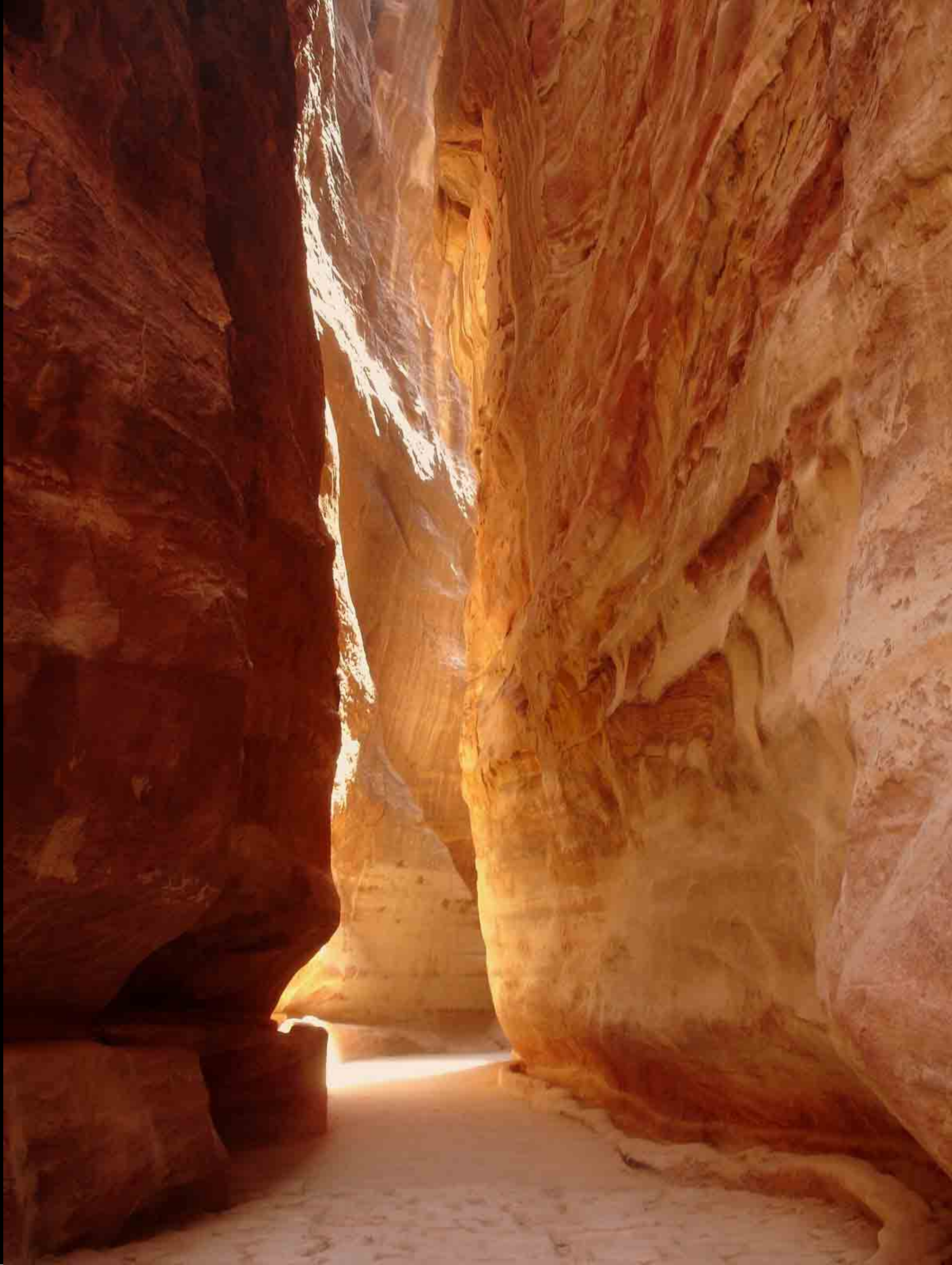
Experiments with Molecular Quantum Gases in Optical Lattices

Gerhard Rempe
MPI of Quantum Optics
Garching



quantum-gas research connects
almost all fields of physics
(AtMolOpt, CondMat, Nucl, ...)
but hardly Quantum Optics
the art of making measurements
and controlling fluctuations,
as introduced by Roy Glauber

quantum light and quantum matter

1960s: laser
1st-order coherence:
→ phase-stable light

1980s: photons
2nd-order coherence:
→ correlated photons

light:
open system
(photons leave)

1990s: BEC
1st-order coherence:
→ phase-stable matter

2000s: atoms
2nd-order coherence:
→ correlated atoms

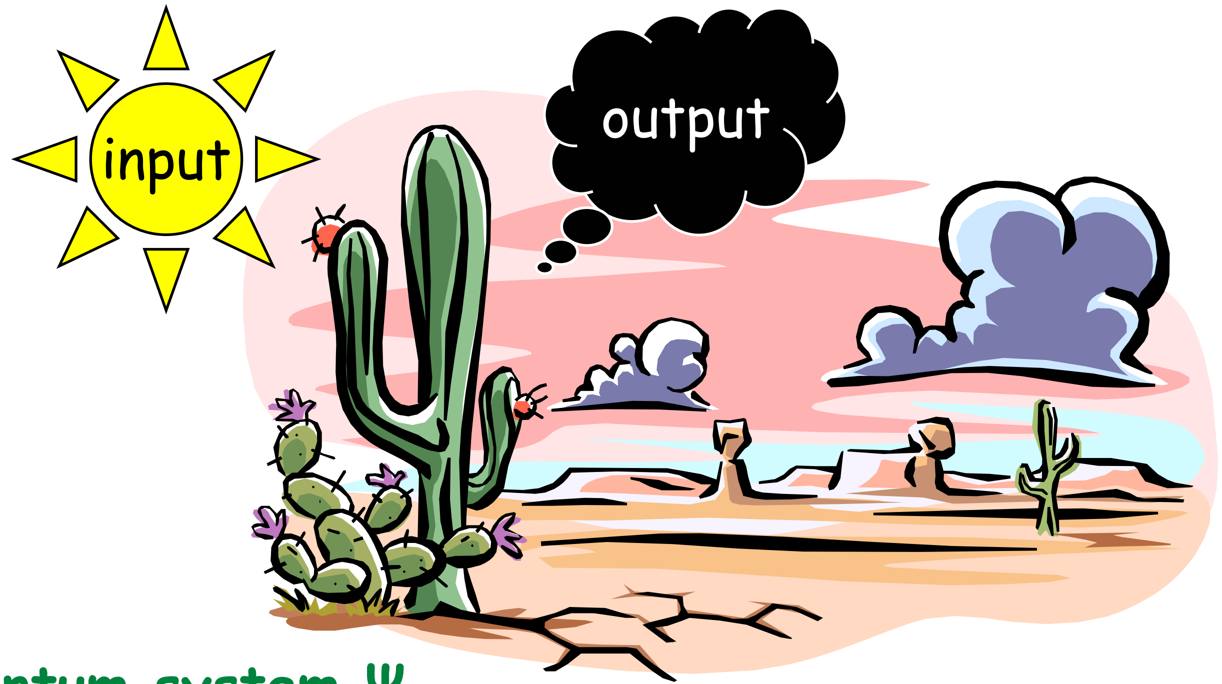
matter:
closed system
(atoms stay)

what happens in dissipative matter systems ?

light systems versus matter systems

solid state physics:
- system structure
- Hamiltonian description
- dissipation = perturbation

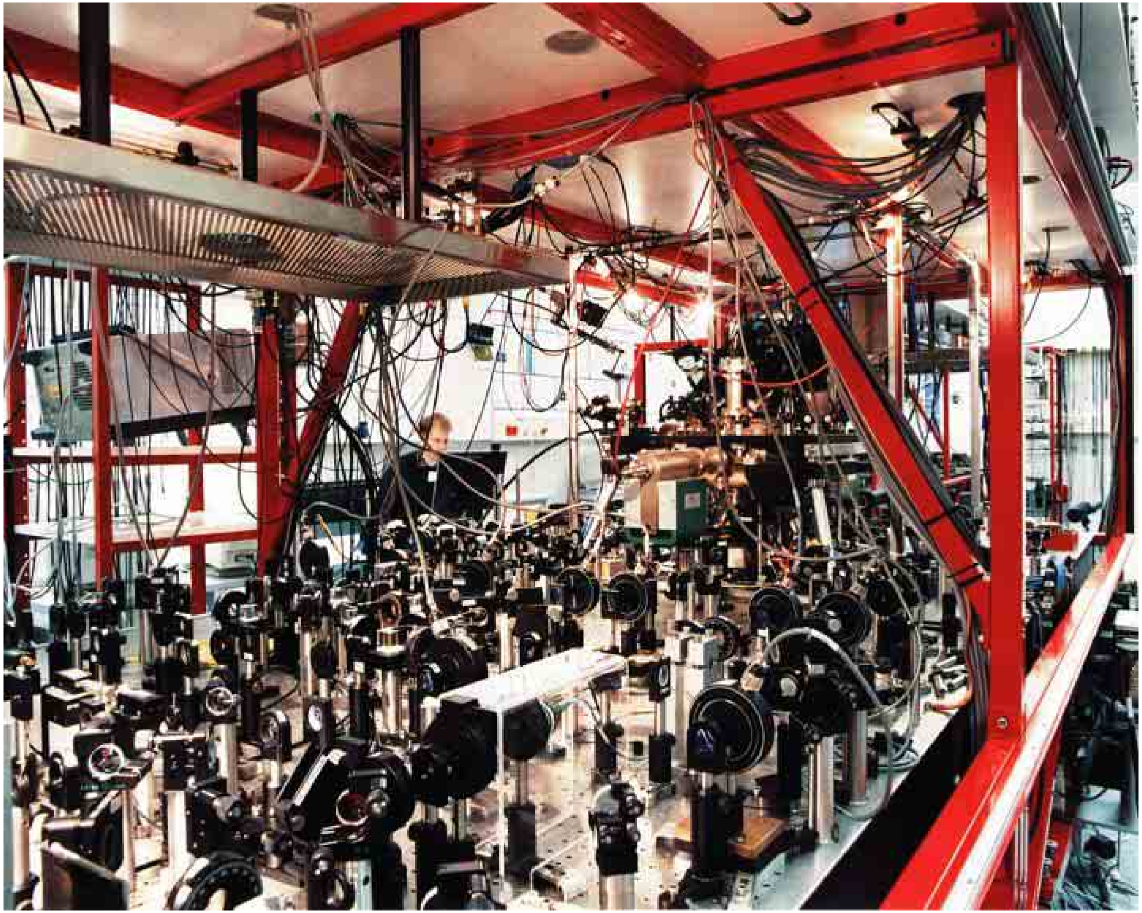
quantum optics:
- system dynamics
- master equation
- measurement = information



quantum system Ψ

outline

1) scattering theory
2) ultracold molecules
3) strong correlations
4) Tonks-Girardeau gas
5) quantum Zeno effect
6) optical control of particle interactions



elements of scattering theory

box potential:

$$V(r) = \begin{cases} -V_0 & : \quad r < R_0 \\ 0 & : \quad r \geq R_0 \end{cases}$$

Schrödinger equation:

$$\left(-\frac{\hbar^2}{2m_r}\vec{\nabla}^2 + V(\vec{r})\right)\Psi_{\vec{k}}(\vec{r}) = E_k \Psi_{\vec{k}}(\vec{r})$$

ansatz (spherical symmetry):

$$\Psi_{\vec{k}}(\vec{r}) = \sum_{l=0}^{\infty} P_l(\cos\theta)\frac{u_{k,l}(r)}{r}$$

elements of scattering theory

Schrödinger equation:

$$-\frac{\hbar^2}{2m_r}\frac{d^2}{dr^2}u_{k,l}(r) + \left(V(r) + \frac{l(l+1)\hbar^2}{2m_r r^2} - E_{\vec{k}}\right)u_{k,l}(r) = 0$$

wave function (l=0):

$$u_{k,0}(r) \propto \begin{cases} \sin(k'r) & : \quad r < R_0 \\ C\sin(kr + \delta_0(k)) & : \quad r \geq R_0 \end{cases}$$

$$k' = \sqrt{2m_r(E+V_0)/\hbar^2}$$
$$k = \sqrt{2m_r E/\hbar^2}$$

elements of scattering theory

matching of wave functions at r = R_0:

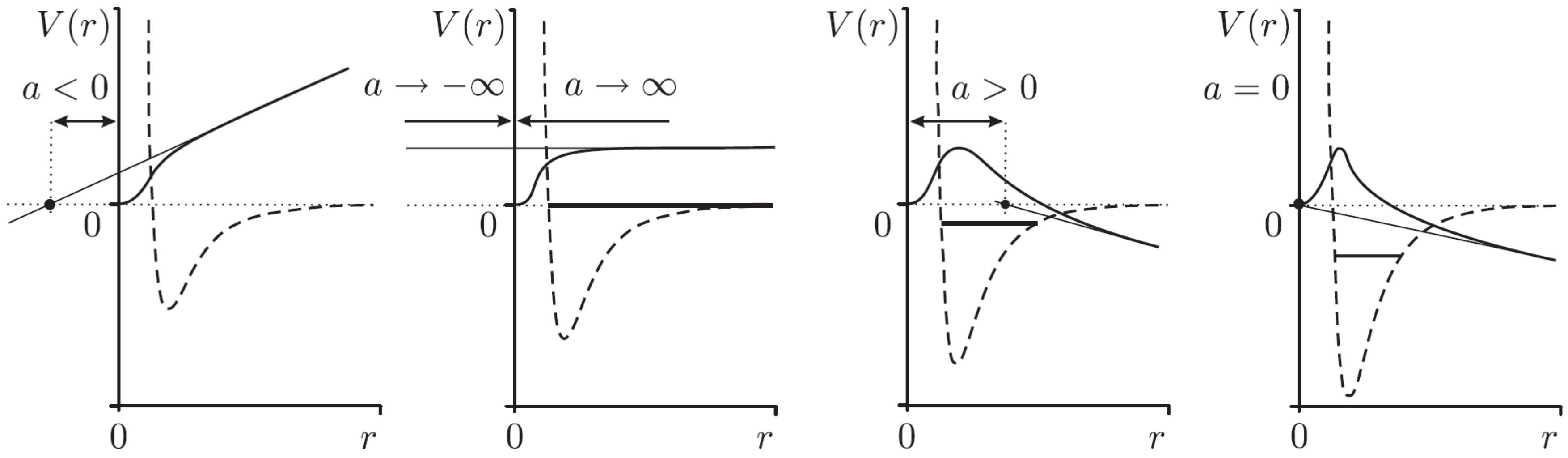
$$u_{k,0}(r) \propto \begin{cases} \sin(k'r) & : \quad r < R_0 \\ C\sin(kr + \delta_0(k)) & : \quad r \geq R_0 \end{cases}$$

$$k' = \sqrt{2m_r(E+V_0)/\hbar^2}$$
$$k = \sqrt{2m_r E/\hbar^2}$$

phase shift:

$$\delta_0(k) = -kR_0 + \arctan\left(\frac{k}{k'}\tan(k'R_0)\right)$$

physical significance of the phase shift



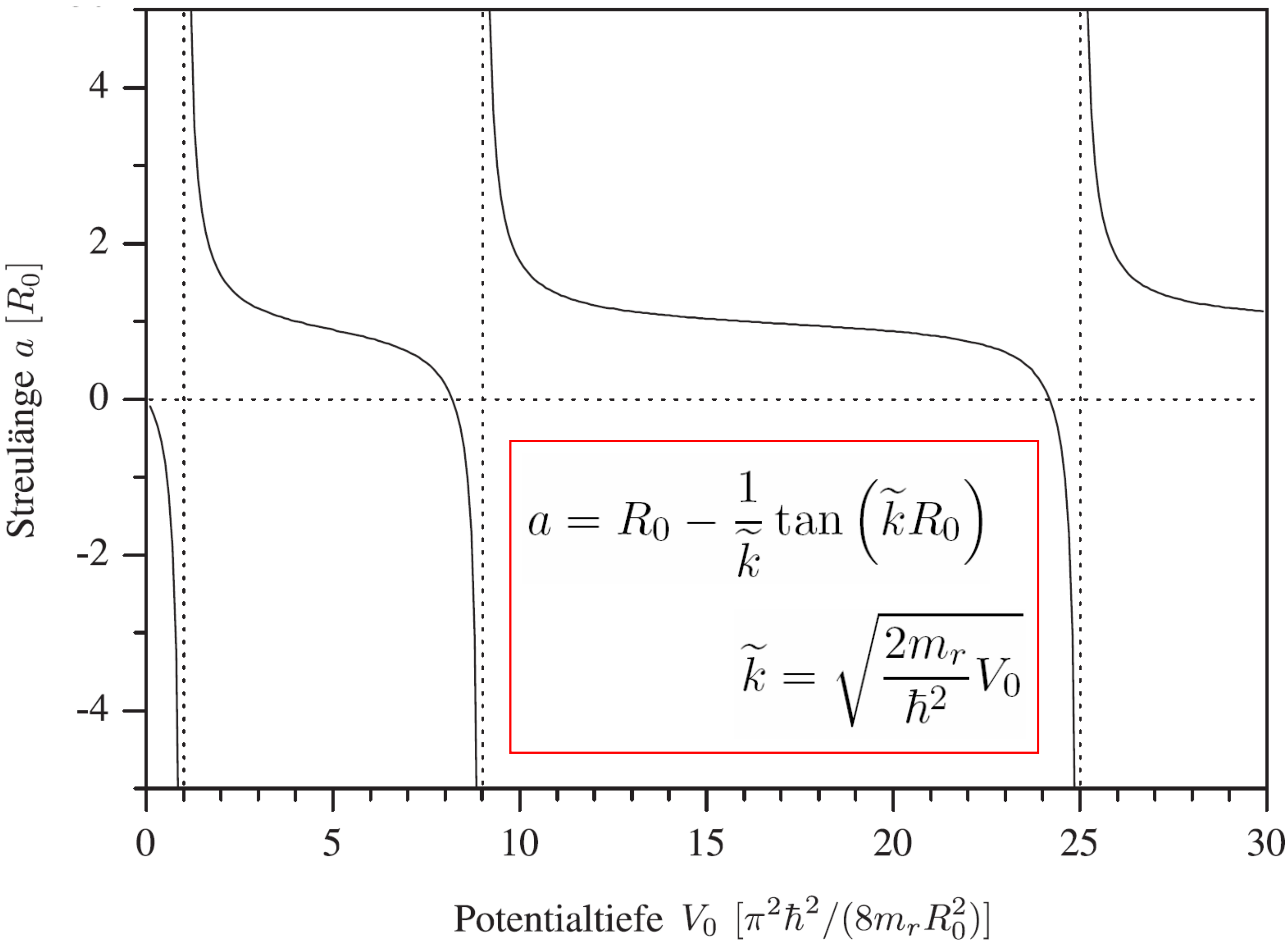
scattering length:

$$a = -\lim_{k \to 0} \frac{\tan \delta_0(k)}{k}$$

cross section:

$$\sigma_s = 8\pi a^2$$

box potential: scattering length

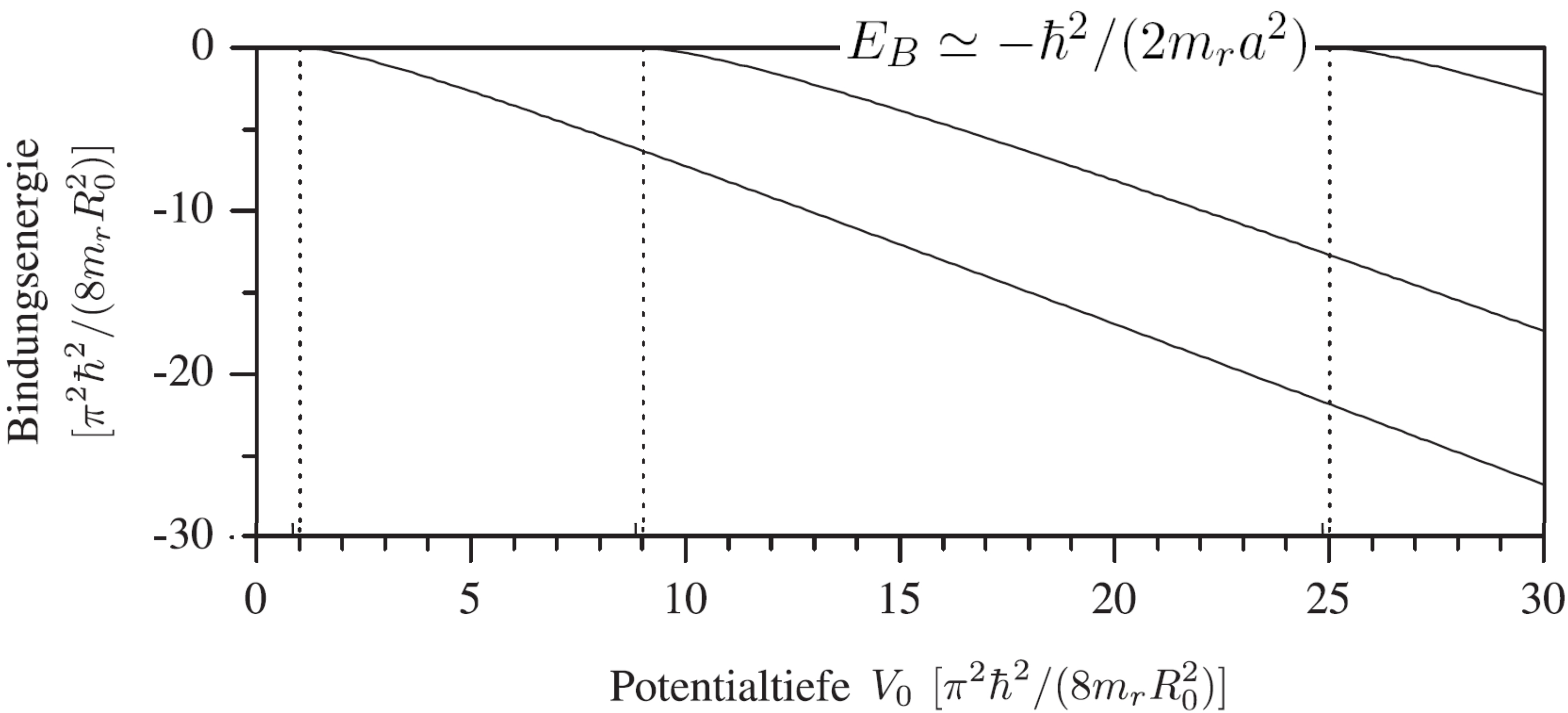


$$a = R_0 - \frac{1}{\widetilde{k}}\tan\left(\widetilde{k}R_0\right)$$

$$\widetilde{k} = \sqrt{\frac{2m_r}{\hbar^2}V_0}$$

box potential: bound states

$$\sqrt{|E_B|} = -\sqrt{V_0 - |E_B|}\ \cot\left(\tilde{k}R_0\sqrt{1-\frac{|E_B|}{V_0}}\right)$$



