From Critical Phenomena to Holographic Duality in Quantum Matter

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2013 Arnold Sommerfeld School

"Gauge-Gravity Duality and Condensed Matter Physics"

Arnold Sommerfeld Center for Theoretical Physics, Munich

5 - 9 August

Lecture 1

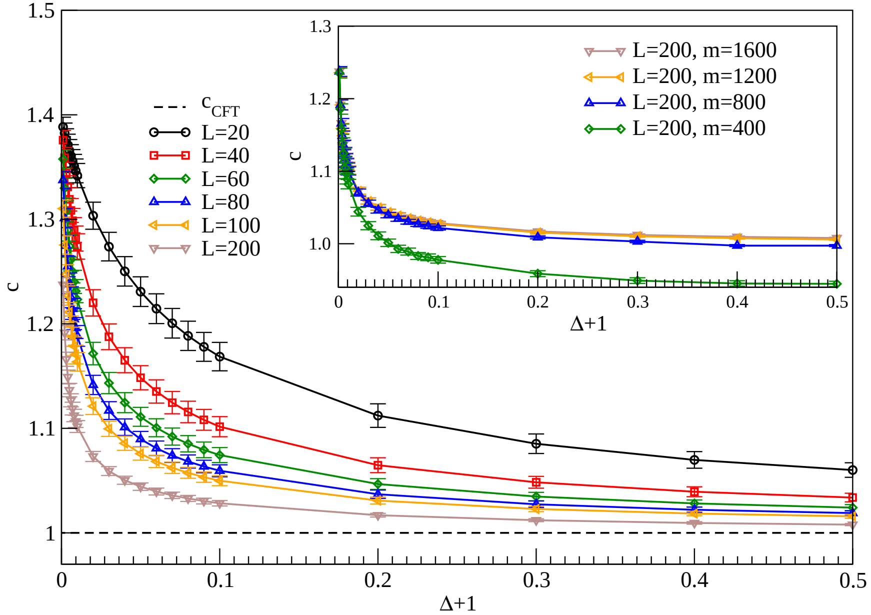
“Introduction to Relativistic QFT & CFT in Condensed Matter”

Accompanying Slides

Central Charge in Gapless Regime XXZ

Chen *et al*, *Entanglement entropy scaling of the XXZ chain*,

`arXiv:1306.5828`



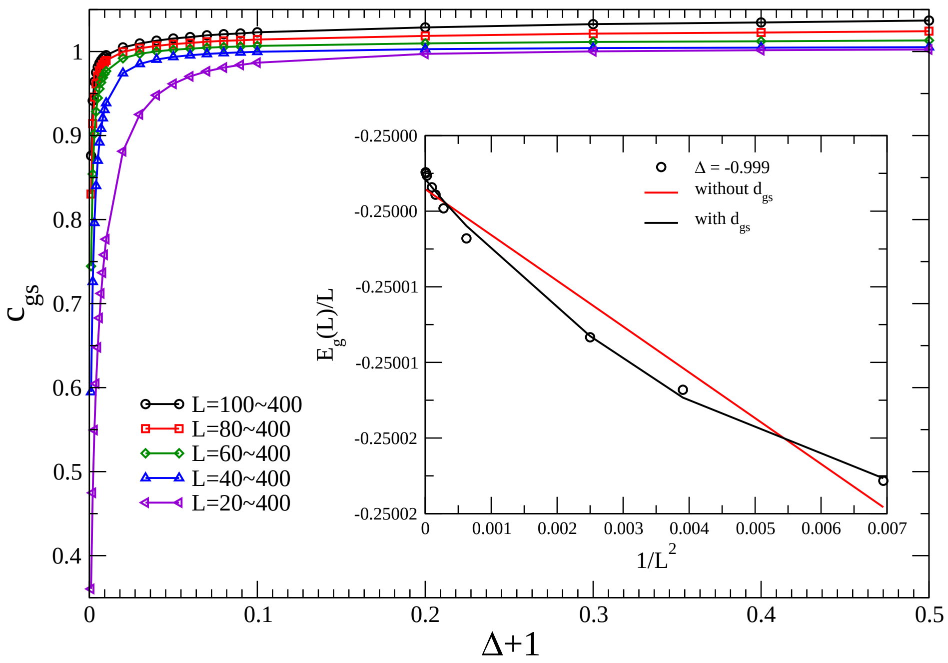
Central charge of XXZ obtained from entanglement entropy in gapless regime $-1 \leq \Delta \leq -0.5$. Results show that $c = 1$ for $\Delta \in (-1, -0.5]$. FM point $\Delta = -1$ needs special treatment.

Entanglement entropy useful for characterising phases <u>and</u> identifying phase transitions

Multiscale Entanglement Renormalization Ansatz (MERA) & AdS/CFT

Central Charge in Gapless Regime XXZ

Chen *et al*, *Entanglement entropy scaling of the XXZ chain*, `arXiv:1306.5828`



Central charge of XXZ obtained from ground state energy in gapless regime $-1 \leq \Delta \leq -0.5$. Results show that $c = 1$ for $\Delta \in (-1, -0.5]$. FM point $\Delta = -1$ needs special treatment.

