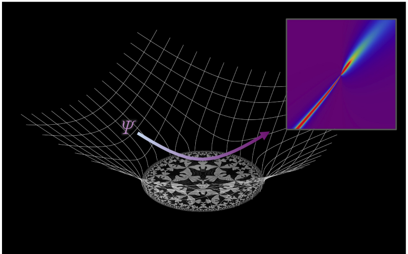
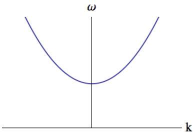
Holography with and without gravity
Lecture 2:
Holographic Fermi Surfaces
and
Quantum Electron Stars

John McGreevy, UCSD



Hierarchy of understoodness

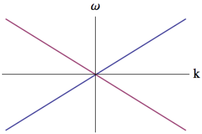
systems with a gap (insulators)



EFT is a topological field theory

(tomorrow's lecture)

systems at critical points or topological insulators with gapless boundary dofs



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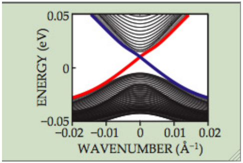
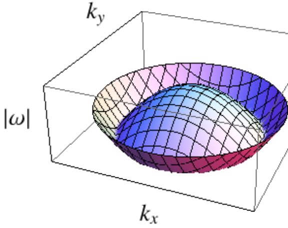
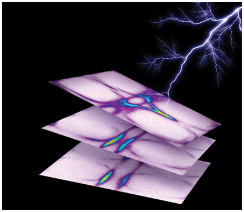


EFT is a CFT

systems with a Fermi surface (metals)





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Outline

1. Introduction: 'post-particle physics of metal'
2. Limit 1: Holographic fermions with too little back-reaction
3. Limit 2: Holographic fermions with too much back-reaction
4. Limit 3: Quantum electron stars in *AdS*
5. How (not) to make a gravitating quantum electron star

Some references for this lecture:

Hong Liu, JM, David Vegh, *Non-Fermi Liquids from Holography*, 0903.2477

Tom Faulkner, HL, JM, DV, *Emergent quantum criticality, Fermi surfaces, and* AdS_2, 0907.2694

TF, Nabil Iqbal, HL, JM, DV, 1003.1728, 1101.0597 and,
at long last, 1306.6396

Sean Hartnoll, *Horizons, holography and condensed matter*, 1106.4324

Andrea Allais, JM, S. Josephine Suh, *A Quantum Electron Star*, 1202.5308

Andrea Allais, JM, *...a Gravitating Quantum Electron Star*, 1306.6075

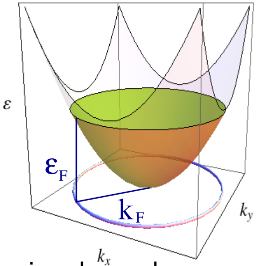
Fermi Liquids

Basic question: What is the effective field theory for a system with a Fermi surface (FS)?

Lore: must be Landau Fermi liquid [Landau, 50s].

Recall :

if we had *free* fermions, we would fill single-particle energy levels $\epsilon(k)$ until we ran out of fermions: $\rightarrow$

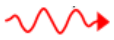


Low-energy excitations:

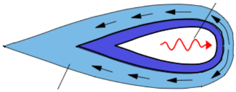
remove or add electrons near the Fermi surface ϵ_F, k_F.

Idea [Landau]: The low-energy excitations of the interacting theory are still weakly-interacting fermionic, charged 'quasiparticles'.

Elementary excitations are free fermions with some dressing:



$\xrightarrow{\textit{in medium}}$



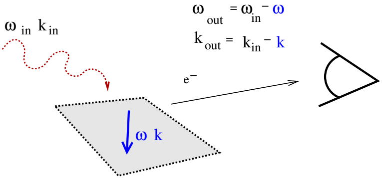
The standard description of metals

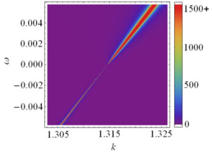
The metallic states that we understand well are described by Landau's Fermi liquid theory.

Landau quasiparticles $\rightarrow$ poles in single-fermion Green function G_R

at $k_\perp \equiv |\vec{k}| - k_F = 0$, $\omega = \omega_\star(k_\perp) \sim 0$: $G_R \sim \dfrac{Z}{\omega - v_F k_\perp + i\Gamma}$

Measurable by ARPES (angle-resolved photoemission):





Intensity $\propto$ spectral density : $A(\omega, k) \equiv \operatorname{Im} G_R(\omega, k) \overset{k_\perp \to 0}{\rightarrow} Z\delta(\omega - v_F k_\perp)$

Landau quasiparticles are long-lived: width is $\Gamma \sim \omega_\star^2$,
residue Z (overlap with external e^-) is finite on Fermi surface.

Reliable calculation of thermodynamics and transport relies on this.

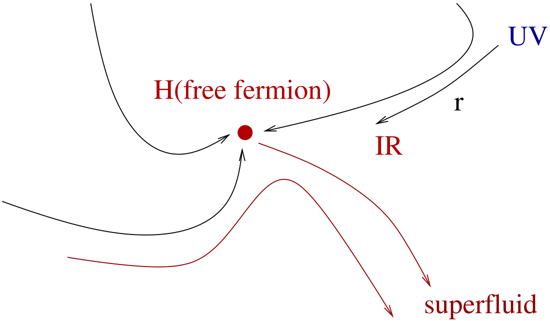
Ubiquity of Landau Fermi liquid

Physical origin of lore:

1. Landau FL successfully describes ^{3}He, metals studied before $\sim 1980s$, ...

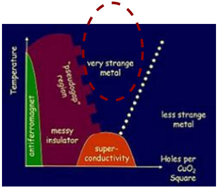
2. RG: Landau FL is stable under almost all perturbations.

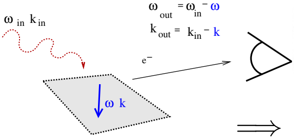
[Shankar, Polchinski, Benfatto-Gallivotti 92]

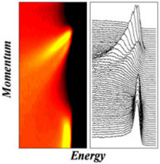


Non-Fermi liquids exist but are mysterious

e.g.: 'normal' phase of optimally-doped cuprates: ('strange metal')







among other anomalies: ARPES shows gapless modes at finite k (FS!)

with width $\Gamma(\omega_\star) \sim \omega_\star$, vanishing residue $Z \overset{k_\perp \to 0}{\to} 0$.

Working defintion of NFL:

Still a sharp Fermi surface

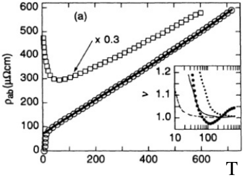
but no long-lived quasiparticles.