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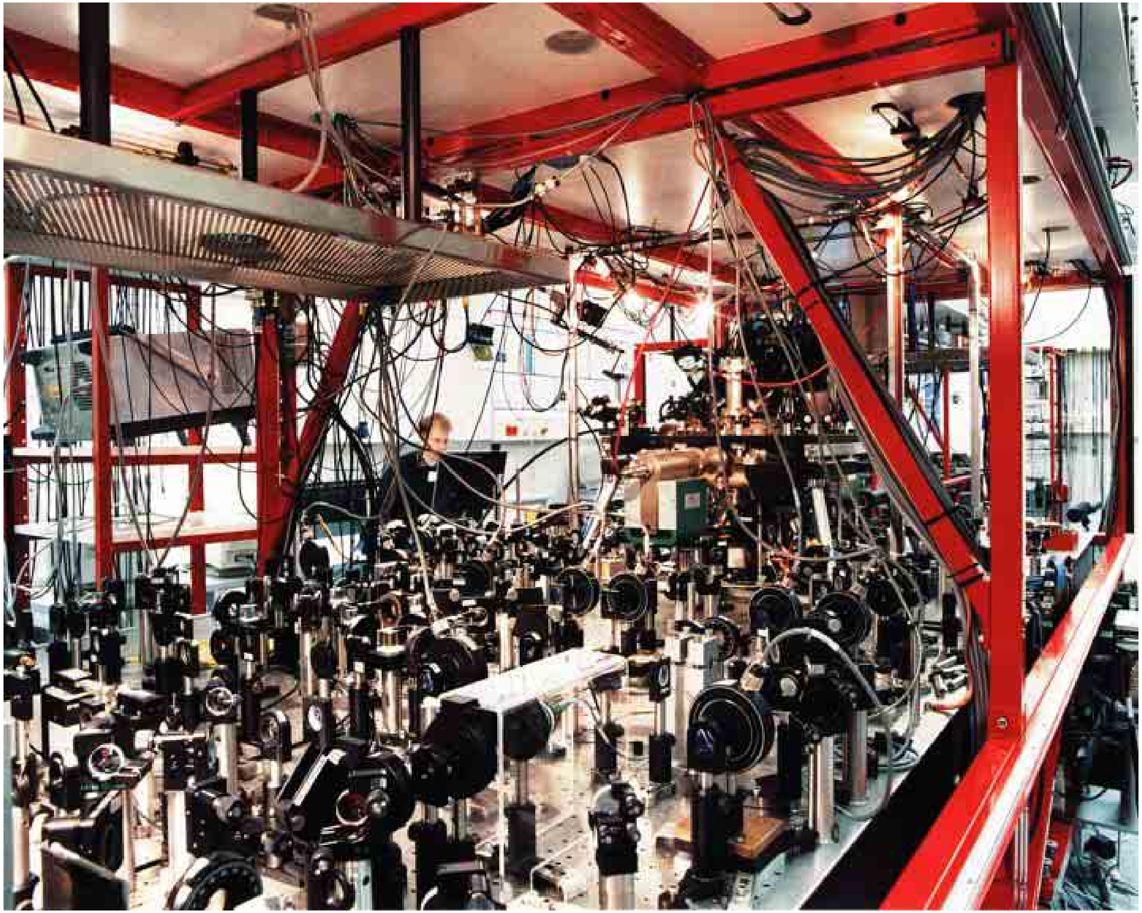
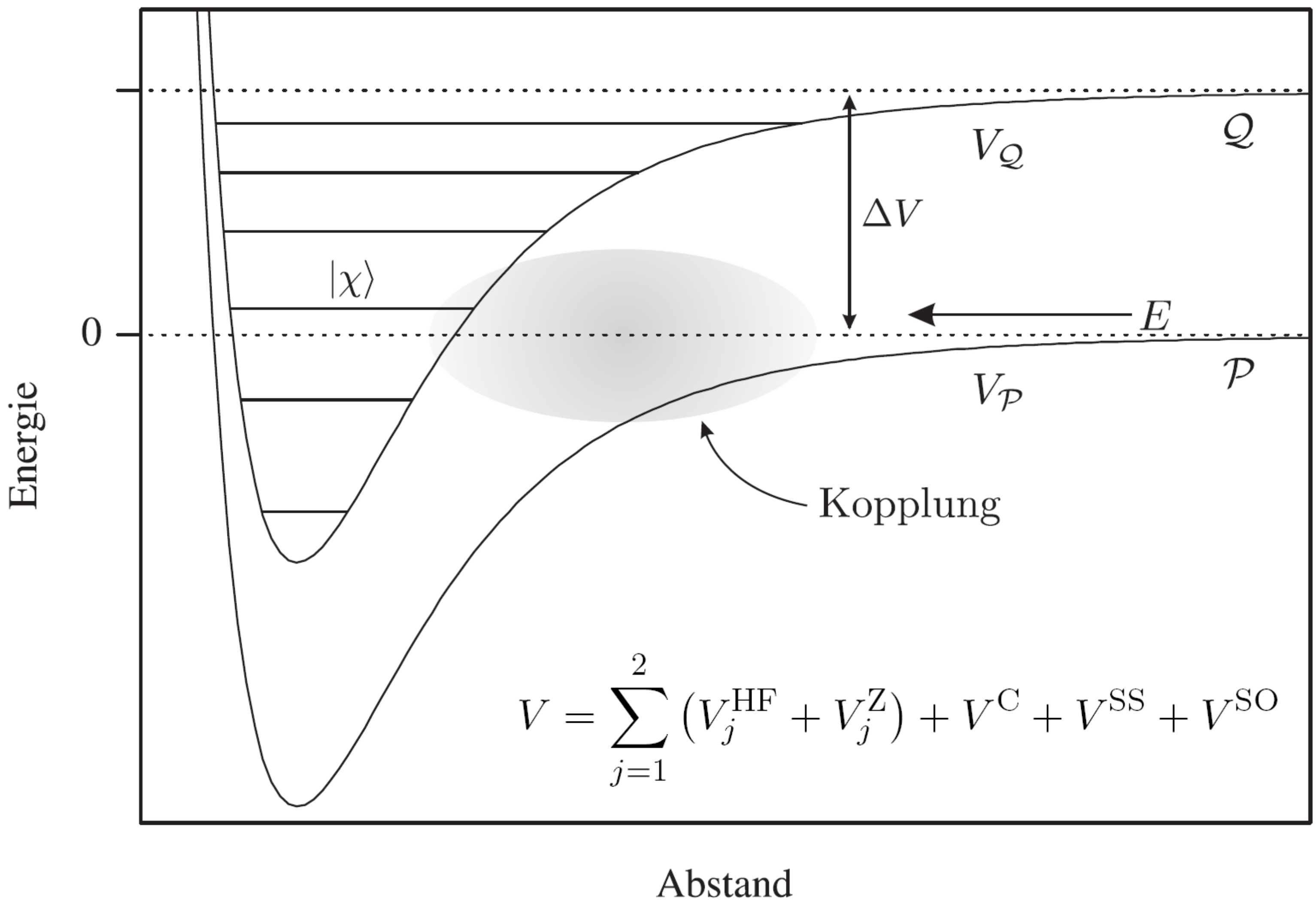
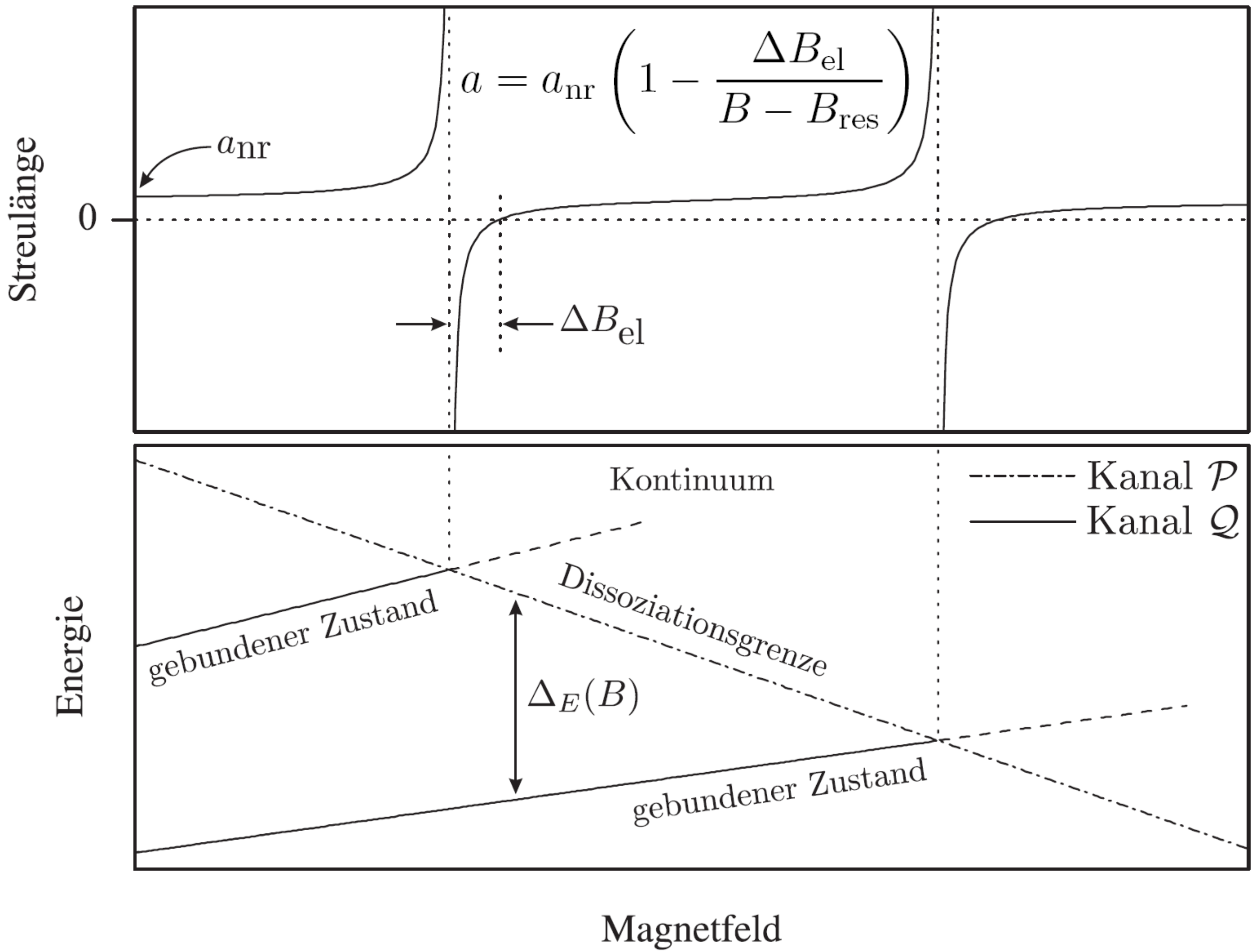


Feshbach resonance: simple picture

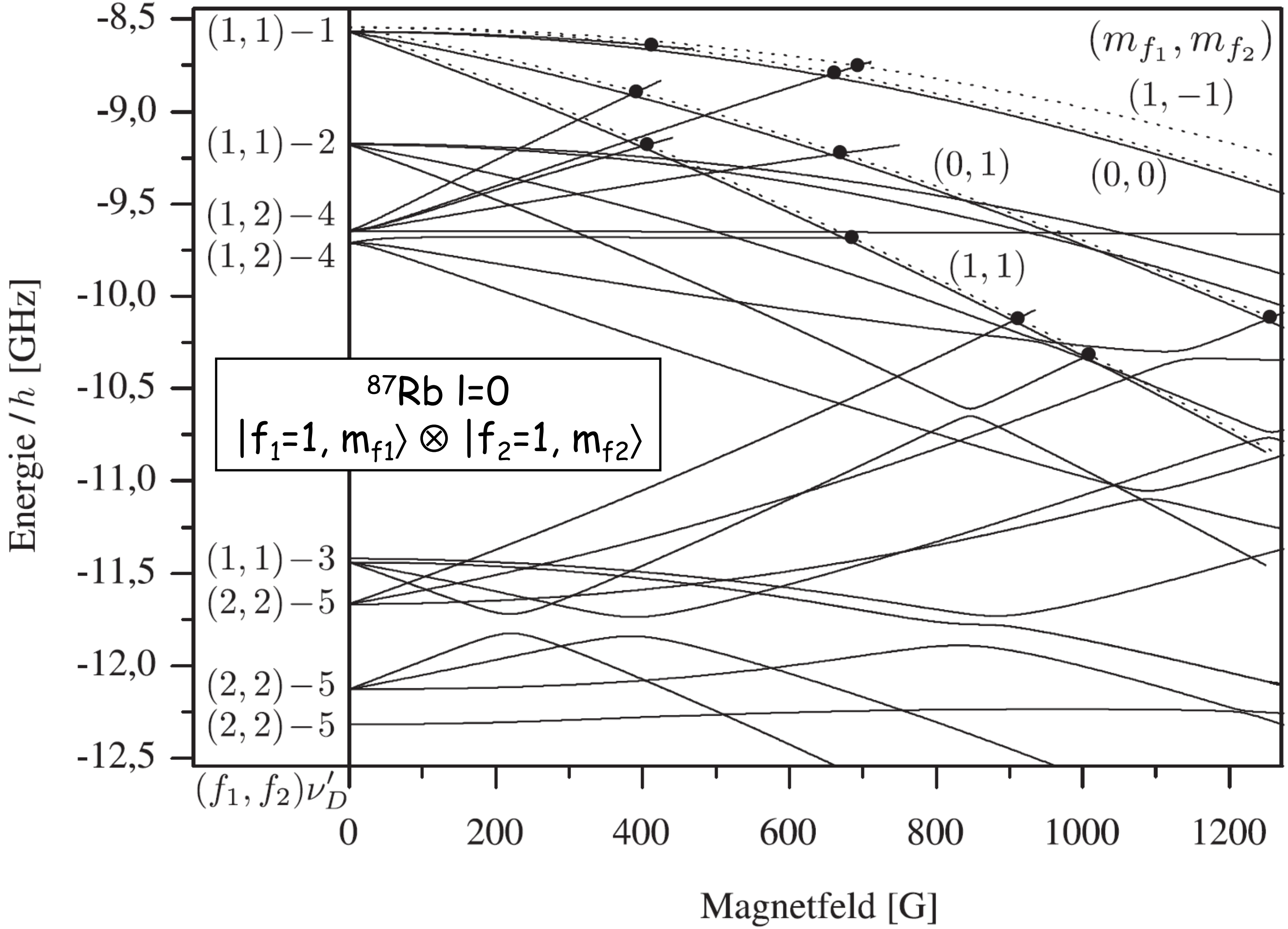


Feshbach resonance: simple picture



Feshbach resonance: real picture

Marte et al., Phys. Rev. Lett. **89**, 283202 (2002)



theoretical prediction for ^{87}Rb

van Kempen et al., Phys. Rev. Lett. **88**, 093201 (2002)

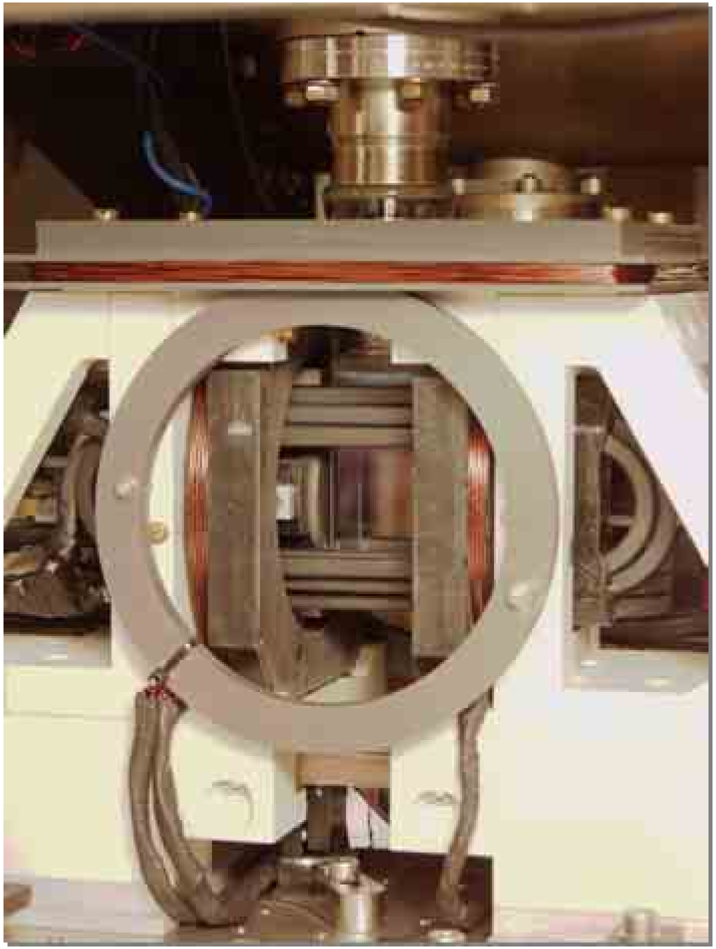
zero-energy resonances
in the atomic ground state $|F=1, m_F=1\rangle$:

TABLE III. Resonance fields B_0 and widths Δ for ^{87}Rb.

B_0 (G)	403(2)	680(2)	899(4)	1004(3)
Δ (mG)	<1	15	<5	216

high-resolution experiments with $\delta B/B \sim 10^{-6}$

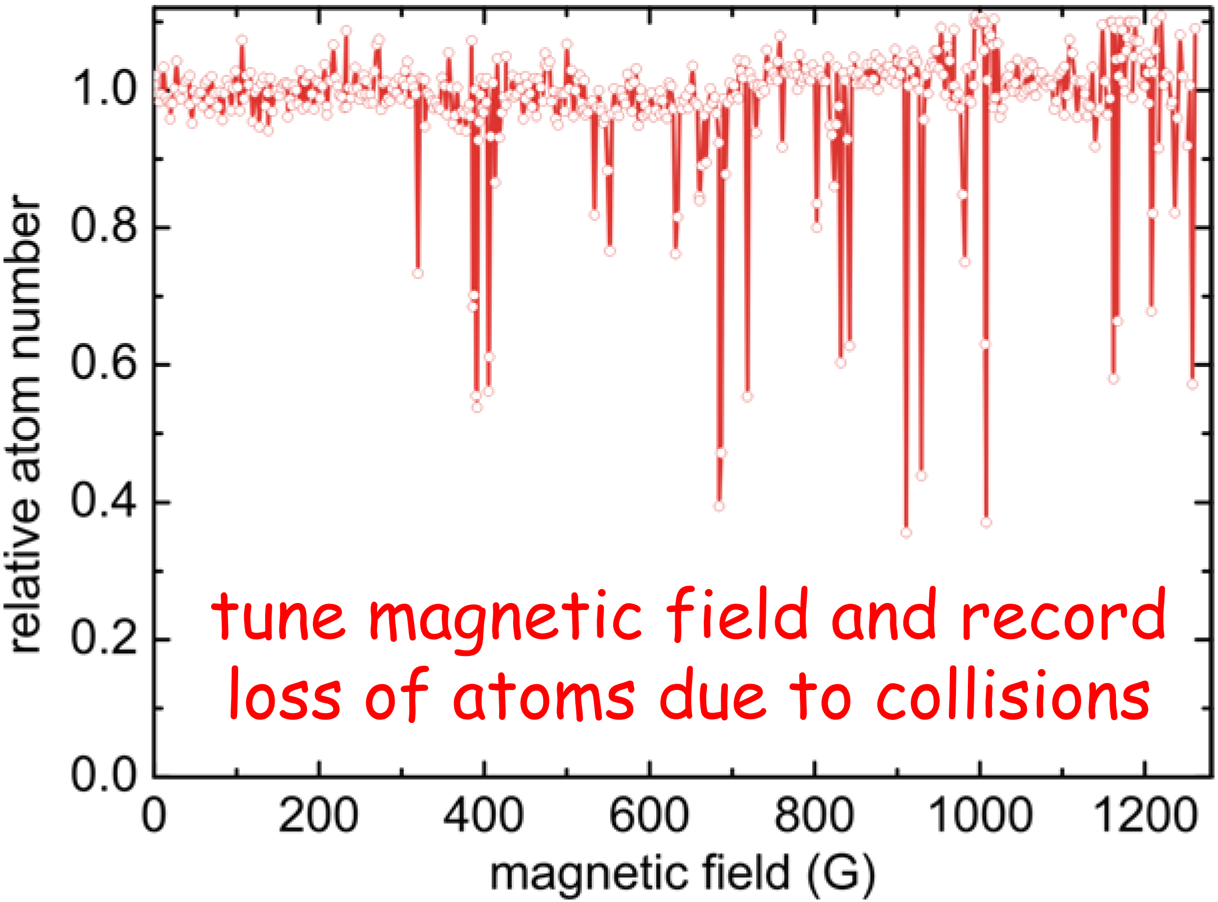
meeting the challenge



electric current:	≤ 1760 A
magnetic field:	≤ 1280 G
absolute stability:	≤ 0.01 G

experimental observation for ^{87}Rb

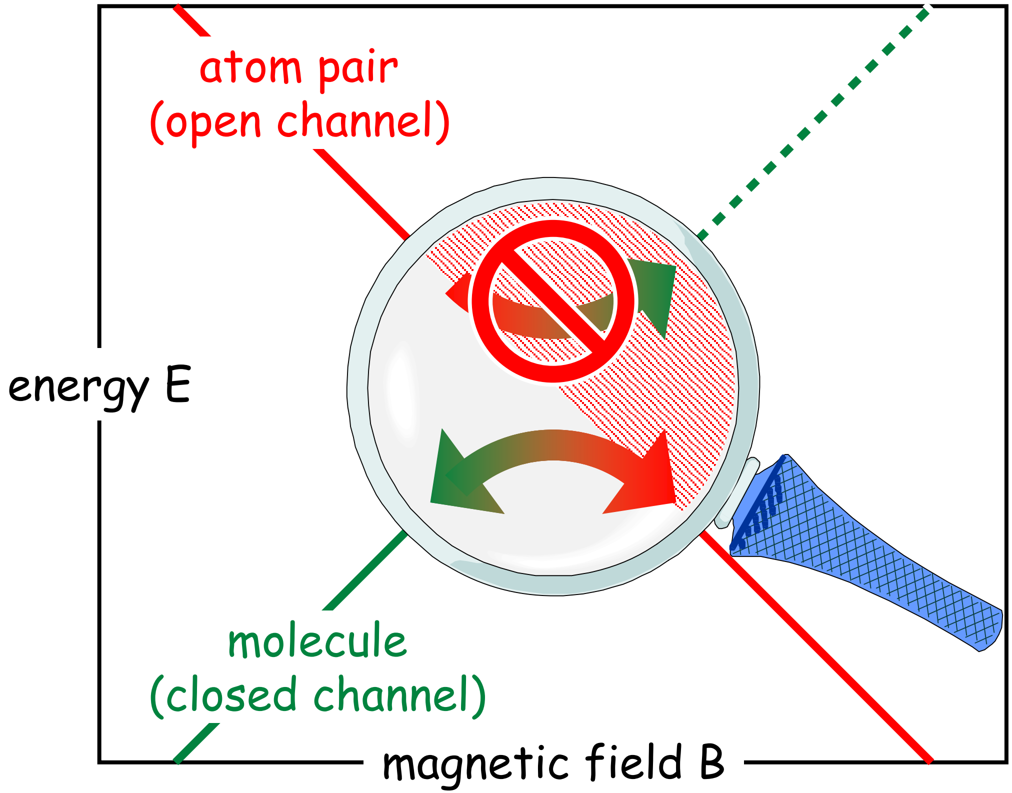
Marte et al., Phys. Rev. Lett. **89**, 283202 (2002)



