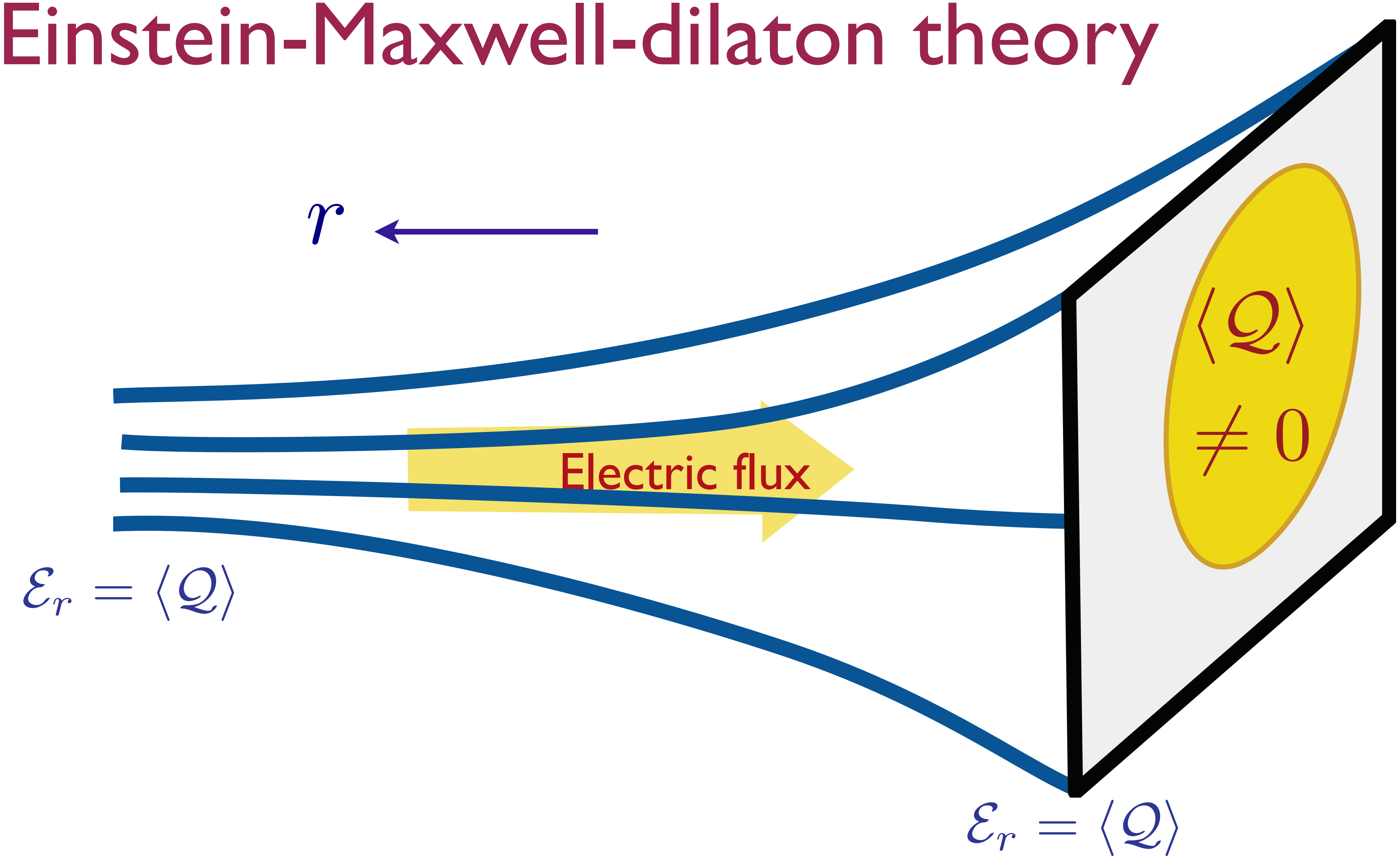


$$\mathcal{S} = \int d^{d+2}x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R - 2(\nabla\Phi)^2 - \frac{V(\Phi)}{L^2} \right) - \frac{Z(\Phi)}{4e^2} F_{ab} F^{ab} \right]$$

with $Z(\Phi) = Z_0 e^{\alpha\Phi}$, $V(\Phi) = -V_0 e^{-\beta\Phi}$, as $\Phi \to \infty$.

C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010).
S. S. Gubser and F. D. Rocha, Phys. Rev. D **81**, 046001 (2010).
N. Iizuka, N. Kundu, P. Narayan and S. P. Trivedi, arXiv:1105.1162 [hep-th].

Einstein-Maxwell-dilaton theory



Leads to metric $ds^2 = L^2 \left(-f(r)dt^2 + g(r)dr^2 + \dfrac{dx^2 + dy^2}{r^2} \right)$

with $f(r) \sim r^{-\gamma}$, $g(r) \sim r^{\delta}$, $\Phi(r) \sim \ln(r)$ as $r \to \infty$.

C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010).
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Holography of a non-Fermi liquid

$$\boxed{ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)}$$

The $r \to \infty$ metric has the above form with

$$\begin{aligned} \theta &= \frac{d^2\beta}{\alpha + (d-1)\beta} \\ z &= 1 + \frac{\theta}{d} + \frac{8(d(d-\theta)+\theta)^2}{d^2(d-\theta)\alpha^2}. \end{aligned}$$

Note $z \geq 1 + \theta/d$.

In the present theory, we have to choose α or β so that $\theta = d - 1$.
Needed: a dynamical quantum analysis which automatically selects this value of θ.

Holography of a non-Fermi liquid

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

$$\boxed{\theta = d - 1}$$

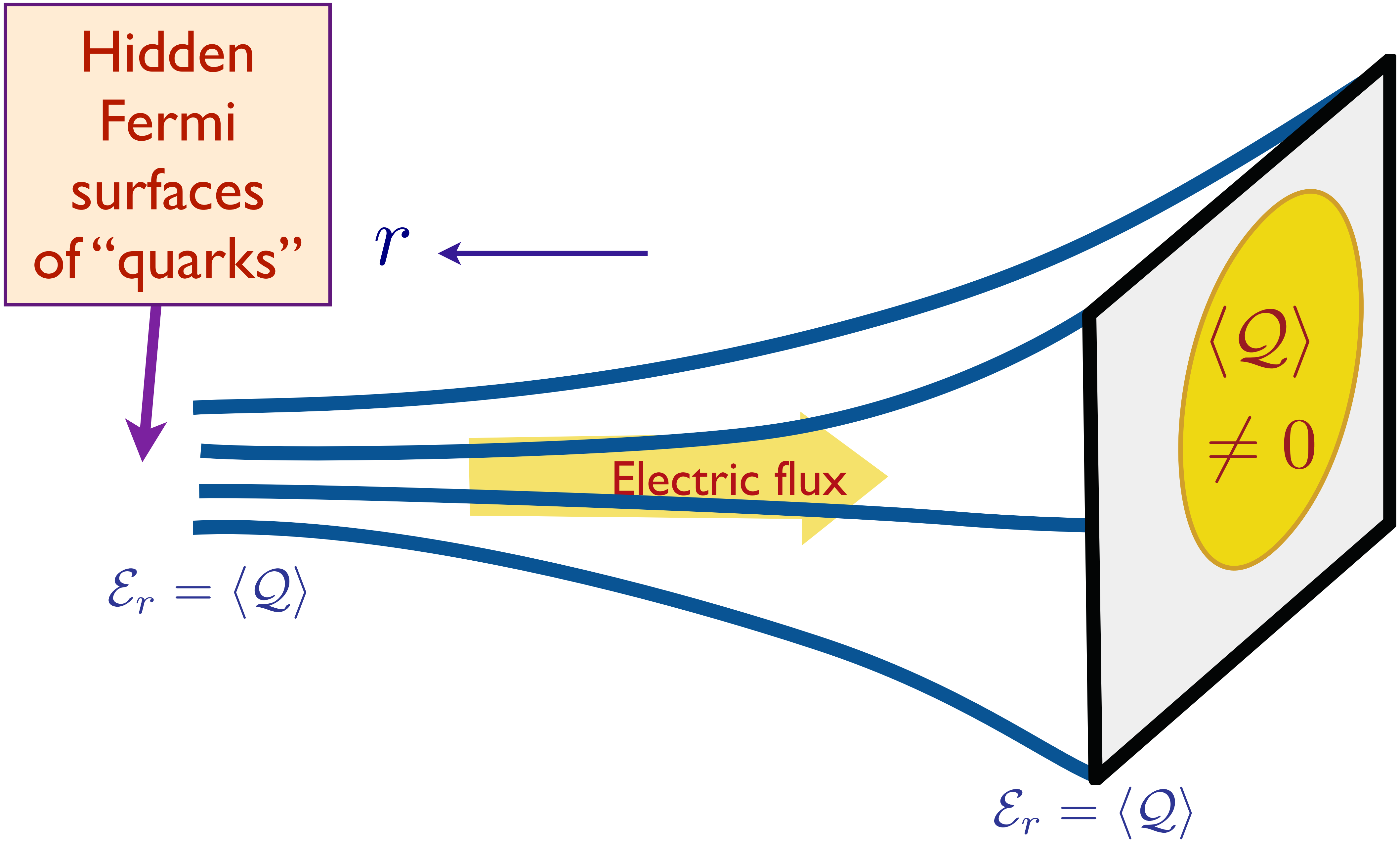
Using the Einstein-Maxwell-dilaton theory we obtain a more precise result for the entanglement entropy

$$S_E = \mathcal{C}_E\, \mathcal{Q}^{(d-1)/d} P \ln(\mathcal{Q}^{(d-1)/d} P)$$

where the co-efficient $\mathcal{C}_E$ is *independent* of all UV details (*e.g.* boundary conditions on the dilaton), but depends on z and other IR characteristics. These properties are just as expected for a circular Fermi surface with a Fermi wavevector obeying $\mathcal{Q} \sim k_F^d$.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holography of a non-Fermi liquid



This is a "bosonization" of the *hidden* Fermi surface

Conclusions

Compressible quantum matter

Evidence for *hidden Fermi surfaces* in compressible states obtained for a class of holographic Einstein-Maxwell-dilaton theories. These theories describe a *non-Fermi liquid* (NFL) state of gauge theories at non-zero density.