Most prominent

mystery of the strange metal phase:

e-e scattering: $\rho \sim T^2$, e-phonon: $\rho \sim T^5$, ...

no known robust effective theory: $\rho \sim T$.



Another source of NFL: how do fermi liquids die?

Some systems have both a Fermi liquid phase, and a phase without a Fermi surface (Mott insulator).

e.g. spin-$\frac{1}{2}$ Hubbard model near half-filling:

$$H = \sum_{\langle ij \rangle} t \; c_i^\dagger c_j + U \sum_i n_i^\uparrow n_i^\downarrow$$

t: kinetic term U: on-site repulsion

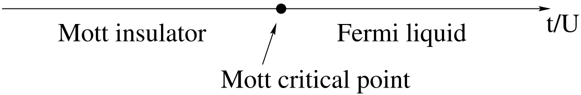


fig from [Senthil, 0803.4009]

$t/U \to \infty$: free electrons, FL.

$t/U \to 0$: each electron picks a site and sits there (Mott insulator).

Critical fermi surfaces

but:

Theorem [Luttinger]: *The volume inside the fermi surface is proportional to the number of electrons, which is conserved.*

It can't just shrink if the number of particles is fixed.

At a continuous transition: "critical fermi surface" [Brinkman-Rice, Senthil]: $Z \to 0$.

$Z =$ jump in momentum space occupation number at the fermi momentum $n(k) = \int \frac{d\omega}{\pi} f(\omega) \mathrm{Im}\, G(\omega, k)$

$f(\omega) \equiv \frac{1}{e^{\beta\omega}+1}$, ω measured from μ.

a) FL

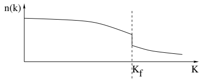
b) mott insulator

c) critical fermi surface

$\partial_k^{(\ell)} n(k) = \infty$ for some ℓ

Z is like an order parameter for the FL phase.

(a)



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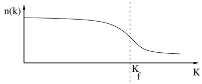
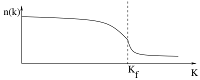


(c)

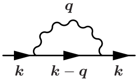


Non-Fermi Liquid from non-Holography

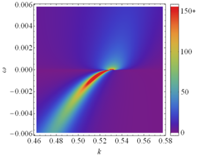
- Luttinger liquid in 1+1 dimensions. ✓
- loophole in RG argument:

couple a Landau FL perturbatively to a bosonic mode

(e.g.: magnetic photon, slave-boson gauge field, statistical gauge field, ferromagnetism, SDW, Pomeranchuk order parameter...)



$\rightarrow$ nonanalytic behavior in $G^R(\omega) \sim \frac{1}{v_F k_\perp + c\omega^{2\nu}}$ at FS: NFL.



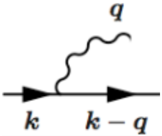
Not strange enough:

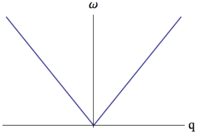
These NFLs are not strange metals in terms of transport.

FL killed by gapless bosons:

small-angle scattering dominates

$\Longrightarrow$ 'transport lifetime' $\neq$ 'single-particle lifetime'





A goal for holography

Can we formulate a tractable effective description of the low-energy physics of a system
with a Fermi surface,
but without long-lived quasiparticles?

Disclaimer about bias: I am defining the problem in terms of single-particle response.
Definition: Fermi surface $\equiv \{k \mid G^{-1}(k, \omega) = 0 \text{ at } \omega = 0\}$

(The kind of function I have in mind is $G \sim \frac{1}{c\omega^{2\nu} + |k| - k_F}$.)

Here G is a two-point correlator of a gauge-invariant fermion operator, like an electron, effectively.

This will be worthwhile even if the toy model has exotic short-distance physics.
Benefit of holography: 0th-order approx far from weakly-coupled particles.

Finite-density states in holography

To describe low-temperature states of holographic matter (not a CFT at all scales), we need more ingredients.

Suppose the CFT has a conserved $U(1)$ current.

$\to$ massless gauge field A_μ in bulk.

Wilson-natural starting point: $\Delta S_{bulk} = -\frac{1}{4g_F^2}\int d^{d+1}x\sqrt{g}F_{AB}F^{AB}$.

$$\text{Max eqn}: \quad 0 = \frac{\delta S_{bulk}}{\delta A_C} \propto \frac{1}{\sqrt{g}}\partial_A\left(\sqrt{g}g^{AB}g^{CD}F_{BD}\right)$$

Max eqn near AdS bdy: $\underline{A} \sim A^{(0)}(x) + \left(\frac{z}{L}\right)^{d-2} A^{(1)}(x)$

$$A_t \sim \underbrace{\mu}_{\text{source}} + \left(\frac{z}{L}\right)^{d-2} \underbrace{\rho}_{\text{response}} .$$

$$\Pi_{A_t} = \frac{\partial\mathcal{L}}{\partial\left(\partial_z A_t\right)} = E_z = A^{(1)} = \rho.$$

Conclusion: $\boxed{\mathrm{tr}\ e^{-\beta(H-\mu N)} \simeq e^{-S_{\text{bulk}}[\underline{g},\underline{A},\ldots]}|_{A\sim\mu\mathrm{d}t}}$

Minimal ingredients for a holographic Fermi surface

Consider any relativistic CFT with a gravity dual $\to g_{\mu\nu}$

a conserved $U(1)$ symmetry proxy for fermion number $\to A_\mu$

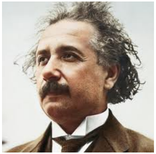
and a charged fermion proxy for bare electrons $\to \psi$.

$\exists$ many examples. Any $d > 1 + 1$, focus on $d = 2 + 1$.

The problem we really want to solve

Wilson tells us to use the following action in the bulk:

$$\mathcal{L}_{d+1} = \frac{\mathcal{R} + \Lambda}{G_N} - \frac{1}{q^2} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} i \left(\not{D} - m \right) \psi$$



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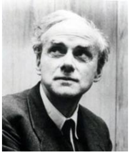
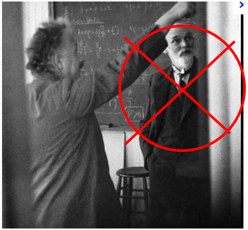




(with AdS boundary conditions,

and a chemical potential: $A_t \equiv \Phi \to \mu$ at the boundary.)

$\kappa^2 \Lambda \ll 1, q \ll 1$ is large N.

Limit 1:

Completely ignore bulk matter fields in constructing the geometry

$$\mathcal{L}_{d+1} = \frac{\mathcal{R} + \Lambda}{G_N} - \frac{1}{q^2} F^2 + \bar{\psi} i \left(\not{D} - m \right) \psi$$

This is correct when $G_N \to 0, q \to 0$ fixing G_N/q.