all 43 but 1 resonances explained

creation of molecules

(oversimplified) naïve model:
two-level system with avoided crossing



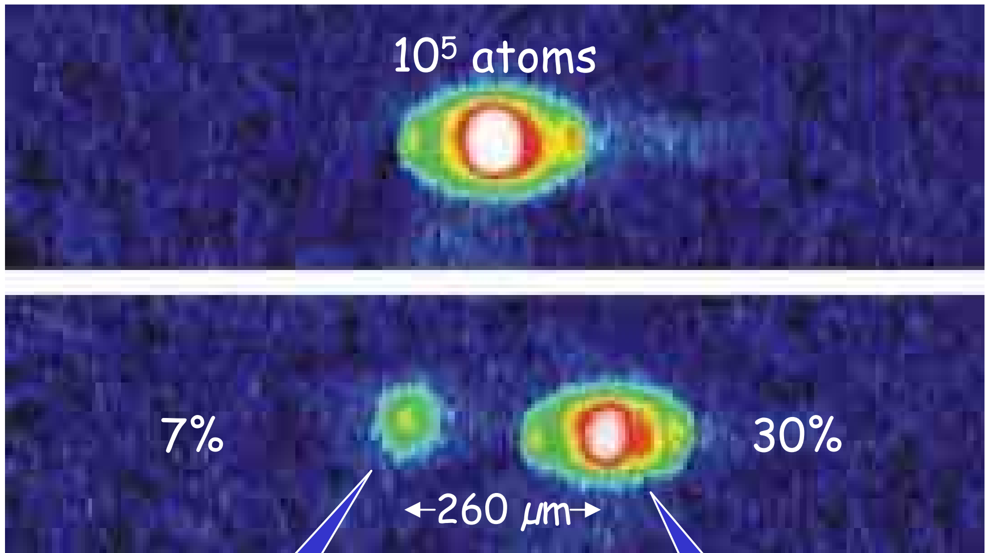
slow magnetic-field ramp:
adiabatic conversion of atoms into molecules and back

Stern-Gerlach separation of atoms and molecules

Dürr et al., Phys. Rev. Lett. **92**, 020406 (2004)

^{87}Rb $|F=1, m_F=1\rangle \otimes |F=1, m_F=1\rangle$ @ B_0=1007 G (Δ=210 mG)

w/o gradient
(reference)



w/gradient
3 ms @ 23 G/cm

hold field
1005.2 G

molecules
(reconverted
to atoms)

atoms
(not converted
to molecules)

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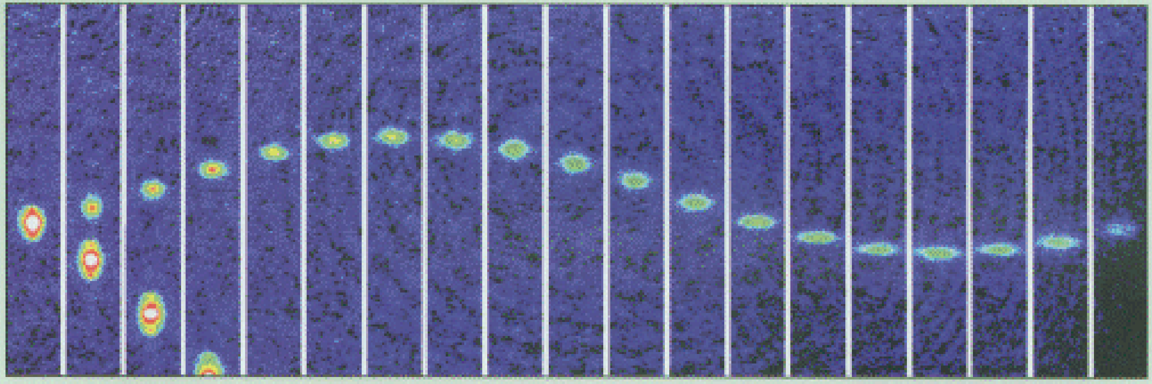

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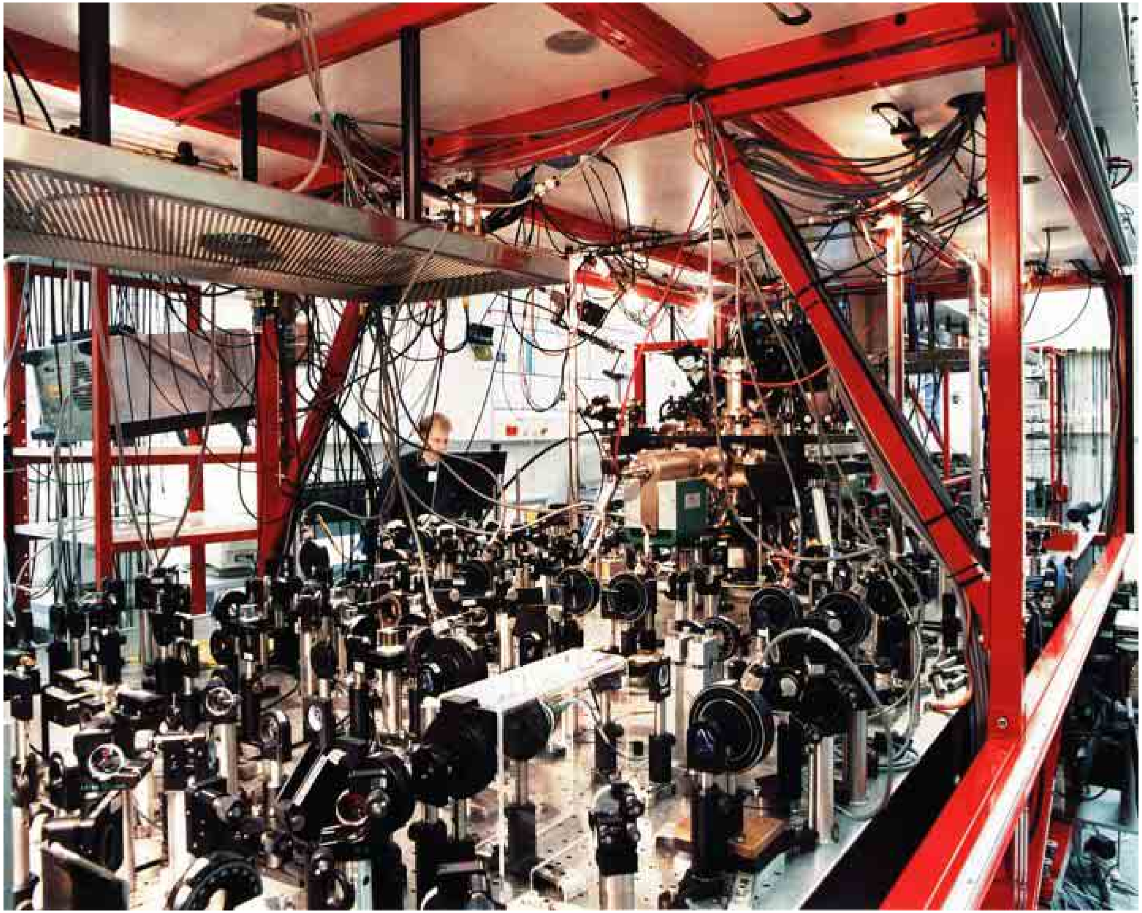
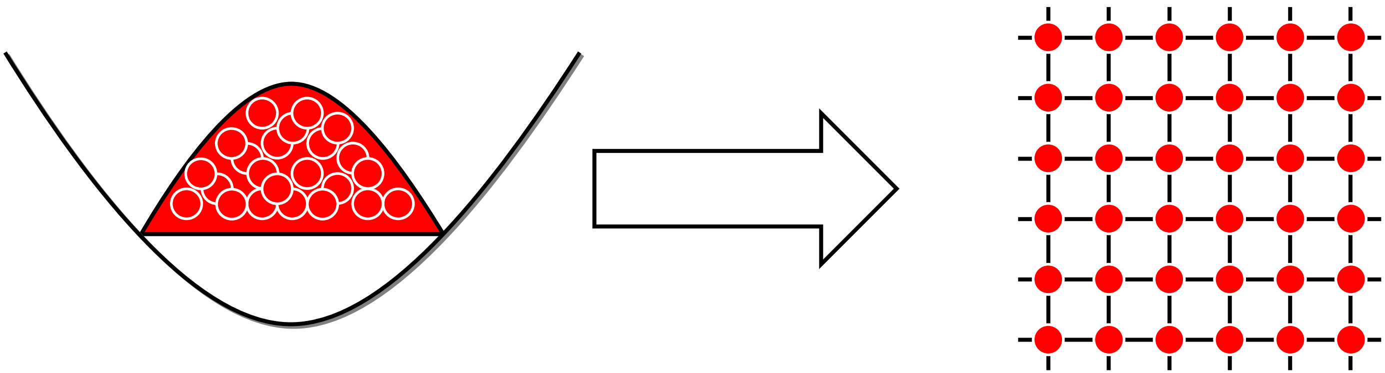


quantum gases

weakly interacting gas
(BEC, ...)

strongly correlated gas
(Mott, Tonks, ...)



inter-particle distance
versus
scattering length

hopping amplitude
versus
on-site interaction energy

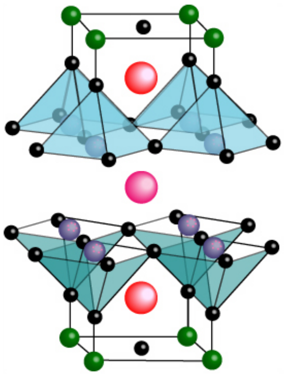
continuum description:
Gross-Pitaevskii equation

discrete description:
Bose-Hubbard Hamiltonian

why are strong correlations interesting ?

solid-state physics:

- high-temperature superconductivity
- fractional quantum Hall effect
- excitations with fractional statistics
- topological quantum computation
- exotic behavior in magnetic systems



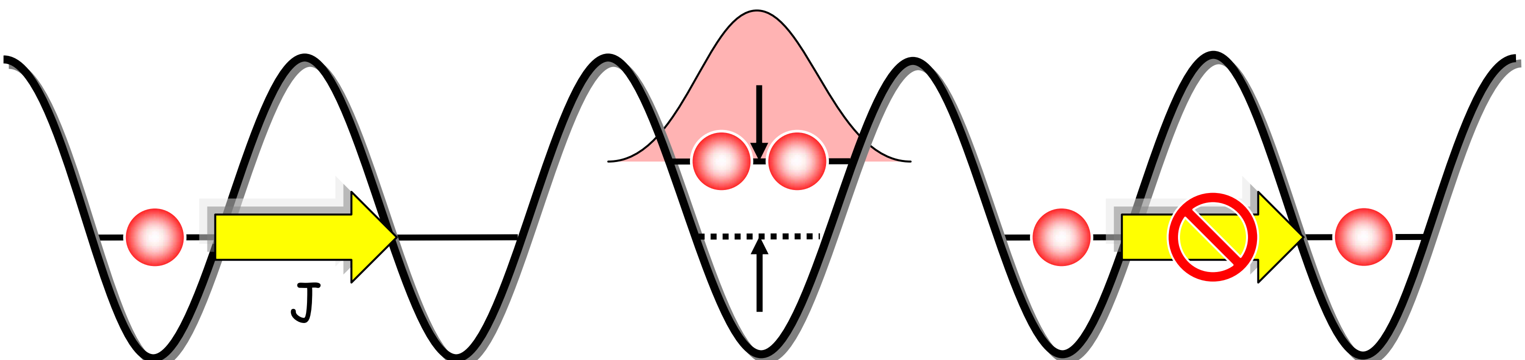
physical origin:

- elastic & repulsive interaction
- wave function vanishes for two particles at the same position

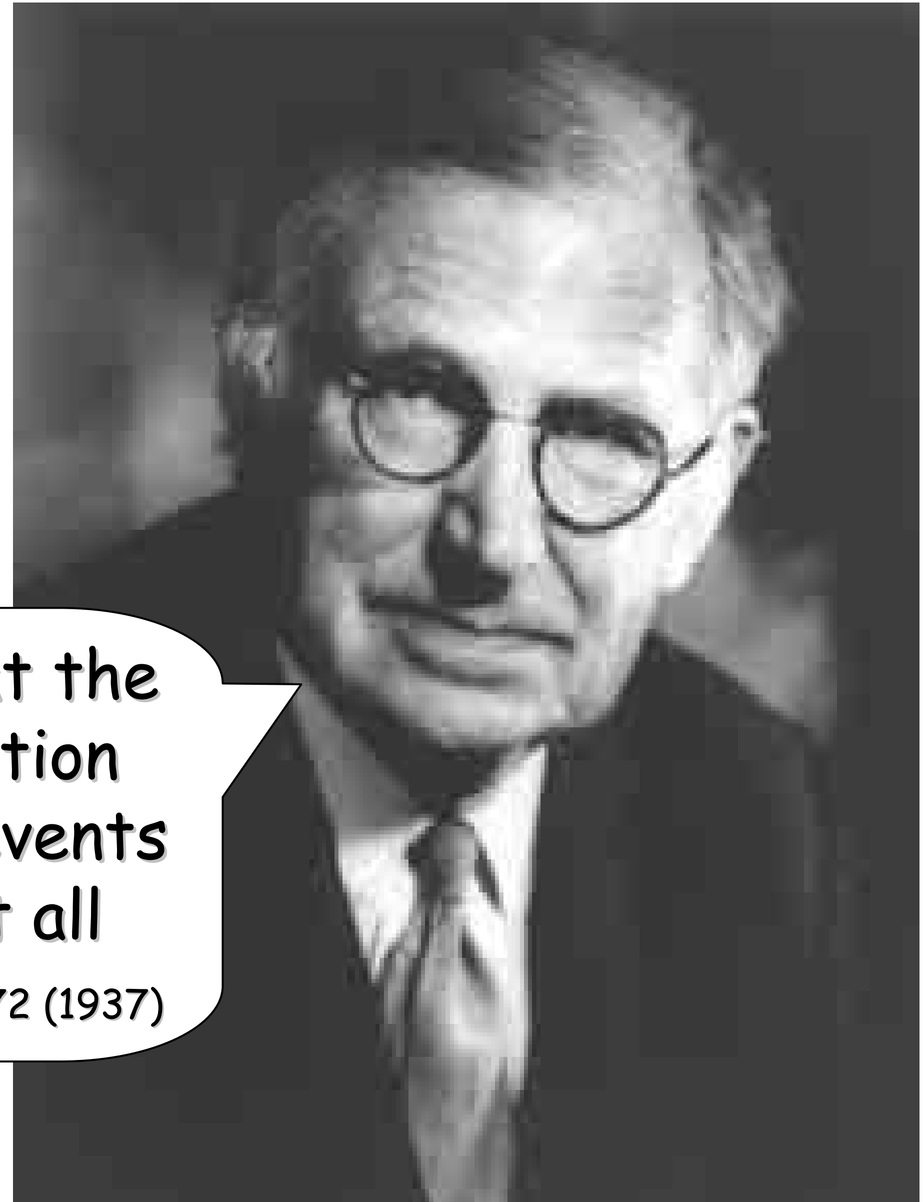
ultracold particles in an optical lattice

$$H = \underbrace{\sum_{i=1}^{N} \varepsilon_i \hat{c}_i^\dagger \hat{c}_i}_{\text{potential energy}} + \underbrace{\tfrac{1}{2} U \sum_{i=1}^{N} \hat{c}_i^{\dagger 2} \hat{c}_i^2}_{\text{onsite interaction}} - \underbrace{J \sum_{<i,j>} \hat{c}_j^\dagger \hat{c}_i}_{\text{tunnelling}}$$

$$U = g \int d^3 r \left| w(\vec{r}) \right|^4 \quad \text{with} \quad g = \frac{4\pi\hbar^2 a}{m}$$



U >> J:
tunneling suppressed
by energy conservation
==> excitation gap

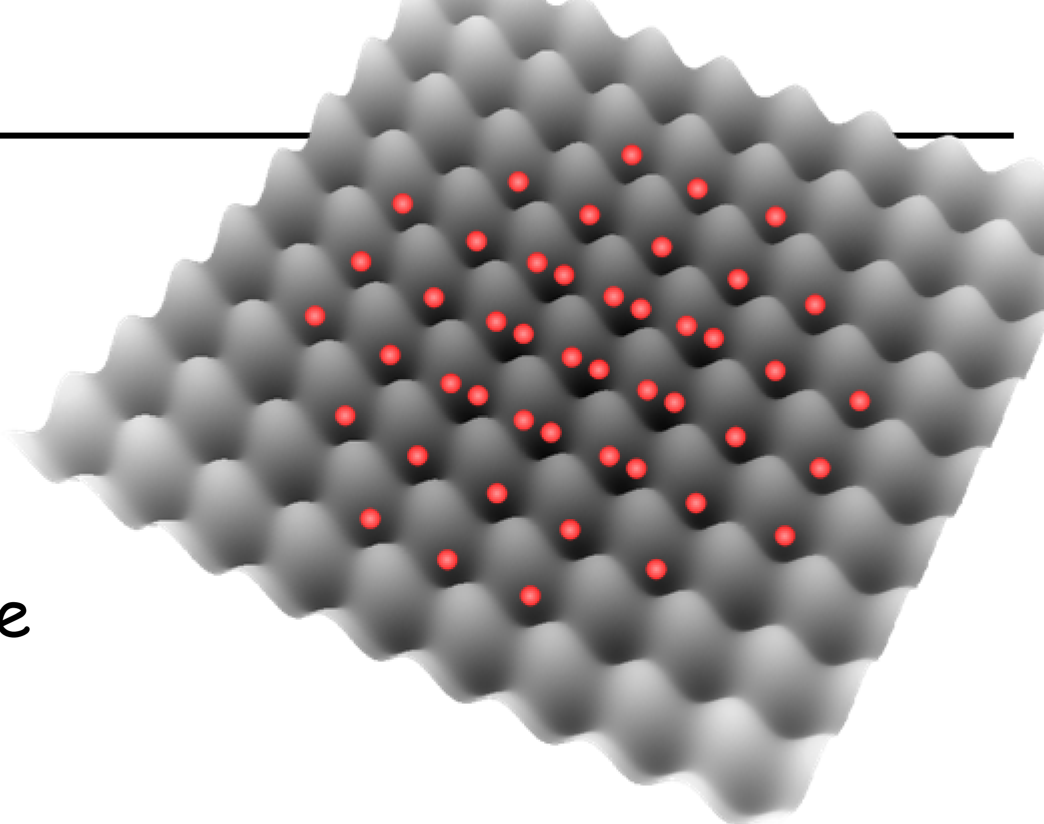


it is quite possible that the electrostatic interaction between electrons prevents them from moving at all

Proc. Phys. Soc. **49**, 72 (1937)

atomic Mott insulator

theory: Jaksch & Zoller
Greiner et al., Nature **415**, 39 (2002)



0D sites of a
3D optical lattice

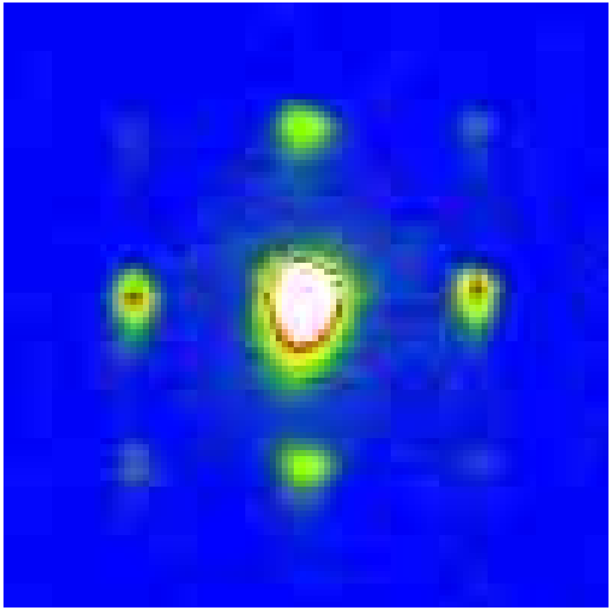
number states for Bosons:

