From Critical Phenomena to Holographic Duality in Quantum Matter

Joe Bhaseen

TSCM Group

King's College London

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5 - 9 August

Lecture 3

“Holographic Approaches to Far From Equilibrium Dynamics”

Accompanying Slides

Outline

- Experimental motivation
- AdS/CMT and far from equilibrium dynamics
- Quenches and thermalization
- Holographic superfluids
- Heat flow between CFTs
- Potential for AdS/CFT to offer new insights
- Higher dimensions and non-equilibrium fluctuations
- Current status and future developments

Acknowledgements

Jerome Gauntlett   Julian Sonner   Toby Wiseman   Ben Simons

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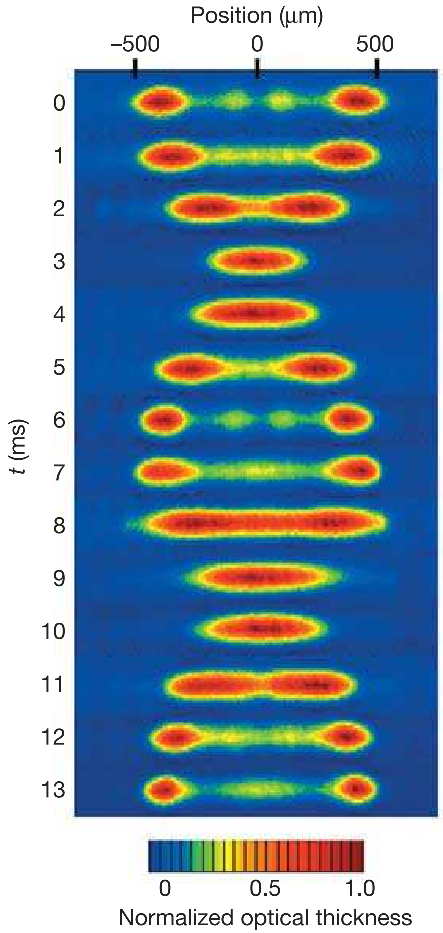
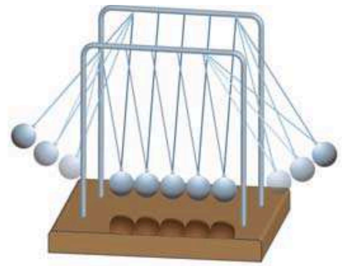
Bartosz Benenowski   Bahman Najian

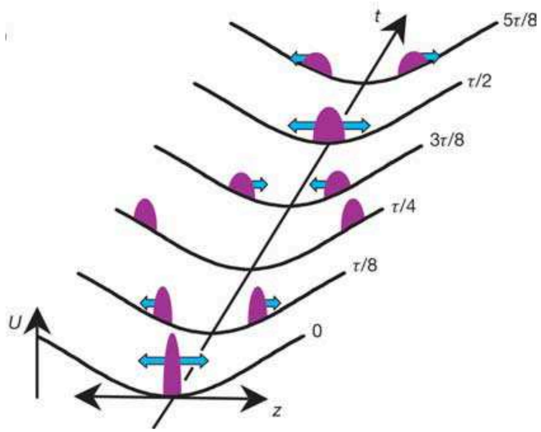
Experiment

Weiss *et al* “A quantum Newton’s cradle”, Nature **440**, 900 (2006)

Non-Equilibrium 1D Bose Gas



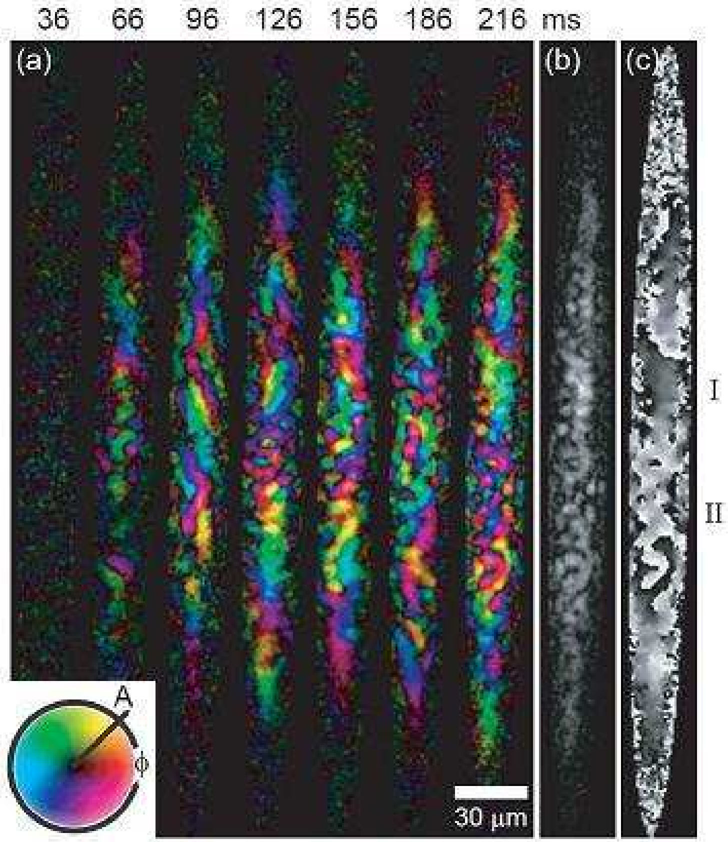




Integrability and Conservation Laws

Experiment

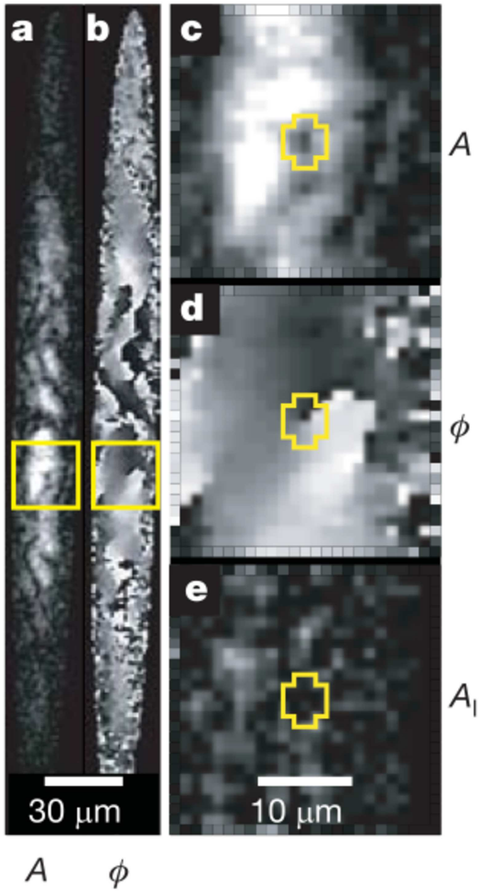
Stamper-Kurn *et al* "Spontaneous symmetry breaking in a quenched ferromagnetic spinor Bose–Einstein condensate", Nature **443**, 312 (2006)



Domain formation

Experiment

Stamper-Kurn *et al* “Spontaneous symmetry breaking in a quenched ferromagnetic spinor Bose–Einstein condensate”, Nature **443**, 312 (2006)

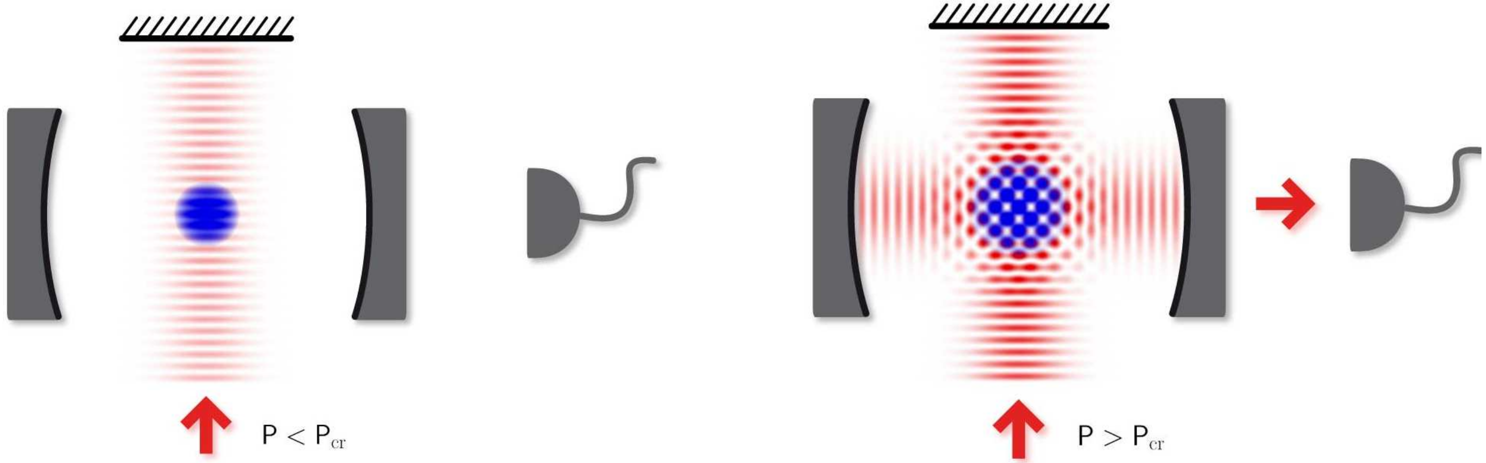


Topological Defects

Dicke quantum phase transition with a superfluid gas in an optical cavity

Kristian Baumann[1], Christine Guerlin[1]†, Ferdinand Brennecke[1] & Tilman Esslinger[1]

Nature **464** 1301 (2010)



Self-Organisation

Domokos & Ritsch, PRL (2002)

Vuletić *et al*, PRL (2003) Nagy *et al*, Eur. Phys. J. D (2008)