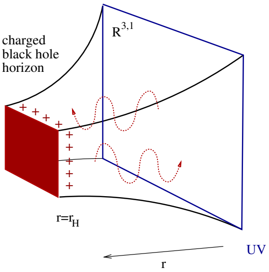
Then the solution of the bulk EoM with the right boundary conditions is the extremal charged black hole in AdS ('Reissner-Nördstrom'):

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + \frac{r^2}{R^2}\left(dx^2 + dy^2\right),$$

$$f(r) = \frac{r^2}{R^2}\left(1 + \frac{Q^2}{r^3} - \frac{M}{r^3}\right), \quad \Phi = \mu\left(1 - \left(\frac{r_H}{r}\right)\right).$$

'Extremal' means $T = 0$. $f \sim (r - r_H)^2$ near the horizon.

Extremal black hole in AdS

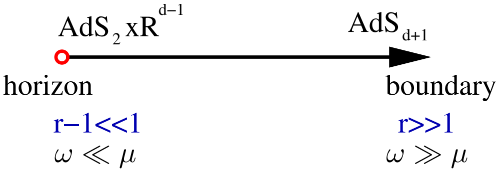


Near-horizon geometry is $AdS_2 \times \mathbb{R}^{d-1}$.

$$ds^2 \sim \frac{-dt^2 + du^2}{u^2} + d\vec{x}^2 \qquad u \equiv \frac{1}{r - r_H}$$

The conformal invariance of this metric is **emergent**.

$t \to \lambda t, x \to \lambda^{1/z} x$ with $z \to \infty$.



AdS/CFT $\implies$ the low-energy physics governed by dual **IR CFT**.

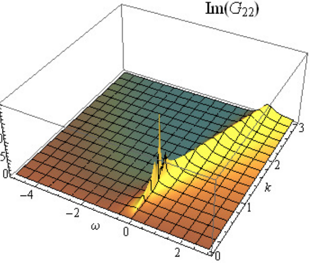
(There is a lot more to say about this IR fixed point.)

Fermi surfaces

To find FS: look for sharp features in fermion Green functions G_R at finite momentum and small frequency. [S-S Lee]

To compute G_R: solve Dirac equation in charged BH geometry.
'Bulk universality': results only depend on q, m.

$$\boxed{G_R(\omega, k) \sim \frac{1}{\mathcal{G}(k,\omega) + k_\perp}}$$



The location of the Fermi surface is determined by short-distance physics (analogous to band structure – $a, b \in \mathbb{R}$ from normalizable sol'n of $\omega = 0$ Dirac equation in full BH)
but the low-frequency scaling behavior near the FS is universal (determined by near-horizon region – IR CFT correlator $\mathcal{G} = c(k)\omega^{2\nu}$).

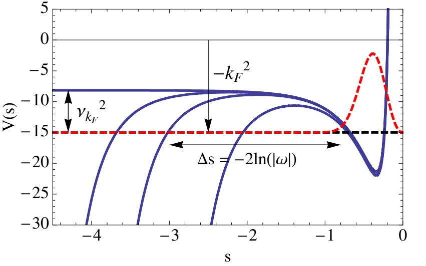
In hindsight: "semi-holographic" interpretation [FLMV, Polchinski-Faulkner]
quasiparticle decays by interacting with $z = \infty$ IR CFT d.o.f.s dual to $AdS_2 \times \mathbb{R}^2$ region.

Death of the quasiparticles

Rewrite spinor equation as
Schrödinger equation (with $E = 0$)
$\left(-\partial_s^2 + V(r(s))\right)\Psi(z(s)) = 0.$

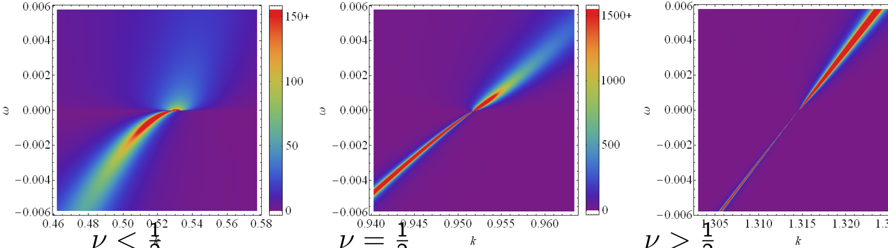
Spinor boundstate at $\omega = 0$ tunnels
into AdS_2 region with rate

$$\Gamma \propto e^{-2\int ds\sqrt{V(s)}} \sim e^{2\nu \ln \omega} = \omega^{2\nu}$$



(WKB approx good at small ω)

FT interpretation: quasiparticle decays by interacting with IR CFT.



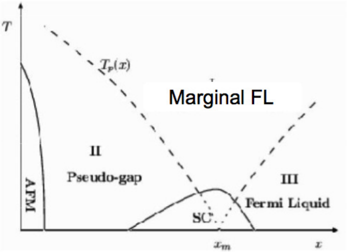
$\nu < \frac{1}{2}$ $\qquad$ $\nu = \frac{1}{2}$ $\qquad$ $\nu > \frac{1}{2}$

$\nu = \frac{1}{2}$: Marginal Fermi liquid

$$G_R \approx \frac{h_1}{k_\perp + \tilde{c}_1 \omega \ln \omega + c_1 \omega}, \quad \tilde{c}_1 \in \mathbb{R}, \quad c_1 \in \mathbb{C}$$

$$\frac{\Gamma(k)}{\omega_\star(k)} \overset{k_\perp \to 0}{\to} \mathrm{const}, \qquad Z \sim \frac{1}{|\ln \omega_\star|} \overset{k_\perp \to 0}{\to} 0.$$

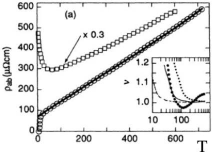
A well-named phenomenological model of high-T_c cuprates near optimal doping



[Varma et al, 1989].

Charge transport by holographic Fermi surfaces

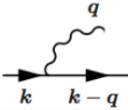
Most prominent mystery
of strange metal phase: $\rho_{\rm DC} \sim T$ $\rightarrow$

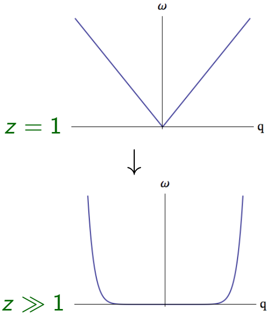


We can compute the contribution
to the conductivity from the Fermi surface

[Faulkner-Iqbal-Liu-JM-Vegh, 1003.1728 and (finally!) 1306.6396]:

$$\rho_{\rm FS} \sim T^{2\nu}$$





Dissipation of current is controlled by
the decay of the fermions into the AdS_2 DoFs.
$\Longrightarrow$ single-particle lifetime controls transport.

marginal Fermi liquid: $\nu = \frac{1}{2} \Longrightarrow \boxed{\rho_{FS} \sim T}$.