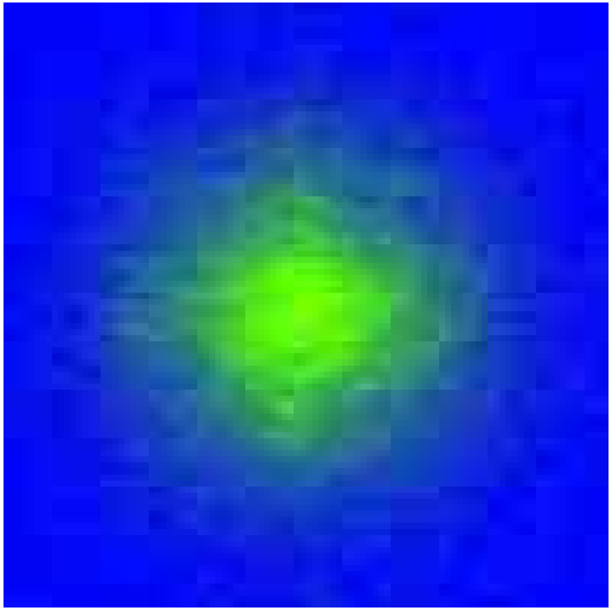
- exactly n=1 (or 2, or 3, ...) atoms per lattice site
- excitation gap determined by onsite energy U_{site}

$$\left|\Psi_{Mott}\right\rangle \propto \prod_{sites\ i} \left(\hat{a}_i^\dagger\right)^n \left|0\right\rangle$$

momentum distribution

Greiner et al., Nature **415**, 39 (2002)
Volz et al., Nature Phys. **2**, 692 (2006)





superfluid state:

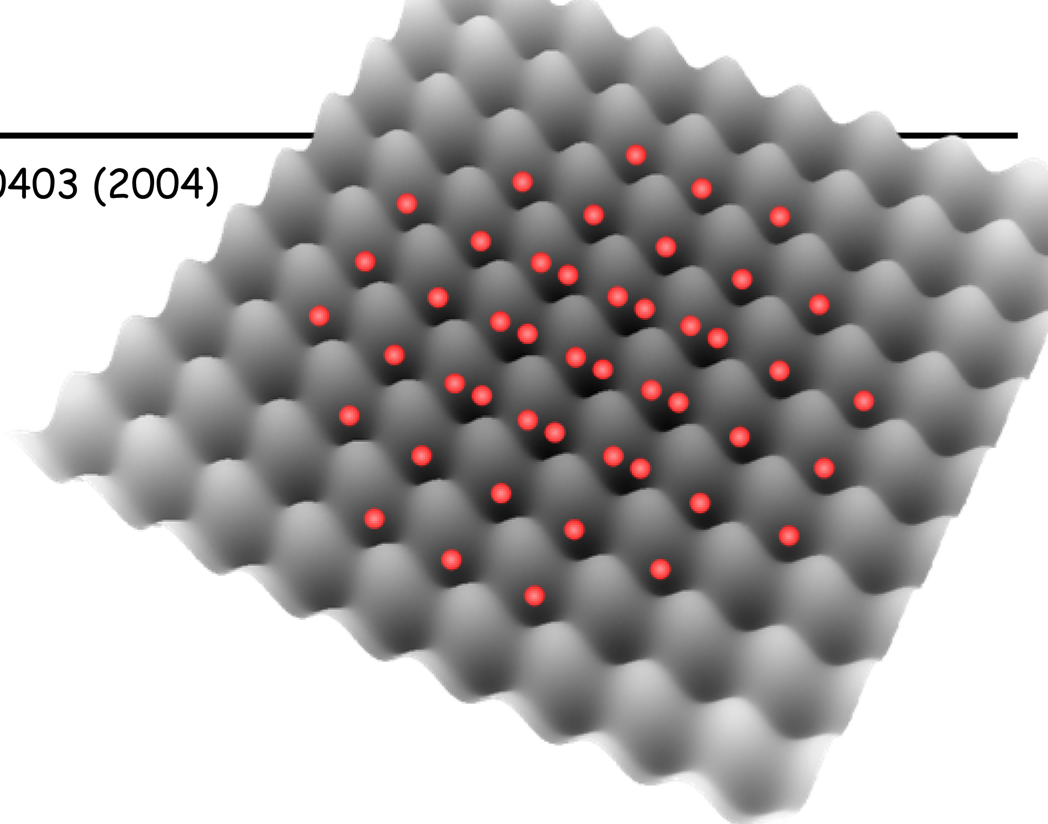
$$\left|\Psi_{SF}\right\rangle \propto \left(\sum_{sites\ i} \hat{a}_i^\dagger\right)^N \left|0\right\rangle$$

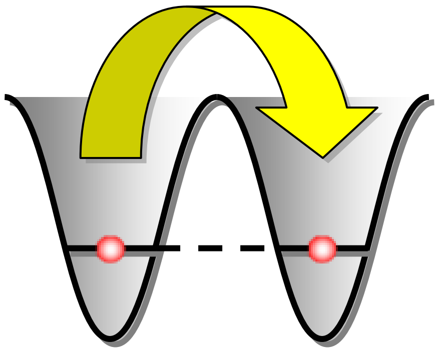
insulating state:

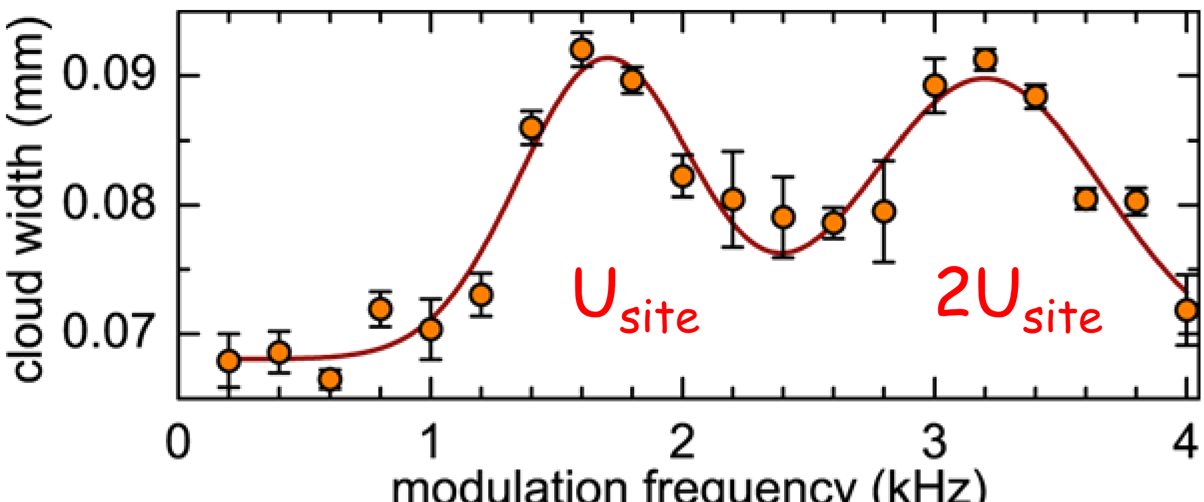
$$\left|\Psi_{Mott}\right\rangle \propto \prod_{sites\ i} \left(\hat{a}_i^\dagger\right)^n \left|0\right\rangle$$

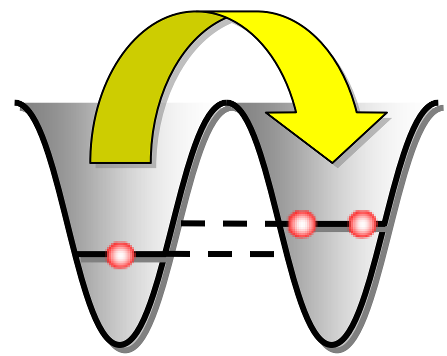
excitation gap

Stöferle et al., Phys. Rev. Lett. **92**, 130403 (2004)
Volz et al., Nature Phys. **2**, 692 (2006)



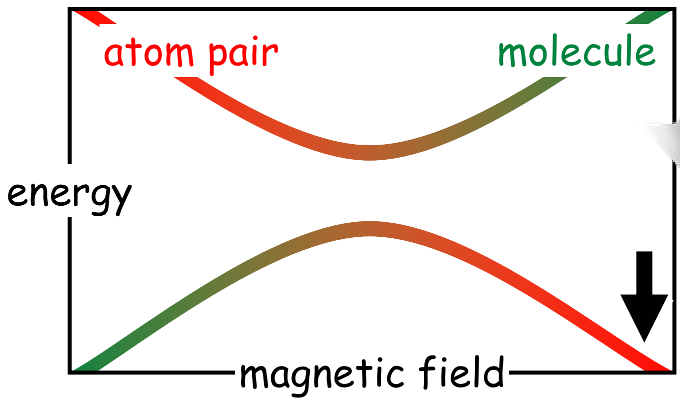


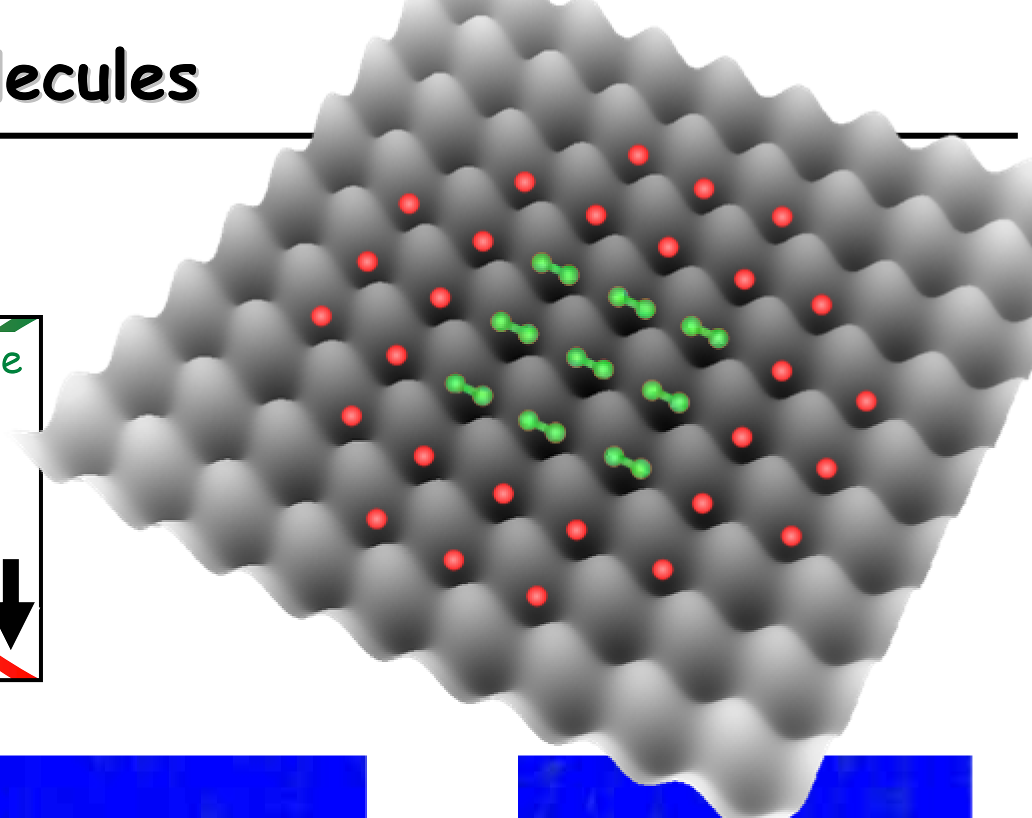


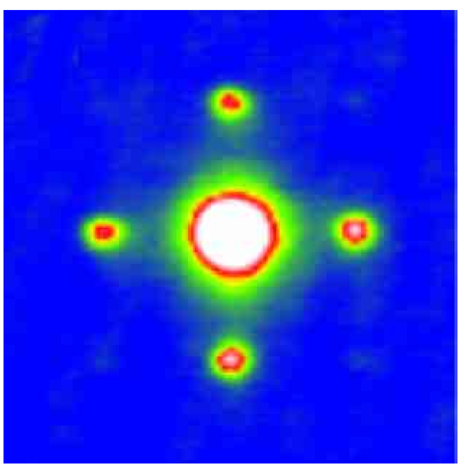


Mott-like state of molecules

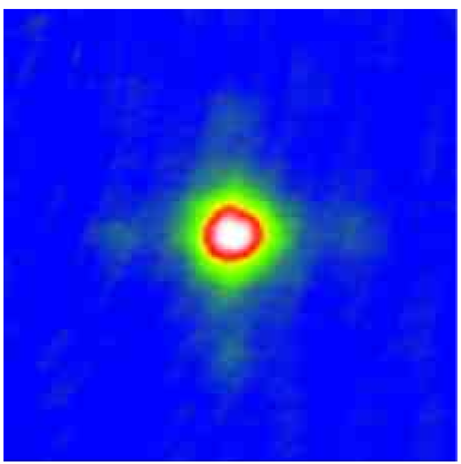
Volz et al., Nature Phys. **2**, 692 (2006)



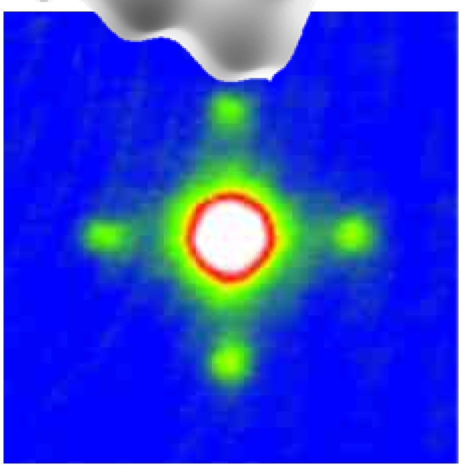




number := 100%
visibility = 93%



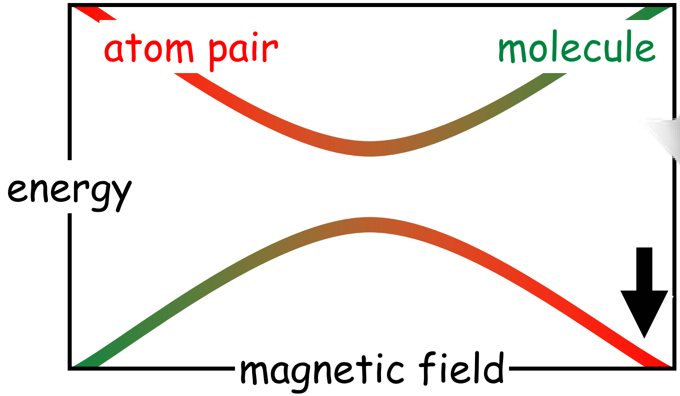
number ≈ 47%
visibility = 80%

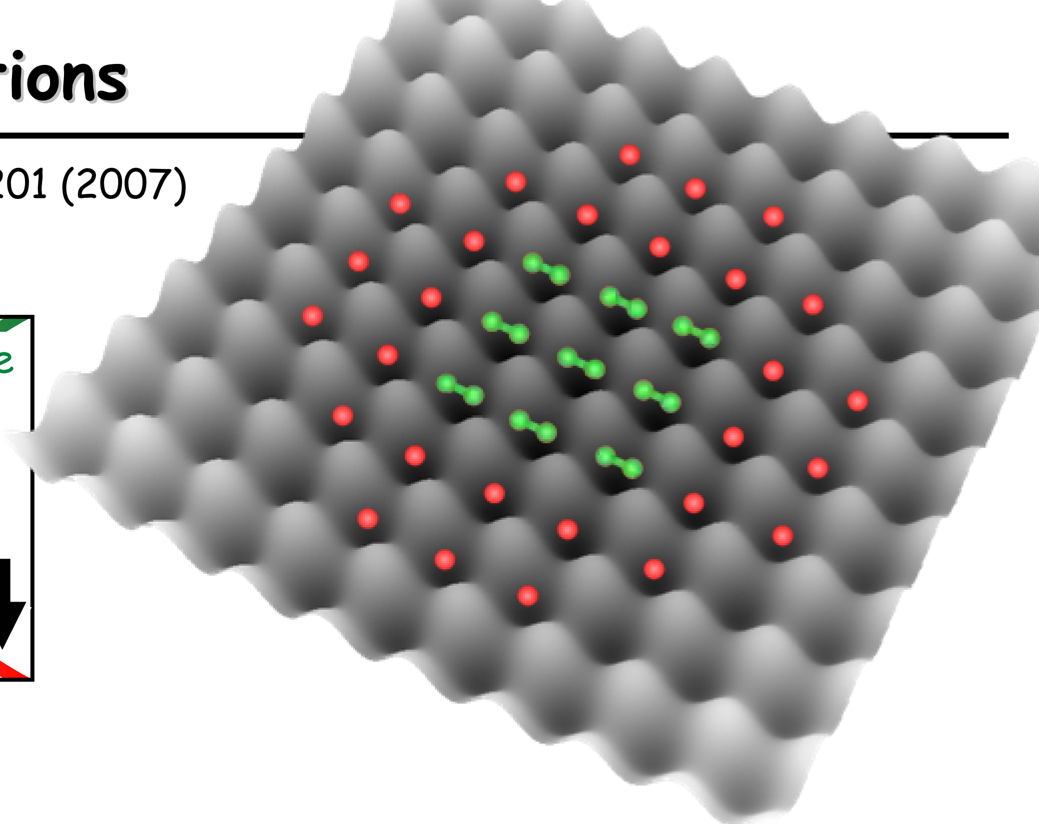


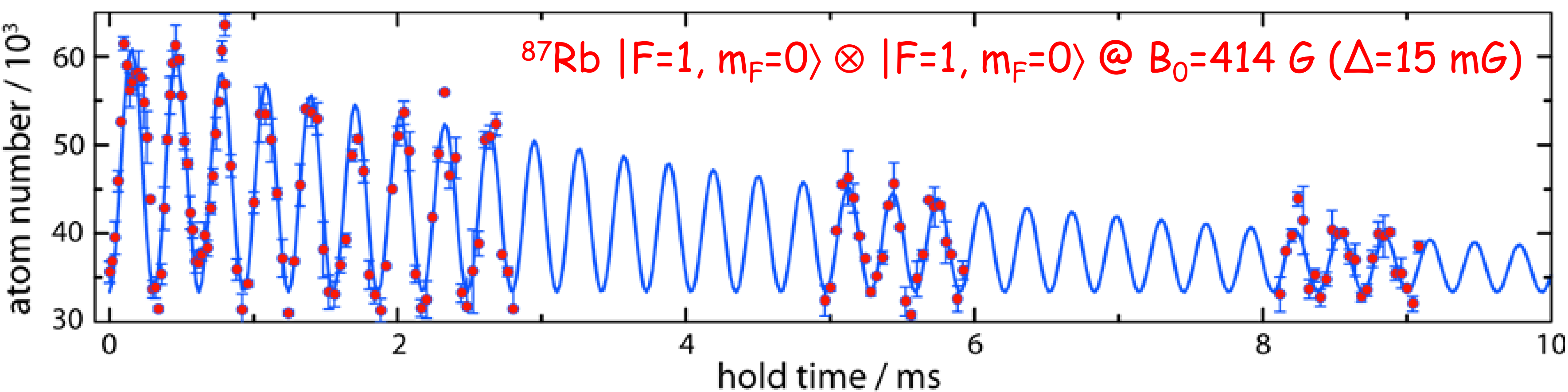
number ≈ 85%
visibility = 86%

atoms-molecule oscillations

Syassen et al., Phys. Rev. Lett. **99**, 033201 (2007)