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- Evidence for Luttinger theorem in prefactor of S_E.
- Monopole operators lead to crystalline state, and have the correct features to yield Friedel oscillations of a Fermi surface.

Holographic theory of a compressible state

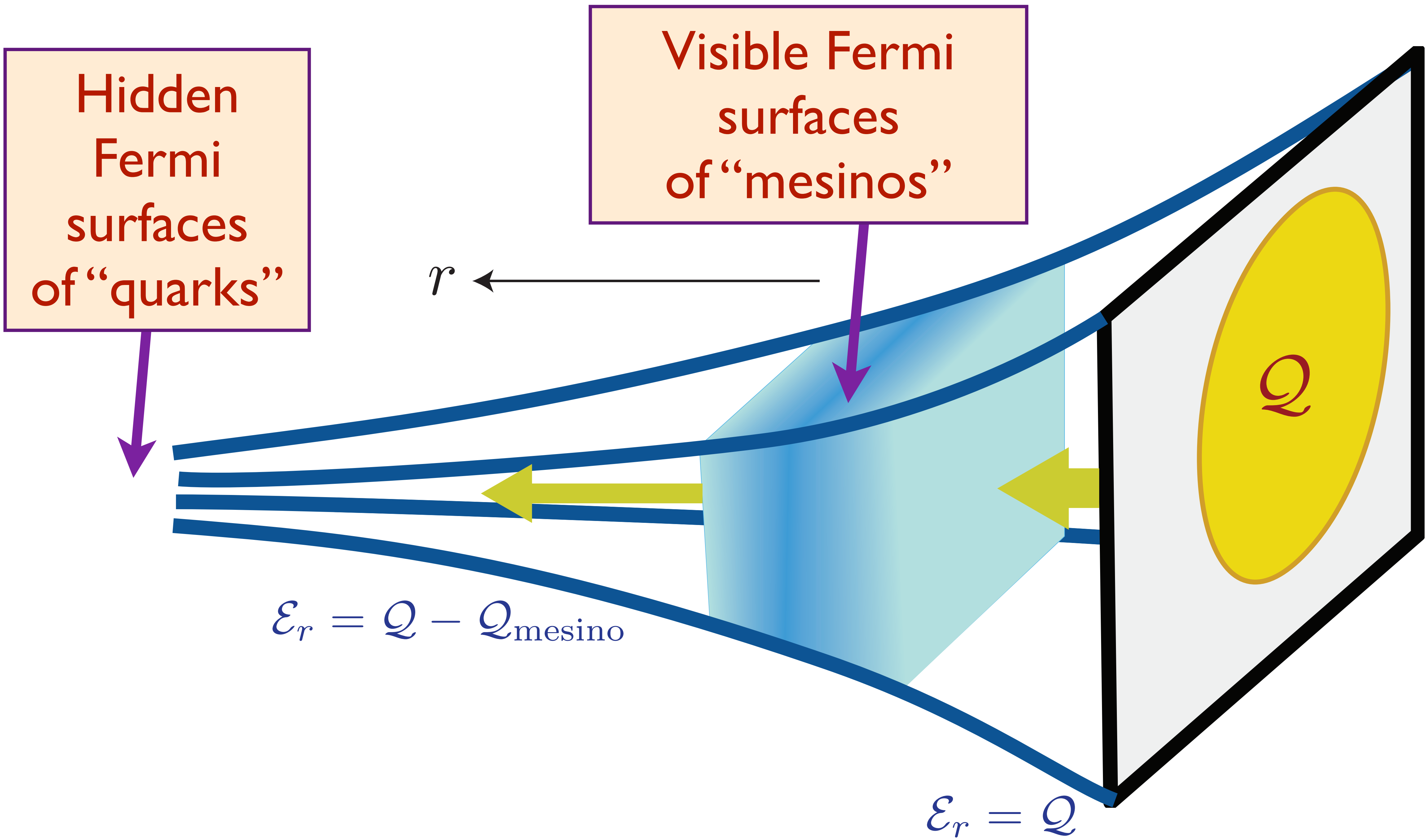
Add a fermonic field ψ to the bulk effective action, carrying the U(1) charge of the bulk gauge field: consequently, this field corresponds to a boundary fermion which carries charge $\mathcal{Q}$, but is *neutral* w.r.t to any gauge fields in the boundary theory. We refer to such fermions as *mesinos*.

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right) - \frac{1}{4g_M^2}F_{ab}F^{ab} + i(\overline{\psi}\Gamma^M D_M\psi + m\overline{\psi}\psi)\right]$$

For a finite density state, we impose the boundary condition $A_t(r \to 0) = \mu$. Procedure to solve the bulk theory:

1. Assume some reasonable form for the electric potential $A_t(r)$ and the metric $g_{\mu\nu}(r)$.
2. Solve Dirac equation for fermions in this background.
3. Occupy negative energy fermions states.
4. Compute the U(1) density and $T_{\mu\nu}$ of the occupied states.
5. Use Poisson's equation and Einstein's equations to recompute $A_t(r)$ and the metric $g_{\mu\nu}(r)$.
6. Return to step 2.

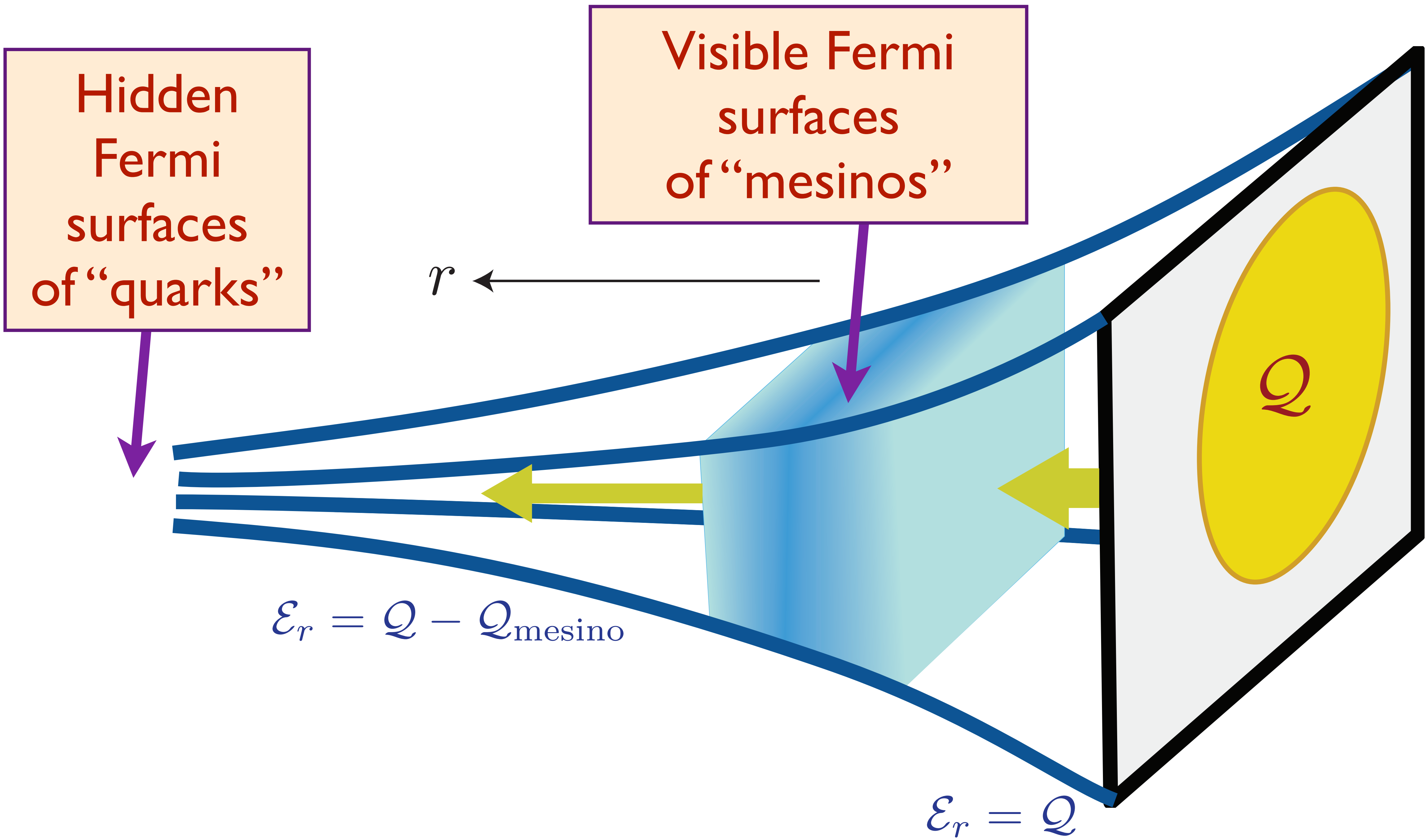
Holographic theory of a fractionalized-Fermi liquid (FL*)



A state with partial fractionalization, and partial electric flux exiting horizon