Quasi-1D Quantum Antiferromagnets

$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i . \mathbf{S}_j$$

Tennant *et al*, PRB **52**, 13368 (1995)

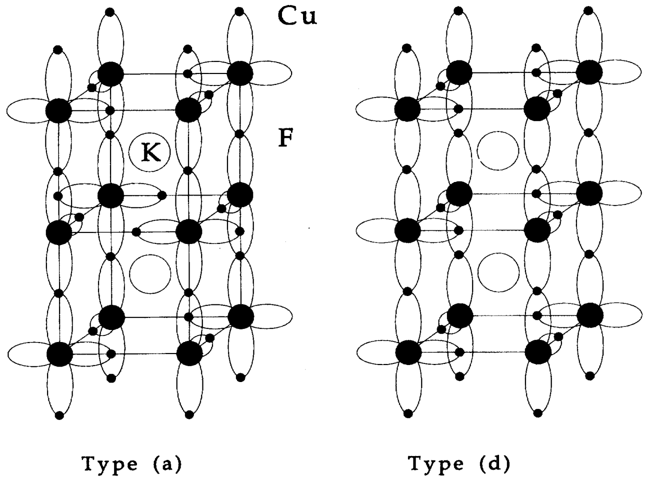


FIG. 2. The crystal structure of $KCuF_3$. The two polytype structures (a) and (d) are shown. These are distinguished by the different displacements of fluorine ions in the *a*-*b* plane.

$KCuF_3$ $\quad S = 1/2 \quad J \simeq 34\,\mathrm{meV} \quad J_\perp \simeq -1.6\,\mathrm{meV}$

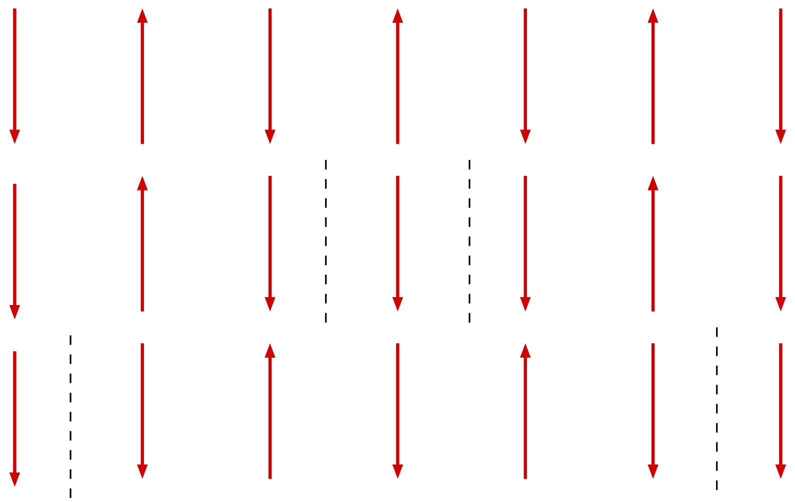
Spinons

Novel disordered ground state

$S_i^+ S_j^-$ causes spin flips

Excitations are **spinons** *not conventional spin waves*

Cartoon



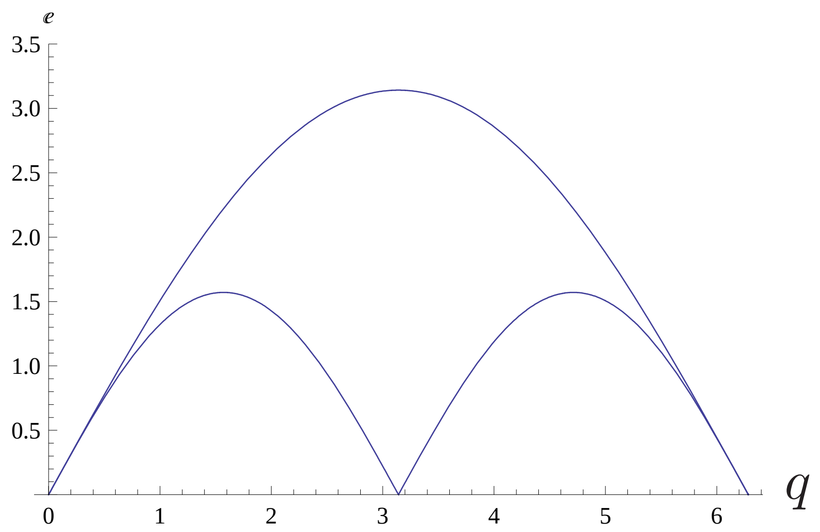
Excitations are produced in **pairs**

Mobile domain walls

Consequence

Excitations are produced in pairs

Neutron scattering ought to see a continuum of states



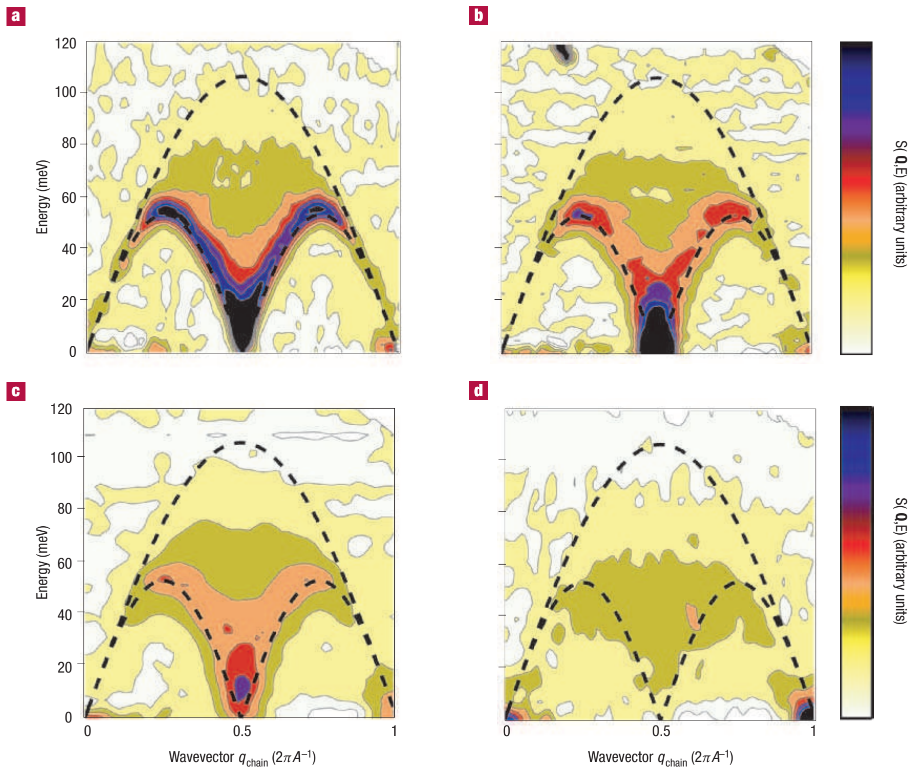
des Cloizeaux & Pearson, Phys. Rev. **128**, 2131 (1962)

$$E_{\rm L} = \frac{\pi J}{2}|\sin(q)| \qquad E_{\rm U} = \pi J|\sin(q/2)|$$

Gapless Critical Theory

Neutrons $S(q,\omega)$

Lake *et al*, *Quantum criticality and universal scaling of a quantum antiferromagnet*, Nature Materials **4**, 329 (2005)