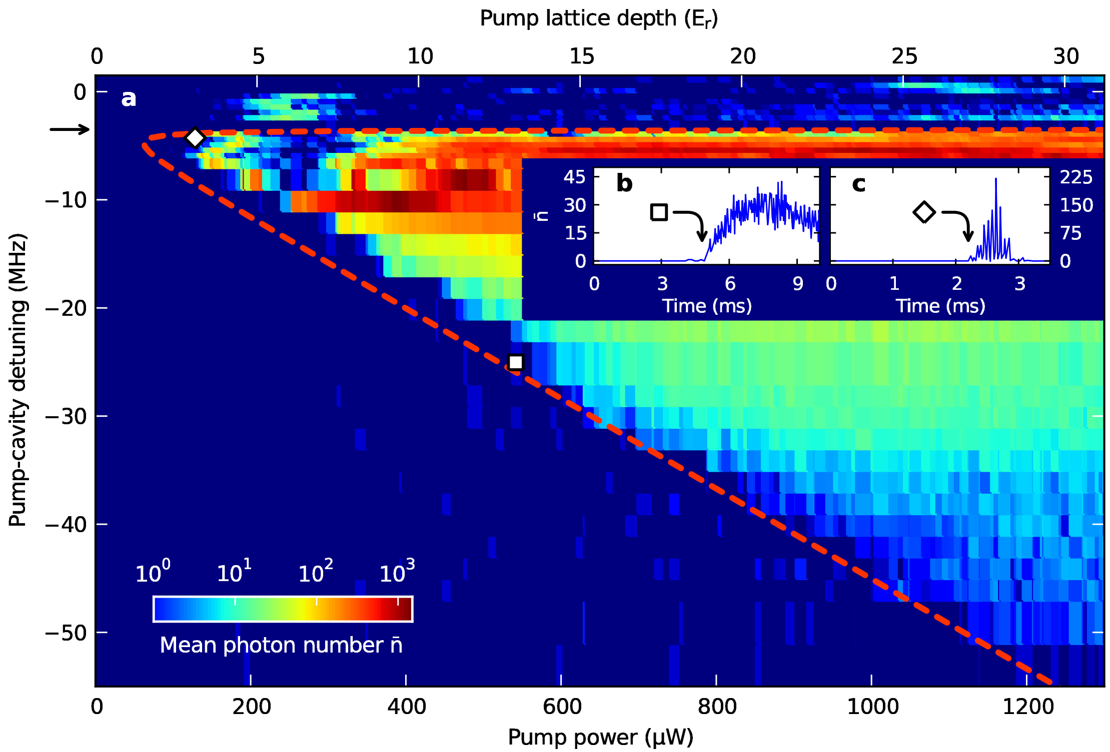
$\Downarrow = |\text{Zero Momentum}\rangle \qquad \Uparrow = |\text{Finite Momentum}\rangle$

"Spins" Coupled to Light

Observation of Dicke Transition

$$H = \omega\psi^\dagger\psi + \omega_0 \sum_i S_i^z + g\sum_i(\psi^\dagger S_i^- + \psi S_i^+)$$

$$+ g' \sum_i(\psi^\dagger S_i^+ + \psi S_i^-) + U \sum_i S_i^z \psi^\dagger\psi$$



Baumann, Guerlin, Brennecke & Esslinger, Nature **464**, 1301 (2010)

Driven Open System

Utility of Gauge-Gravity Duality

Quantum dynamics | Classical Einstein equations

Finite temperature | Black holes

Real time approach to finite temperature quantum dynamics in interacting systems, with the possibility of anchoring to $1+1$ and generalizing to higher dimensions

Non-Equilibrium Beyond linear response

Temporal dynamics in strongly correlated systems

Combine analytics with numerics

Dynamical phase diagrams

Organizing principles out of equilibrium

Progress on the CMT Side

Simple protocals and integrability

Methods of integrability and CFT have been invaluable in classifying equilibrium phases and phase transitions in 1+1

Do do these methods extend to non-equilibrium problems?

Quantum quench Parameter in H abruptly changed

$$H(g) \rightarrow H(g')$$

System prepared in state $|\Psi_g\rangle$ but time evolves under $H(g')$

Quantum quench to a CFT

Calabrese & Cardy, PRL **96**, 136801 (2006)

Quantum quenches in quantum spin chains

e.g. Calabrese, Essler & Fagotti, PRL **106**, 227203 (2011)

Quantum quench in BCS pairing Hamiltonian

Andreev, Gurarie, Radzihovsky, Barankov, Levitov, Yuzbashyan, Caux...

Thermalization **Integrability** **Generalized Gibbs**

Quantum Quench to a CFT

Calabrese and Cardy, *"Time Dependence of Correlation Functions Following a Quantum Quench"*, PRL **96**, 136801 (2006)

$$\langle \Phi(t) \rangle = A_b^\Phi \left[\frac{\pi}{4\tau_0} \frac{1}{\cosh[\pi t/2\tau_0]} \right]^x \sim e^{-\pi x t/2\tau_0}$$

Scaling dimension x Non-universal decay $\tau_0 \sim m^{-1}$

Ratios of observables exhibit universality

$$\langle \Phi(r,t)\Phi(0,t) \rangle \sim e^{-\pi x r/2\tau_0} \qquad t > r/2$$

Emergent length scale or temperature scale

AdS/CFT gives access to higher dimensional interacting critical points and the possibility of universal results

Quenches in Transverse Field Ising

Recent work: Calabrese, Essler, Fagottini, PRL **106**, 227203 (2011)

$$H(h) = -\frac{1}{2}\sum_{l=-\infty}^{\infty}[\sigma_l^x\sigma_l^x + h\sigma_l^z]$$

QPT   Ferromagnetic $h < 1$ Paramagnetic $h > 1$

Quantum quench $h_0 \to h$ within the ordered phase $h_0, h \leq 1$

Exact results for long time asymptotics of order parameter

$$\langle\sigma_l^x(t)\rangle \propto \exp\left[t\int_0^\pi \frac{dk}{\pi}\epsilon_h'(k)\ln(\cos\Delta_k)\right]$$

$$\epsilon_h(k) = \sqrt{h^2 - 2h\cos k + 1}$$

$$\cos\Delta_k = \frac{hh_0 - (h+h_0)\cos k + 1}{\epsilon_h(k)\epsilon_{h_0}(k)}$$

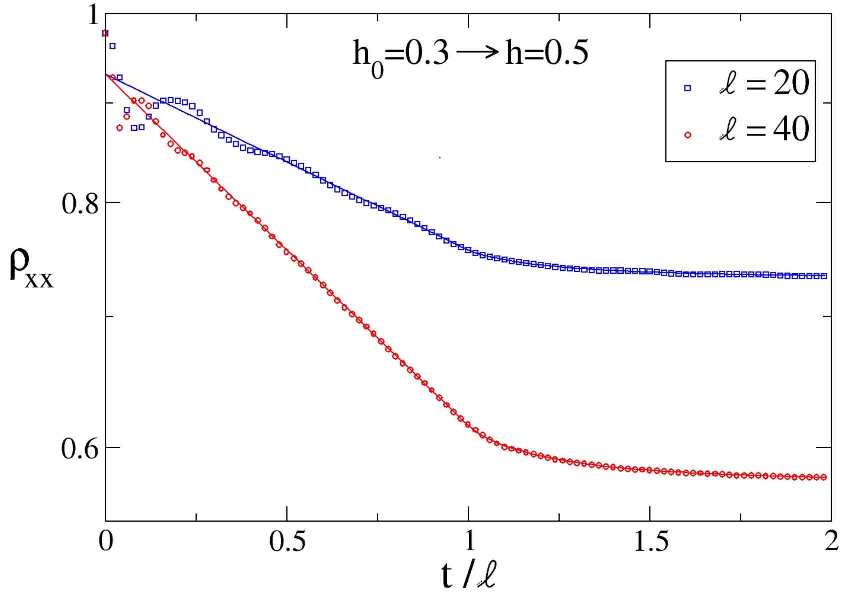
Exact results for asymptotics of two-point functions

$$\langle\sigma_l^x(t)\sigma_{1+l}^x(t)\rangle \propto \exp\left[l\int_0^\pi \frac{dk}{\pi}\ln(\cos\Delta_k)\theta(2\epsilon_h'(k)t - l)\right]$$

$$\times\exp\left[2t\int_0^\pi \frac{dk}{\pi}\epsilon_h'(k)\ln(\cos\Delta_k)\theta(l - 2\epsilon_h'(k)t)\right]$$

Quenches in Transverse Field Ising

Recent work: Calabrese, Essler, Fagottini, PRL **106**, 227203 (2011)



Good agreement between numerics and analytics

Determinants **Form Factors**

Integrable quenches in interacting field theories

BCS Quench Dynamics

Barankov, Levitov and Spivak, *"Collective Rabi Oscillations and Solitons in a Time-Dependent BCS Pairing Problem"*, PRL **93**, 160401 (2004)

Time dependent BCS Hamiltonian

$$H = \sum_{\mathbf{p},\sigma} \epsilon_{\mathbf{p}} a^\dagger_{\mathbf{p},\sigma} a_{\mathbf{p},\sigma} - \frac{\lambda(t)}{2} \sum_{\mathbf{p},\mathbf{q}} a^\dagger_{\mathbf{p},\uparrow} a^\dagger_{-\mathbf{p},\downarrow} a_{-\mathbf{q},\downarrow} a_{\mathbf{q},\uparrow}$$

Pairing interactions turned on abruptly

$$\lambda(t) = \lambda\theta(t)$$

Generalized time-dependent many-body BCS state

$$|\Psi(t)\rangle = \prod_{\mathbf{p}} \left[u_{\mathbf{p}}(t) + v_{\mathbf{p}}(t) a^\dagger_{\mathbf{p},\uparrow} a^\dagger_{-\mathbf{p},\downarrow} \right] |0\rangle$$

Integrable

Collective Oscillations

Barankov, Levitov and Spivak, PRL **93**, 160401 (2004)

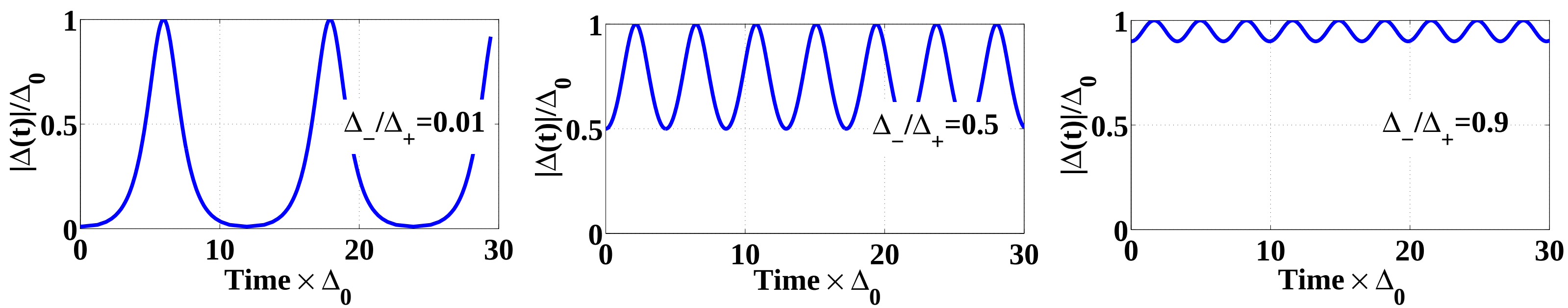
Time-dependent pairing amplitude

$$\Delta(t) \equiv \lambda \sum_{\mathbf{p}} u_{\mathbf{p}} v_{\mathbf{p}}^{*}(t) = e^{-i\omega t}\Omega(t)$$