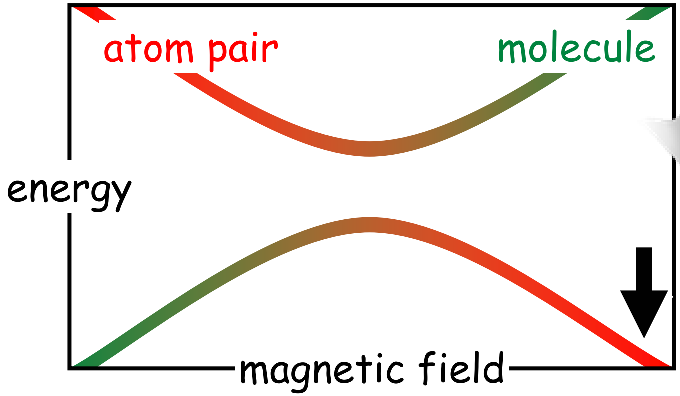


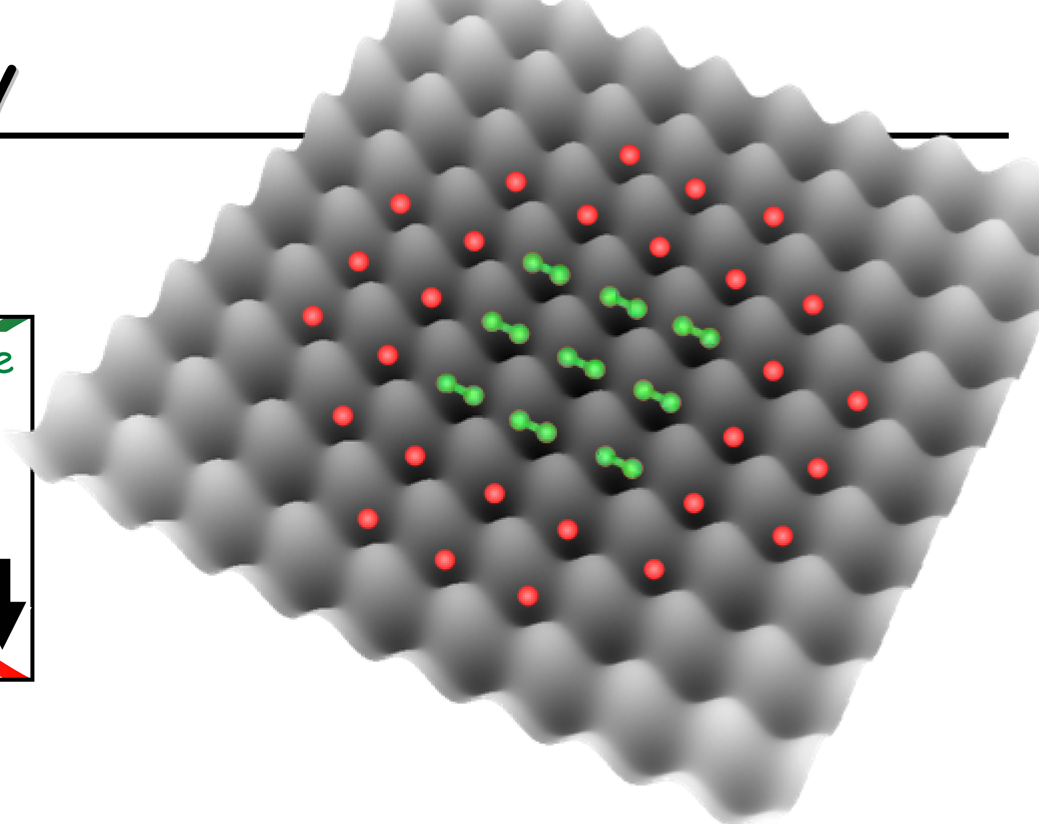


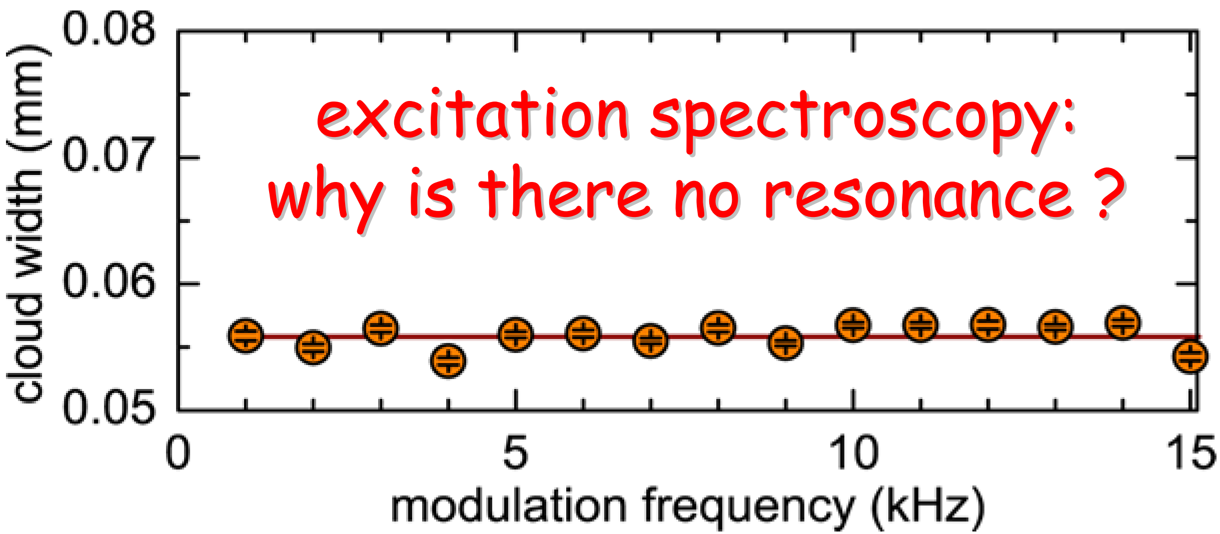


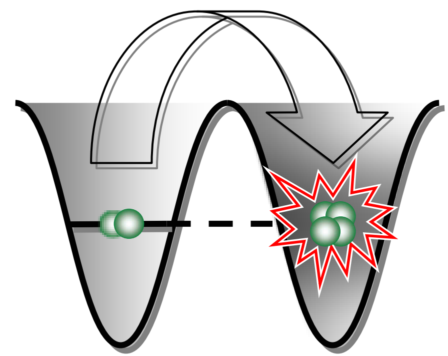
excitation spectroscopy

Dürr et al., ICAP **20**, 278 (2006)









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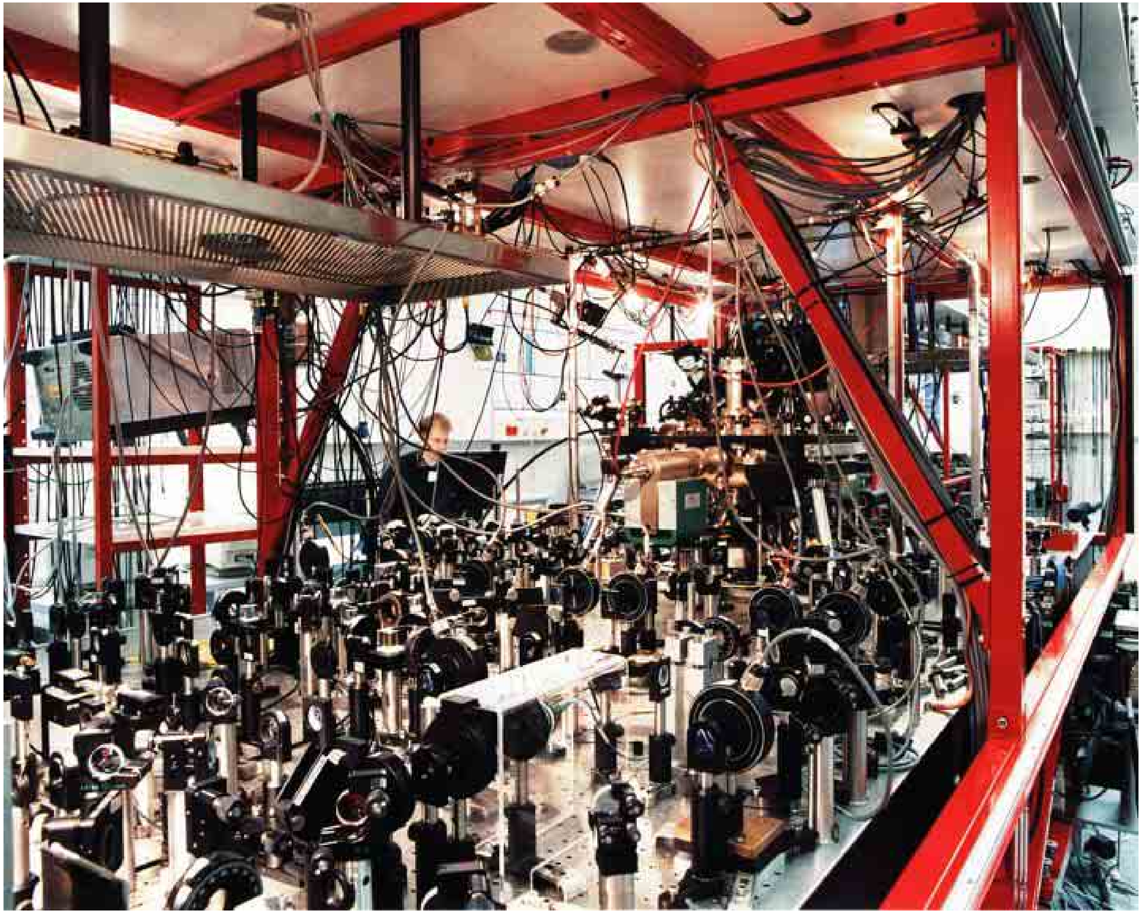
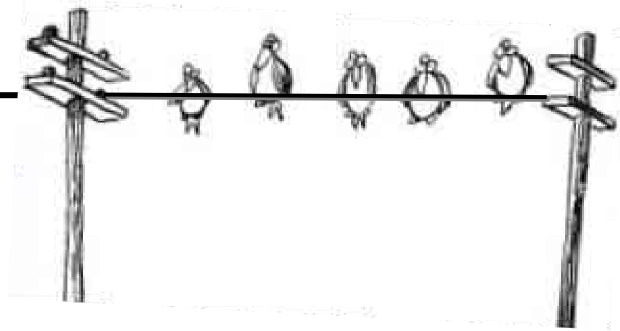
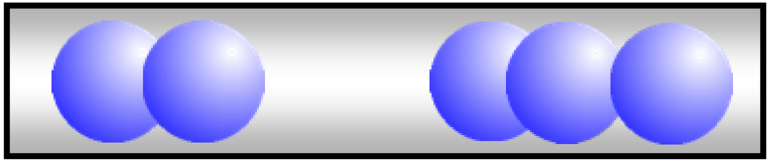


Tonks-Girardeau gas

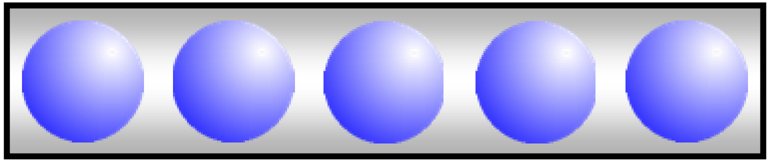
Tonks, Phys. Rev. **50**, 955 (1936)
Girardeau, J. Math. Phys. **1**, 516 (1960)



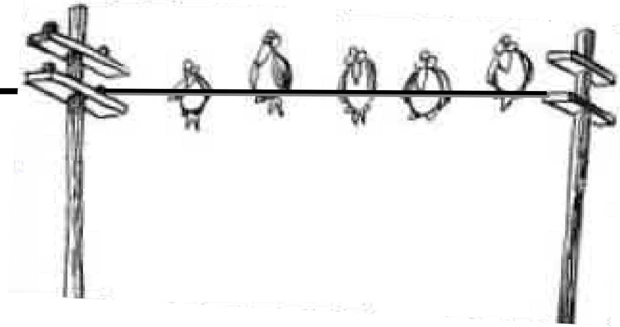
ideal gas of Bosons in one dimension:



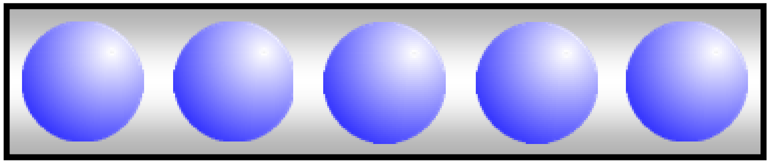
hard-sphere repulsion mimics Pauli principle:



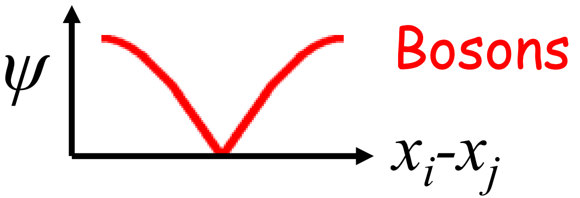
Tonks-Girardeau gas

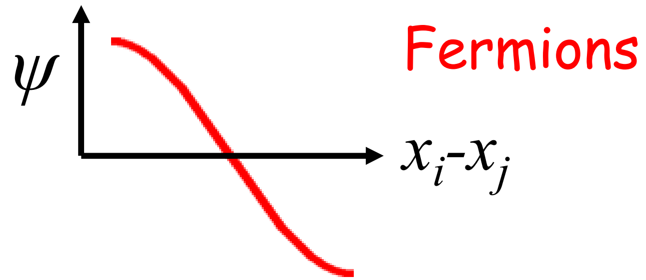


Tonks, Phys. Rev. **50**, 955 (1936)
Girardeau, J. Math. Phys. **1**, 516 (1960)



$$\underbrace{\psi_{Bose}(x_1 \cdots x_n)}_{\text{symmetric}} = \underbrace{\psi_{Fermi}(x_1 \cdots x_n)}_{\text{antisymmetric}} \underbrace{\prod_{i<j} \mathrm{sgn}(x_i - x_j)}_{\text{antisymmetric}}$$





one-dimensional Bose gas

Lieb & Liniger, Phys. Rev. **130**, 1605 (1963)

Schrödinger equation:

$$E\psi(x) = -\frac{\partial^2}{\partial x^2}\psi(x) + g\delta(x)\psi(x) \qquad x = x_i - x_j$$

ansatz ($x \to 0$):

$$\psi(x) = c_0 + c_1 |x| + c_2 x^2 + \ldots \quad \text{(Bosonic symmetry)}$$

$$\psi'(x) = c_1 \Theta(x) + 2c_2 x + \ldots$$

$$\psi''(x) = c_1 \delta(x) + 2c_2 + \ldots$$

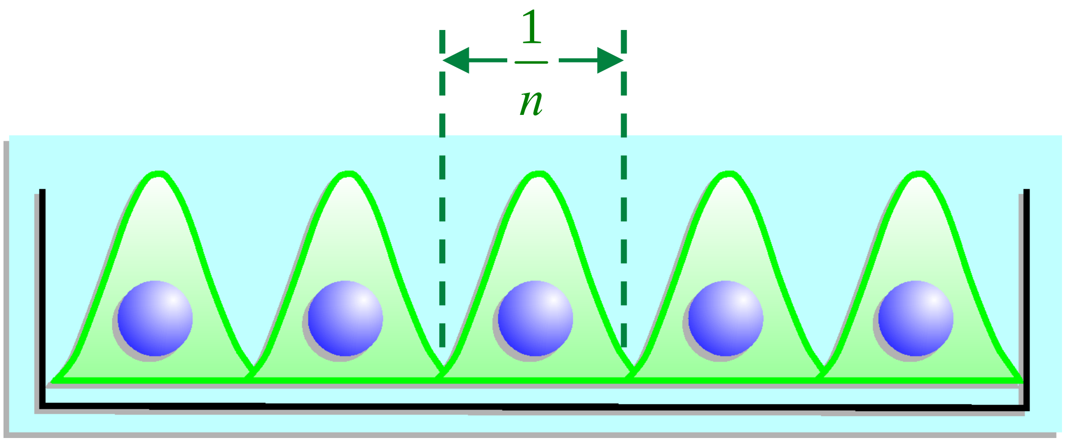
Schrödinger equation:

$$E\psi(x) = \underbrace{(-c_1 + c_0 g)}_{=0}\delta(x) + c_1 g \delta(x)|x| + \ldots$$

solution:

$$\psi(x) = c_1\left[\frac{1}{g} + |x| + \ldots\right] \xrightarrow[g\to\infty]{} |x|$$

low temperature & low density

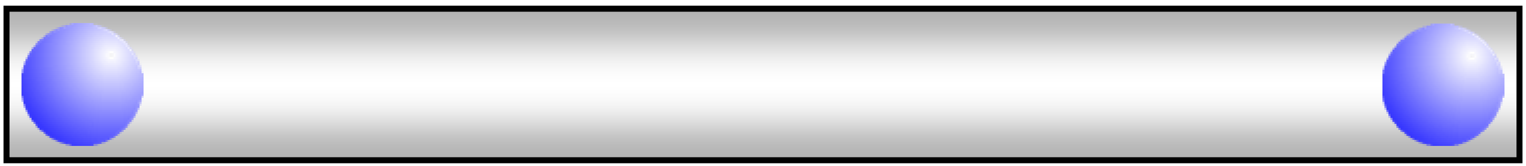


$$\underbrace{U_{int} \sim gn}_{\text{interaction energy}} \quad \gg \quad \underbrace{E_{kin} \sim \frac{\hbar^2}{m}\frac{1}{\lambda^2} \sim \frac{\hbar^2}{m} n^2}_{\text{kinetic energy}}$$

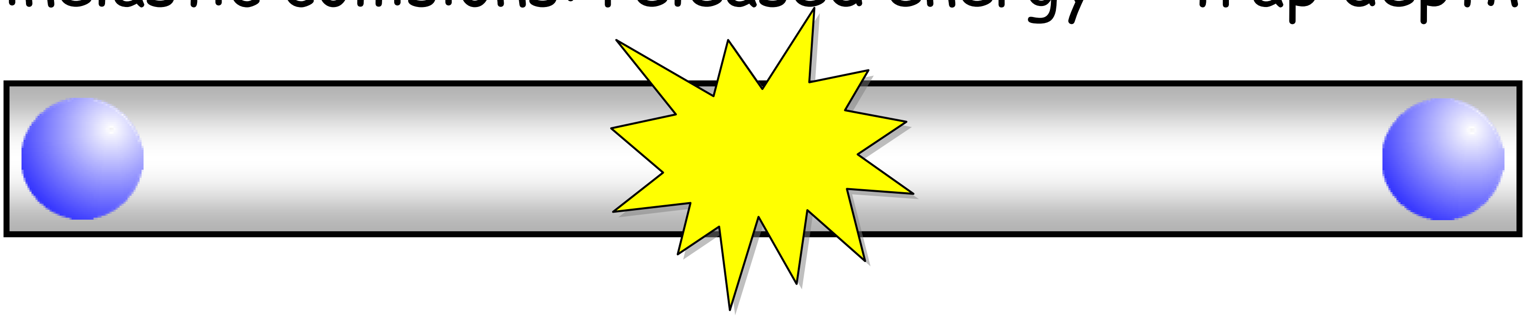
$$\underbrace{\frac{U_{int}}{E_{kin}} \sim \gamma \equiv \frac{mg}{\hbar^2 n}}_{\text{dimensionless interaction parameter}} \gg 1$$

elastic versus inelastic collisions

elastic collisions: number of particles is conserved



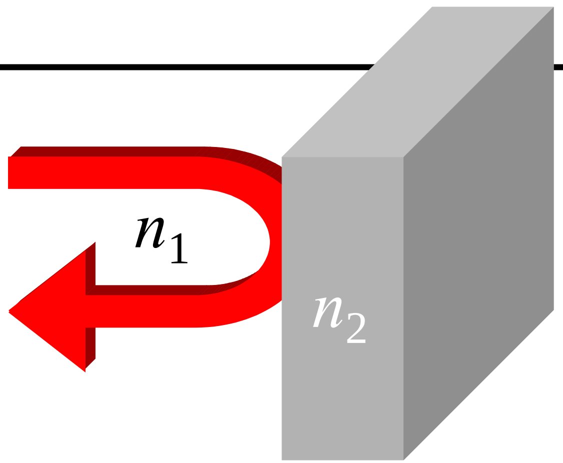
inelastic collisions: released energy >> trap depth



→ loss of particles

strong loss causes reflection

classical optics:

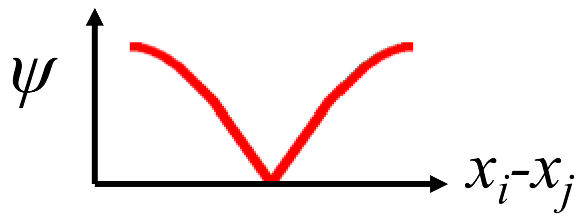


Im(n_2): absorption
= loss of intensity

$$R = \left| \frac{n_1 - n_2}{n_1 + n_2} \right|^2 \xrightarrow{|n_2| \to \infty} 1$$

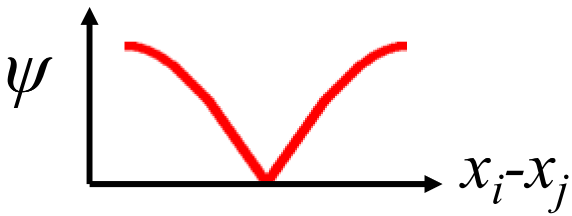
- index mismatch
- even for imaginary n_2

strong dissipation causes reflection:
→ vanishing boundary condition at the surface



strong loss causes reflection

Dürr et al., Phys. Rev. A **79**, 023614 (2009)
Garcia-Ripoll, New J. Phys. **11**, 013053 (2009)



inelastic two-body collisions
$\Rightarrow$ Tonks-Girardeau gas
$\Rightarrow$ suppression of loss

$$\frac{d}{dt}n = -K_2 g^{(2)}(0) n^2$$

$$K_2 = -\frac{2}{\hbar}\mathrm{Im}(g) \quad \text{and} \quad g^{(2)}(0) = \frac{\langle n^2(x)\rangle}{\langle n(x)\rangle^2}$$

from real to imaginary scattering parameters

Dürr et al., Phys. Rev. A **79**, 023614 (2009)

ground-state energy: $$E = N\frac{\hbar^2\pi^2}{6m}n^2\left(\frac{\gamma}{\gamma+2}\right)^2$$

initial loss rate: $$\left.\frac{dN}{dt}\right|_{t=0} = \frac{4}{\hbar}\mathrm{Im}(E)$$

density correlation: $$g^{(2)}(0) = \frac{2\,\mathrm{Im}(E)}{Nn\,\mathrm{Im}(g)}$$