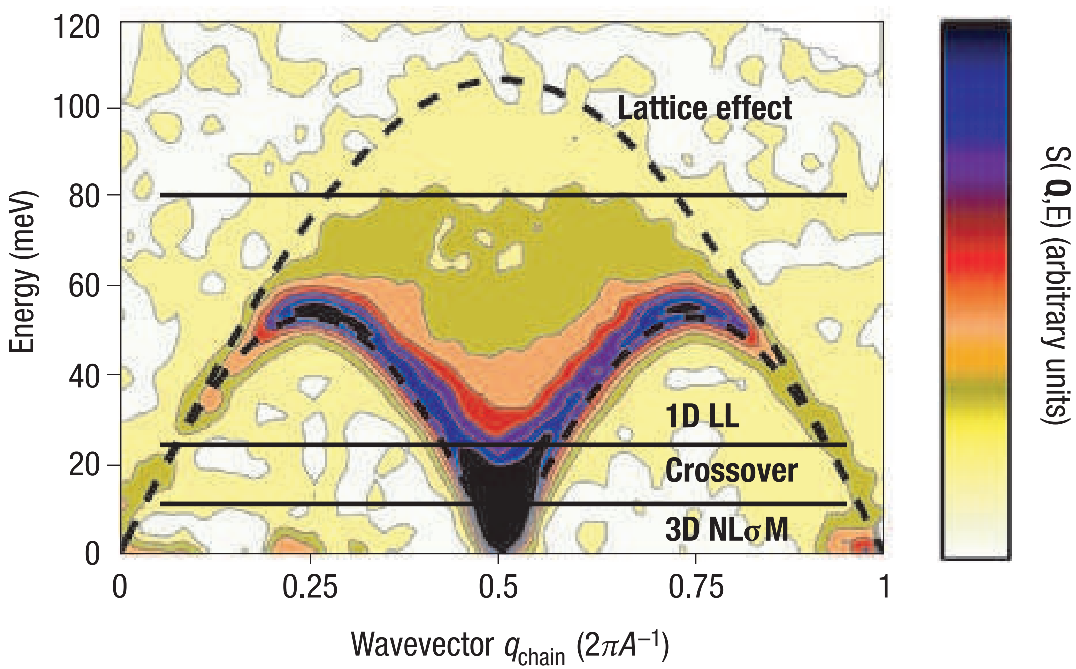


(a) 6 K (b) 50 K (c) 150 K (d) 300 K

T_N=39 K

Dimensional Crossover

Lake *et al*, *Quantum criticality and universal scaling of a quantum antiferromagnet*, Nature Materials **4**, 329 (2005)

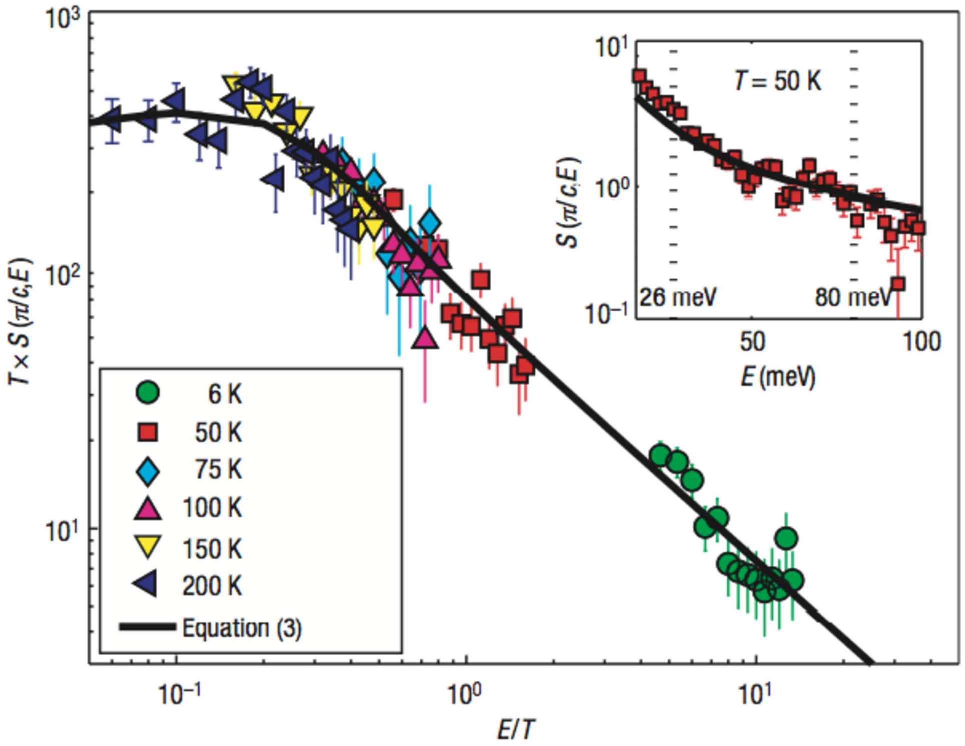


T= 6 K T_N=39 K

1D behavior observed in anisotropic materials in appropriate energy and temperature windows