[Important disclaimer: this is NOT the leading contribution to $\sigma_{\rm DC}$!]

Drawbacks of this construction

1. The Fermi surface degrees of freedom are a small part ($o(N^0)$) of a large system ($o(N^2)$). (More on this in a moment.)
2. *Too much* universality! If this charged black hole is inevitable, how do we see the myriad possible dual states of matter (*e.g.* superconductivity...)?
3. The charged black hole violates the 3rd Law of Thermodynamics (Nernst's version): $S(T = 0) \neq 0$ – it has a groundstate degeneracy.

   This is a manifestation of the black hole information paradox: classical black holes seem to eat quantum information.

Problems 2 and 3 solve each other: degeneracy $\implies$ instability.
The charged black hole describes an intermediate-temperature phase.
There are many possible IR endpoints (superconductor, density waves, things we haven't thought of...).
The dominant one depends on the operator content of the CFT.

Stability of the groundstate

Charged bosons: In many explicit dual pairs, $\exists$ charged scalars.
• At small T, they can condense spontaneously breaking the $U(1)$ symmetry, changing the background [Gubser, Hartnoll-Herzog-Horowitz].

spinor: $G_R(\omega)$ has poles only in LHP of ω [Faulkner-Liu-JM-Vegh, 0907]

scalar: $\exists$ poles in UHP $\quad \langle \mathcal{O}(t) \rangle \sim e^{i\omega_\star t} \propto e^{+\mathrm{Im}\,\omega_\star t}$

$\Longrightarrow$ growing modes of charged operator: holographic superconductor

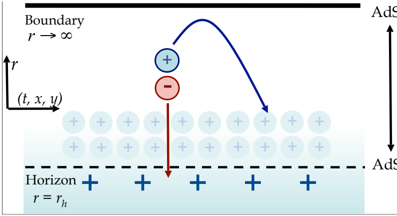
why: black hole *spontaneously* emits charged particles [Starobinsky, Unruh, Hawking].

AdS is like a box: they can't escape.

Fermi: negative energy states get filled.

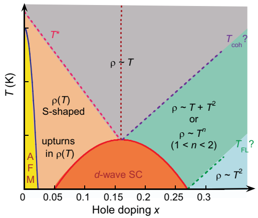
Bose: the created particles then cause *stimulated emission* (superradiance).

A holographic superconductor is a "black hole laser".



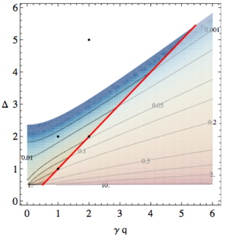
Stability of the groundstate, cont'd

- Many systems
to which we'd like to apply this
also have a superconducting region.



- If the boson's bulk mass/charge
is big enough, they don't condense.

[Denef-Hartnoll 09]:
(vs: a weakly-coupled charged boson
at $\mu \neq 0$ will condense.)
Finding such string vacua
is like moduli stabilization.



Idea: make the bulk fermions more important (solves problem 1).

They will back-react on the geometry (solves problems 2 and 3).

[Hartnoll-Polchinski-Silverstein-Tong 09]

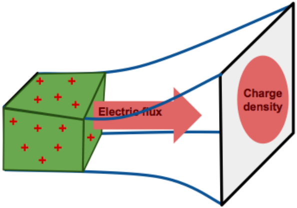
Perhaps there is a stable geometry sourced by the fermions.

Problem: it's hard.

Limit 2:

Very heavy fermions in the bulk

Electron stars



[Hartnoll and collaborators, de Boer-Papadodimas-Verlinde]

Choose q, m to reach a regime where the bulk fermions can be treated as a (gravitating) fluid

(Oppenheimer-Volkov aka Thomas-Fermi approximation).

$\longrightarrow$ "electron star"

But:

- In the regime of parameters studied here (large mass) the dual Green's function exhibits *many* Fermi surfaces.

[Hartnoll-Hofman-Vegh, Iqbal-Liu-Mezei 2011]

- Large mass $\implies$ lots of backreaction $\implies$ kills IR CFT

$\implies$ stable quasiparticles at each FS.

Thomas-Fermi electron stars, cont'd

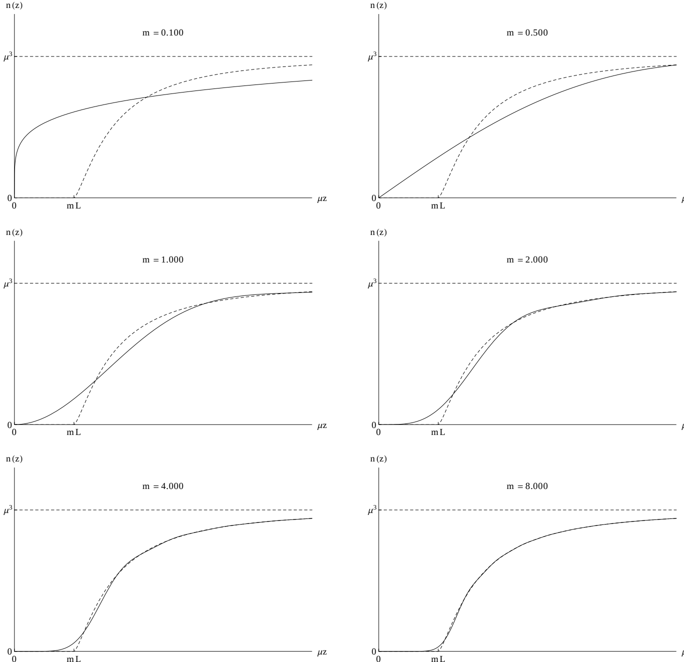
Comment in bitter hindsight:

The motivation for the large-mass limit was to guarantee validity of the Thomas-Fermi approximation.

The actual regime of validity of this approximation is not clear (and is larger).

To do better, we need to take into account the wavefunctions of the bulk fermion states: a *quantum* electron star.

The Thomas-Fermi approximation can be wrong



Solid curves: Exact charge density in *AdS* with $\Phi(z) = \mu$.

Dashed curves: TF approximation. $\mu_{\text{local}}(z) = \frac{\Phi}{\sqrt{g_{tt}}} = \mu z$.

For $m \sim 1$, TF gives wrong UV physics ($z \to 0$).

Limit 3:

Find back-reaction of fermions on the gauge field, but ignore gravitational back-reaction of both fermions *and* gauge fields.