S. Sachdev, *Physical Review Letters* **105**, 151602 (2010); S. Sachdev, *Physical Review D* **84**, 066009 (2011)

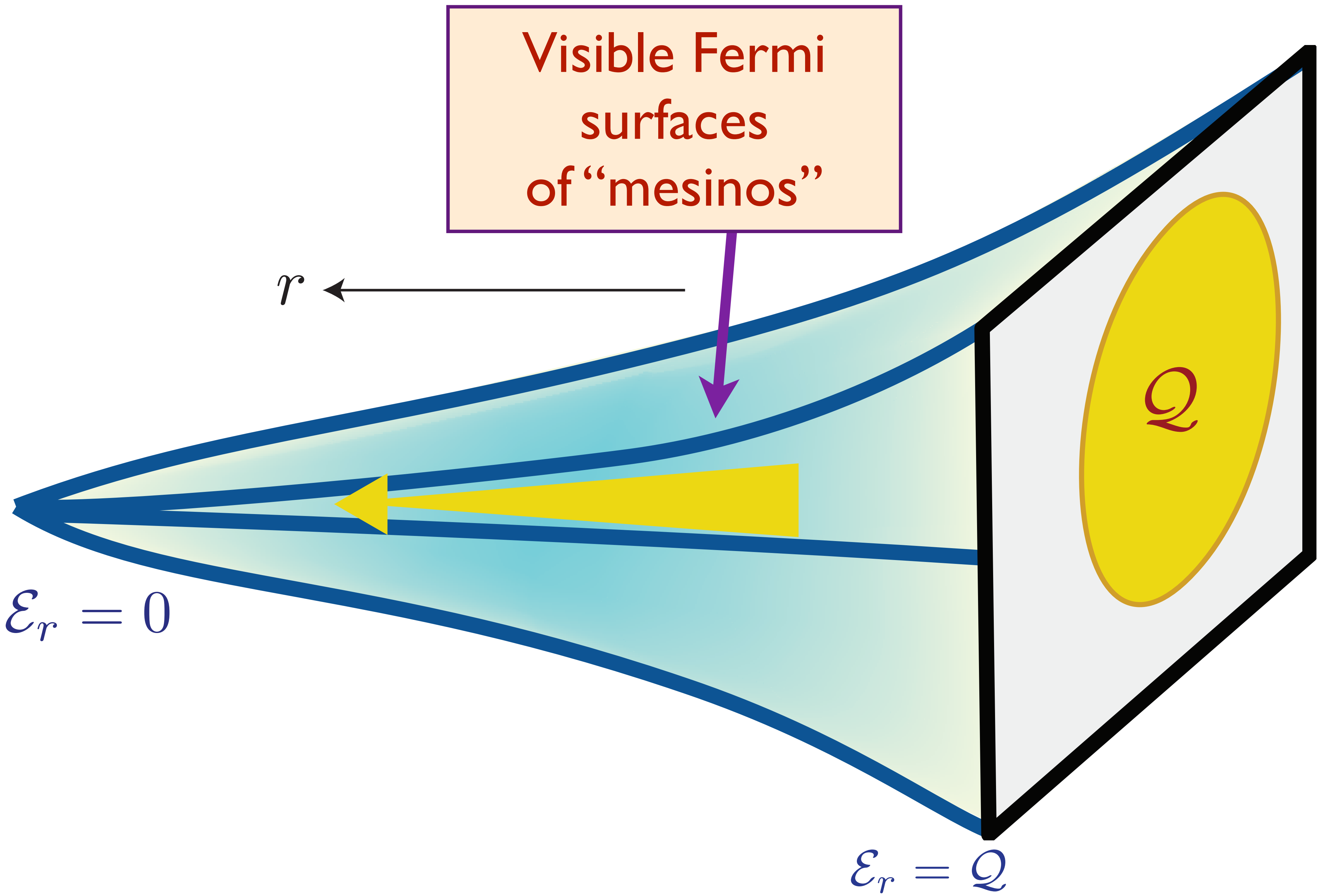
Holographic theory of a fractionalized-Fermi liquid (FL*)



The "mesinos" corresponds to the Fermi surfaces obtained in the early probe fermion computation (S.-S. Lee, Phys. Rev. D **79**, 086006 (2009); H. Liu, J. McGreevy, and D. Vegh, arXiv:0903.2477; M. Čubrović, J. Zaanen, and K. Schalm, Science **325**, 439 (2009)).

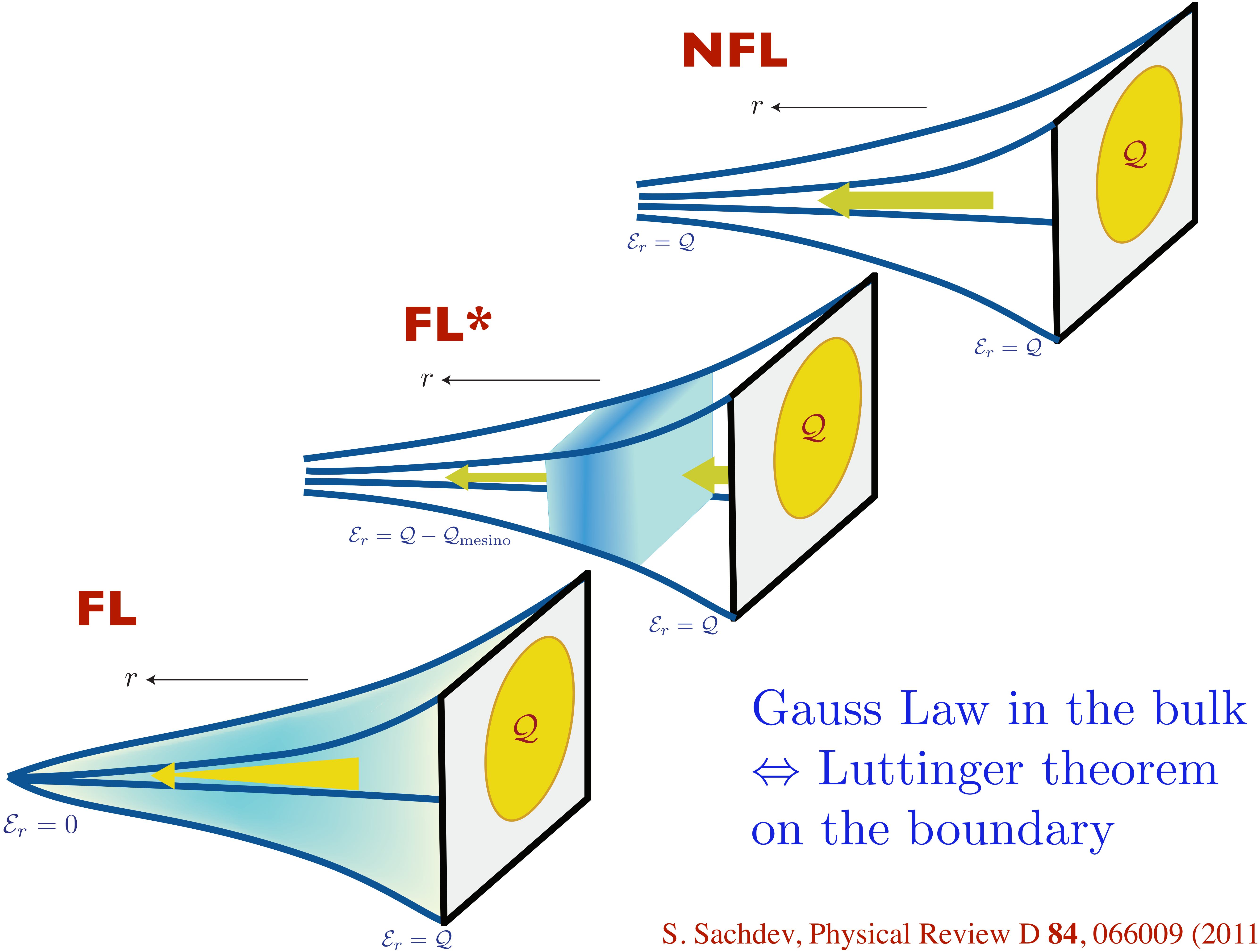
These are spectators, and are expected to have well-defined quasiparticle excitations.

Holographic theory of a Fermi liquid (FL)



- Confining geometry leads to a state which has all the properties of a Landau Fermi liquid.

S. Sachdev, Physical Review D **84**, 066009 (2011)



Gauss Law in the bulk
$\Leftrightarrow$ Luttinger theorem
on the boundary

S. Sachdev, Physical Review D **84**, 066009 (2011)

Compressible quantum matter

the holographic perspective

- Fermi liquid (**FL**): the entire charge $\mathcal{Q}$ is contained in the bulk, and there is no electric flux leaking to infinity.

- Bose metal (**NFL**): All the electric flux leaks to infinity, and this is linked to hidden Fermi surface of gauge-charged 'quarks'.

- Fractionalized Fermi liquid (**FL***): Part of the electric flux leaks to infinity, and remainder is within visible Fermi surfaces in the bulk.