Equation of Motion

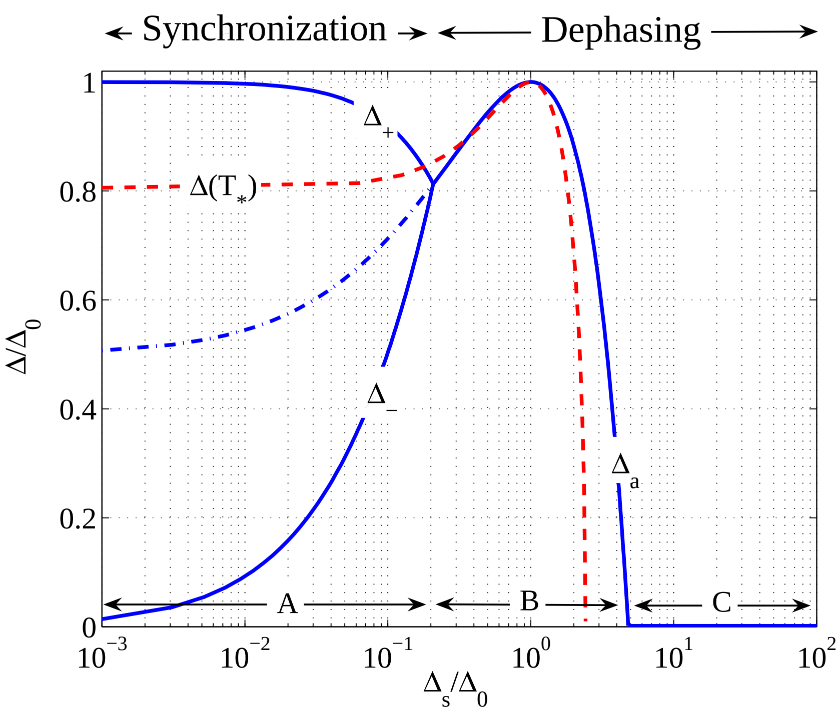
$$\dot{\Omega}^2 + (\Omega^2 - \Delta_-^2)(\Omega^2 - \Delta_+^2) = 0$$

Oscillations between Δ_- and Δ_+



Regimes of BCS Quench Dynamics

Barankov and Levitov, *"Synchronization in the BCS Pairing Dynamics as a Critical Phenomenon"*, PRL **96**, 230403 (2006)



Initial pairing gap $\boxed{\Delta_s}$ Final pairing gap $\boxed{\Delta_0}$

(A) Oscillations between $\Delta_\pm$ (B) Underdamped approach to Δ_a

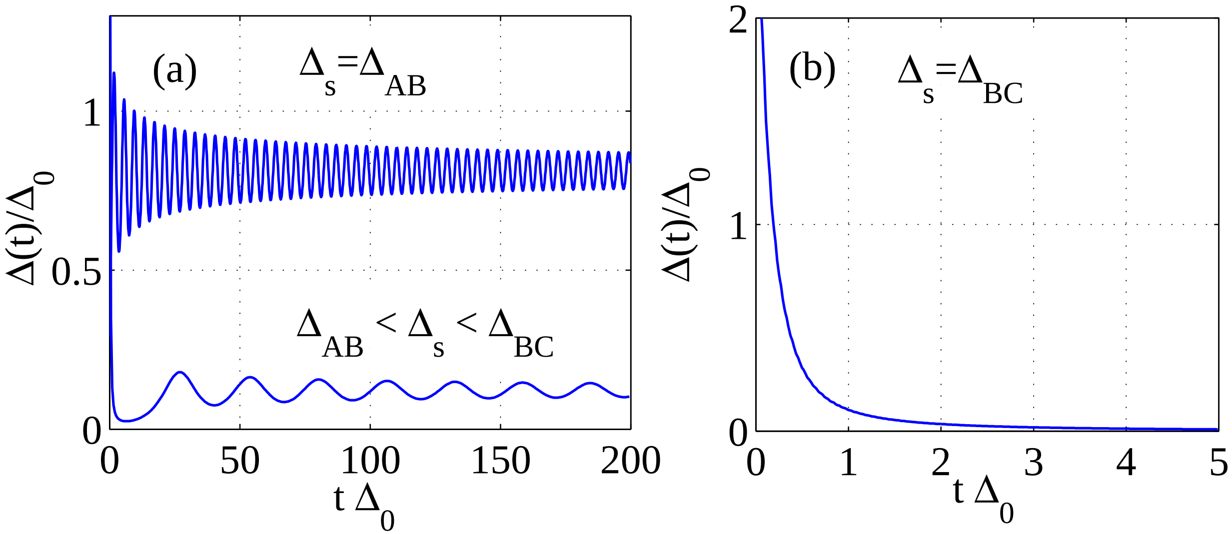
(C) Overdamped approach to $\Delta = 0$

Emergent temperature T^* and gap $\Delta(T^*)$

Bloch Dynamics

Anderson Pseudo-Spins

$$H = -\sum_{\mathbf{p}} 2\epsilon_{\mathbf{p}} s^z_{\mathbf{p}} - \lambda(t) \sum_{\mathbf{pq}} s^-_{\mathbf{p}} s^+_{\mathbf{q}}$$



$$\Delta(t) = \Delta_a + A(t)\sin(2\Delta_a t + \alpha)$$

$$A(t) \propto t^{-1/2}$$

$$\Delta(t) \propto (\Delta_s t)^{-1/2} e^{-2\Delta_s t}$$

Non-Equilibrium Dynamical Phase Diagram

Yuzbashyan *et al* Andreev *et al*

AdS Quenches

Chesler & Yaffe, *"Horizon formation and far-from-equilibrium isotropization in supersymmetric Yang–Mills plasma"*, PRL (2009)

$$\boxed{ds^2 = -dt^2 + e^{B_0(t)} d\mathbf{x}_\perp^2 + e^{-2B_0(t)} d\mathbf{x}_\parallel^2}$$

The dependent shear of the geometry $\quad B_0(t) = \frac{1}{2} c \,[1 - \tanh(t/\tau)]$

Jan de Boer & Esko Keski-Vakkuri *et al*, *"Thermalization of Strongly Coupled Field Theories"*, PRL (2011)

$$\boxed{ds^2 = \frac{1}{z^2}\left[-(1 - m(v)) z^d dv^2 - 2dzdv + d\mathbf{x}^2\right]}$$

Vaidya metric quenches $\quad m(v) = \frac{1}{2} M [1 + \tanh(v/v_0)]$

Aparício & López, *"Evolution of Two-Point Functions from Holography"*, arXiv:1109.2571

Albash & Johnson, *"Evolution of Holographic Entanglement Entropy after Thermal and Electromagnetic Quenches"*, NJP (2011)

Basu & Das, *"Quantum Quench across a Holographic Critical Point"*, arXiv:1109.3309

Holographic Superconductor

Gubser, *"Breaking an Abelian Gauge Symmetry Near a Black Hole Horizon"*, Phys. Rev. D. **78**, 065034 (2008)

Hartnoll, Herzog, Horowitz, *"Building a Holographic Superconductor"*, PRL **101**, 031601 (2008)

Abelian Higgs Coupled to Einstein–Hilbert Gravity

$$\boxed{S = \int d^D x \sqrt{-g} \left[R - 2\Lambda - \frac{1}{4} F_{ab} F^{ab} - |D_a \psi|^2 - m^2 |\psi|^2 \right]}$$

$$F_{ab} = \partial_a A_b - \partial_b A_a \quad D_a = \partial_a - iqA_a \quad \Lambda = -\tfrac{D(D-1)}{4L^2}$$

Finite temperature **Black hole**

Below critical temperature black hole with $\psi = 0$ unstable

Black hole with charged scalar hair $\psi \neq 0$ becomes stable

Spontaneous U(1) symmetry breaking

Non-Equilibrium AdS Superconductors

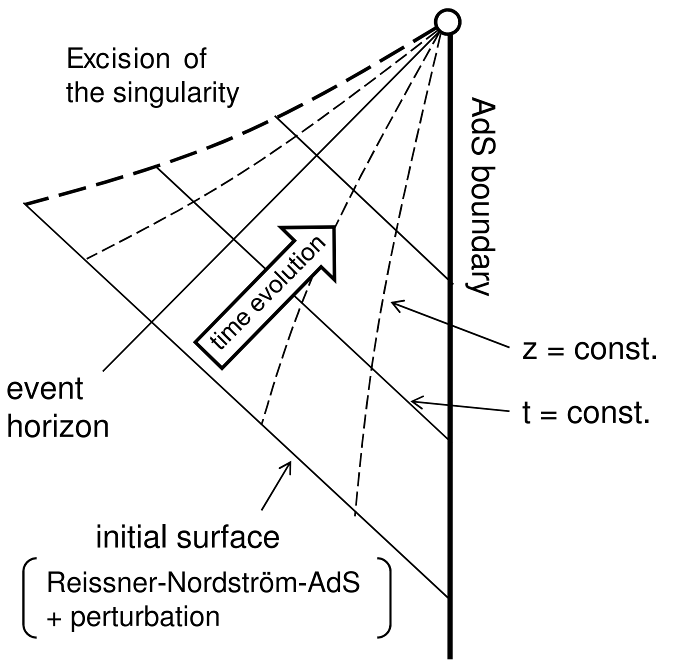
Murata, Kinoshita & Tanahashi, *"Non-Equilibrium Condensation Process in a Holographic Superconductor"*, JHEP 1007:050 (2010)

Start in unstable normal state described by a Reissner–Nordström black hole and perturb

Temporal dynamics from supercooled to superconducting

AdS Geometry

Murata, Kinoshita & Tanahashi, JHEP 1007:050 (2010)



$$\psi(t,z) = \psi_1(t)z + \psi_2(t)z^2 + \psi_3(t)z^3 + \cdots \equiv \psi_1(t)z + \tilde{\psi}(t,z)z^2$$

Gaussian perturbation of Reissner–Nordström–AdS

$$\tilde{\psi}(t=0,z) = \frac{\mathcal{A}}{\sqrt{2\pi}\delta} \exp\left[-\frac{(z-z_m)^2}{2\delta^2}\right]$$

$$\mathcal{A} = 0.01,\ \delta = 0.05,\ z_m = 0.3$$

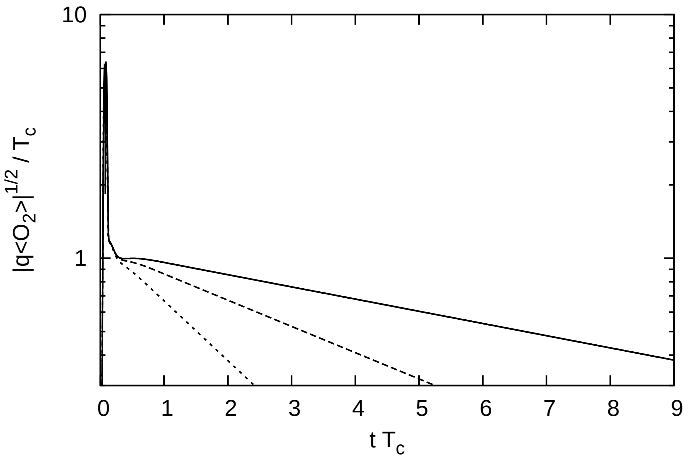
Time Evolution of the Order Parameter

Murata, Kinoshita & Tanahashi, JHEP 1007:050 (2010)