Universal Scaling

Lake *et al*, *Quantum criticality and universal scaling of a quantum antiferromagnet*, Nature Materials **4**, 329 (2005)



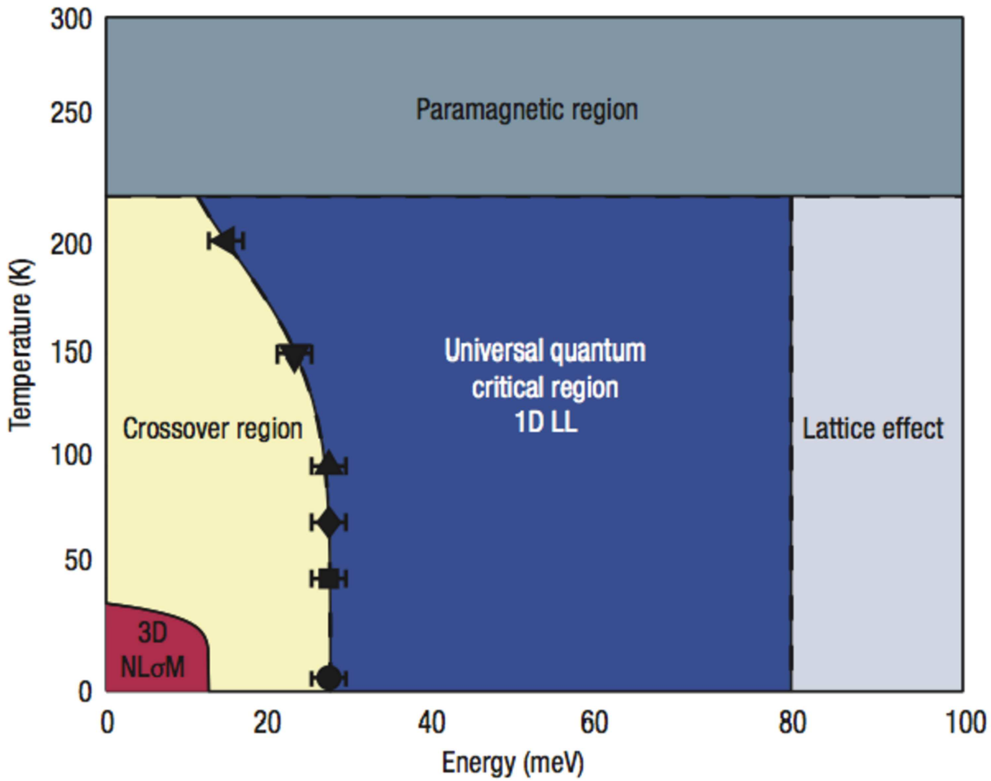
$$S(\pi, E) = \frac{e^{E/kT}}{e^{E/kT}-1}\,\frac{A}{T}\,\mathrm{Im}\left[\rho\left(\frac{E}{4\pi T}\right)^2\right]$$

$$\rho(x) = \frac{\Gamma(\frac{1}{4}-ix)}{\Gamma(\frac{3}{4}-ix)}$$

Critical scaling in quantitative agreement with theory

Regimes

Lake *et al*, *Quantum criticality and universal scaling of a quantum antiferromagnet*, Nature Materials **4**, 329 (2005)



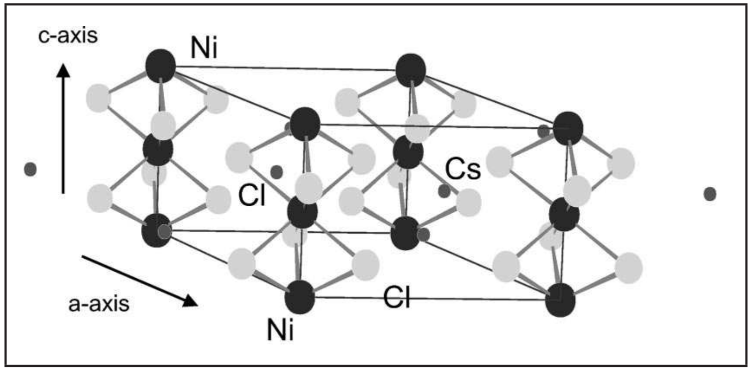
1D critical behavior observed in anisotropic materials in appropriate energy and temperature windows

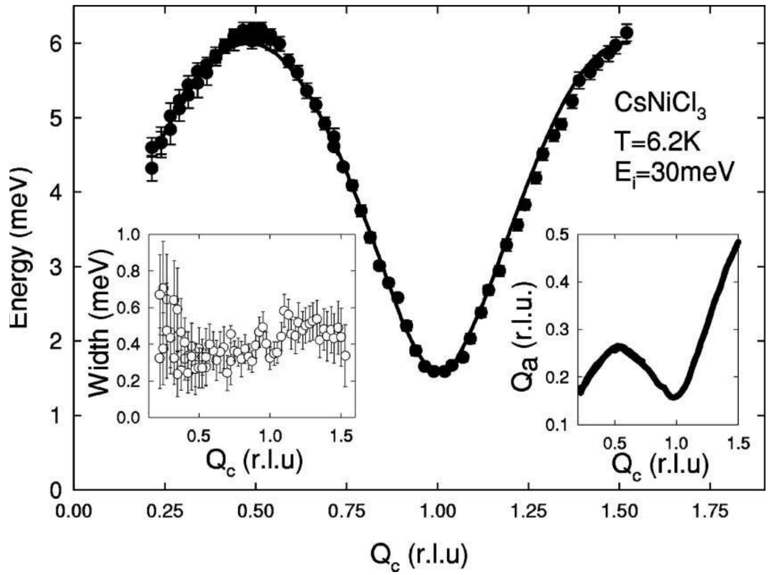
c.f. avoided quantum criticality

Integer v Half Integer Spin Chains

$CsNiCl_3$ $\quad S = 1 \quad J \simeq 2.28\text{meV} \quad J_\perp \simeq 0.044\,\text{meV}$

Kenzelmann *et al*, PRB **66**, 024407 (2002)





Haldane Gap

Ising Model in a Magnetic Field

2D classical Ising Model solved exactly by Onsager

Fermions

In a **magnetic field** remains **unsolved** on the lattice

However, close to T_c, continuum perturbed CFT

A. B. Zamolodchikov, Int. J. Mod. Phys. A **4**, 4235 (1989)

$$\mathcal{A} = \mathcal{A}_{\mathbb{Z}_2} + h \int d^2x \, \sigma(\mathbf{x})$$

No longer critical $\sim$ massive

E_8 Mass Spectrum

248 generators!