Compressible quantum matter

the cond-mat perspective

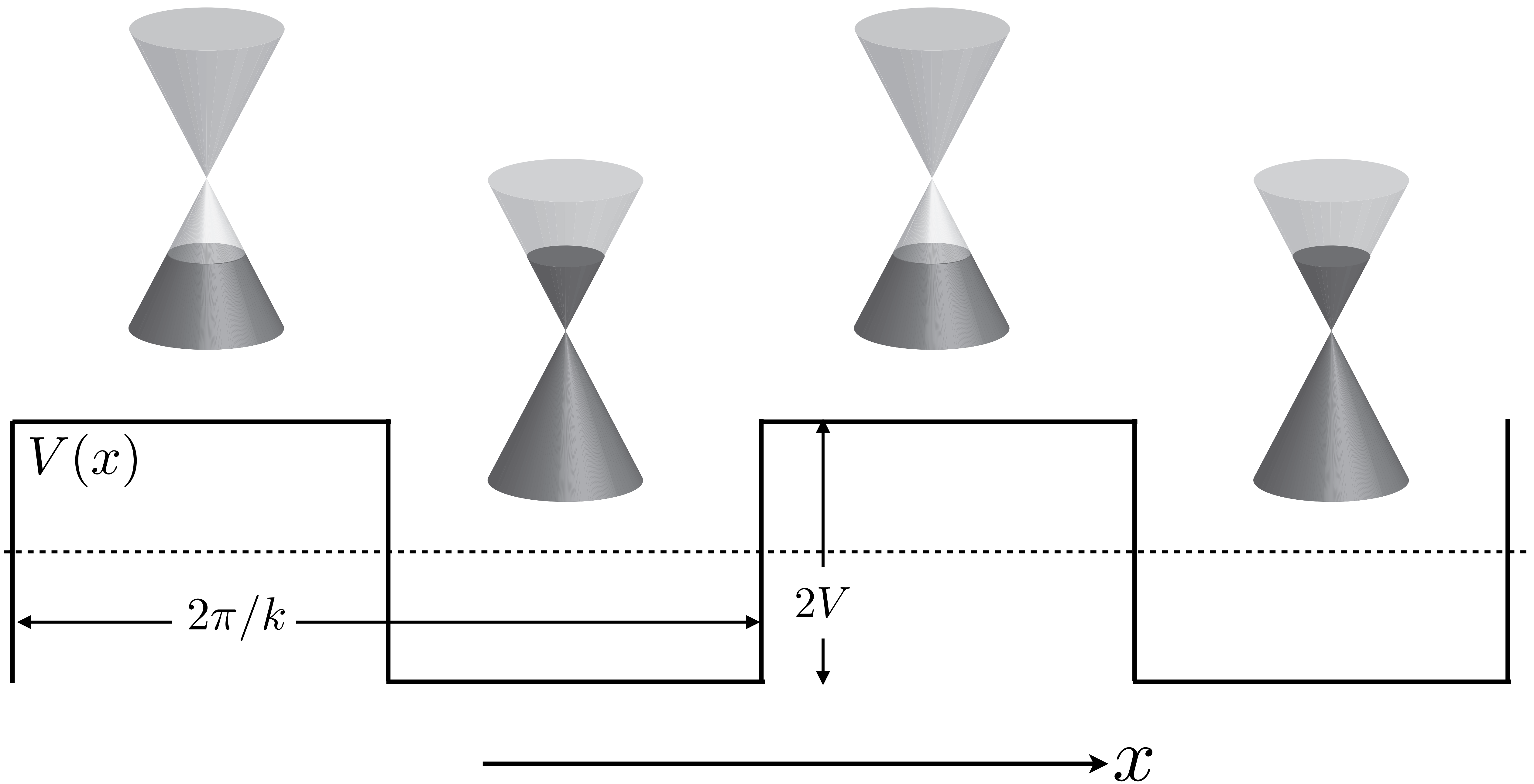
- Fermi liquid (**FL**): the entire charge $\mathcal{Q}$ is contained within visible Fermi surfaces

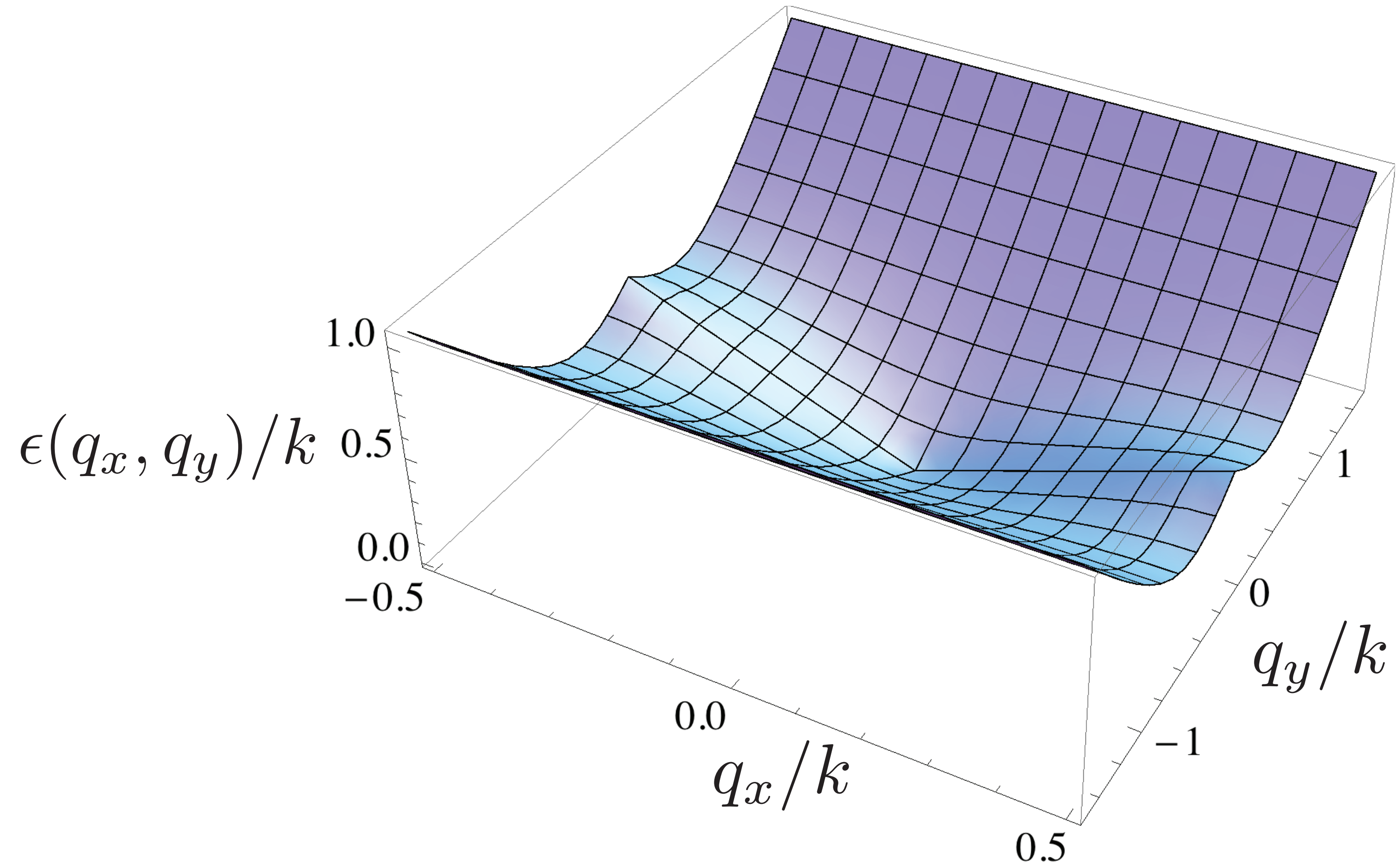
- Bose metal (**NFL**): the entire charge $\mathcal{Q}$ is contained within hidden Fermi surfaces of gauge-charged fermions.

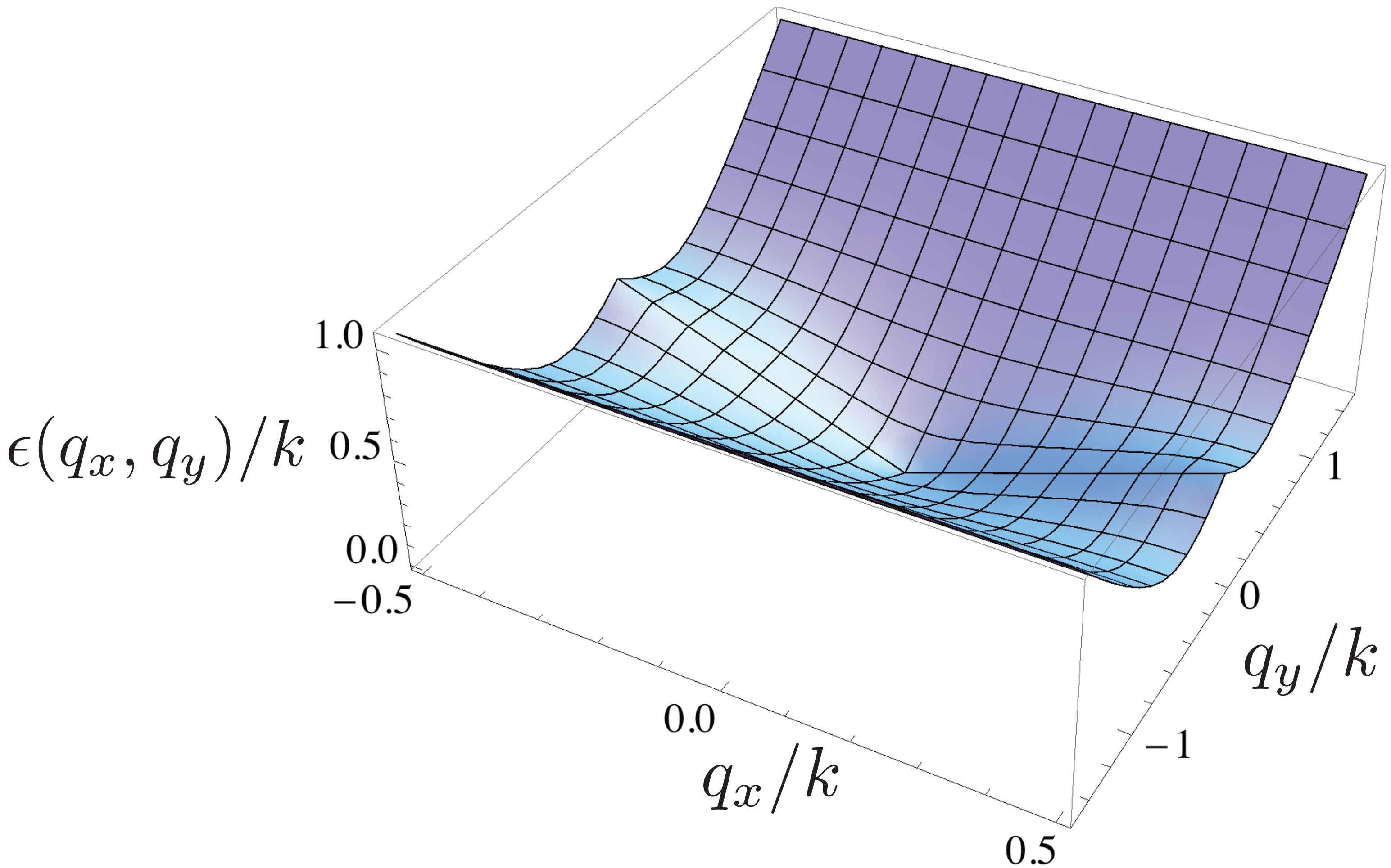
- Fractionalized Fermi liquid (**FL***): the charge $\mathcal{Q}$ is divided between visible and hidden Fermi surfaces.