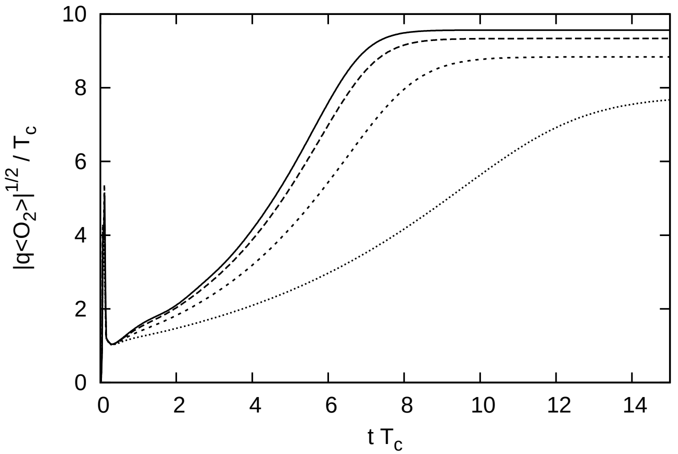
Start in normal state (Reissner–Nordström) & perturb

Numerical solution of Einstein equations

$$\langle \mathcal{O}_2(t) \rangle \equiv \sqrt{2}\psi_2(t)$$



$T/T_\mathrm{c} = 1.1, 1.2, 1.4$



$T/T_\mathrm{c} = 0.2, 0.4, 0.6, 0.8$

$T > T_\mathrm{c}$ **Relaxation to N** $|\langle \mathcal{O}_2(t) \rangle| = C \exp(-t/t_\mathrm{relax})$

$T < T_\mathrm{c}$ **Evolution to SC** $|\langle \mathcal{O}_2(t) \rangle| = C_1 \exp(-t/t_\mathrm{relax}) + C_2$

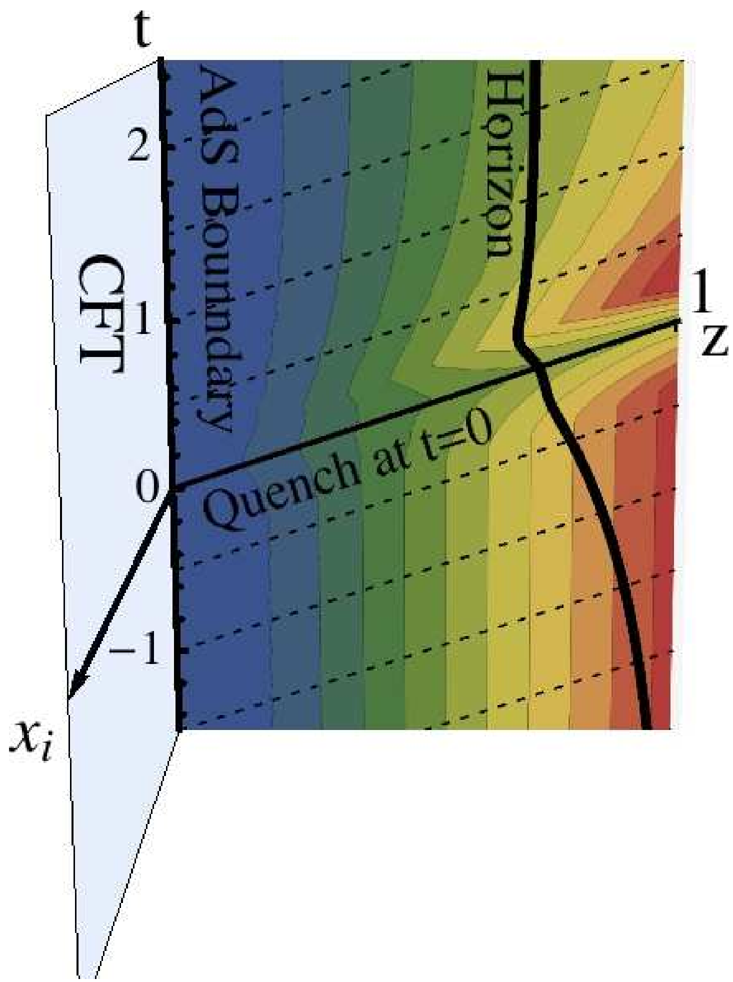
Questions

Where are the oscillations?

- Is there always damping or are there different regimes?
- Are the dynamics of holographic superconductors related to condensed matter systems or intrinsically different?
- What is the role of coupling to a large number of critical degrees of freedom?
- What is the role of large N and strong coupling?
- What is their influence on the emergent timescales?
- Do thermal fluctuations reduce the amplitude of oscillations?
- Beyond BCS and mean field dynamics using AdS/CFT?
- What is the dynamics at short, intermediate and long times?
- What happens if one quenches from a charged black hole?

AdS/CMT far from equilibrium?

Setup



Homogeneous isotropic dynamics of CFT from Einstein Eqs

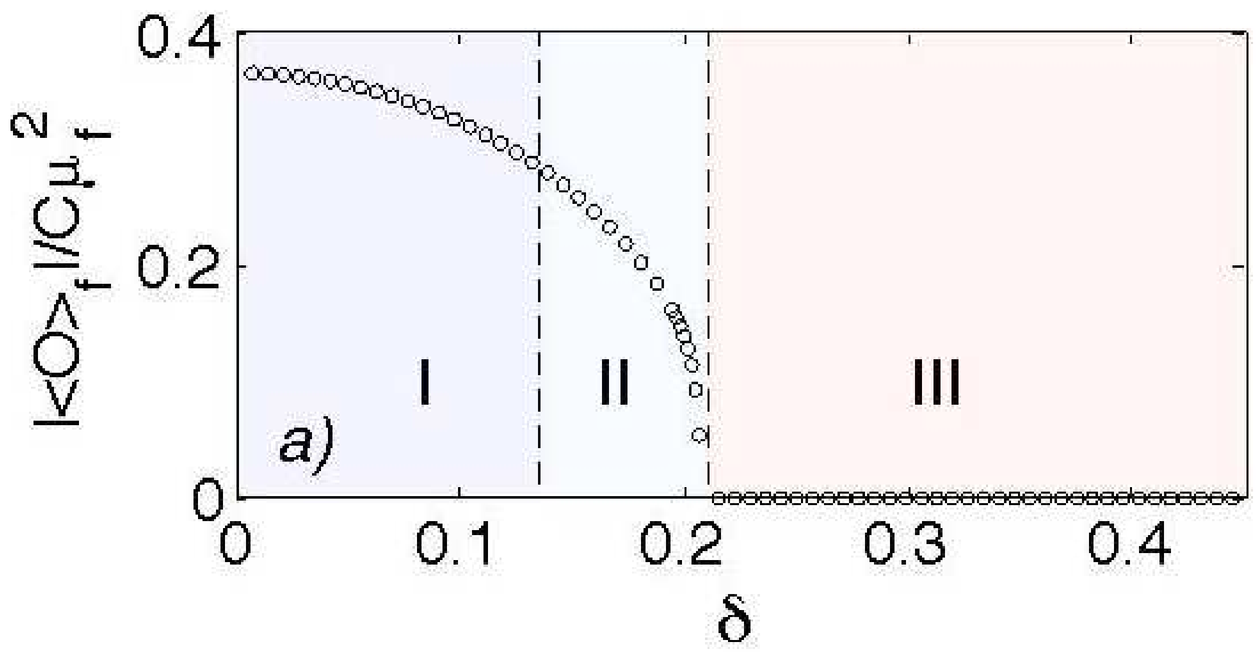
Dynamical Phase Diagram

Conjugate field pulse

AdS$_4$/CFT$_3$ $\quad \psi(t,z) = \psi_1(t) z + \psi_2(t) z^2 + \ldots$

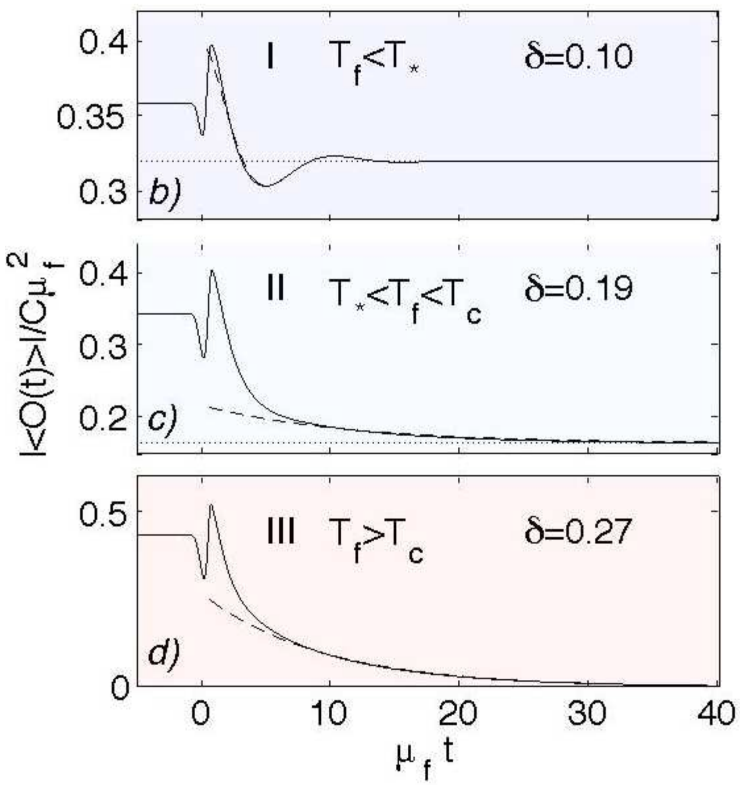
$$\psi_1(t) = \bar{\delta} e^{-(t/\bar{\tau})^2}$$

$$\delta = \bar{\delta}/\mu_i \quad \tau = \bar{\tau}/\mu_i \quad \tau = 0.5 \quad T_i = 0.5 T_c$$



Three Dynamical Regimes

Bhaseen, Gauntlett, Simons, Sonner & Wiseman, *"Holographic Superfluids and the Dynamics of Symmetry Breaking"* PRL (2013)