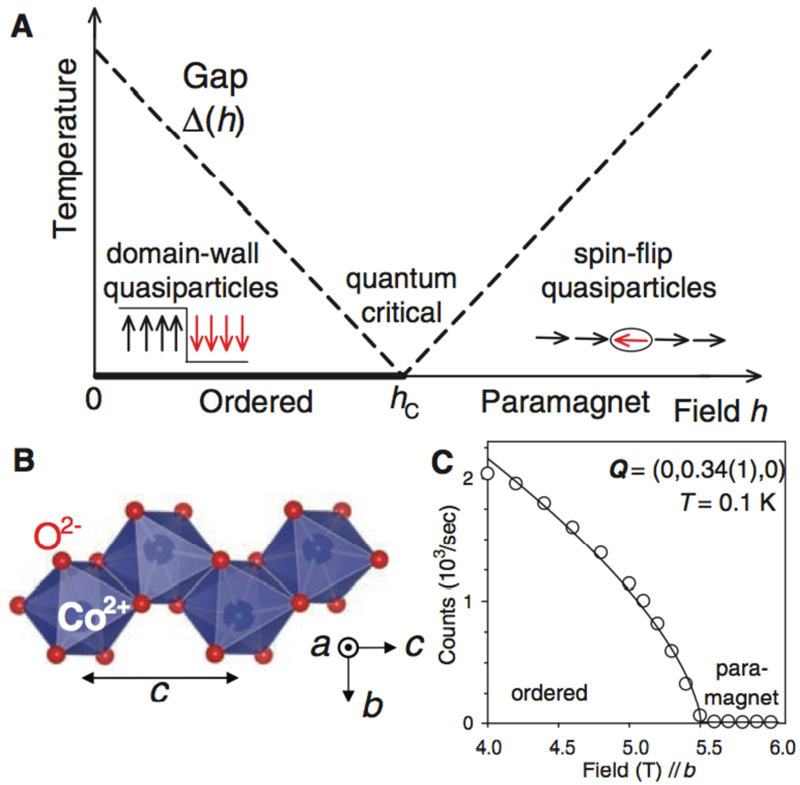
$$m_1 = m \quad m_2 = 2m \cos \pi/5 \quad m_3 = 2m \cos \pi/30 \quad \ldots$$

Analogue of quark confinement

Experiments on $CoNb_2O_6$ by Coldea *et al*

Quantum Criticality in an Ising Chain: Experimental Evidence for Emergent E_8 Symmetry, Science **327**, 177 (2010)

$$H = -J \sum_i S_i^z S_{i+1}^z - h_x \sum_i S_i^x - h_z \sum_i S_i^z$$



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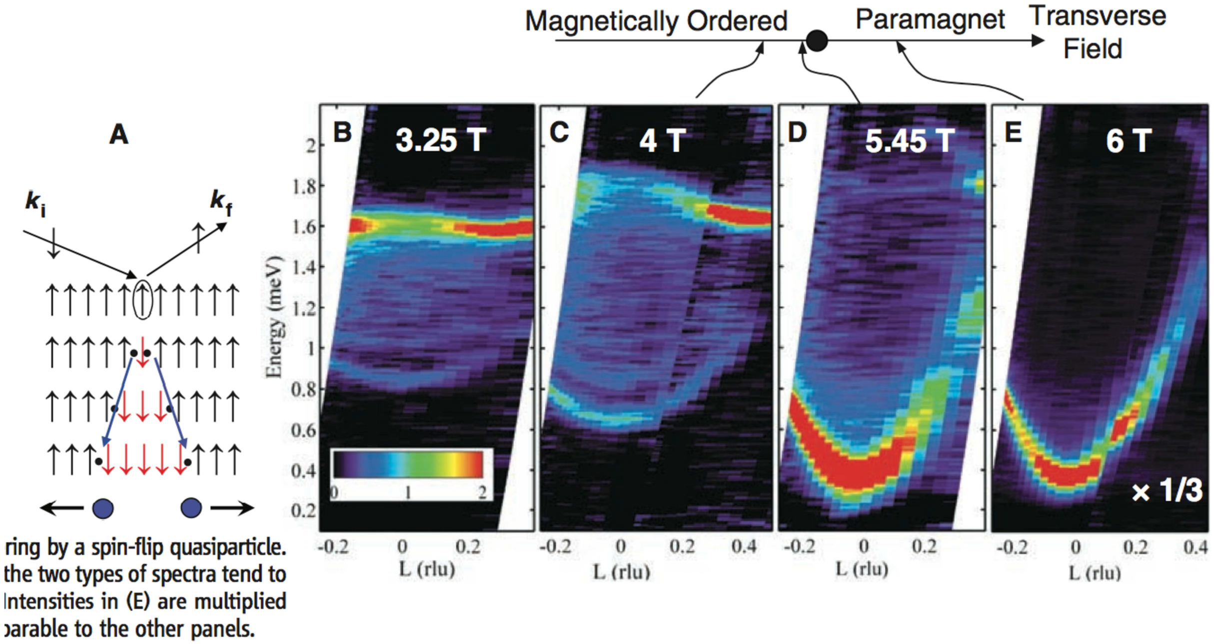