Conformal field theories in a periodic chemical potential

A. Lucas, P. Chesler, and S. Sachdev arXiv:1308.0329




1.0
0.5
0.0
$\epsilon(q_x, q_y)/k$
−0.5
0.0
0.5
q_x/k
1
0
−1
q_y/k


1.0
0.5
0.0
$\epsilon(q_x, q_y)/k$
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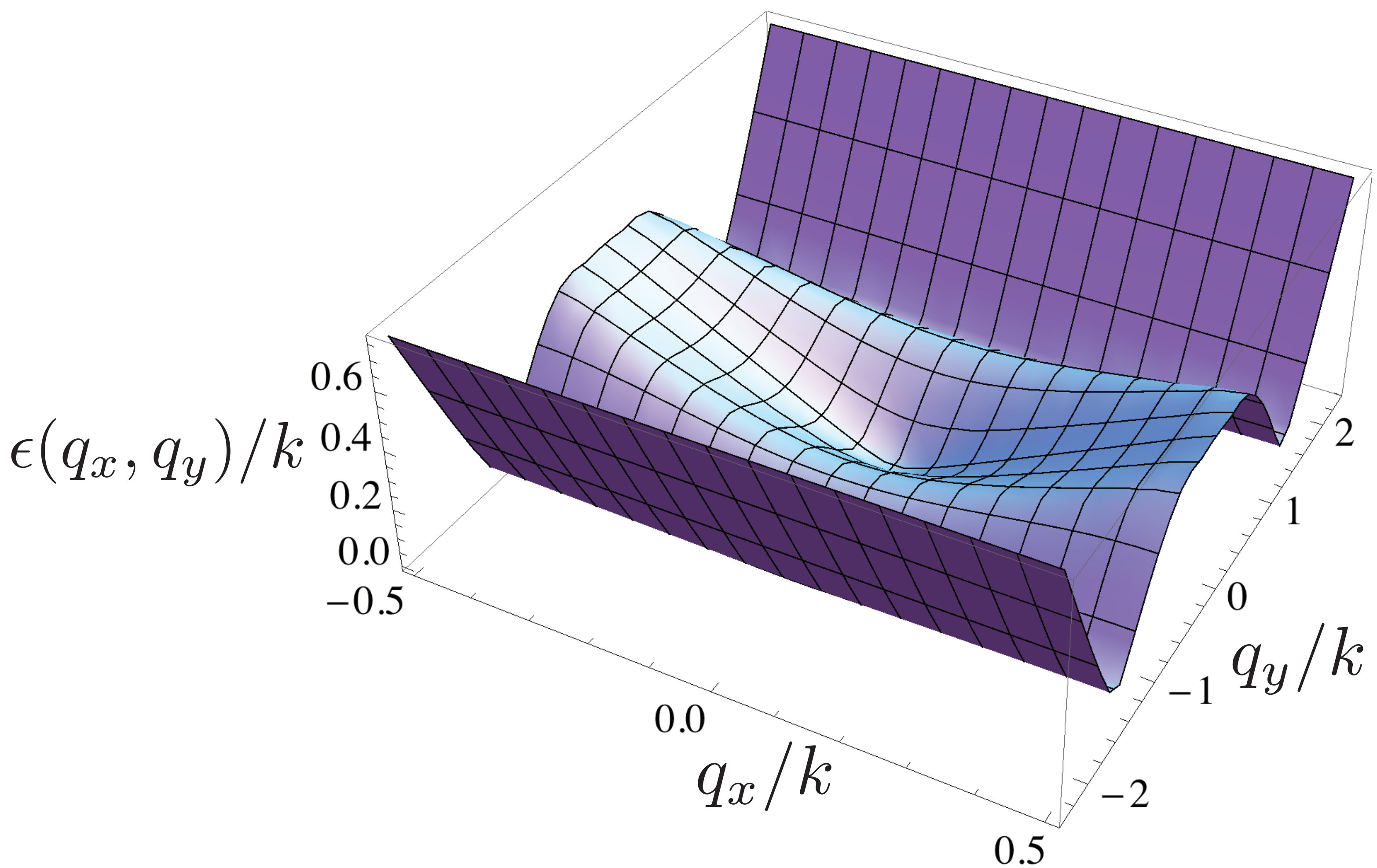
0.0
0.5
q_x/k
1
0
−1
q_y/k