(I) Damped Oscillatory to SC

$$\langle \mathcal{O}(t)\rangle \sim a + be^{-kt}\cos[l(t-t_0)]$$

(II) Over Damped to SC

$$\langle \mathcal{O}(t)\rangle \sim a + be^{-kt}$$

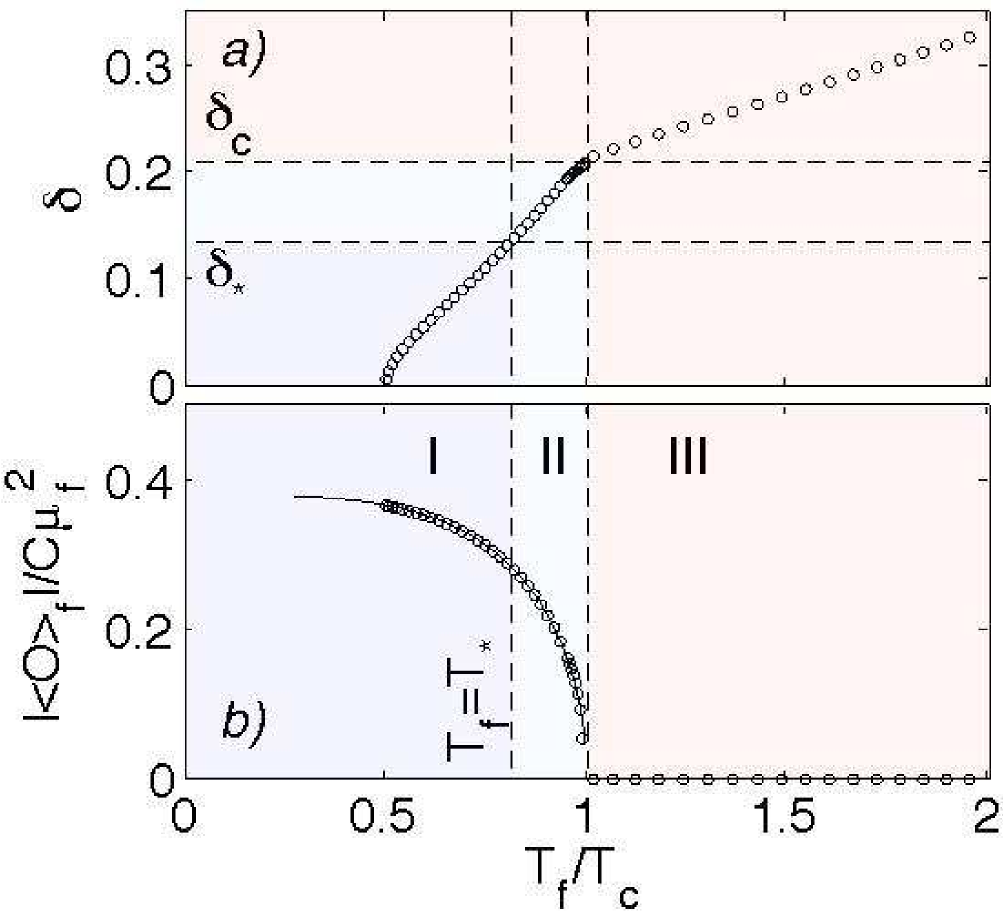
(III) Over Damped to N

$$\langle \mathcal{O}(t)\rangle \sim be^{-kt}$$

Asymptotics described by black hole quasi normal modes

Far from equilibrium → close to equilibrium

Approach to Thermal Equilibrium

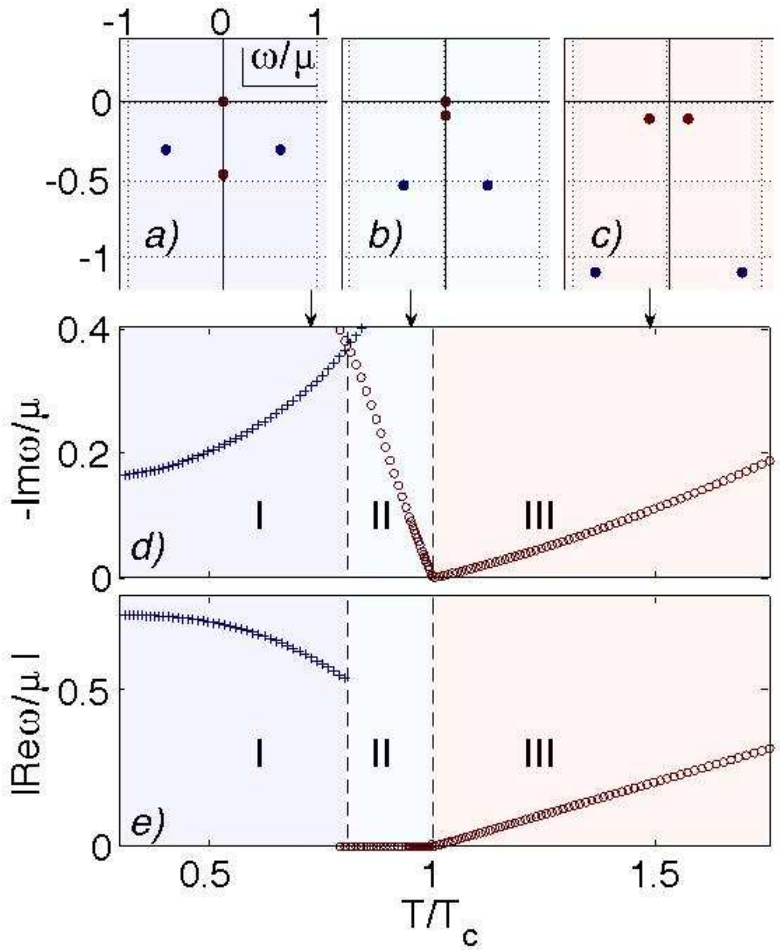


Collapse on to Equilibrium Phase Diagram

Emergent temperature scale T_* within superfluid phase

Quasi Normal Modes

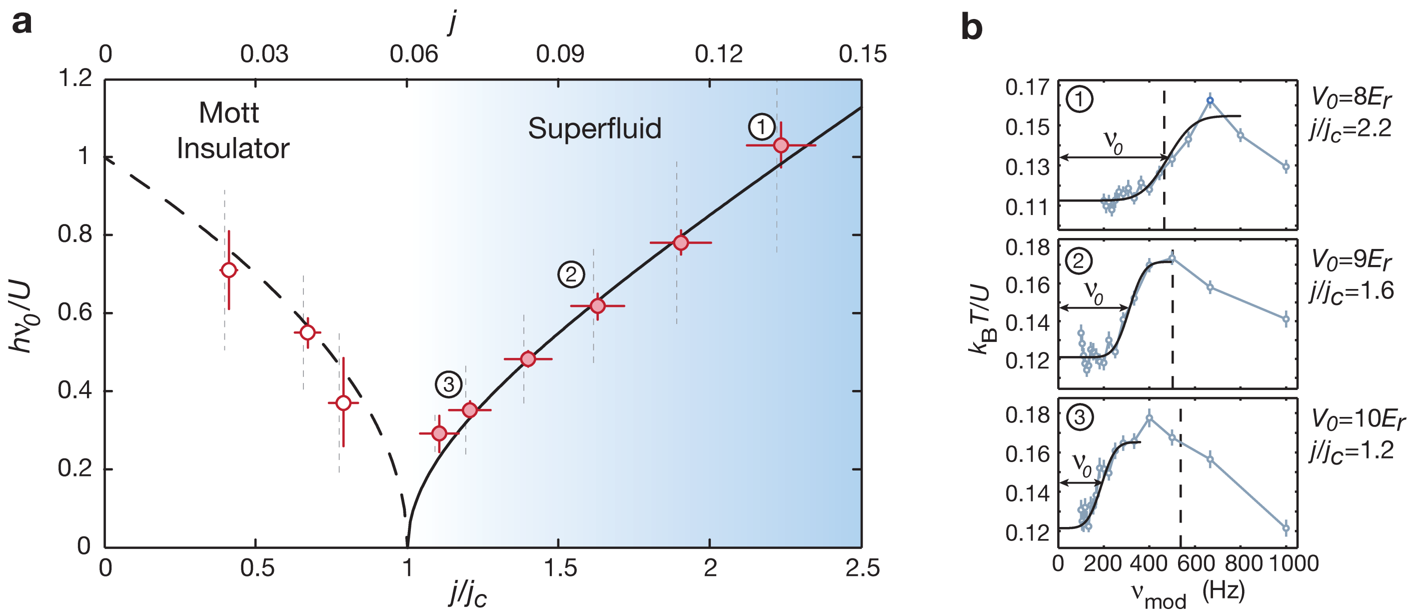
$$|\langle \mathcal{O}(t) \rangle| \simeq |\langle \mathcal{O} \rangle_f + \mathcal{A} e^{-i\omega t}|$$



Three regimes and an emergent T_*

Recent Experiments

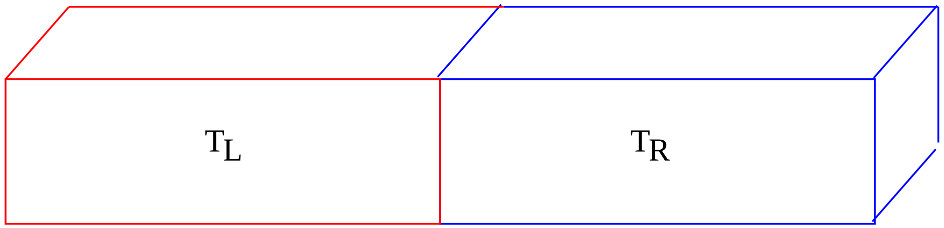
Endres *et al*, *"The 'Higgs' Amplitude Mode at the Two-Dimensional Superfluid-Mott Insulator Transition"*, Nature **487**, 454 (2012)



Time-dependent experiments can probe excitations

What is the pole structure in other correlated systems?

Thermalization

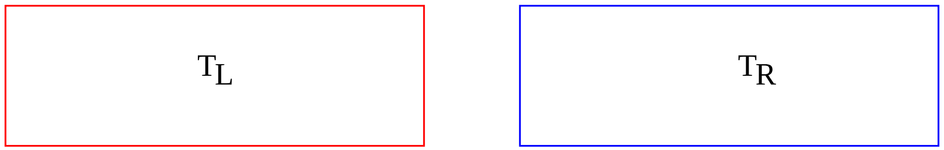


Why not connect two strongly correlated systems together and see what happens?