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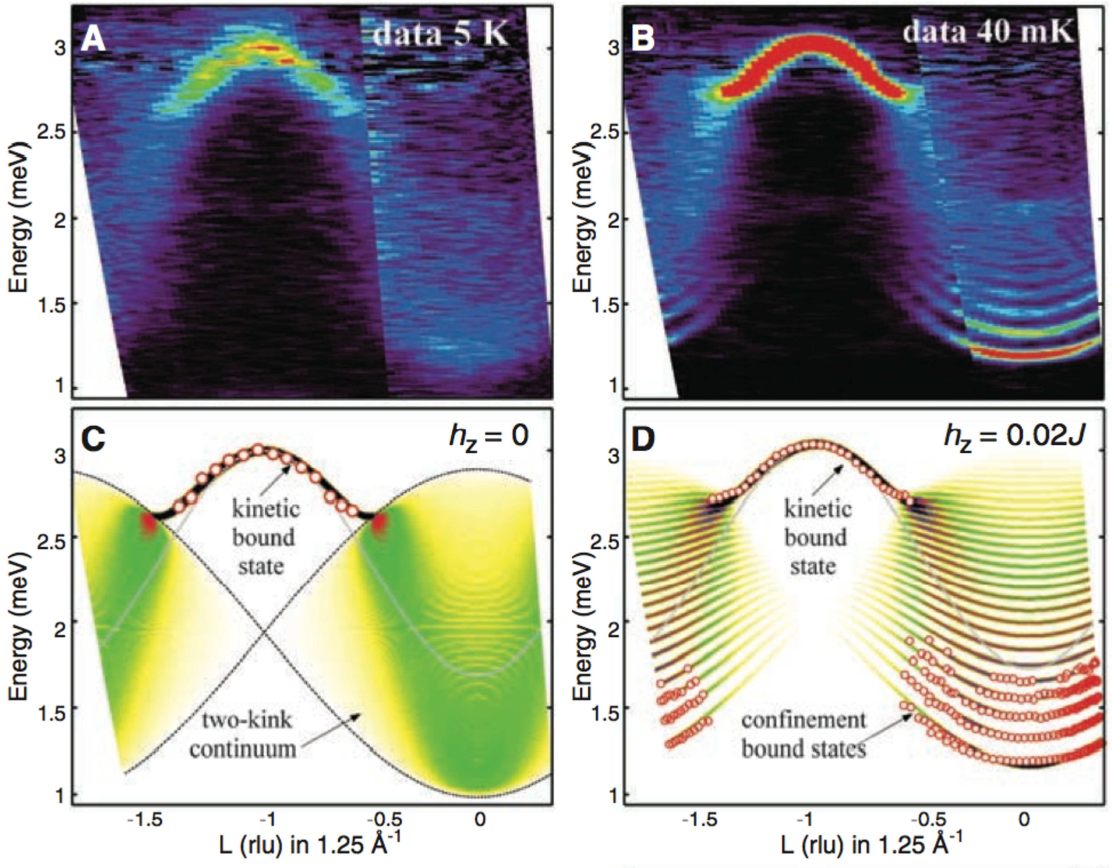

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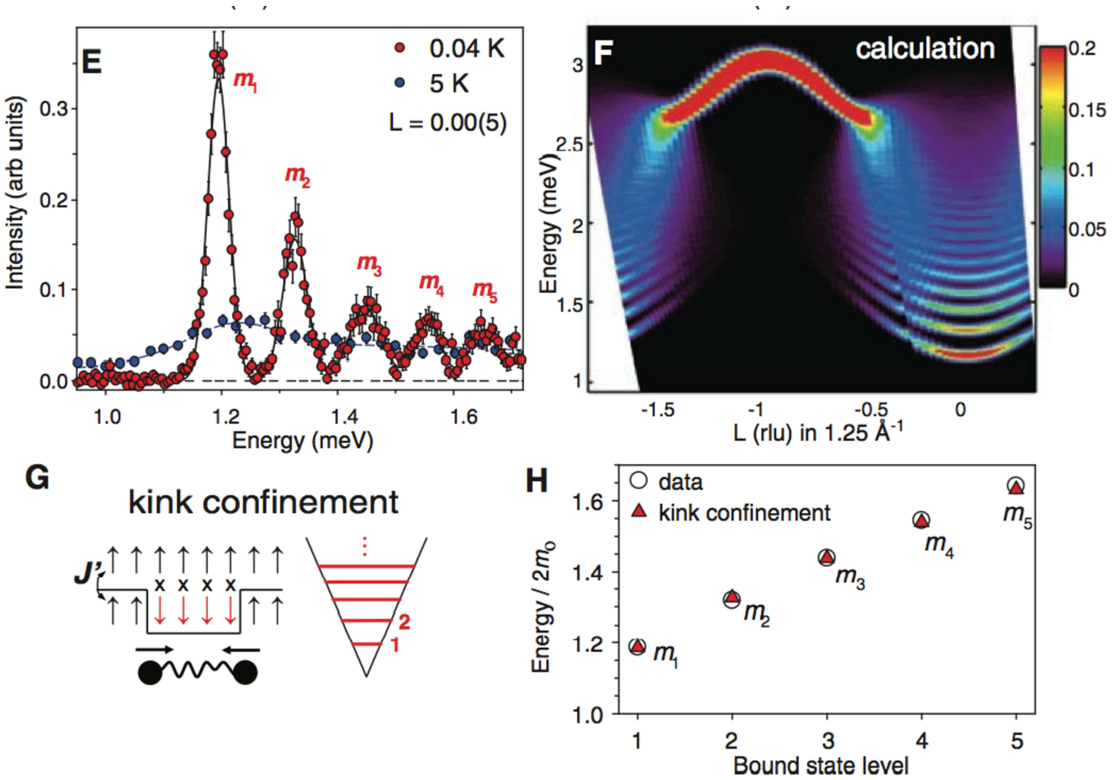