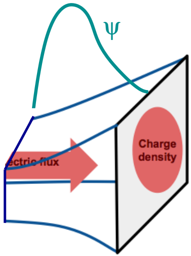
$$\mathcal{L}_{d+1} = \frac{\mathcal{R} + \Lambda}{G_N} - \frac{1}{q^2} F^2 + \bar{\psi} i \left(\not{D} - m \right) \psi$$

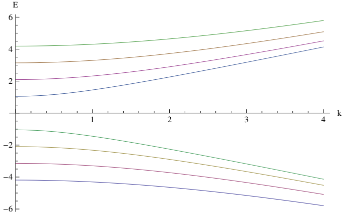
Probe limit: $G_N \to 0$ [like HHH 0803]

QFT Interpretation: most CFT dofs are neutral. $(\underbrace{c \sim \frac{L^2}{G_N}}_{\propto \langle TT \rangle} \gg \frac{1}{q^2} \propto \langle jj \rangle)$

A solution of QED in AdS [A. Allais, JM, S. J. Suh 12].

Towards a quantum electron star





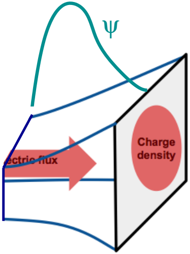
[Sachdev, 2011]: A holographic model of a Fermi liquid.

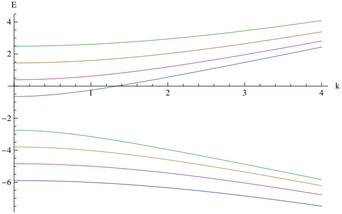
Like AdS/QCD: a toy model of the groundstate of a confining gauge theory from a hard cutoff in AdS.

Add chemical potential.
Compute spectrum of Dirac field, solve for backreaction on A_μ. Repeat as necessary. (Hartree-Fock)

The system in the bulk *is* a Fermi liquid (in a box determined by the dual gauge dynamics).

Towards a quantum electron star





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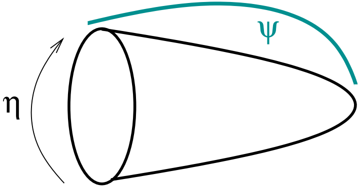
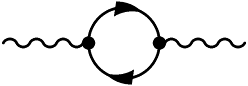
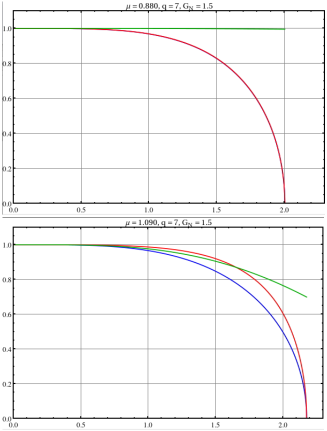
Towards a fermion-driven deconfinement transition



Lots of low-E charged dofs screen gauge interactions.





Effect of fermions on the gauge dynamics = gravitational backreaction.

A real holographic model of confinement: AdS soliton

first attempt: $\rightarrow$

What's the endpoint of this transition?

A quantum electron star in AdS

Zeroth-order problem: what can the state of the bulk fermions be if the geometry has a horizon?

Probe limit ($G_N \to 0$):
Fix the geometry to be AdS with an IR cutoff.

$$\psi =: (-gg^{zz})^{-\frac{1}{4}} e^{-i\omega t + ik_i x^i} \chi$$

Normalizable BCs at $z = 0$,
hard-wall BC at $z = z_m$

$$\not{D}_\Phi \psi = 0$$

$$\begin{pmatrix} \Phi(z) + k & \frac{\partial}{\partial z} - \frac{m}{z} \\ -\frac{\partial}{\partial z} - \frac{m}{z} & \Phi(z) - k \end{pmatrix} \chi_n = \omega_n \chi_n$$

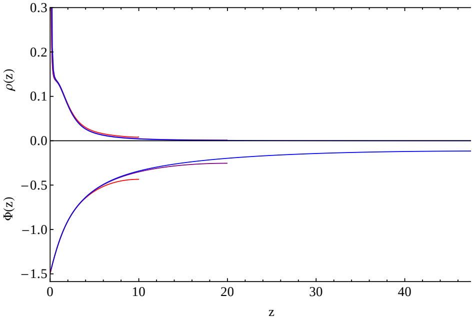
$$\Phi'' = -q^2 \rho$$

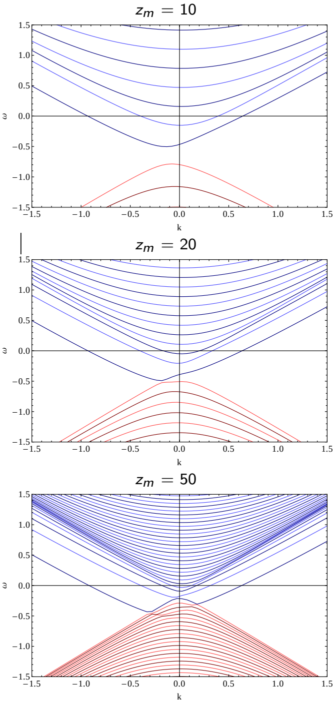
$$\Phi''(z) = -q^2 \left(\rho(z) - \rho(z)|_{\Phi=0} \right),$$

$$\rho(z) \equiv \sum_{n, \omega_n < 0} \psi_n^2(z)$$

A quantum electron star in *AdS*

The limit $z_m \to \infty$ exists! :





Electrostatic and Pauli repulsion supports the fermions against falling into the AdS gravitational well.

The padded room

Compute charge density:

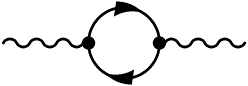
$$\langle n(z)\rangle = \langle \psi^\dagger(z)\psi(z)\rangle = \sum_k n_k(z) \sim \int^\Lambda d^2k \frac{1}{k^2}\Phi''(z) + \text{finite}$$

Cutoffs everywhere: UV cutoff on AdS radial coordinate, bulk UV cutoff (lattice), UV cutoff on k integral, IR cutoff on AdS radial coordinate: z_m.

Charge renormalization.

Define charge susceptibility by linear response:

$$\chi \equiv \sum_k \chi(k), \;\; \chi(k) = \frac{\Delta\rho_k(z_\star)}{\Phi''(z_\star)}$$



$$q_R^2 = q_0^2 \frac{1}{1 - q_0^2\chi}$$

Physics check

Chiral anomaly.

Each k mode is a $D = 1+1$ fermion field $S_k = \int drdt\ i\bar{\psi}_k \left(\not{D} + m + i\gamma^5 k\right)\psi_k$

$\overset{?}{\Longrightarrow}$ $\partial_r n_k \to 0$ when $m, k \to 0$.