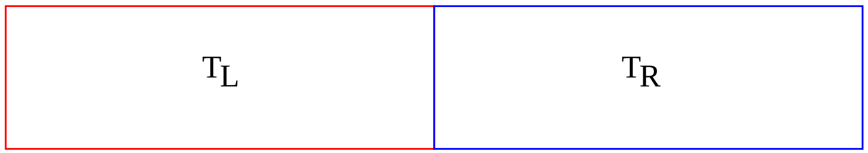
Non-Equilibrium CFT

Bernard & Doyon, *Energy flow in non-equilibrium conformal field theory*, J. Phys. A: Math. Theor. **45**, 362001 (2012)

Two critical 1D systems (central charge c)
at temperatures T_L & T_R



Join the two systems together



Alternatively, take one critical system and impose a step profile

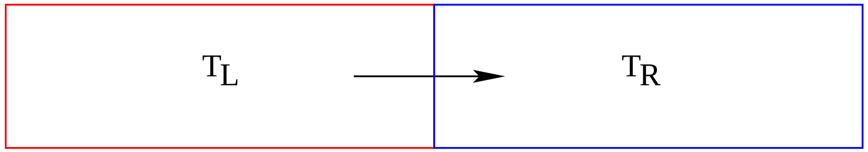
Local Quench

Steady State Heat Flow

Bernard & Doyon, *Energy flow in non-equilibrium conformal field theory*, J. Phys. A: Math. Theor. **45** 362001 (2012)

If systems are very large ($L \gg vt$) they act like heat baths

For times $t \ll L/v$ a steady heat current flows



Non-equilibrium steady state

$$J = \frac{c\pi^2 k_B^2}{6h}(T_L^2 - T_R^2)$$

Universal result out of equilibrium

Direct way to measure central charge; velocity doesn't enter

Sotiriadis and Cardy. J. Stat. Mech. (2008) P11003.

Linear Response

Bernard & Doyon, *Energy flow in non-equilibrium conformal field theory*, J. Phys. A: Math. Theor. **45**, 362001 (2012)

$$J = \frac{c\pi^2 k_B^2}{6h}(T_L^2 - T_R^2)$$

$$T_L = T + \Delta T/2 \qquad T_R = T - \Delta T/2 \qquad \Delta T \equiv T_L - T_R$$

$$J = \frac{c\pi^2 k_B^2}{3h} T\Delta T \equiv g\Delta T \qquad g = cg_0 \qquad g_0 = \frac{\pi^2 k_B^2 T}{3h}$$

Quantum of Thermal Conductance

$$g_0 = \frac{\pi^2 k_B^2 T}{3h} \approx (9.456 \times 10^{-13}\,\mathrm{WK}^{-2})\,T$$

Free Fermions

Fazio, Hekking and Khmelnitskii, PRL **80**, 5611 (1998)

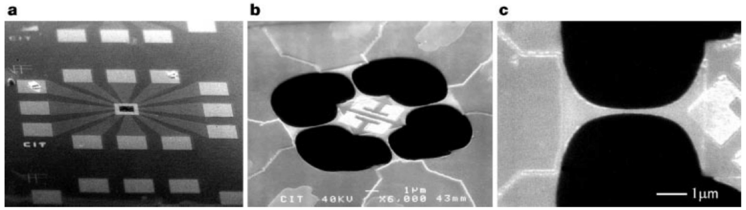
Wiedemann–Franz $\quad \frac{\kappa}{\sigma T} = \frac{\pi^2}{3e^2} \qquad \sigma_0 = \frac{e^2}{h} \qquad \kappa_0 = \frac{\pi^2 k_B^2 T}{3h}$

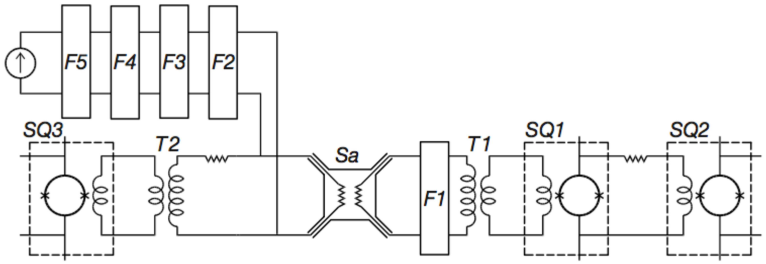
Conformal Anomaly

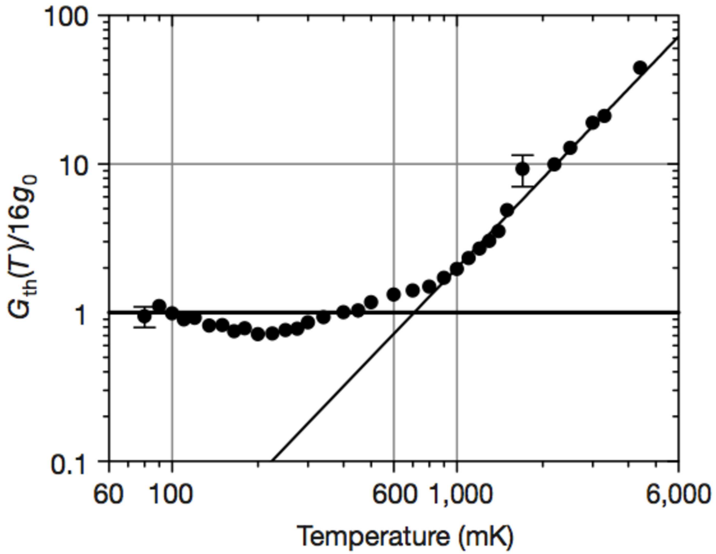
Cappelli, Huerta and Zemba, Nucl. Phys. B **636**, 568 (2002)

Experiment

Schwab, Henriksen, Worlock and Roukes, *Measurement of the quantum of thermal conductance*, Nature **404**, 974 (2000)







Quantum of Thermal Conductance

Heuristic Interpretation of CFT Result

$$J = \sum_m \int \frac{dk}{2\pi} \hbar\omega_m(k) v_m(k)[n_m(T_L) - n_m(T_R)]\mathbb{T}_m(k)$$

$$v_m(k) = \partial\omega_m/\partial k \quad n_m(T) = \frac{1}{e^{\beta\hbar\omega_m} - 1}$$

$$J = f(T_L) - f(T_R)$$

Consider just a single mode with $\omega = vk$ and $\mathbb{T} = 1$

$$f(T) = \int_0^\infty \frac{dk}{2\pi} \frac{\hbar v^2 k}{e^{\beta\hbar v k} - 1} = \frac{k_B^2 T^2}{h} \int_0^\infty dx \frac{x}{e^x - 1} = \frac{k_B^2 T^2}{h} \frac{\pi^2}{6} \qquad x \equiv \frac{\hbar v k}{k_B T}$$

Velocity cancels out

$$J = \frac{\pi^2 k_B^2}{6h}(T_L^2 - T_R^2)$$

For a 1+1 critical theory with central charge c

$$J = \frac{c\pi^2 k_B^2}{6h}(T_L^2 - T_R^2)$$

Stefan–Boltzmann

Cardy, *The Ubiquitous 'c': from the Stefan-Boltzmann Law to Quantum Information*, `arXiv:1008.2331`

Black Body Radiation in $3+1$ dimensions

$$dU = TdS - PdV$$

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - P = T\left(\frac{\partial P}{\partial T}\right)_V - P$$

$$\boxed{u = T\left(\frac{\partial P}{\partial T}\right)_V - P}$$

For black body radiation $P = u/3$

$$\frac{4u}{3} = \frac{T}{3}\left(\frac{\partial u}{\partial T}\right)_V \qquad \frac{du}{4u} = \frac{dT}{T} \qquad \frac{1}{4}\ln u = \ln T + \text{const.}$$

$$u \propto T^4$$

Stefan–Boltzmann and CFT

Cardy, *The Ubiquitous 'c': from the Stefan-Boltzmann Law to Quantum Information*, `arXiv:1008.2331`

Energy-Momentum Tensor in $d+1$ Dimensions

$$T_{\mu\nu} = \begin{pmatrix} u & & & \\ & P & & \\ & & P & \\ & & & \ldots \end{pmatrix} \qquad \text{Traceless} \qquad P = u/d$$

Thermodynamics

$$u = T\left(\frac{\partial P}{\partial T}\right)_V - P \qquad \boxed{u \propto T^{d+1}}$$

For $1+1$ Dimensional CFT

$$\boxed{u = \frac{\pi c k_B^2 T^2}{6\hbar v} \equiv \mathcal{A} T^2} \qquad \boxed{J = \frac{\mathcal{A} v}{2}(T_L^2 - T_R^2)}$$

Stefan–Boltzmann and AdS/CFT

Gubser, Klebanov and Peet, *Entropy and temperature of black 3-branes*, Phys. Rev. D **54**, 3915 (1996).

Entropy of $SU(N)$ SYM = Bekenstein–Hawking $S_{\rm BH}$ of geometry

$$\boxed{S_{\rm BH} = \frac{\pi^2}{2} N^2 V_3 T^3}$$

Entropy at Weak Coupling = $8N^2$ free massless bosons & fermions

$$\boxed{S_0 = \frac{2\pi^2}{3} N^2 V_3 T^3}$$

Relationship between strong and weak coupling

$$\boxed{S_{\rm BH} = \tfrac{3}{4} S_0}$$

Gubser, Klebanov, Tseytlin, *Coupling constant dependence in the thermodynamics of $\mathcal{N} = 4$ supersymmetric Yang-Mills Theory*, Nucl. Phys. B **534** 202 (1998)

Energy Current Fluctuations

Bernard & Doyon, *Energy flow in non-equilibrium conformal field theory*, J. Phys. A: Math. Theor. **45**, 362001 (2012)

Generating function for all moments

$$\boxed{\mathrm{F}(\lambda) \equiv \lim_{t\to\infty} t^{-1} \ln\langle e^{i\lambda \Delta_t Q}\rangle}$$

Exact Result

$$\boxed{F(\lambda) = \frac{c\pi^2}{6h}\left(\frac{i\lambda}{\beta_l(\beta_l - i\lambda)} - \frac{i\lambda}{\beta_r(\beta_r + i\lambda)}\right)}$$

Denote $z \equiv i\lambda$

$$F(z) = \frac{c\pi^2}{6h}\left[z\left(\frac{1}{\beta_l^2} - \frac{1}{\beta_r^2}\right) + z^2\left(\frac{1}{\beta_l^3} + \frac{1}{\beta_r^3}\right) + \dots\right]$$

$$\boxed{\langle J\rangle = \frac{c\pi^2}{6h} k_B^2 (T_L^2 - T_R^2)} \qquad \boxed{\langle \delta J^2\rangle \propto \frac{c\pi^2}{6h} k_B^3 (T_L^3 + T_R^3)}$$

Poisson Process $\quad \int_0^\infty e^{-\beta\epsilon}(e^{i\lambda\epsilon} - 1)d\epsilon = \frac{i\lambda}{\beta(\beta - i\lambda)}$

Full Counting Statistics

A large body of results in the mesoscopic literature

Free Fermions

Levitov & Lesovik, *"Charge Distribution in Quantum Shot Noise"*, JETP Lett. **58**, 230 (1993)

Levitov, Lee & Lesovik, *"Electron Counting Statistics and Coherent States of Electric Current"*, J. Math. Phys. **37**, 4845 (1996)

Luttinger Liquids and Quantum Hall Edge States

Kane & Fisher, *"Non-Equilibrium Noise and Fractional Charge in the Quantum Hall Effect"*, PRL **72**, 724 (1994)