Linear Confinement

Experiments on $CoNb_2O_6$ by Coldea *et al*

Quantum Criticality in an Ising Chain: Experimental Evidence for Emergent E_8 Symmetry, Science **327**, 177 (2010)

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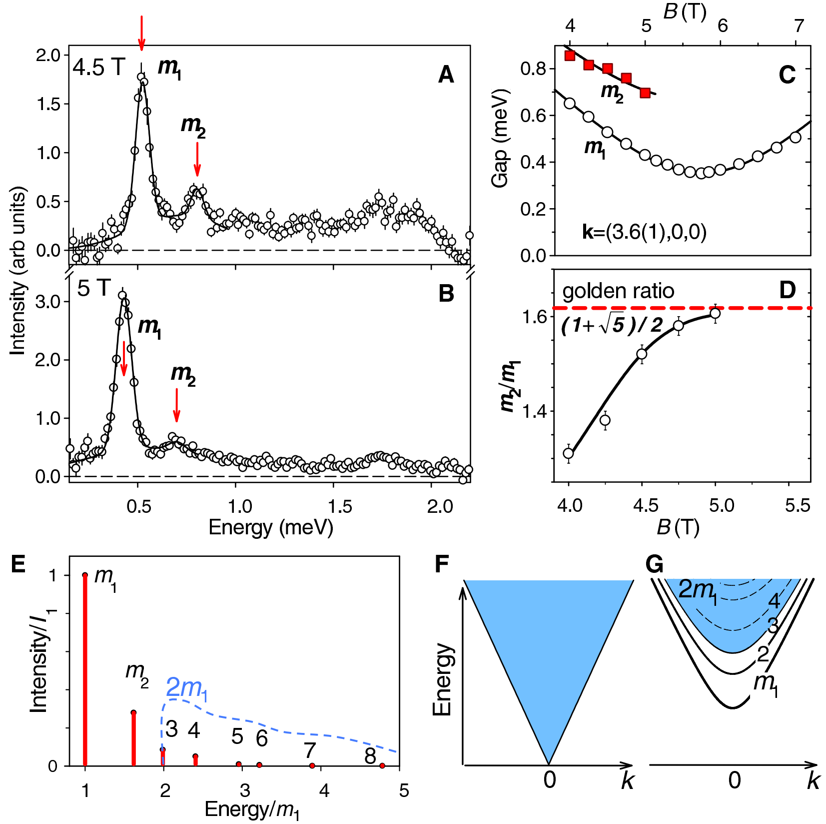
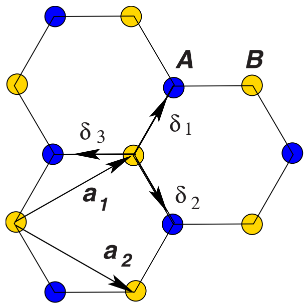
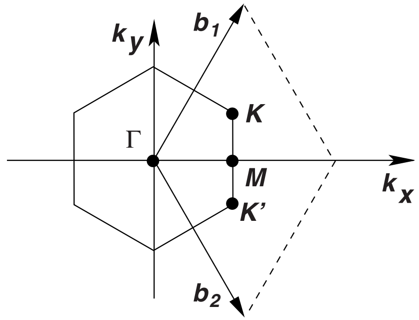
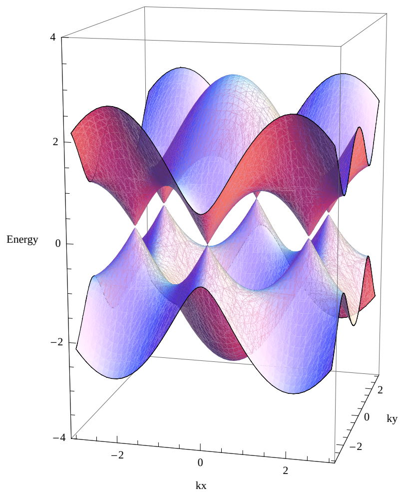


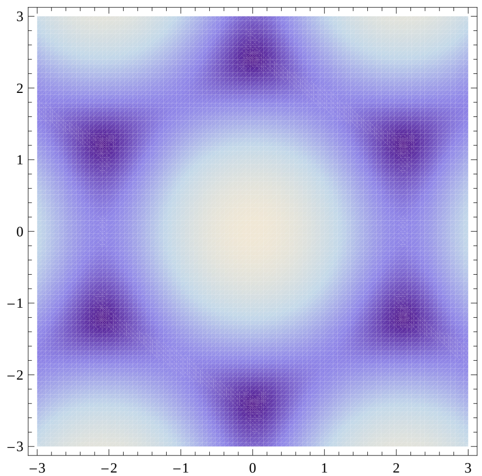
Graphene

Castro Neto *et al*, *"The electronic properties of graphene"*, RMP (2009)









Emergent Lorentz invariance with effective speed of light

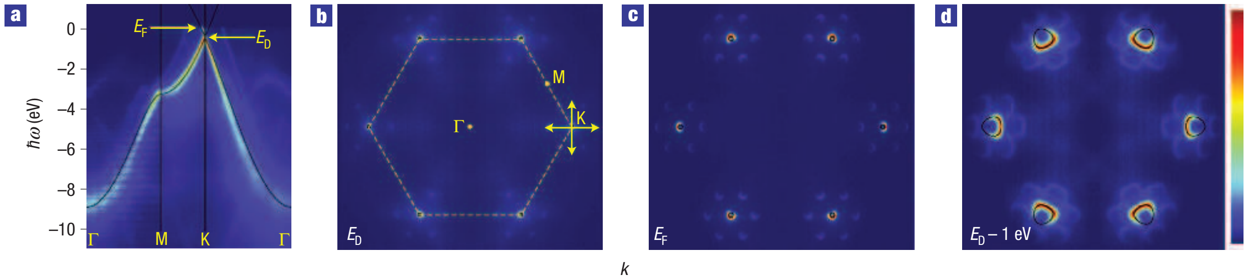
$$c \approx 1 \times 10^6 \mathrm{ms}^{-1}$$