Not so in numerics:

$$\partial_\mu j_5^\mu = \frac{1}{2\pi}\epsilon_{\mu\nu}F^{\mu\nu} = -\frac{1}{\pi}\Phi' \qquad \checkmark$$

Semi-holographic interpretation

In retrospect, the dual system can be regarded as
a Fermi Surface coupled to relativistic CFT (with gravity dual)

$\Phi(z)$: how much of the chemical potential is seen by the dofs of wavelength $\sim z$.

Convergence of EOM requires $\Phi(\infty) = 0$, complete screening in far IR.

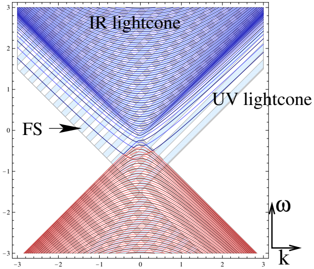
$\Phi(\infty) = 0$ means FS survives this coupling to CFT:
FS at $\{\omega = 0, |\vec{k}| = k_F \neq 0\}$ is outside IR lightcone $\{|\omega| \geq |\vec{k}|\}$.

Interaction is kinematically forbidden.
[Landau: minimum damping velocity in SF;

Gubser-Yarom; Faulkner et al 0911]

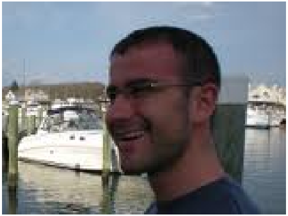
In probe limit, quasiparticles survive.

With "Landau damping," IR speed of light smaller, maybe not.



Electron stars minus zero/no limit

[Andrea Allais, JM, 1306.6075]



Towards gravitating quantum electron stars

When we include gravitational backreaction
(dual to effects of FS on gauge theory dynamics)
the IR geometry can be different from the UV AdS.

Optimism: happy medium between

AdS_2 (no fermions)
and
classical electron star (heavy fermions).

How to construct a gravitating quantum electron star

[Andrea Allais, JM 1306.6075]

Covariant IR regulator: spatial sections are S^2s of radius R.

Other virtue: unambiguous IR boundary conditions.

$$\mathrm{d}s^2 = \frac{1}{\beta^2(z)}\left[-\mathrm{d}t^2 + \mathrm{d}z^2 + \alpha^2(z)\mathrm{d}\Omega_2^2\right],\ A = \Phi(z)\mathrm{d}t\,.$$

$$\text{UV: } \beta(z) \overset{z\to 0}{\sim} z/L \qquad \text{IR: } \alpha(z) \overset{z\to z_m}{\sim} (z - z_m)\ .$$

Radius of the CFT's S^2 is $\boxed{R = \alpha(0)}$.

Increase z_m to increase $R = \alpha(0)$.

Physics depends on μR. WLOG $\mu = 1$.

Brief comments on methods

[Andrea Allais, JM 1306.6075]

Again, two steps:

```
10 Given geometry, find sources
20 Given sources, find geometry
30 GOTO 10
```

- The Dirac Hamiltonian with this ansatz is compact and Hermitian: discrete spectrum.
- Adapted spectral methods were required to diagonalize it efficiently.
- The renormalization of the bulk stress tensor is much harder than the charge density. $\langle T \rangle \sim \frac{a}{s^4} + \frac{b}{s^2} + c \ln s +$ finite.
- Covariant regulators (heat kernel, zeta function, Pauli-Villars) are not feasible numerically.
- We had to resort to a point-splitting regulator and *adiabatic subtraction* [Birrell-Davies].
- Any deviation from the true and righteous path was fatal.

Construction of stress tensor

Point split:

$$J_0^\mu(x) = \langle \bar{\psi}(x')\gamma^\mu \psi(x) \rangle$$
$$T_0^{\mu\nu}(x) = \langle \bar{\psi}(x')\gamma^{(\mu} \mathbf{i} D^{\nu)} \psi(x) \rangle \, .$$

x' is a point along a geodesic from x a distance s in the direction t^μ.

$$T_0^{\mu\nu}(x) = \underbrace{-\frac{1}{\pi^2 s^4}\left(g^{\mu\nu} - 4t^\mu t^\nu\right) + \ldots}_{\text{local, divergent, geometric}} + \underbrace{T^{\mu\nu}_{\text{finite}}(t)}_{\text{nonlocal, finite, cutoff-dependent}} + \mathcal{O}(s^2)$$

$$T^{\mu\nu}_{\text{ren}} \equiv T_0^{\mu\nu}(t) - \underbrace{T^{\mu\nu}_{\text{adiabatic}}}_{\text{geometric, finite, cutoff-dependent}}$$

$T_{\text{adiabatic}}$ is the *derivative expansion* of the bare stress tensor.

It completely misses the non-local finite response, but completely cancels the symmetry-breaking effects of the point-splitting.

(determined analytically.)

For $t^\mu = (\Delta t, \vec{0})$, both $T_0^{\mu\nu}$ and $T^{\mu\nu}_{\text{adiabatic}}$ are covariantly conserved.

Adiabatic subtraction

$$\langle T\rangle \sim \frac{a}{s^4} + \frac{b}{s^2} + c\ln s + \text{finite}.$$

UV divergences come from local contributions. To compute the contribution at each point, Taylor expand around that point [Schwinger, de Wit, Birrell-Davies].

$a = \delta\Lambda\ g_{\mu\nu}$

$b = \delta\left(\frac{1}{G_N}\right)\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right)$

It works!

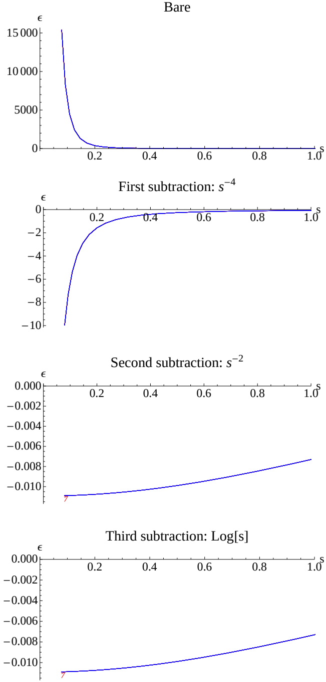
$\rightarrow$ a covariant, conserved $T_{\mu\nu}$, with

$T^{\lambda}{}_{\lambda} =$

$\frac{1}{2880\pi^2}\left(-\frac{7}{4}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 2R_{\mu\nu}R^{\mu\nu} + \frac{5}{4}R^2 - 3\Box R\right.$

the correct Weyl anomaly.