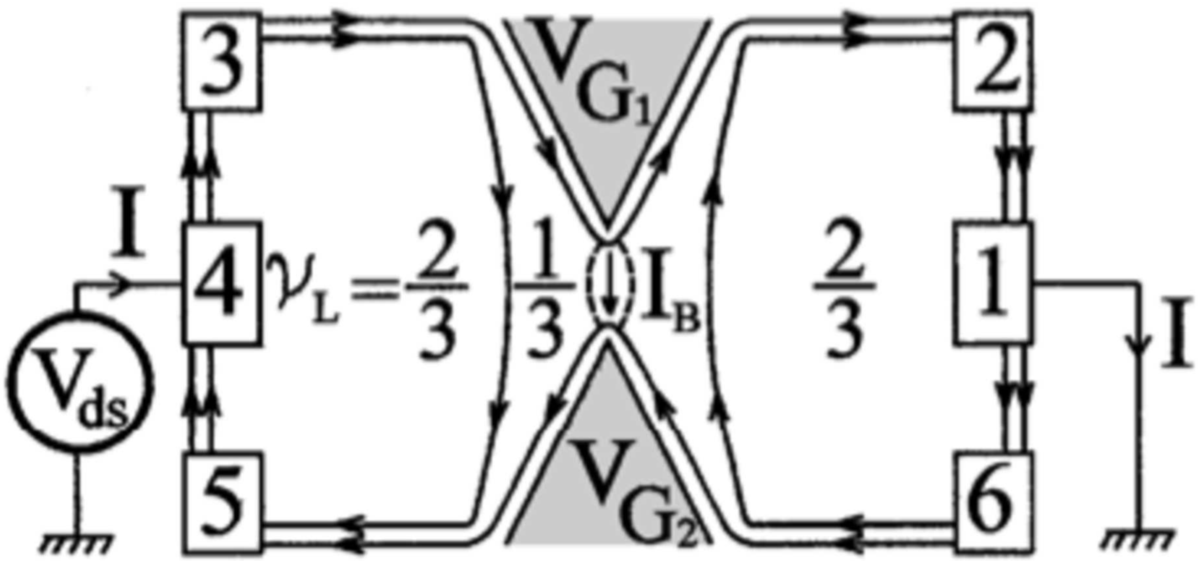
Fendley, Ludwig & Saleur, *"Exact Nonequilibrium dc Shot Noise in Luttinger Liquids and Fractional Quantum Hall Devices"*, PRL (1995)

Quantum Impurity Problems

Komnik & Saleur, *"Quantum Fluctuation Theorem in an Interacting Setup: Point Contacts in Fractional Quantum Hall Edge State Devices"*, PRL **107**, 100601 (2011)

Experimental Setup

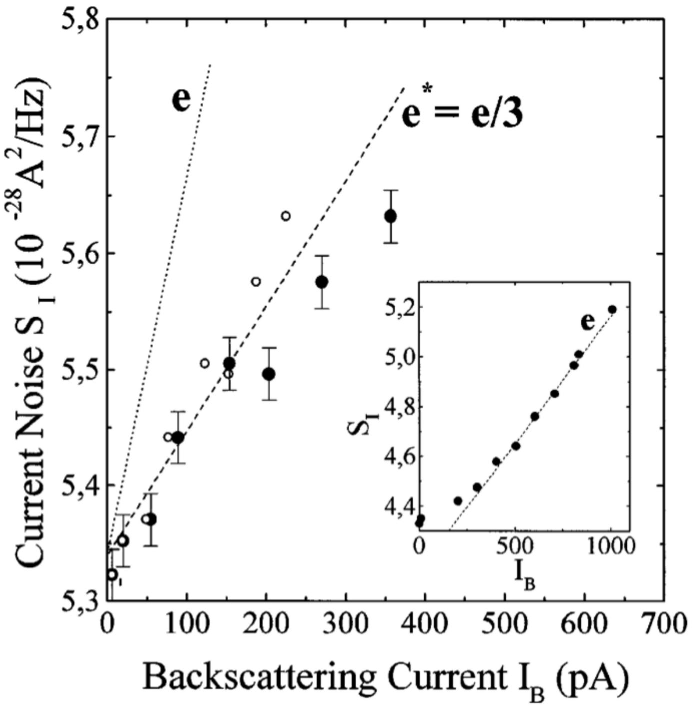
Saminadayar *et al*, PRL **79**, 2526 (1997)

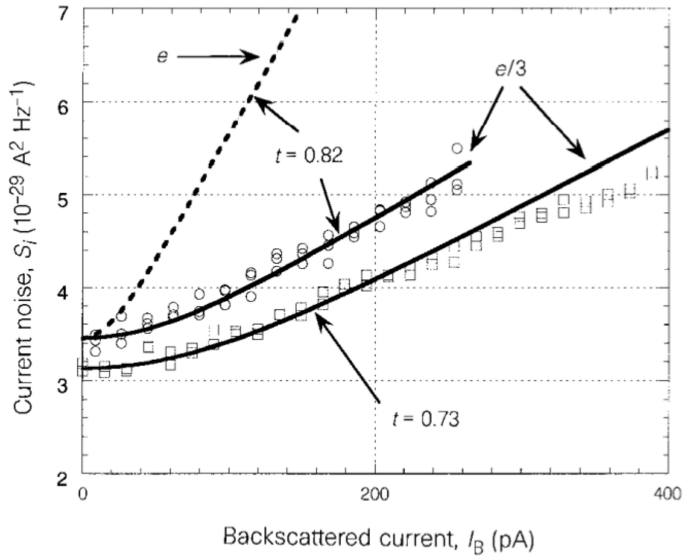


Shot Noise in the Quantum Hall Effect

Saminadayar *et al*, *"Observation of the e/3 Fractionally Charged Laughlin Quasiparticle"*, PRL **79**, 2526 (1997)

R. de-Picciotto *et al*, *"Direct observation of a fractional charge"*, Nature **389**, 162 (1997)





$S_I = 2QI_B$ $Q = e/3$

Non-Equilibrium Fluctuation Relation

Bernard & Doyon, *Energy flow in non-equilibrium conformal field theory*, J. Phys. A: Math. Theor. **45**, 362001 (2012)

$$F(\lambda) \equiv \lim_{t\to\infty} t^{-1} \ln\langle e^{i\lambda\Delta_t Q}\rangle = \frac{c\pi^2}{6h}\left(\frac{i\lambda}{\beta_l(\beta_l - i\lambda)} - \frac{i\lambda}{\beta_r(\beta_r + i\lambda)}\right)$$

$$F(i(\beta_r - \beta_l) - \lambda) = F(\lambda)$$

Irreversible work fluctuations in isolated driven systems

Crooks relation $$\frac{P(W)}{\tilde{P}(-W)} = e^{\beta(W-\Delta F)}$$

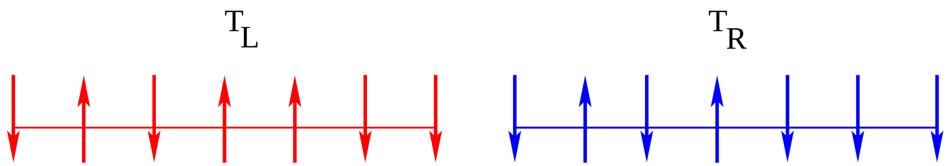
Jarzynski relation $$\langle e^{-\beta W}\rangle = e^{-\beta\Delta F}$$

Entropy production in non-equilibrium steady states

$$\frac{P(S)}{P(-S)} = e^{S}$$

Esposito *et al*, *"Nonequilibrium fluctuations, fluctuation theorems, and counting statistics in quantum systems"*, RMP **81**, 1665 (2009)

Lattice Models



Quantum Ising Model

$$H = J \sum_{\langle ij \rangle} S_i^z S_j^z + \Gamma \sum_i S_i^x$$

$\Gamma = J/2$ Critical $c = 1/2$

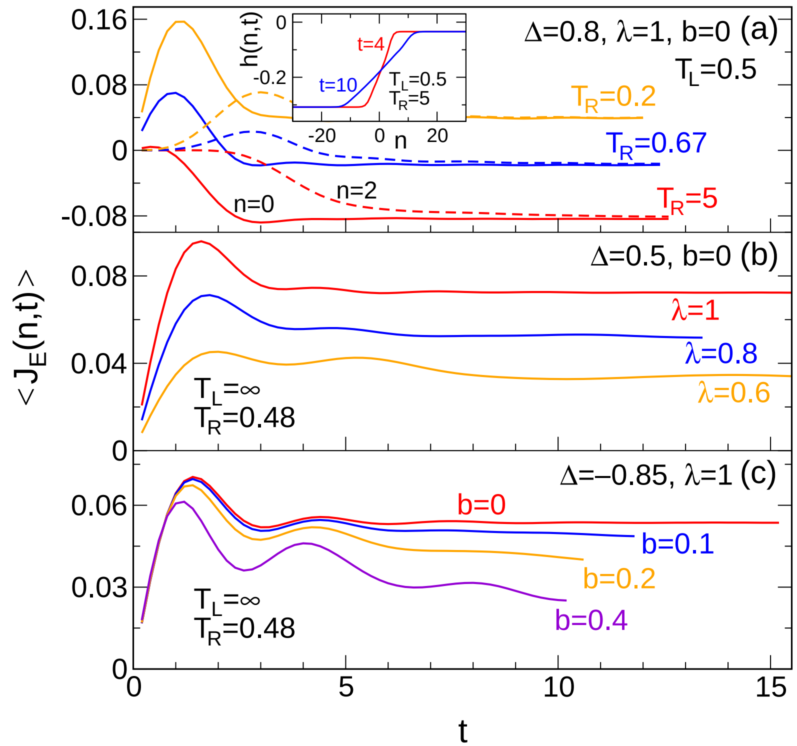
Anisotropic Heisenberg Model (XXZ)

$$H = J \sum_{\langle ij \rangle} \left(S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z \right)$$