ARPES

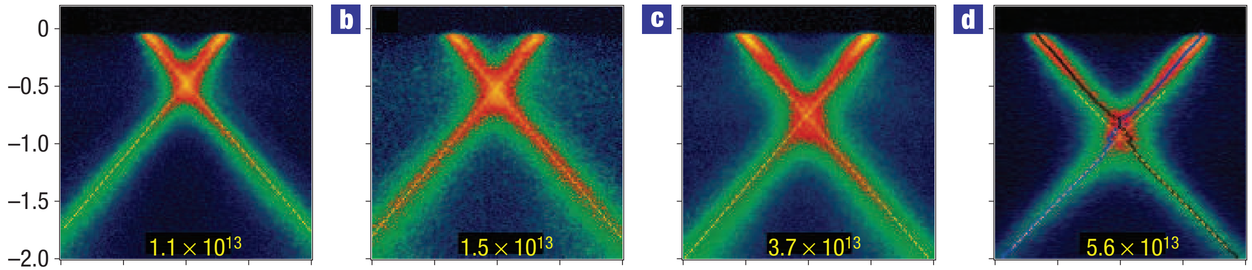
Angle Resolved Photo Emission

Bostwick *et al*, Nature Physics **3**, 36 (2007)

Solid line fit to single particle dispersion ($k_x \leftrightarrow k_y$)



E v k along a line through K-point parallel to ΓM

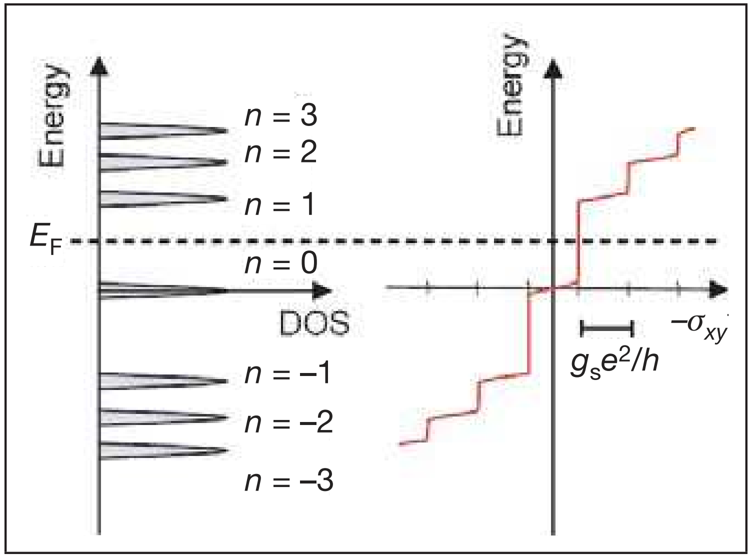


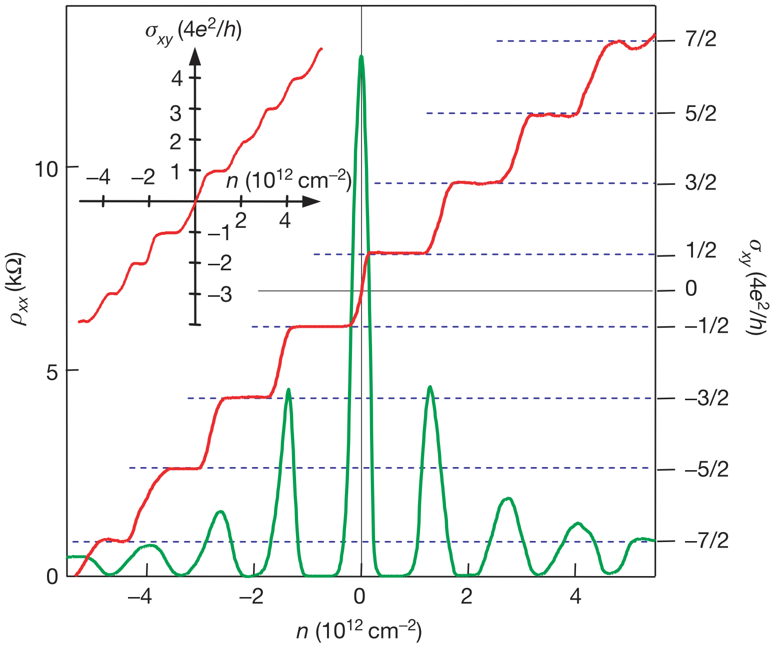
Increasing electron density (per cm^2)

Relativistic QHE in Graphene

Novoselov *et al*, *Two–Dimensional gas of massless Dirac fermions in graphene*, Nature **438**, 197 (2005)

Zhang *et al*, *Experimental observation of the quantum Hall effect and Berry's phase in graphene*, Nature **438**, 201 (2005)





Disorder   Plateau Transitions   SUSY NLσM

Summary

Relativistic QFT plays a useful role in describing quantum systems

Strong links between high energy & condensed matter

Lorentz invariance, Dirac fermions, spin chains, Graphene

CFT & central charge, entanglement & numerics

Emergent excitations, fractionalization & confinement, topology

Evidence for scaling close to a quantum critical point

Perturbed CFTs & novel symmetries

Theory and Experiment

Ideas & themes recur in condensed matter physics

Can we describe emergent phenomena in higher D?

Can we use gauge-gravity duality to treat them?

Universality