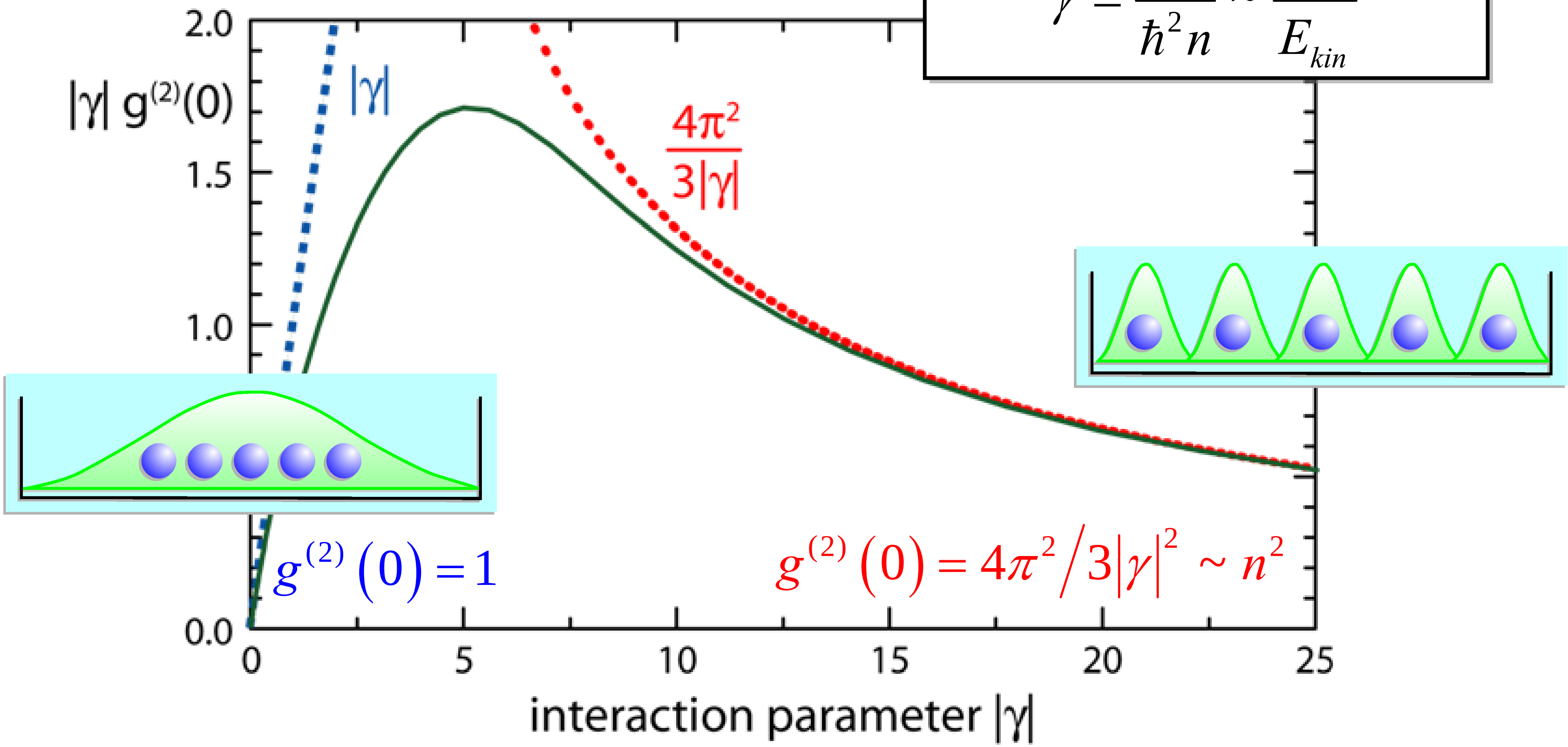
$$a_\perp = \sqrt{\hbar/m\omega_\perp} \qquad g = \frac{2\hbar^2 a}{ma_\perp^2}\left(1+\frac{a}{\sqrt{2}a_\perp}\zeta\left(\frac{1}{2}\right)\right)^{-1}$$

loss suppressed by loss

Dürr et al., Phys. Rev. A **79**, 023614 (2009)

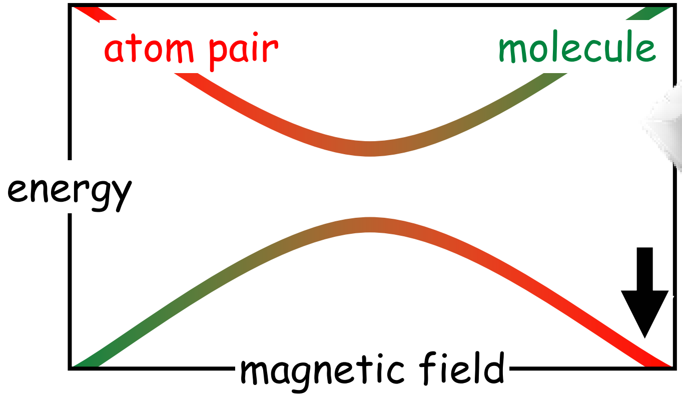
interaction parameter:

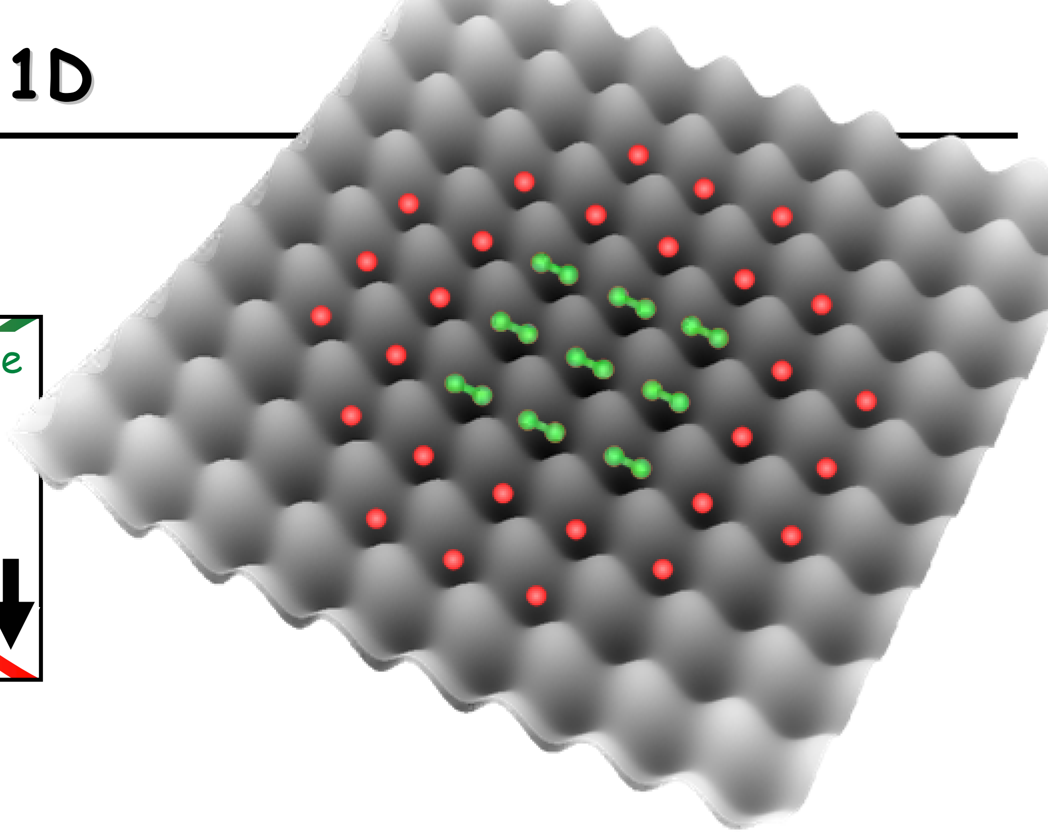
$$\gamma = \frac{mg}{\hbar^2 n} \sim \frac{U_{int}}{E_{kin}}$$



inelastic collisions → fermionization → reduced losses

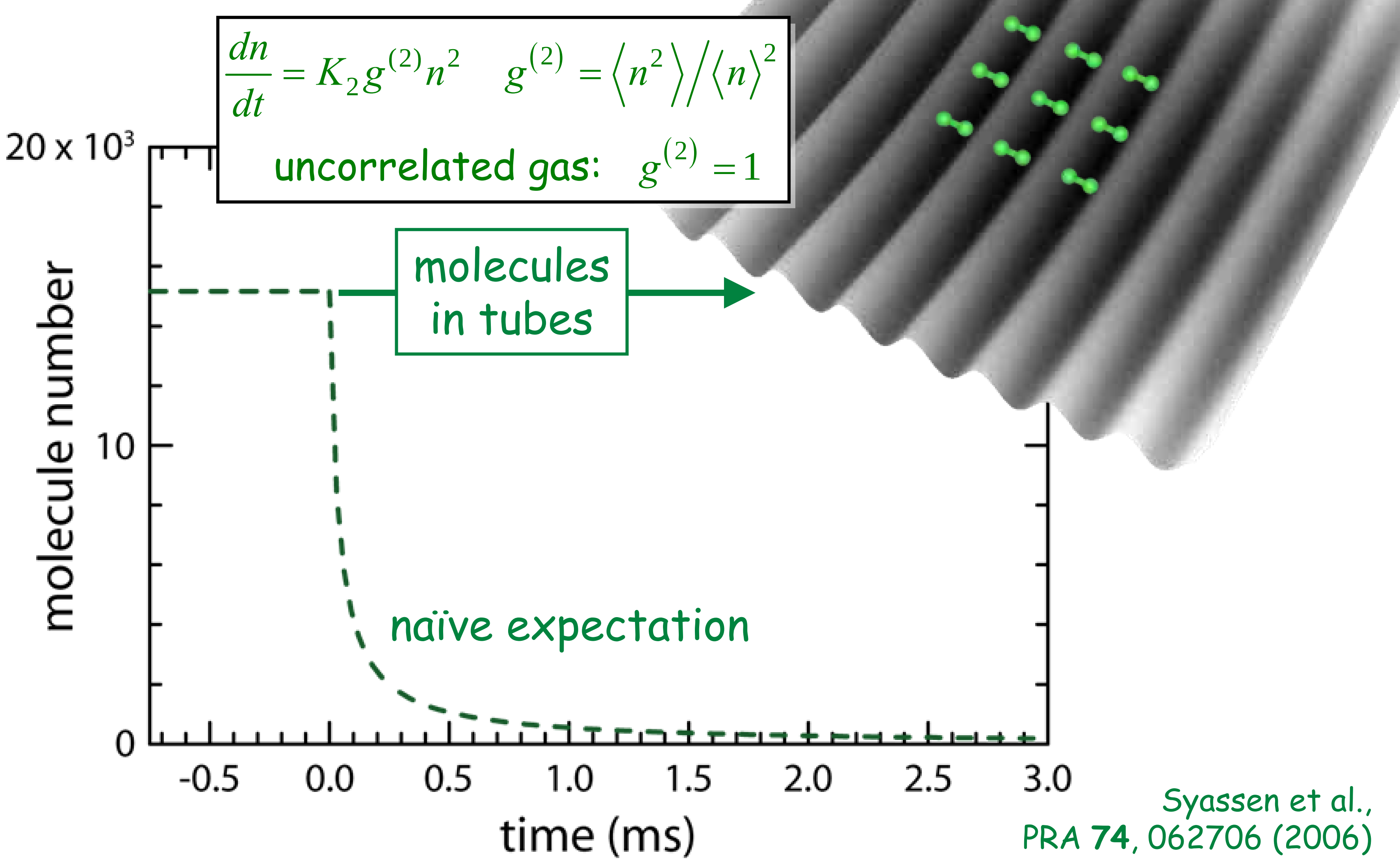
Feshbach molecules in 1D





time-dependent loss

Syassen et al., Science **320**, 1329 (2008)



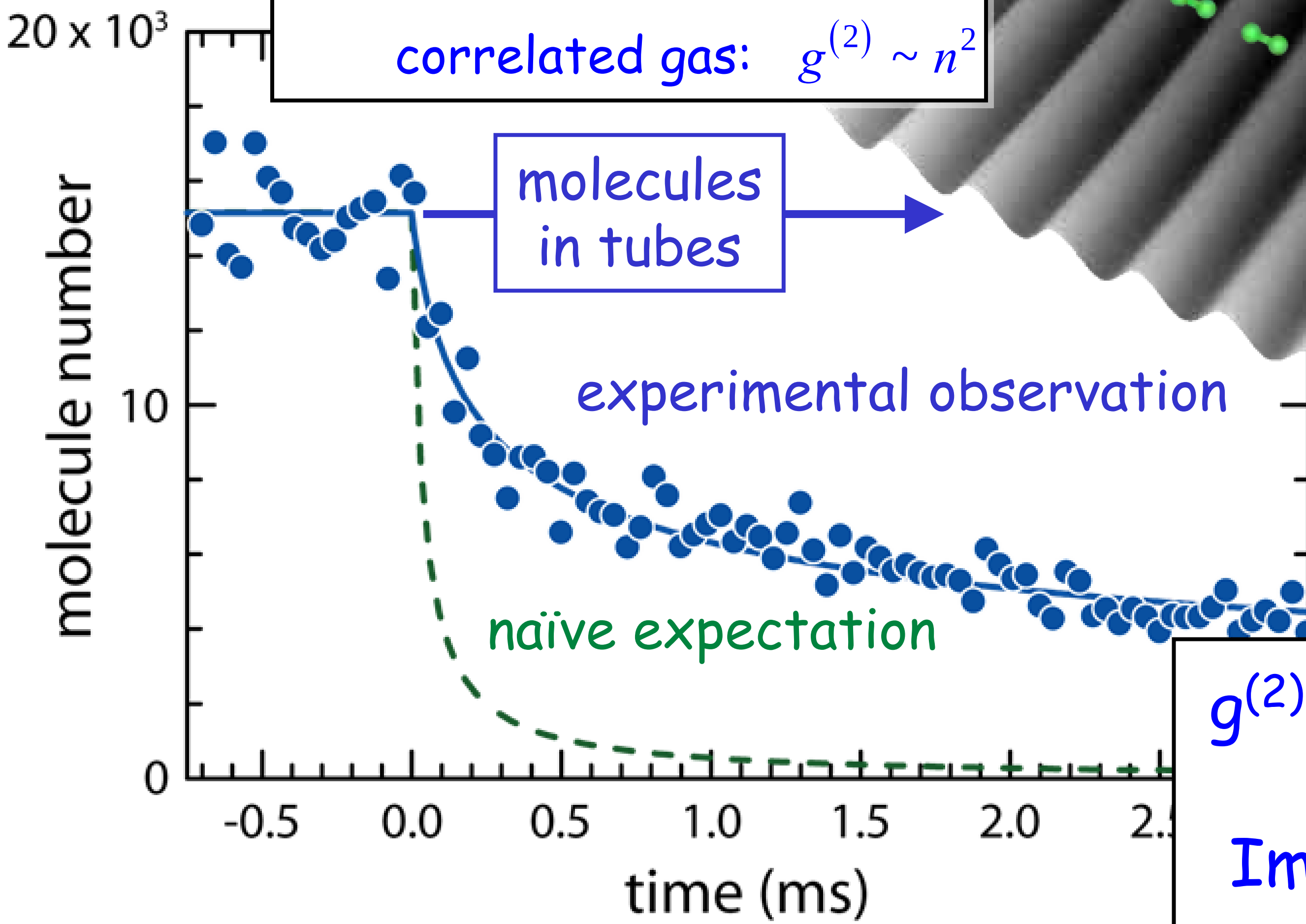
Syassen et al.,
PRA **74**, 062706 (2006)

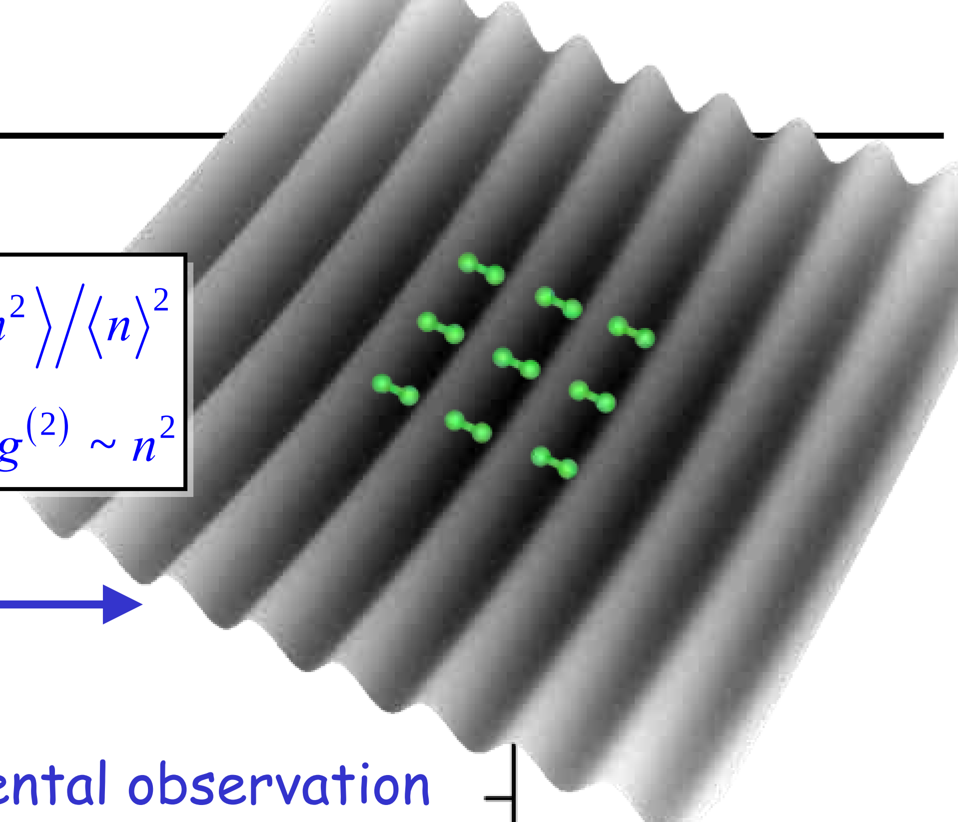
time-dependent loss

Syassen et al., Science **320**, 1329 (2008)

$$\frac{dn}{dt} = K_2 g^{(2)} n^2 \qquad g^{(2)} = \langle n^2 \rangle / \langle n \rangle^2$$

correlated gas: $g^{(2)} \sim n^2$





$g^{(2)}(0) = 0.11$

$|\gamma| = 11$

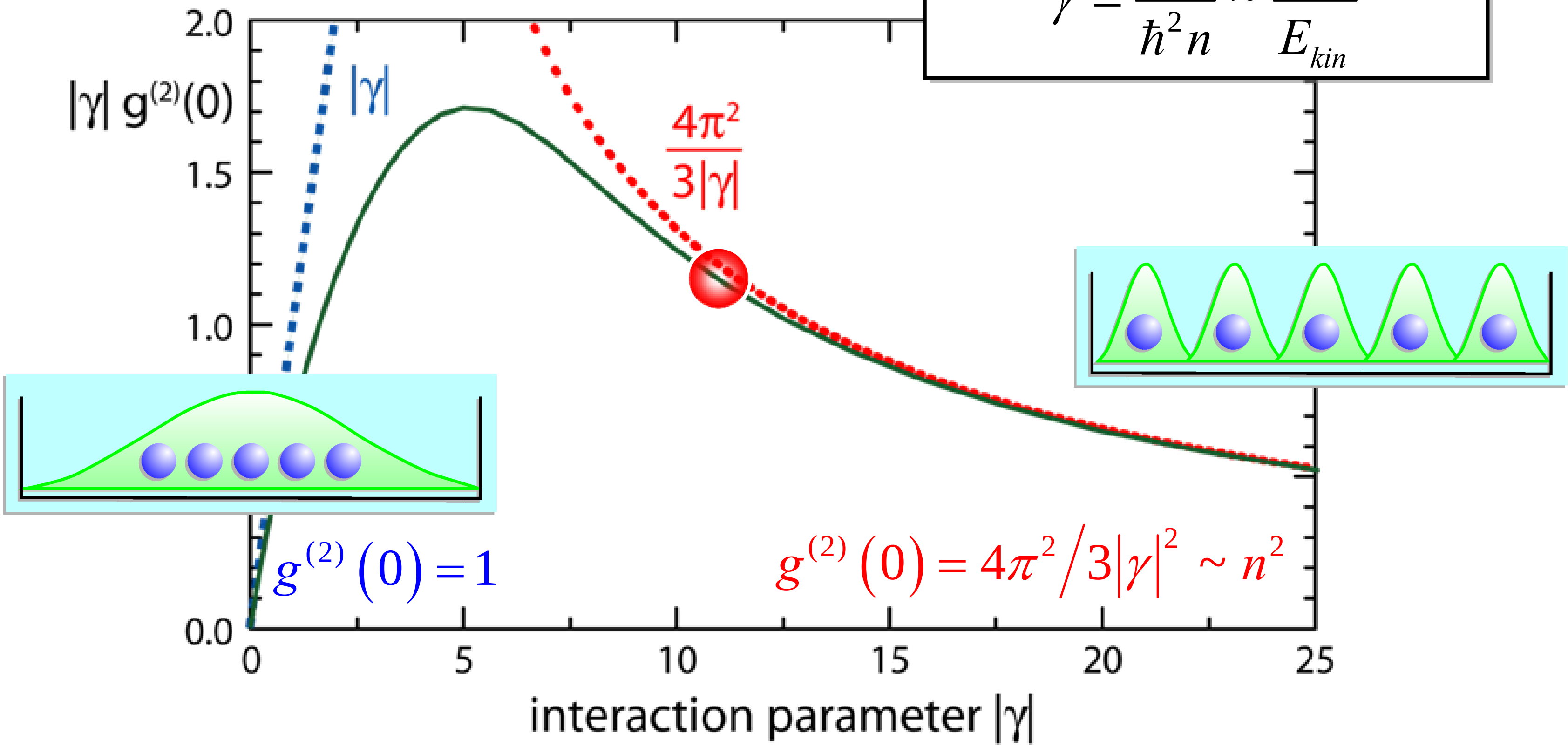
$\mathrm{Im}(a) = -450\ a_0$

loss suppressed by loss

Dürr et al., Phys. Rev. A **79**, 023614 (2009)

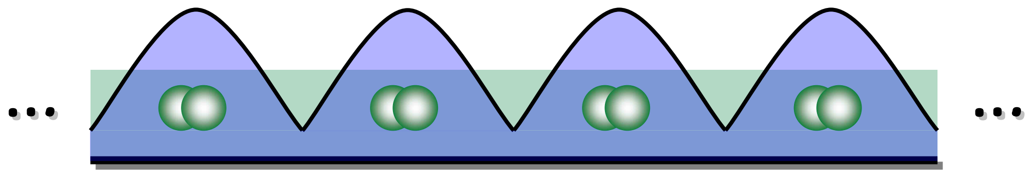
interaction parameter:

$$\gamma = \frac{mg}{\hbar^2 n} \sim \frac{U_{int}}{E_{kin}}$$



increase of the correlations

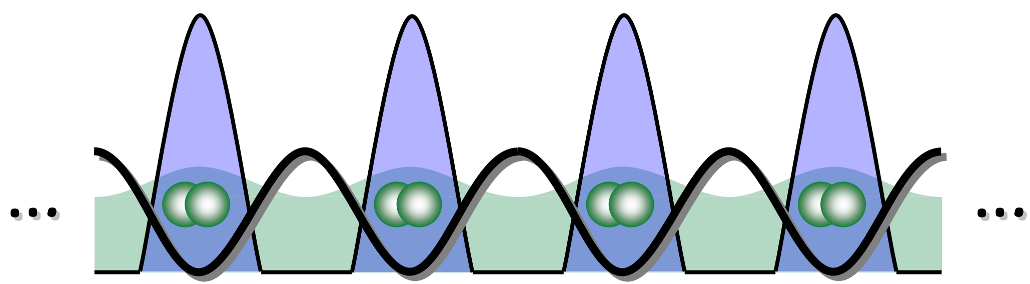
one-dimensional tubes:



$g^{(2)}(0) = 0.1$

optical lattice → effective mass increases ...