0.6
0.4
0.2
0.0
$\epsilon(q_x, q_y)/k$
−0.5
0.0
0.5
q_x/k
2
1
0
−1
−2
q_y/k

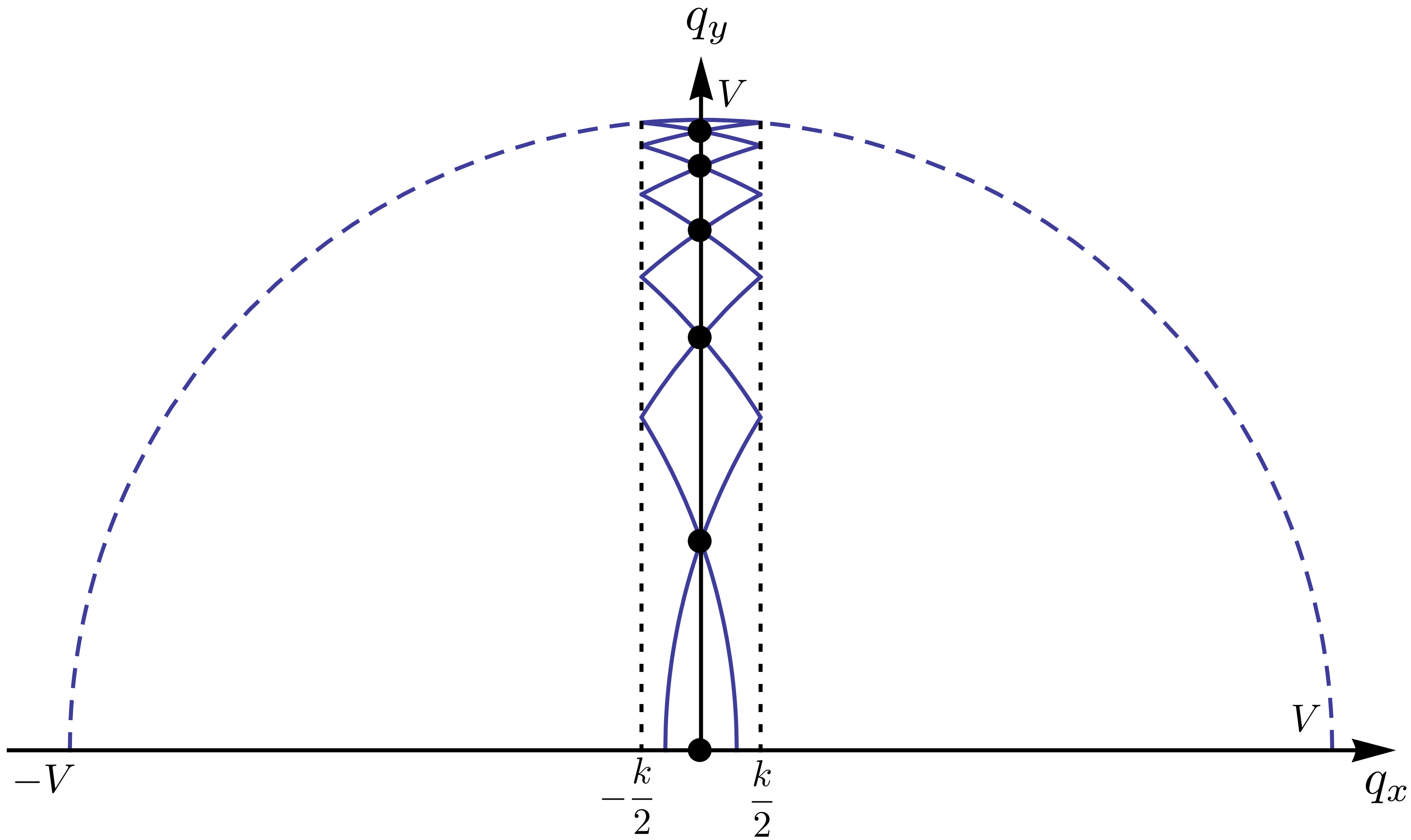
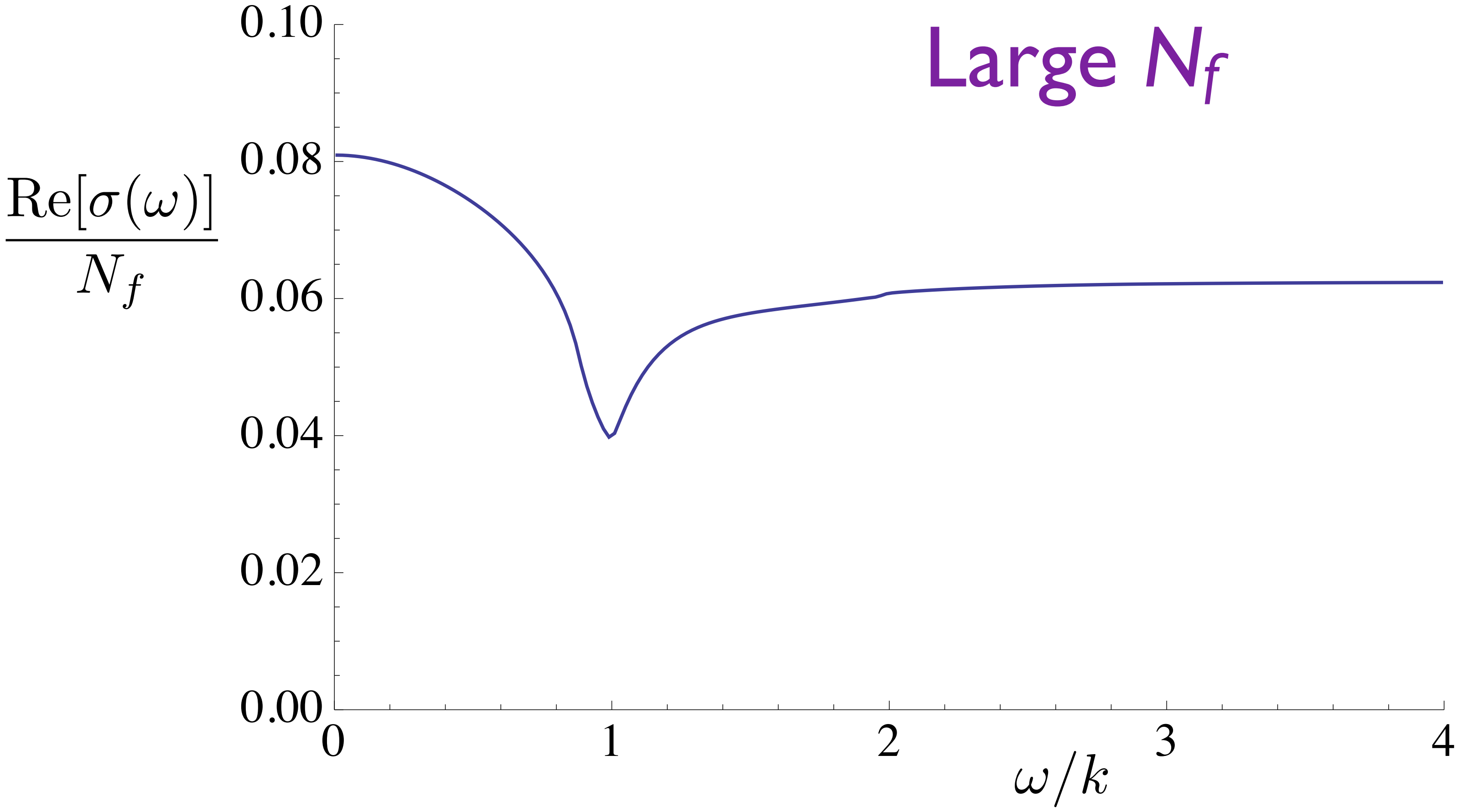


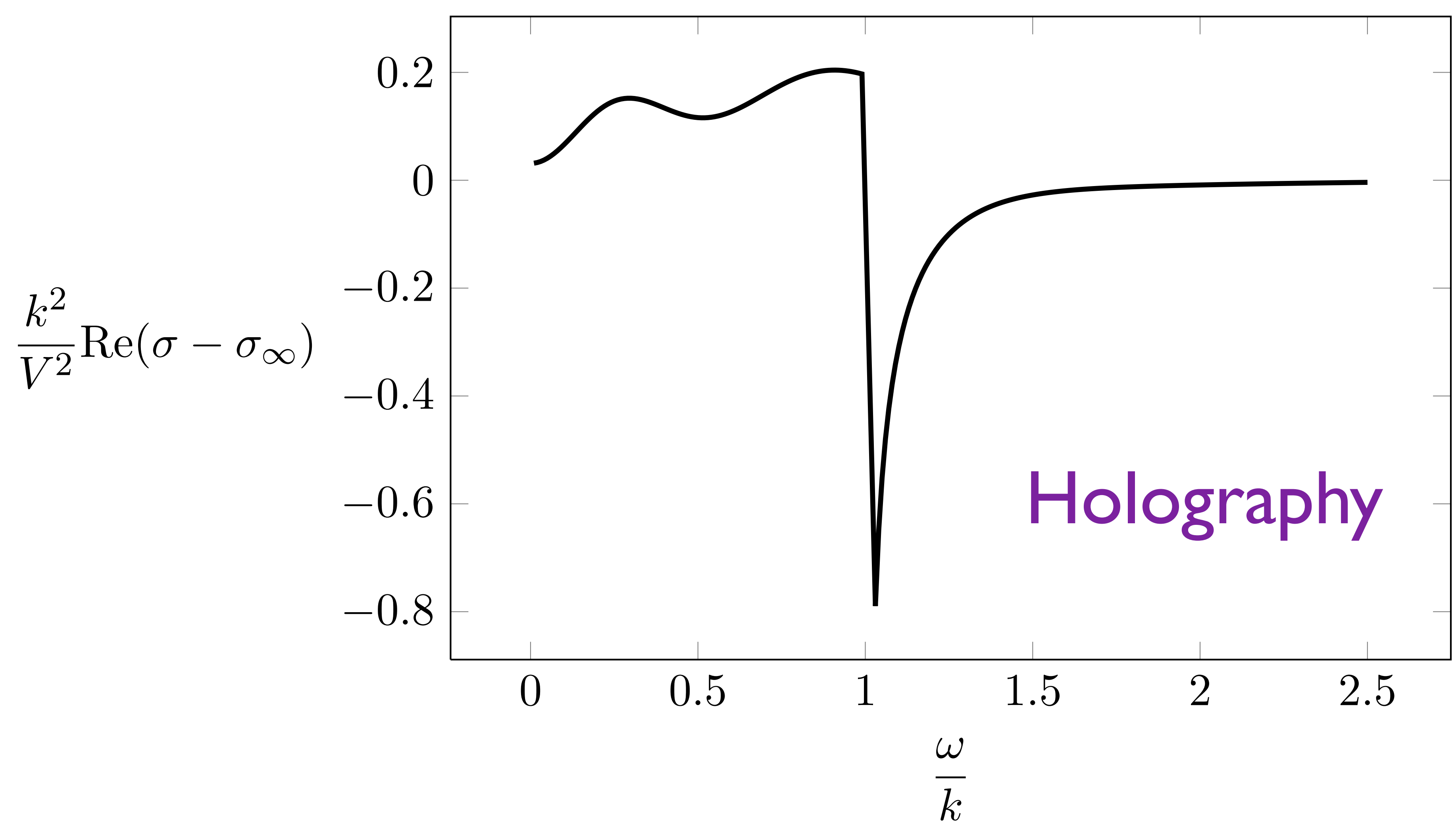
FIGURE 19: Illustration of the positions of the Dirac points with positive q_D for $V/k = 5.3$. The dashed line is the location of the electron and hole Fermi surfaces of Fig. 17. These are folded back into the first Brillouin zone $-k/2 < q_x < k/2$ and shown as the full lines. The Dirac points are the filled circles at the positions in Eq. (69), and these appear precisely at the intersection points of the folded Fermi surfaces in the first Brillouin zone.

IR behavior is described by a CFT whose "central charge" changes in discrete steps as a function of *V/k*, every time pairs of Dirac zero modes appear.