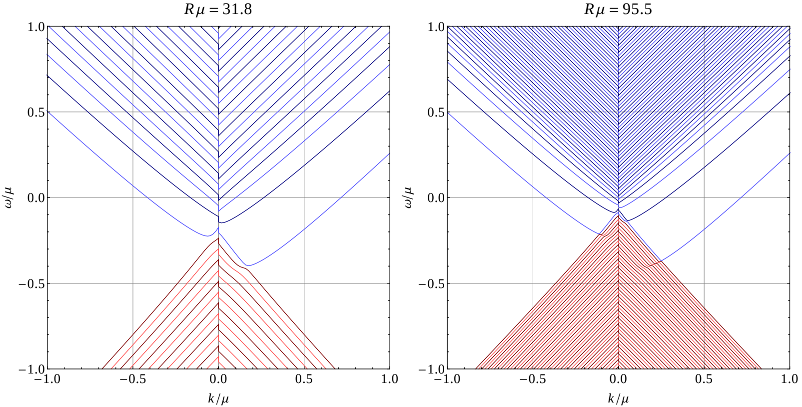
Spinor energy density in empty AdS : $\longrightarrow$



Results

[Andrea Allais, JM 1306.6075]

- $G_N = 0$: we reproduce our previous hard-wall results.



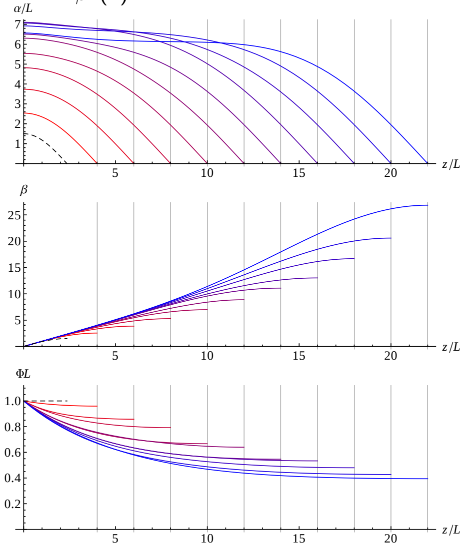
(Note that we only calculate spherical harmonics with discrete ℓ; approaches continuous $k \sim \frac{\ell}{R}$ at large R.)

- For $R < \frac{3}{2}L$: $\mu <$ gap. No fermions. Global AdS (dashed line below):

$$\mathrm{d}s^2 = \frac{L^2}{R^2 \sin^2 \frac{z}{R}} \left[-\mathrm{d}t^2 + \mathrm{d}z^2 + R^2 \cos^2 \frac{z}{R} \mathrm{d}\Omega_2^2 \right] \ , \quad A = \frac{1}{L}\mathrm{d}t \, .$$

Results, cont'd

$$ds^2 = \frac{1}{\beta^2(z)}\left(-dt^2 + dz^2 + \alpha^2(z)d\Omega_2^2\right) A = \Phi dt\,.$$



[Andrea Allais, JM 1306.6075]

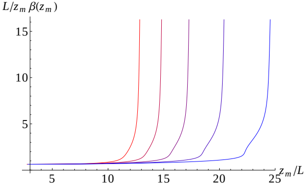
- Gravitating electron stars:

red to blue: various z_m

Bad news: for $G_N > 0$, as we increase z_m, R stops growing!

Results, cont'd

Instead, $\boxed{\beta(z_m) \overset{z_m \to z_m^c(q)}{\longrightarrow} \infty}$! $\longrightarrow$

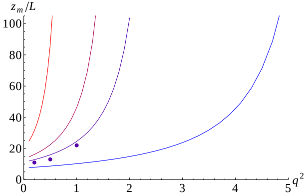


(Personally) Worse news: TF approximation works well in this regime. Can be used to study approach to transition.

(breaks down *at* transition)

Location of the critical z_m for various q: $\longrightarrow$

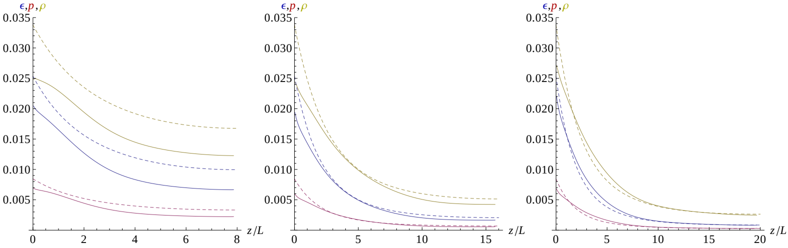
(predicted accurately by TF)



$\boxed{+}$ We've found a deconfinement-like quantum phase transition, driven by the fermion density (!):
the bulk $\mathbf{H}_{\text{Dirac}}$ becomes non-compact. (Recall: holographic picture of confinement.)

$\boxed{-}$ This transition is an obstacle to the thermodynamic limit $\mu \gg 1/R$ with our present methods (compact $\mathbf{H}_{\text{Dirac}}$ is crucial.).

Accuracy of TF at $m = 0$



Dashed: Hartree-Fock answer.

$q^2 = 1.0,$, $\kappa^2/L^2 = 0.1$, $mL = 0.0$

$z_m/L = 8,\ 16,\ 20$

This is a mystery to me at the moment.

Concluding comments

1. This may seem like a lot of effort, but it's still a lot more tractable than directly solving a strongly-coupled quantum many body problem (which has a sign problem).
2. Radial dependence of the bulk fields encode running couplings in the dual QFT
   (along with information about the state).
   Q: How should we interpret holographically
   the (quantum) information in bulk fermion fields?
   [possible answer: Sung-Sik Lee 1305.3908]
3. Open Q: How to get beyond this 'deconfinement' transition?
   (Directly study non-compact Dirac operator? Other IR regulator?)
   More general Q: What other Fermi surface states can arise holographically?

Public service announcement

Please practice holography responsibly.

Please Practice Holography Responsibly

Holography gives us tractable toy models of strongly correlated systems.

Toy models are only useful if we ask the right questions.

- critical exponents depend on 'landscape issues'
  (parameters in bulk action)
- thermodynamics doesn't distinguish weak and strong coupling
  (in examples: $\mathcal{N} = 4$ SYM, lattice QCD)
- transport is very different
  transport by weakly-interacting quasiparticles is less effective

$$\left(\frac{\eta}{s}\right)_{\text{weak}} \sim \frac{1}{g^4 \ln g} \quad \gg \quad \left(\frac{\eta}{s}\right)_{\text{strong}} \sim \frac{1}{4\pi}.$$

- EFT of strange metals: ?
- far from equilibrium physics: ?
- source of optimism: Humans are dumb. Simple pictures can be very useful.