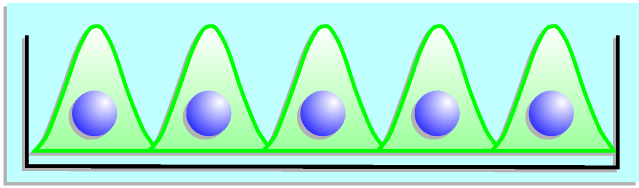
one-dimensional lattice:



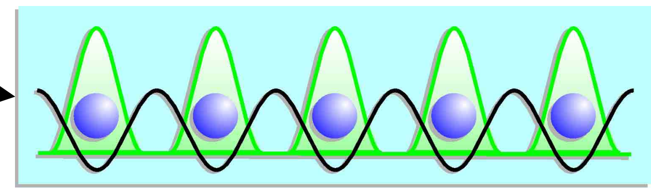
$g^{(2)}(0) \ll 1$

... correlation increases, $g^{(2)}(0) \sim 1/m^2$

lattice systems with dissipation



Tonks-Girardeau box → Bose-Hubbard lattice

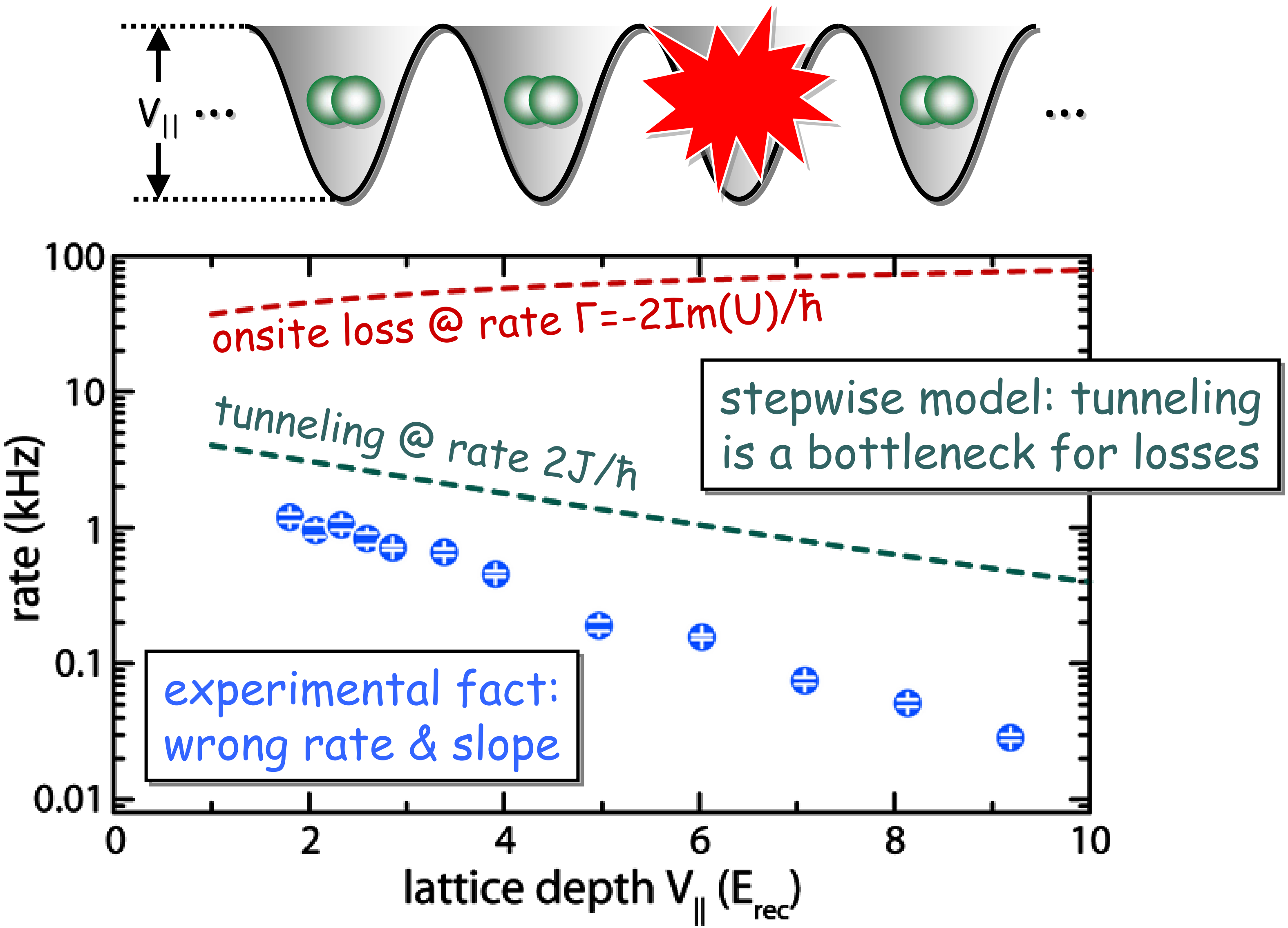


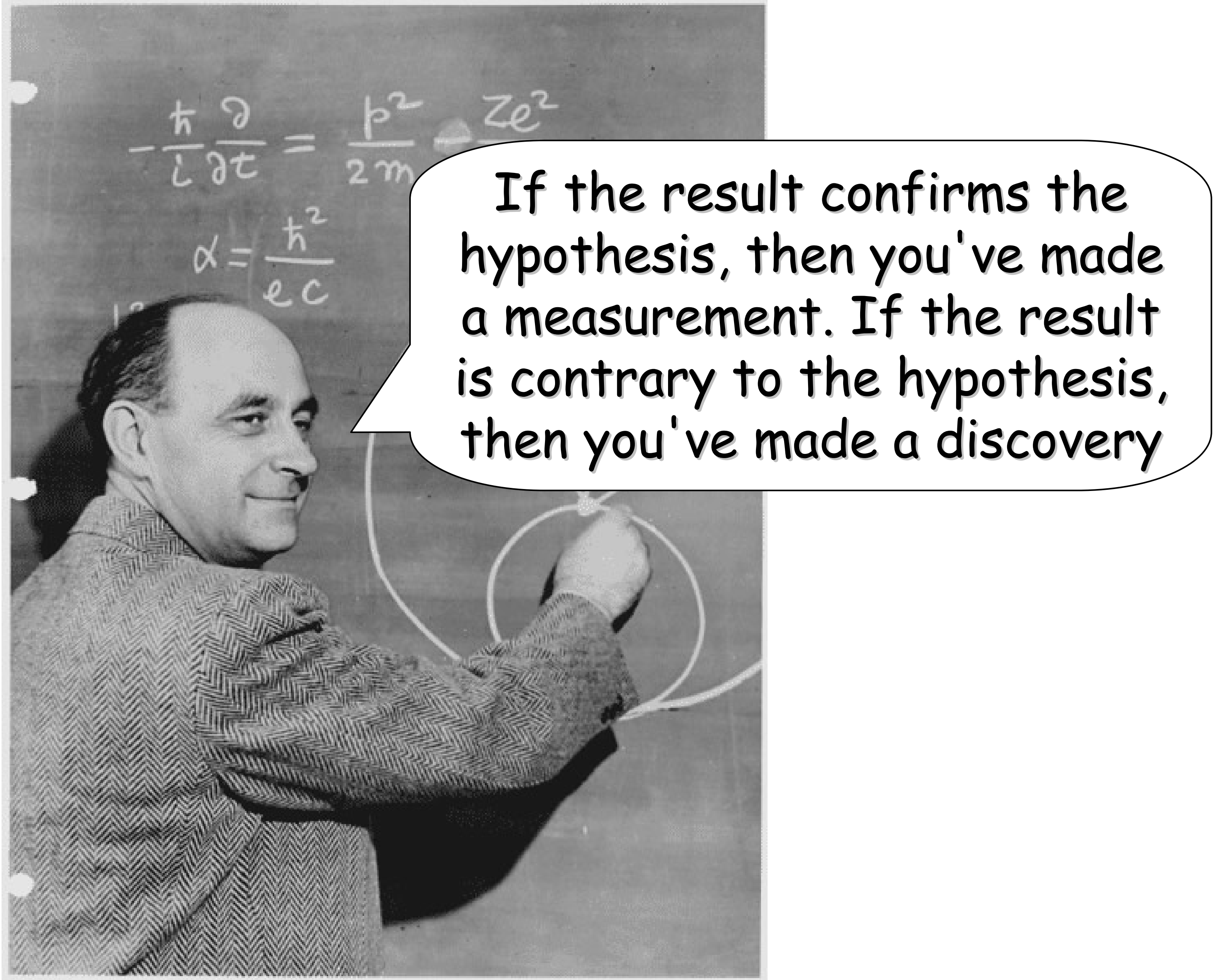
from a closed Hamiltonian system …

$$H = \underbrace{\sum_{i=1}^{N} \varepsilon_i \hat{c}_i^\dagger \hat{c}_i}_{\text{potential energy}} + \underbrace{\tfrac{1}{2} U \sum_{i=1}^{N} \hat{c}_i^{\dagger 2} \hat{c}_i^{2}}_{\text{onsite interaction}} - \underbrace{J \sum_{<i,j>} \hat{c}_j^\dagger \hat{c}_i}_{\text{tunnelling}}$$

… to an open dissipative system

tunneling of molecules




If the result confirms the hypothesis, then you've made a measurement. If the result is contrary to the hypothesis, then you've made a discovery

outline

1)
scattering theory
2)
ultracold molecules
3)
strong correlations
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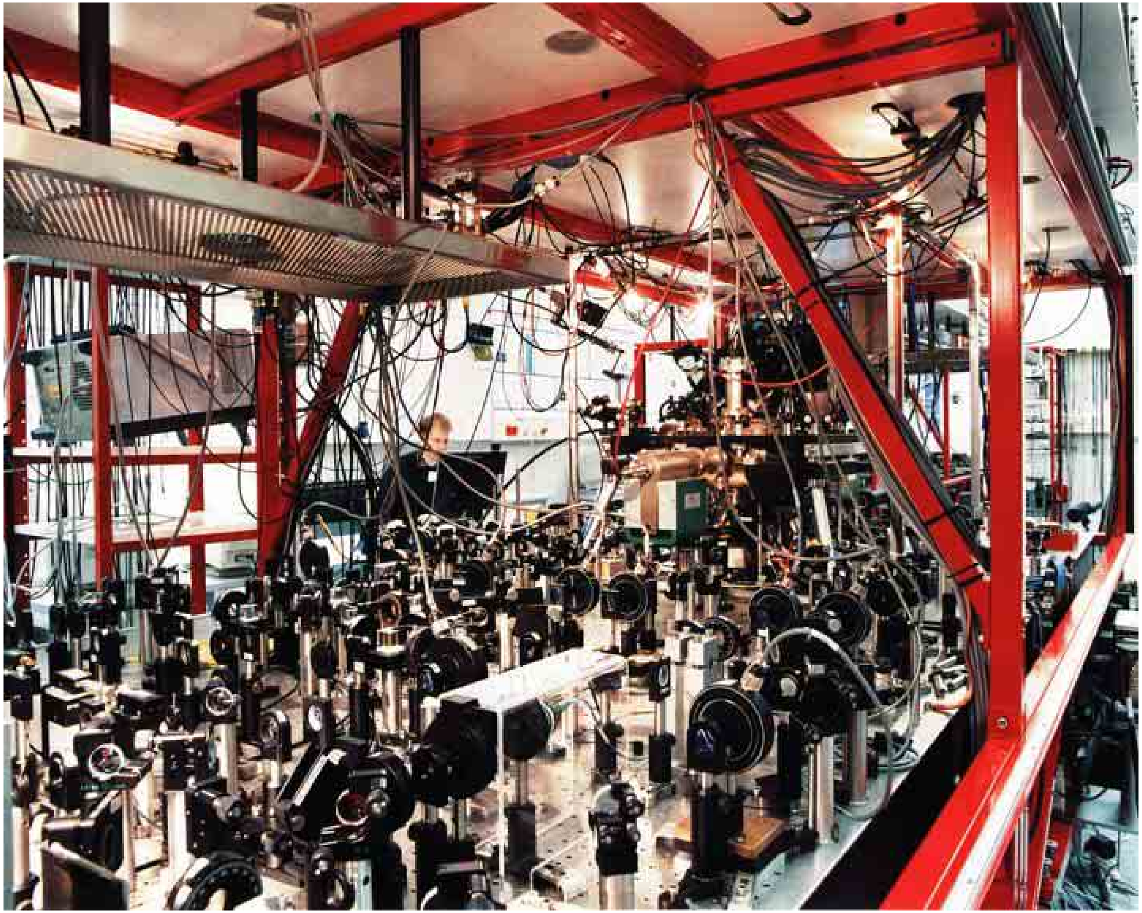
Tonks-Girardeau gas
5)
quantum Zeno effect
6)
optical control of
particle interactions



continuous measurements in quantum physics

quantum theory provides an
algorithm to calculate for
specific instants of time
the probability distributions
either for the free evolution
or a measurement (collapse),
but not both together

from free evolution ...

Misra & Sudarshan, J. Math. Phys. **18**, 756 (1977)

state preparation:

$$|\psi(t=0)\rangle = |\psi_0\rangle$$

unitary time evolution:

$$|\psi(t>0)\rangle = U(t)|\psi_0\rangle = e^{-iHt}|\psi_0\rangle$$

survival probabilty:

$$p_1(t\to 0) = \left|\langle\psi_0|U(t)|\psi_0\rangle\right|^2$$

$$= 1-\left(\langle\psi_0|H^2|\psi_0\rangle - \langle\psi_0|H|\psi_0\rangle^2\right)t^2 + \ldots$$

$$= 1-(\Delta H)^2 t^2 + \ldots = 1 - t^2/t_z^2 + \ldots$$

$$= e^{-t^2/t_z^2}$$

Zeno time $1/t_z = \Delta H$

… to repeated observations

Misra & Sudarshan, J. Math. Phys. **18**, 756 (1977)

during time interval $[0,T]$, perform $N = T/\tau$ measurements every τ seconds, i.e. at times

T/N, $2T/N$, ..., $(N-1)T/N$, T

survival probability after N measurements:

$$p_N(T) = p_1(\tau)^N = e^{N \ln p_1(\tau)} = e^{-N\tau^2/t_Z^2} = e^{-T^2/N t_Z^2}$$

continuous observation ($N \to \infty$):

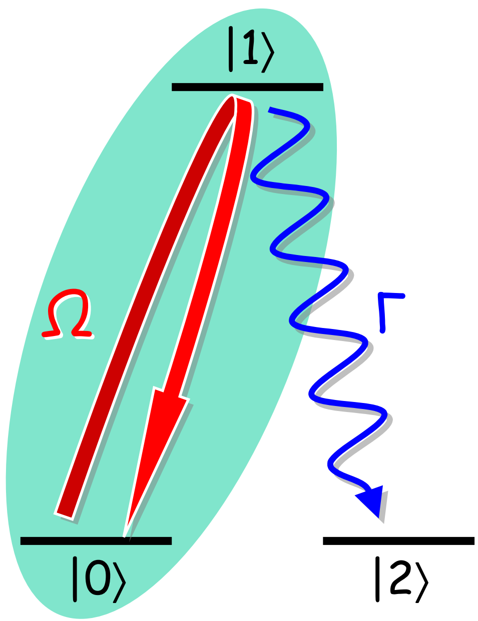
$$p_N(T) = 1$$

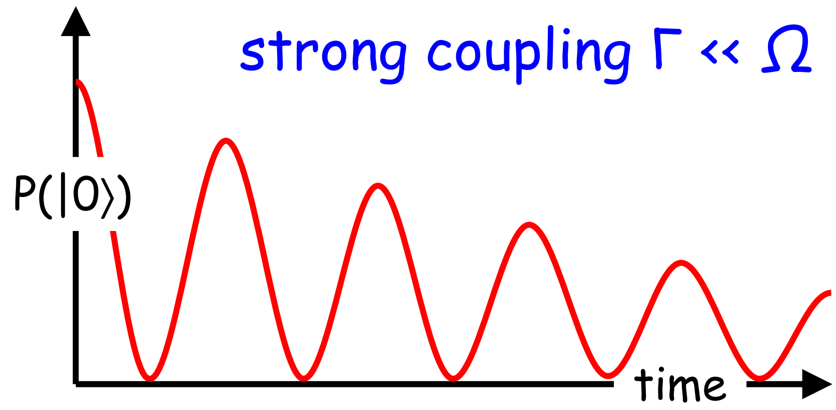
an observed system never evolves: Zeno effect

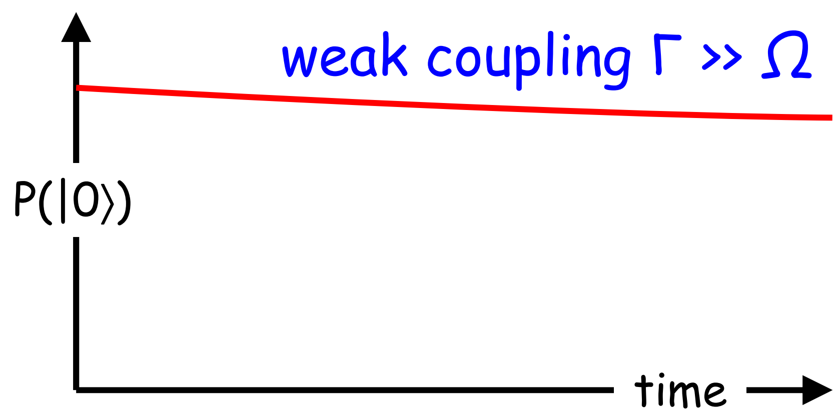
open quantum system

Itano et al., Phys. Rev. A **41**, 2295 (1990)