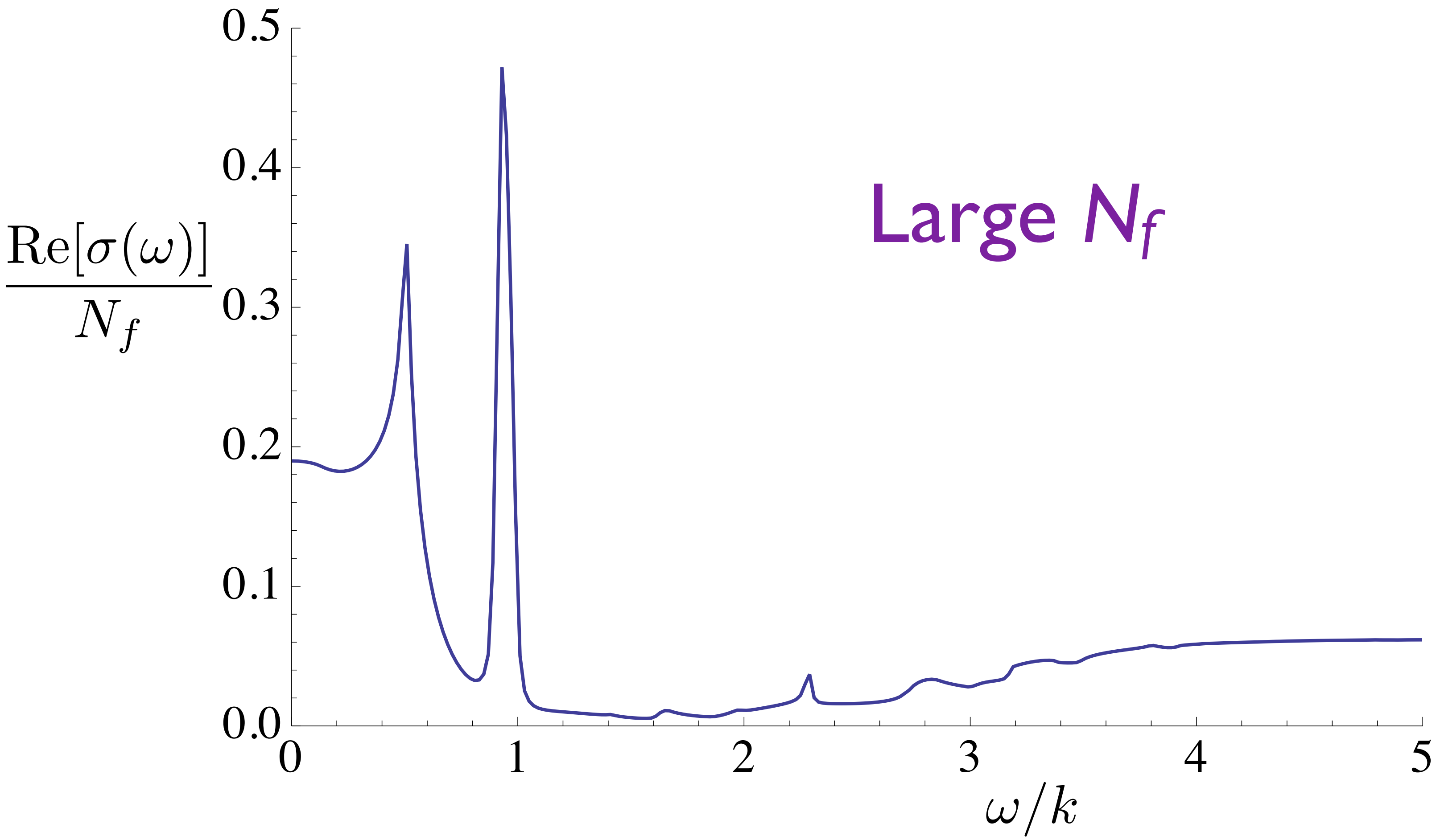
Large N_f
$\frac{\mathrm{Re}[\sigma(\omega)]}{N_f}$
0.10
0.08
0.06
0.04
0.02
0.00
0
1
2
3
4
ω/k

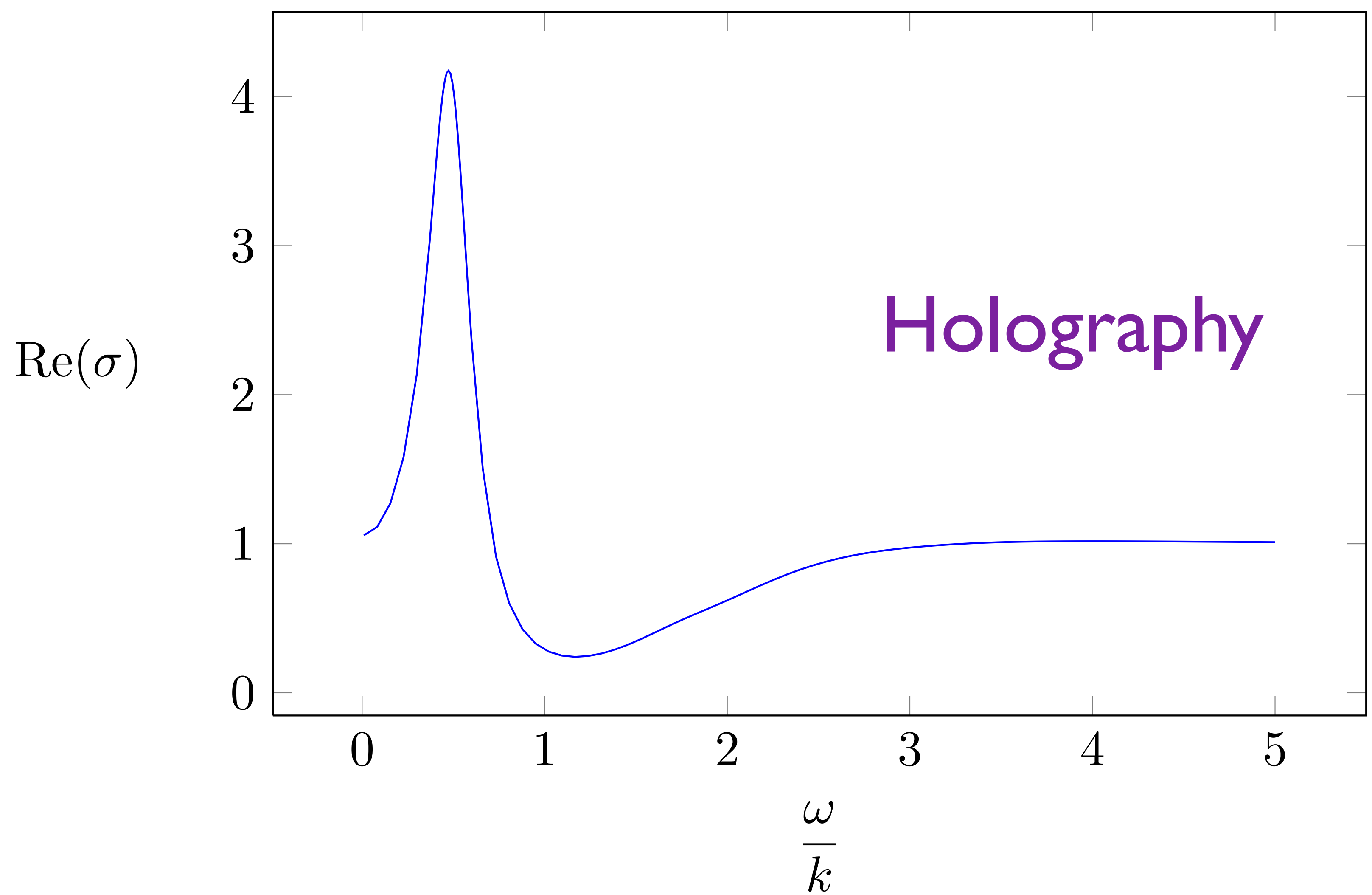
Small V/k

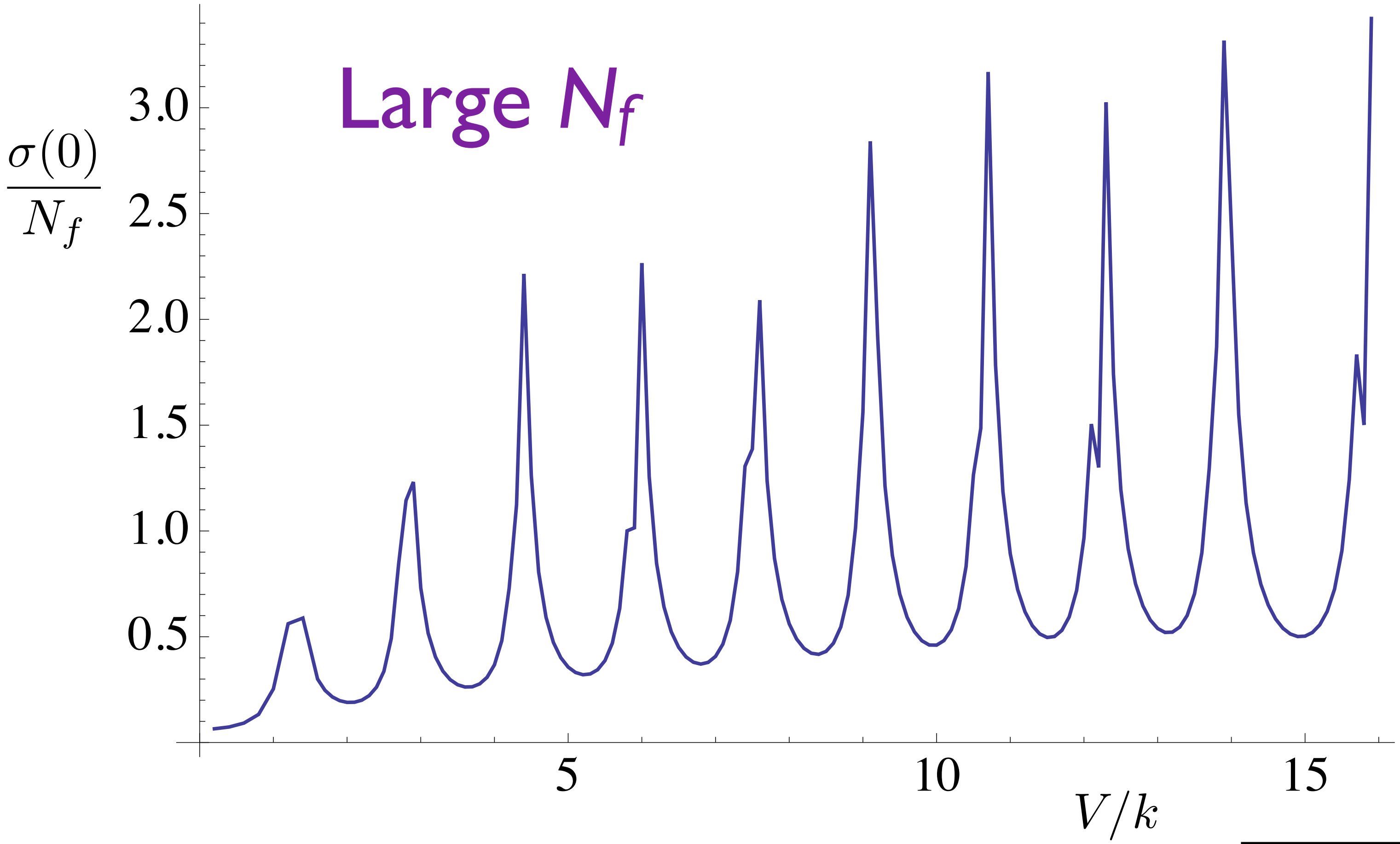


$\frac{k^2}{V^2}\mathrm{Re}(\sigma - \sigma_\infty)$
0.2
0
−0.2
−0.4
−0.6
−0.8
0
0.5
1
1.5
2
2.5
$\frac{\omega}{k}$
Holography