End of second lecture.

Appendices

Frameworks for non-Fermi liquid

- a Fermi surface coupled to a critical boson field

[Recent work: S-S Lee, Metlitski-Sachdev, Mross-JM-HL-Senthil, 1003.0894]

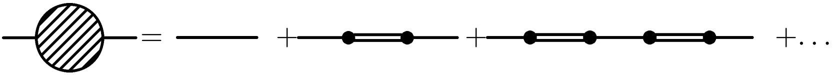
$$L = \bar{\psi}\,(\omega - v_F k_\perp)\,\psi + \bar{\psi}\psi a + L(a)$$

small-angle scattering dominates $\Longrightarrow$ transport is not that of strange metal.

- a Fermi surface mixing with a bath of critical fermionic fluctuations with large dynamical exponent

[FLMV 0907.2694, Faulkner-Polchinski 1001.5049, FLMV+Iqbal 1003.1728]

$$L = \bar{\psi}\,(\omega - v_F k_\perp)\,\psi + \bar{\psi}\chi + \psi\bar{\chi} + \bar{\chi}\mathcal{G}^{-1}\chi \qquad \chi\text{: IR CFT operator}$$



$$\langle\bar{\psi}\psi\rangle = \frac{1}{\omega - v_F k_\perp - \mathcal{G}} \qquad \mathcal{G} = \langle\bar{\chi}\chi\rangle = c(k)\omega^{2\nu}$$

$\nu \le \frac{1}{2}$: $\bar{\psi}\chi$ coupling is a relevant perturbation.

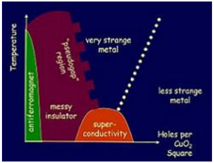
The large z of the IR CFT allows efficient current dissipation.

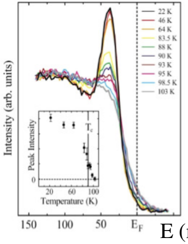
Photoemission ‘exp’ts’ on holographic superconductors

$S_{BH}(T=0) \Longrightarrow$ instability.

With charged scalars in bulk, groundstate is superconducting. [Gubser; Hartnoll et al 2008]

In SC state: a sharp peak forms in $A(k,\omega)$.



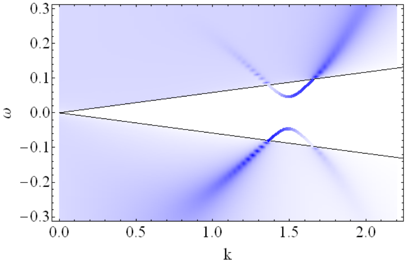


With a suitable coupling between ψ and φ, the superconducting condensate opens a gap in the fermion spectrum.

[Faulkner, Horowitz, JM, Roberts, Vegh, 0911.3402]

For $q_\varphi = 2q_\psi$

$$L_{\text{bulk}} \ni \eta_5 \varphi \bar{\psi} \mathcal{C} \Gamma^5 \bar{\psi}^T + \text{h.c}$$



The (gapped) quasiparticles are exactly stable in a certain kinematical regime

(outside the lightcone of the IR CFT) – the condensate lifts the IR CFT modes into which they decay.

Obstacles to covariant bulk fermion stress tensor

Consider a massless Dirac fermion in 1+1 dimensions, $\Phi = 0$.

$$\text{Fixed metric:} \qquad ds^2 = -f_t(z)dt^2 + f_z(z)dz^2$$

For convenience take $z \simeq z + 2\pi$. WLOG $f_z = 1$ gauge.
Conformal anomaly:

$$T^\mu_\mu = \frac{1}{4\pi}\mathcal{R}(z) = \frac{1}{4\pi}\left(\frac{1}{2}\left(\frac{f_t'^2}{f_t}\right)^2 - \frac{f_t''}{f_t}\right)$$

$$H = \begin{pmatrix} 0 & -\left(\frac{f_t}{f_z}\right)^{1/4}\partial_z\left(\frac{f_t}{f_z}\right)^{1/4} \\ \left(\frac{f_t}{f_z}\right)^{1/4}\partial_z\left(\frac{f_t}{f_z}\right)^{1/4} & 0 \end{pmatrix}, \qquad H\psi_a = \omega_a\psi_a.$$

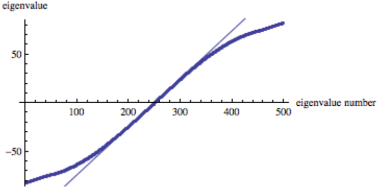
Latticize, add up:

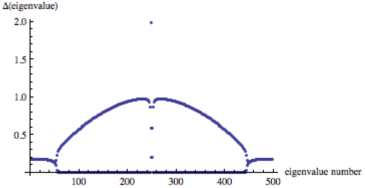
$$T^\mu_\mu = \sum_{a \,\in\, \text{spectrum of H}} \theta(-\omega_a)\psi_a^\dagger\,(\ldots)\,\psi_a = \frac{1}{4\pi}\left(\frac{3}{4}\left(\frac{f_t'^2}{f_t}\right)^2 - \frac{f_t''}{f_t}\right)$$

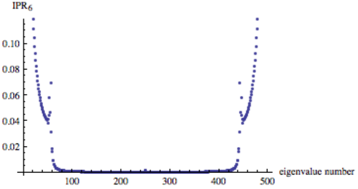
Not a scalar!

Obstacles, cont'd

Why not covariant?

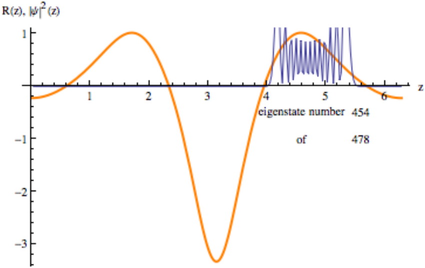






e.g.: $f_t = 1 + .3\cos z + .2\cos 2z$,

$n = 249$ sites.



$$IPR_k \equiv \int dz\ |\psi|^{2k}$$

Solution, pt 1: Even more regulators!

- An additional (bulk UV) regulator is required:

$$\boldsymbol{\rho}_{\text{bare}}(s) \equiv \sum_a \theta(-\omega_a)\psi_a^\dagger \psi_a \; e^{-s|\omega_a|}$$

Cuts off (exponentially) the contribution of the localized modes.

Must have: $1/s \ll 1/a$ to keep the lattice artifacts out.

(This is point-splitting in t. Not covariant.
Hamiltonian Pauli-Villars would also work in principle, and *is* covariant. But it only kills the UV bits by a power-law suppression: $1/p^2 - 1/(p^2 + M^2)$.

Not fast enough.)