$-1 < \Delta < 1$ Critical $c = 1$

Time-Dependent DMRG

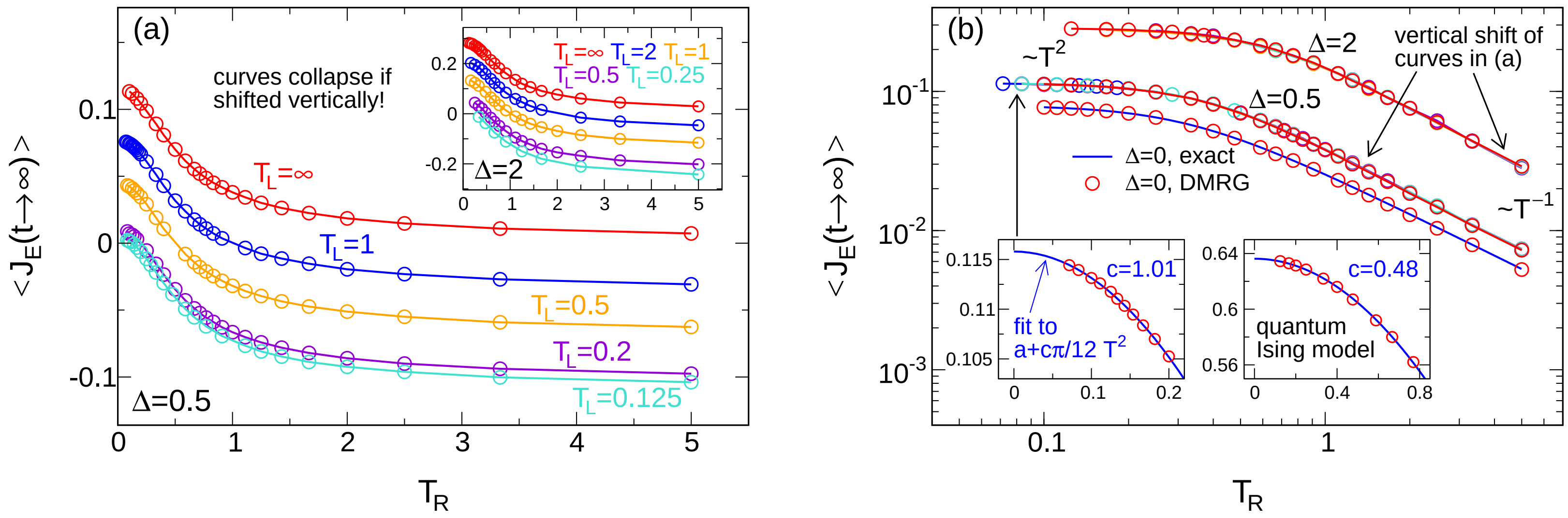
Karrasch, Ilan and Moore, *Non-equilibrium thermal transport and its relation to linear response*, `arXiv:1211.2236`



Dimerization $J_n = \begin{cases} 1 & n\,\text{odd} \\ \lambda & n\,\text{even} \end{cases}$ $\quad \Delta_n = \Delta \quad$ Staggered $b_n = \frac{(-1)^n b}{2}$

Time-Dependent DMRG

Karrasch, Ilan and Moore, *Non-equilibrium thermal transport and its relation to linear response*, `arXiv:1211.2236`



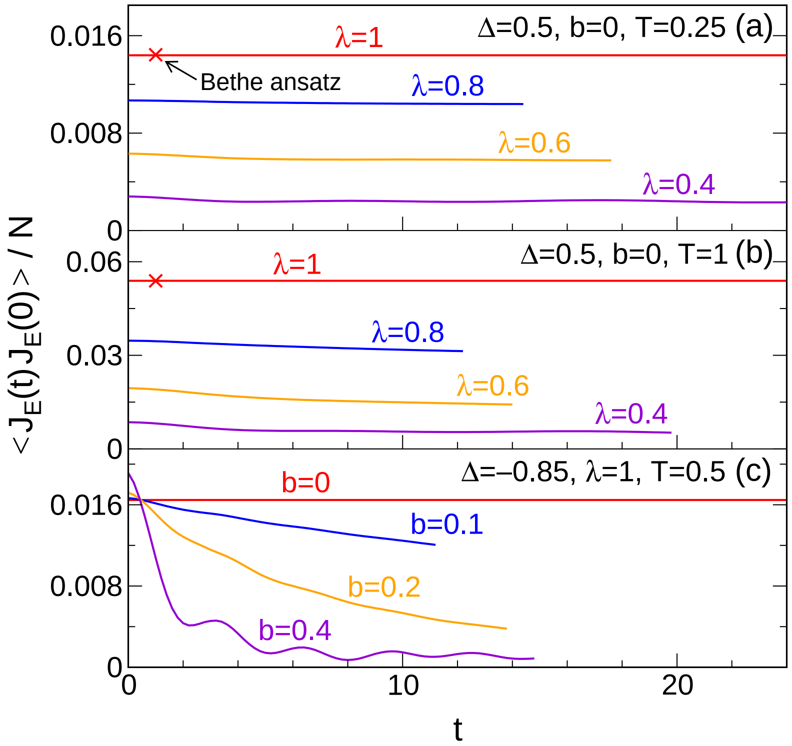
$$\lim_{t\to\infty}\langle J_E(n,t)\rangle = f(T_L) - f(T_R)$$

$$f(T) \sim \begin{cases} T^2 & T \ll 1 \\ T^{-1} & T \gg 1 \end{cases}$$

Beyond CFT to massive integrable models (Doyon)

Energy Current Correlation Function

Karrasch, Ilan and Moore, *Non-equilibrium thermal transport and its relation to linear response*, `arXiv:1211.2236`

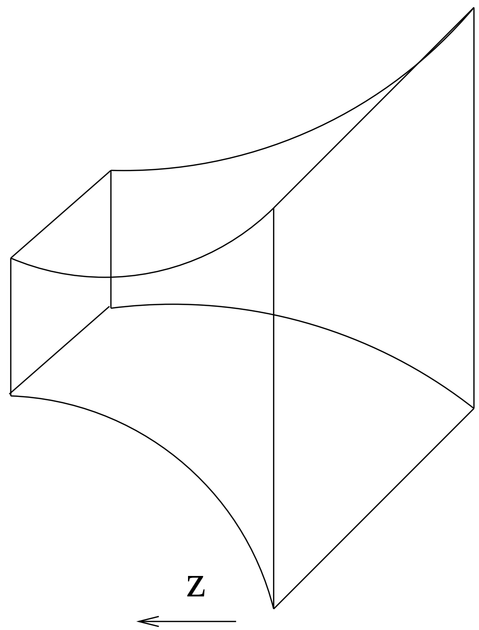


Beyond Integrability

Importance of CFT for pushing numerics and analytics

AdS/CFT

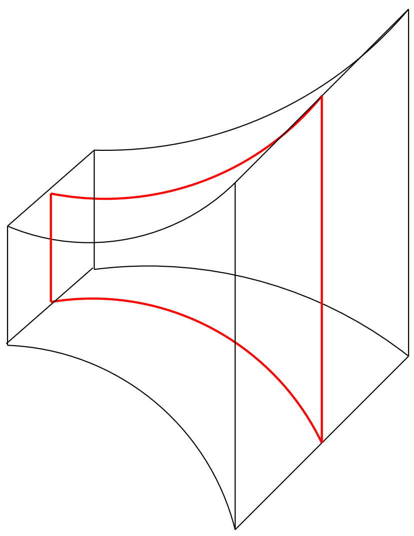
Heat flow may be studied within pure Einstein gravity

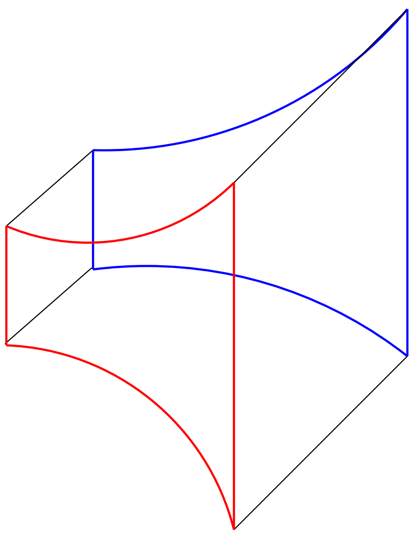


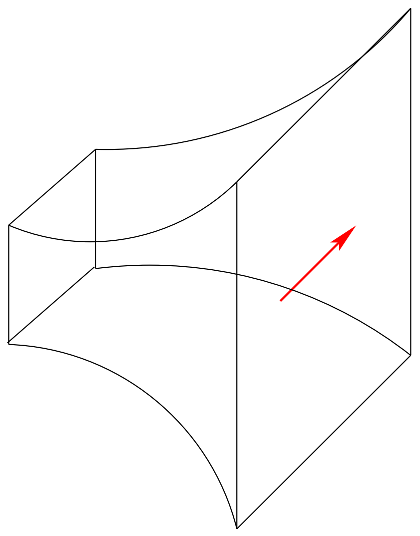
$$g_{\mu\nu} \leftrightarrow T_{\mu\nu}$$

Possible Setups

Local Quench **Driven Steady State** **Spontaneous**







General Considerations

$$\partial_\mu T^{\mu\nu} = 0 \quad \partial_0 T^{00} = -\partial_x T^{x0} \quad \partial_0 T^{0x} = -\partial_x T^{xx}$$

Stationary heat flow $\Longrightarrow$ Constant pressure

$$\partial_0 T^{0x} = 0 \Longrightarrow \partial_x T^{xx} = 0$$

In a CFT

$$P = u/d \Longrightarrow \partial_x u = 0$$

No energy/temperature gradient

Stationary homogeneous solutions

without local equilibrium

AdS/CFT

Pure Einstein Gravity $S = \int d^3x\sqrt{-g}(R - 2\Lambda)$

Unique homogeneous solution in AdS_3 is boosted BTZ

$$ds^2 = \frac{r^2}{L^2}\left[-\left(1 - \frac{r_0^2}{r^2}\cosh^2\eta\right)dt^2 + \frac{r_0^2}{r^2}\sinh(2\eta)dtdx + \frac{L^4}{(1-\frac{r_0^2}{r^2})}dr^2 + \left(1 - \frac{r_0^2}{r^2}\sinh^2\eta\right)dx^2\right]$$

(neglect rotating black hole solutions)

$r_0 = 2\pi T$ ***unboosted*** **temp** L **AdS radius** η **boost param**

$$\boxed{\langle T_{ij}\rangle = \frac{L^3}{8\pi G_N}g_{ij}^{(0)}} \qquad \langle T_{tx}\rangle = \frac{L^3}{8\pi G_N}\frac{r_0^2}{2L^2}\sinh(2\eta) = \frac{\pi L}{4G_N}T^2\sinh(2\eta)$$

$$c = 3L/2G_N$$

$$\langle T_{tx}\rangle = \frac{\pi c}{6}T^2\sinh(2\eta) = \frac{\pi c}{12}(T^2e^{2\eta} - T^2e^{-2\eta})$$

$T_L = Te^{\eta}$ $T_R = Te^{-\eta}$ $\langle T_{tx}\rangle = \frac{c\pi^2k_B^2}{6h}(T_L^2 - T_R^2)$

Work In Progress & Future Directions

Firmly establish 1D result for average energy flow

Uniqueness in AdS_3 and identification of T_L and T_R

Energy current fluctuations

Exact results in $1 + 1$ suggests simplifications in AdS_3

Higher dimensions

Conjectures for average heat flow and fluctuations

Absence of left-right factorization at level of CFT

Free theories   Poisson process

Generalizations

Other types of charge noise   Non-Lorentz invariant situations

Different central charges   Fluctuation theorems   Numerical GR