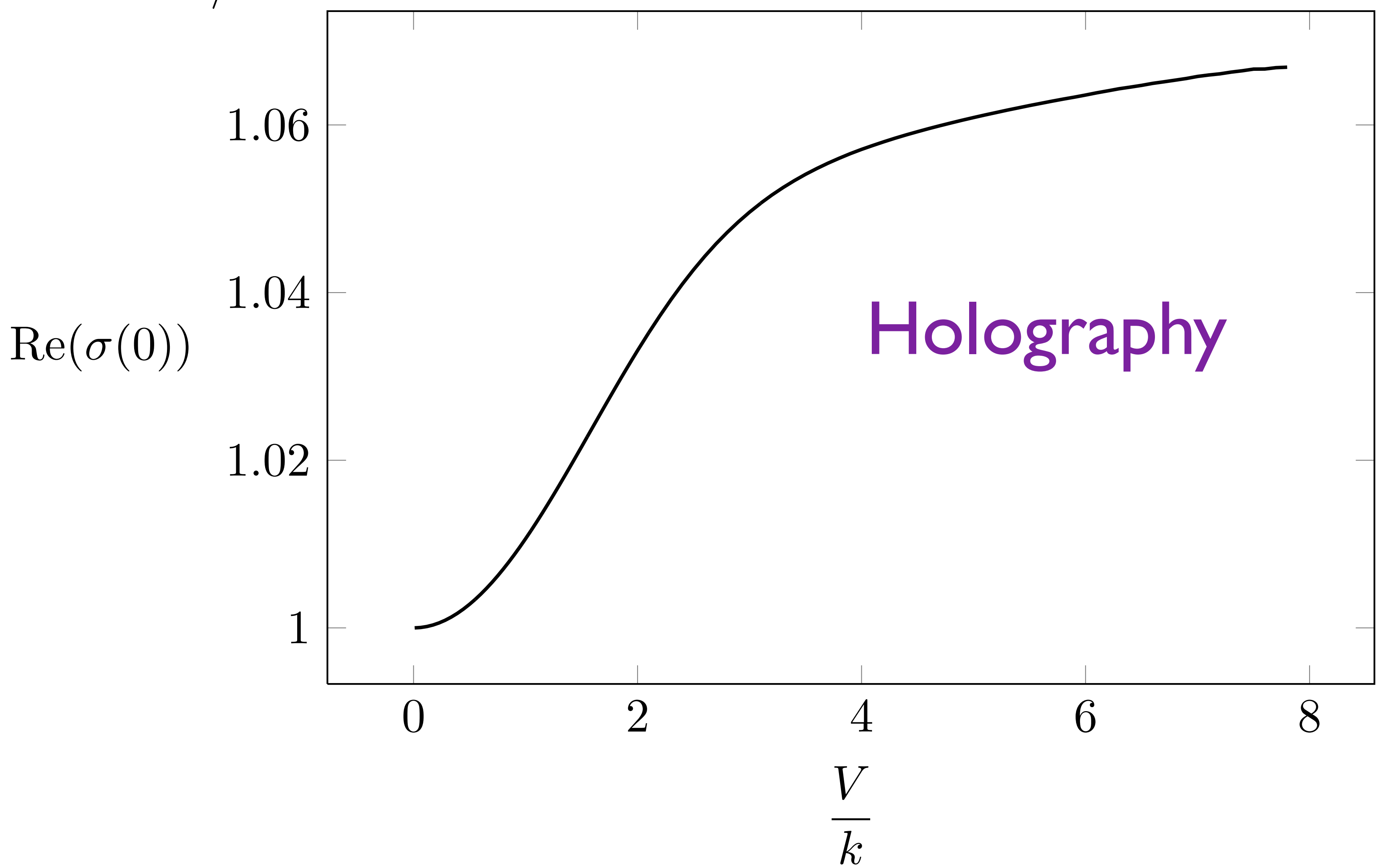
Large V/k


Large N_f
$\frac{\mathrm{Re}[\sigma(\omega)]}{N_f}$
0.0
0.1
0.2
0.3
0.4
0.5
0
1
2
3
4
5
ω/k


Holography
$\mathrm{Re}(\sigma)$
0
1
2
3
4
0
1
2
3
4
5
$\frac{\omega}{k}$



d.c. conductivity



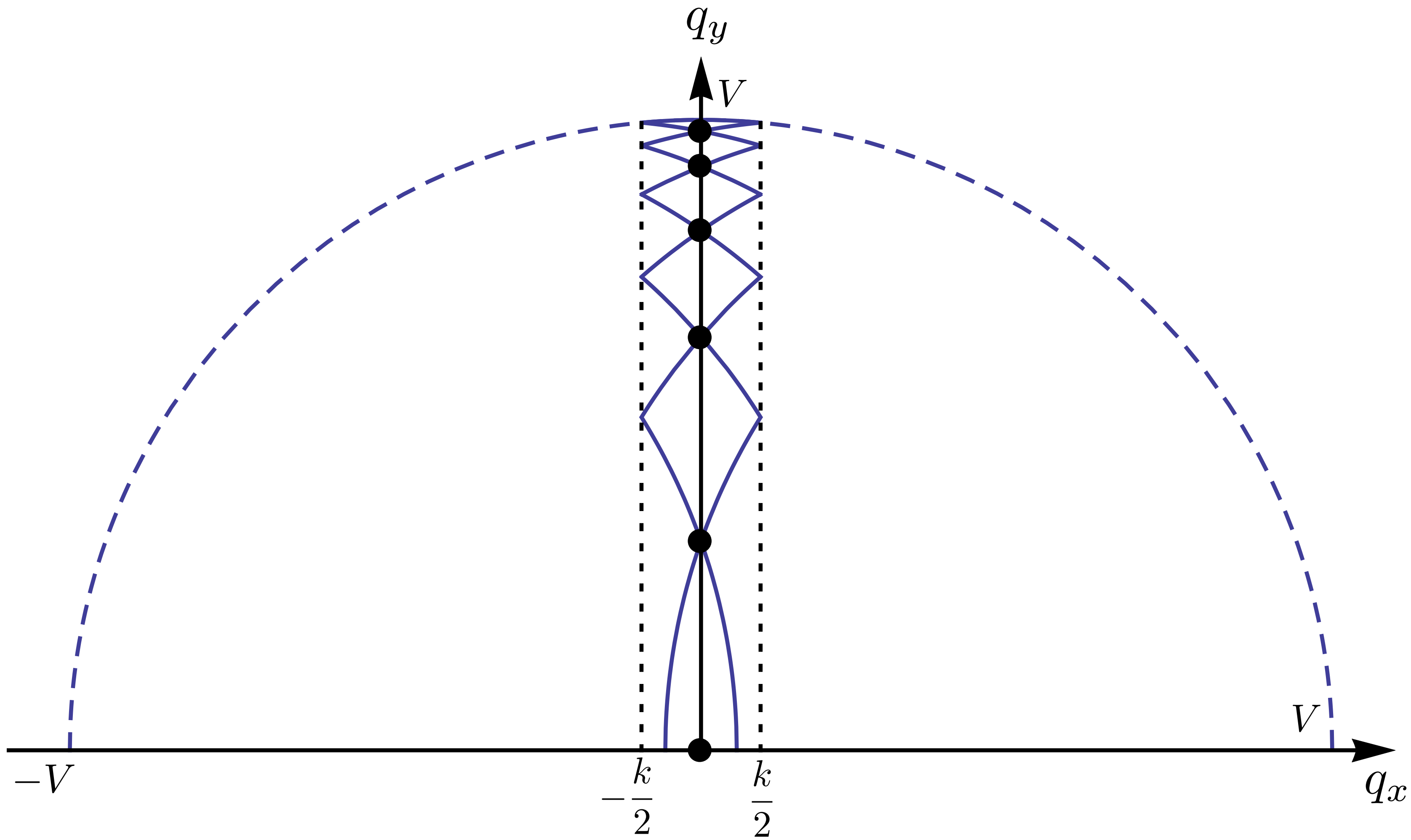


FIGURE 19: Illustration of the positions of the Dirac points with positive q_D for $V/k = 5.3$. The dashed line is the location of the electron and hole Fermi surfaces of Fig. 17. These are folded back into the first Brillouin zone $-k/2 < q_x < k/2$ and shown as the full lines. The Dirac points are the filled circles at the positions in Eq. (69), and these appear precisely at the intersection points of the folded Fermi surfaces in the first Brillouin zone.

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