- internal dynamics Ω
- external damping Γ





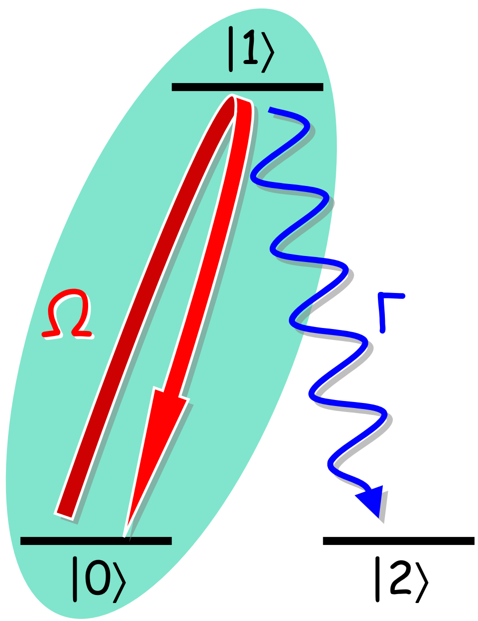


effective decay rate

Ω^2/Γ

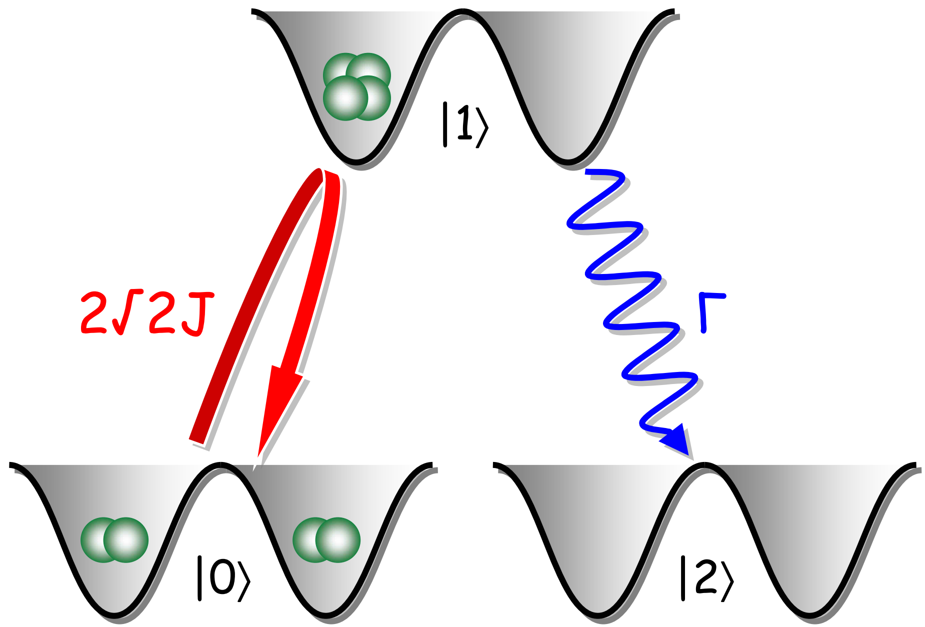
open quantum system

- internal dynamics Ω
- external damping Γ



effective decay rate
Ω^2/Γ

- tunneling $2J$
- Bose enhancement $\sqrt{2}$
- onsite loss rate Γ

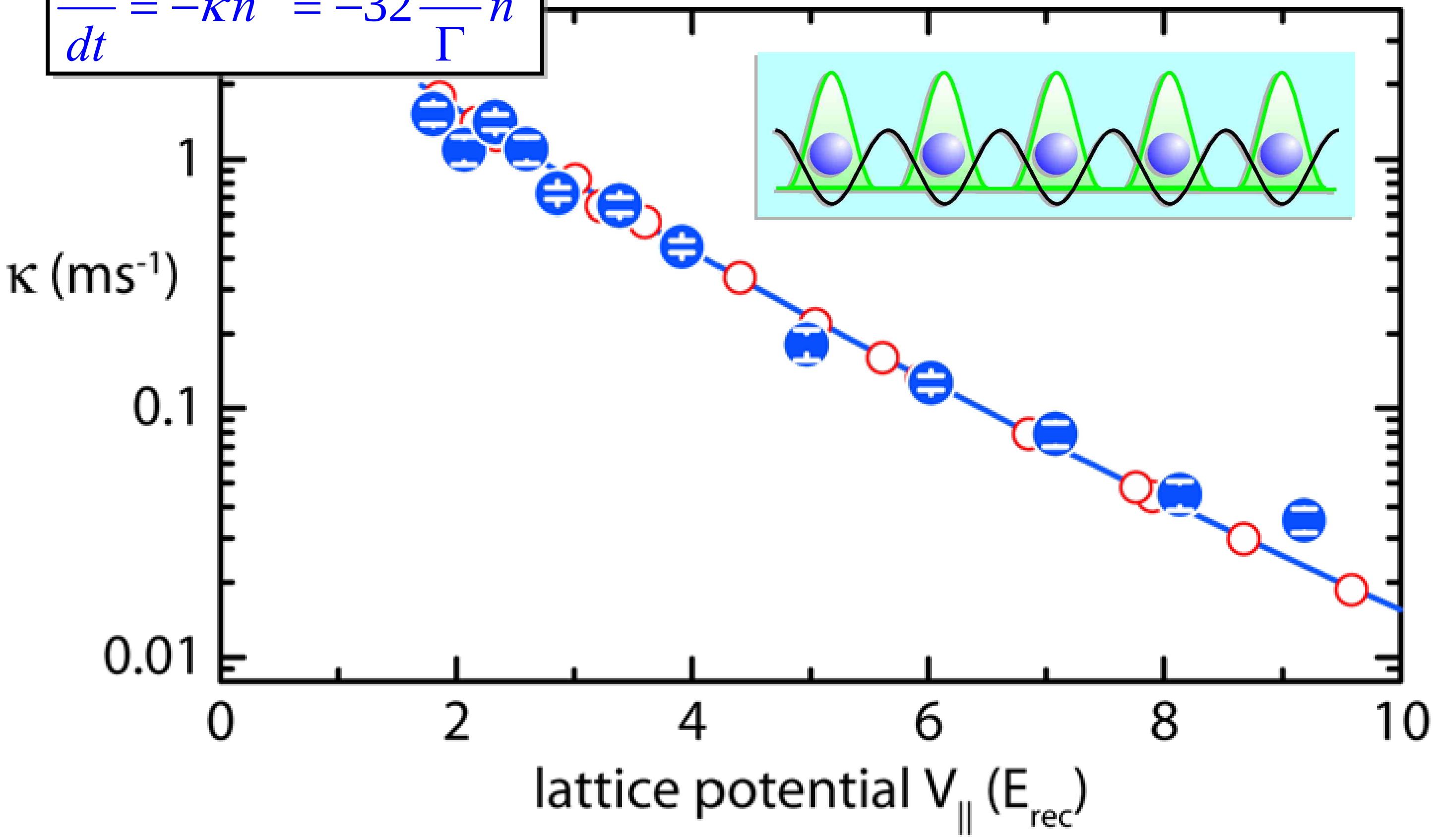


periodic system:
effective decay rate
$\kappa = 8J^2/\Gamma$
$32J^2/\Gamma$

tunneling suppressed by dissipation

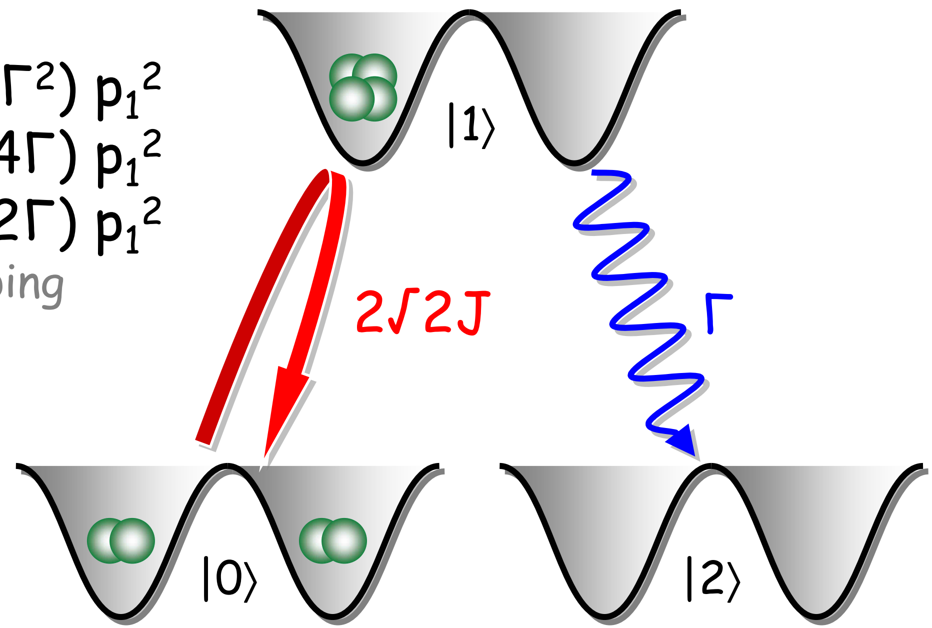
Syassen et al., Science **320**, 1329 (2008)

$$\frac{dn}{dt} = -\kappa n^2 = -32\frac{J^2}{\Gamma}n^2$$



pair correlation

probability $p_2 = (8J^2/\Gamma^2)\, p_1^2$
$= (\kappa/4\Gamma)\, p_1^2$
$= (\kappa/2\Gamma)\, p_1^2$
right or left hopping



probability p_1^2

pair correlation:

$$g^{(2)}(0) = \frac{\langle n(n-1)\rangle}{\langle n\rangle^2} = \frac{0\times(-1)\times p_0 + 1\times 0\times p_1 + 2\times 1\times p_2 + \ldots}{(0\times p_0 + 1\times p_1 + \ldots)^2} = \frac{2p_2}{p_1^2} = \frac{\kappa}{\Gamma}$$

wanted: dissipation

correlated state versus superfluid state

correlated state $(g^{(2)}<1)$: $$\frac{dn}{dt} = -\kappa n^2 = -32\frac{J^2}{\Gamma}n^2$$

decay suppressed by onsite loss

superfluid state $(g^{(2)}=1)$: $$\frac{dn}{dt} = -\Gamma n^2$$

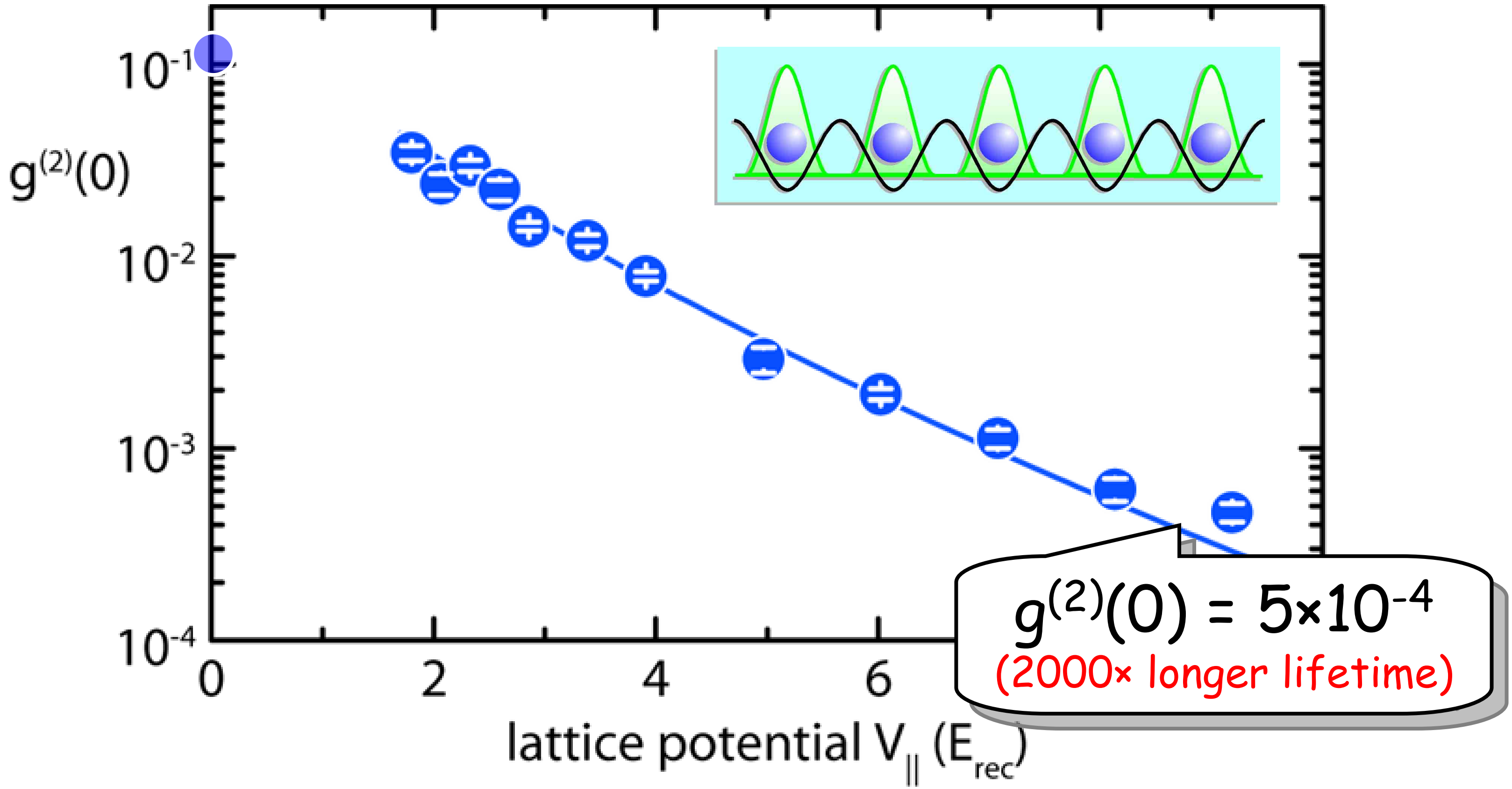
decay due to onsite loss

$\Rightarrow$ pair correlation: $$g^{(2)}(0) = \frac{\kappa}{\Gamma} = 32\frac{J^2}{\Gamma^2}$$

wanted: dissipation

giant (anti-) correlations

Syassen et al., Science **320**, 1329 (2008)




Matthias Lettner
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theory:
J Ignacio Cirac
JJ García-Ripoll
technique:
Thomas Volz
Dominik Bauer
Daniel Dietze

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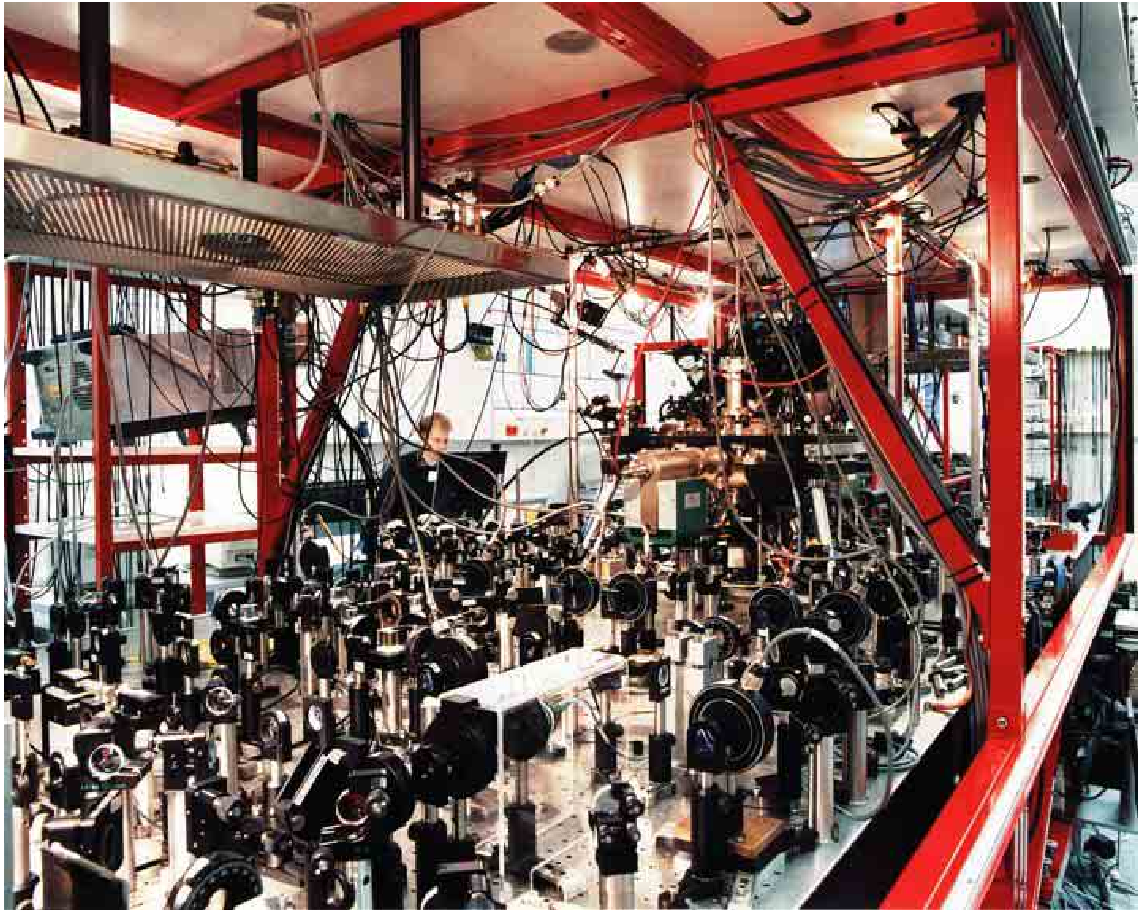
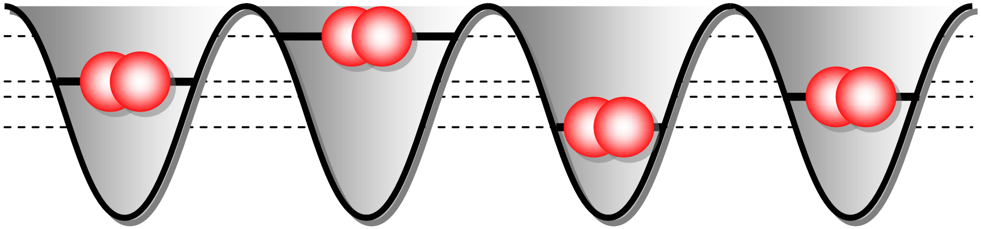


a dream to come true ?

imagine one could spatially control/address
the interaction strength ... :

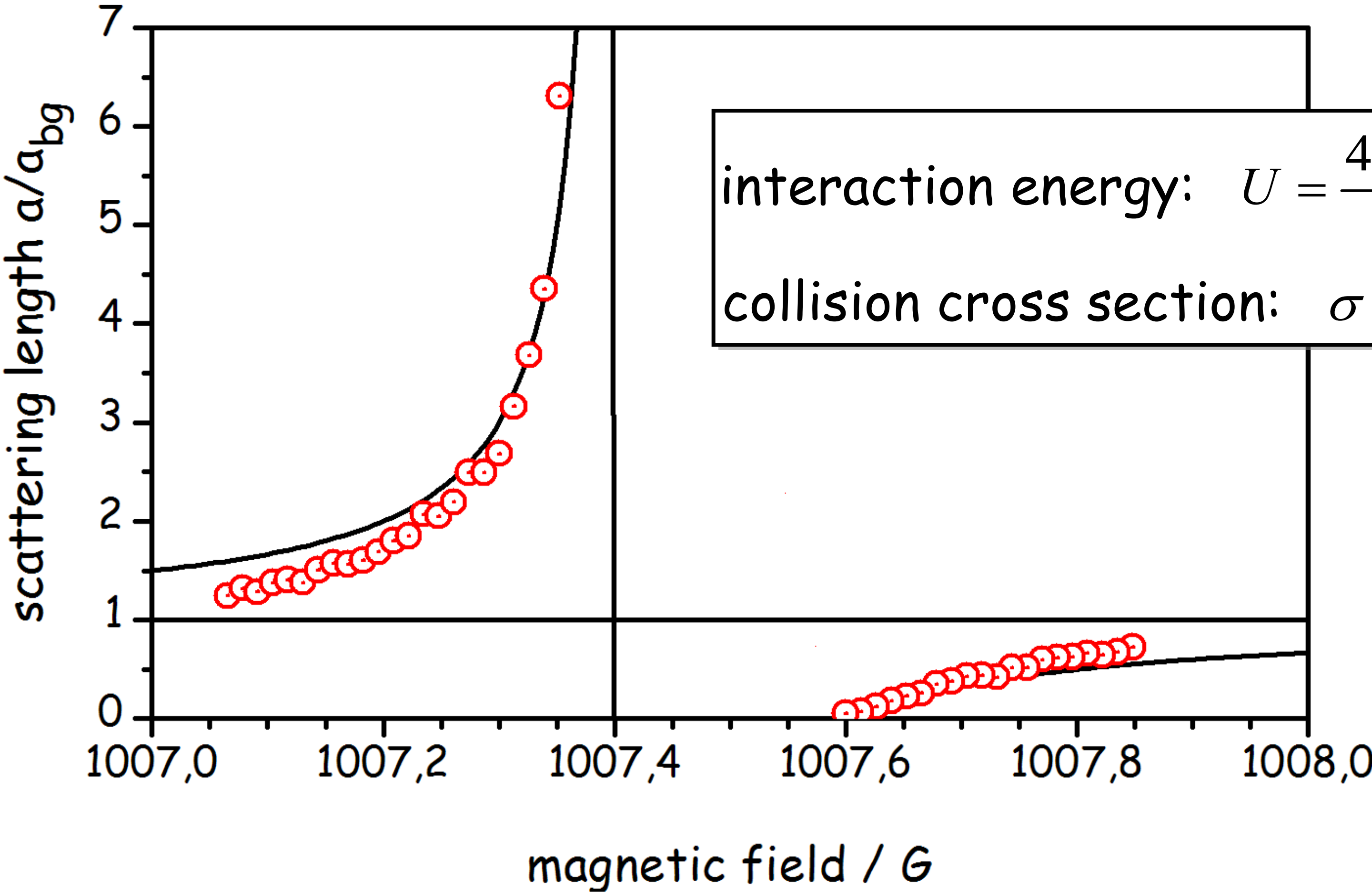


transport properties (sound, localization, ...)
simulations (superlattice, sonic black holes, ...)

... ...

magnetic Feshbach resonance

Volz et al., Phys. Rev. A **68**, 010702(R) (2003)



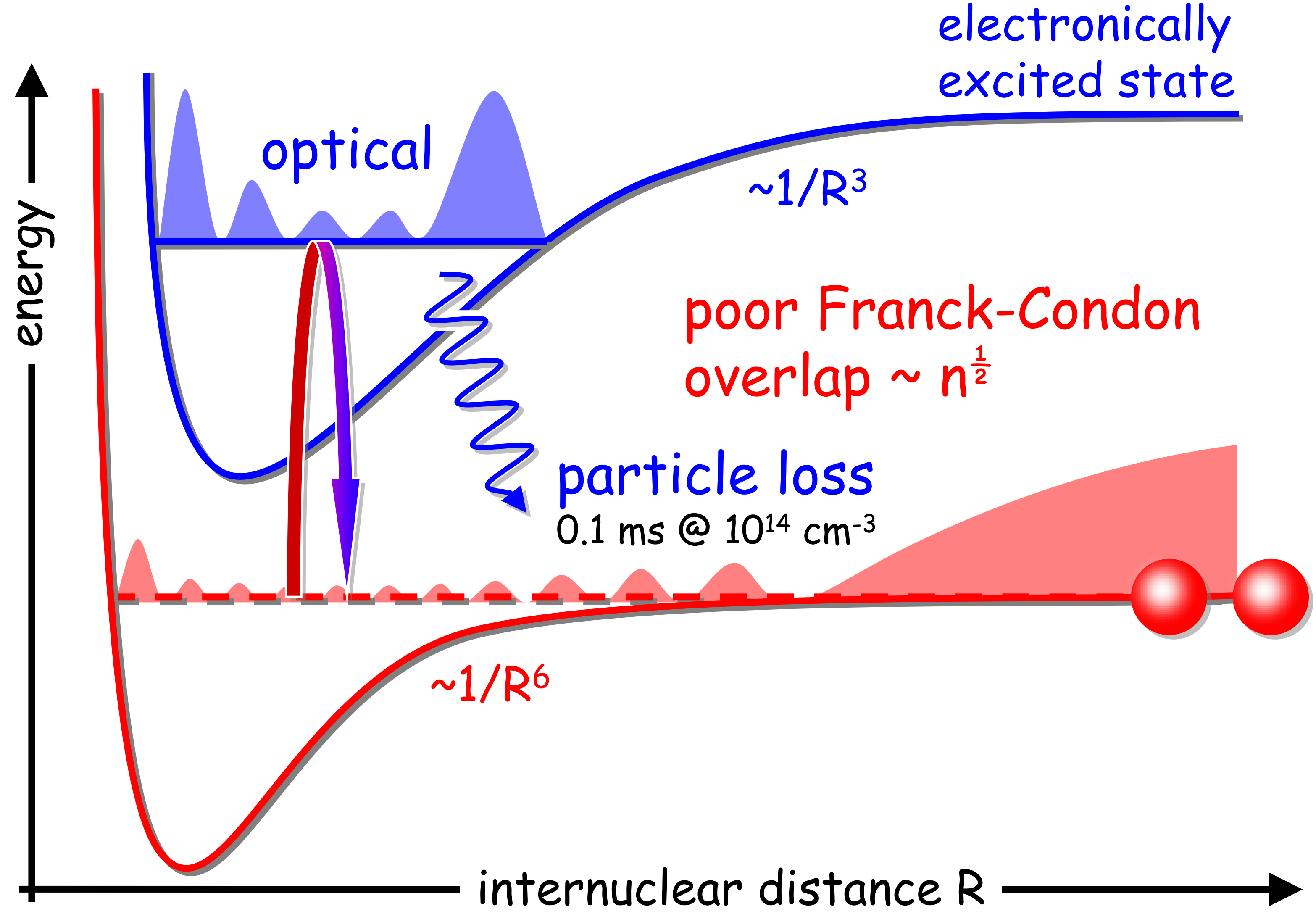
interaction energy: $U = \frac{4\pi\hbar^2 a}{m}|\psi|^2$

collision cross section: $\sigma = 8\pi a^2$

control of interaction is possible, but only global

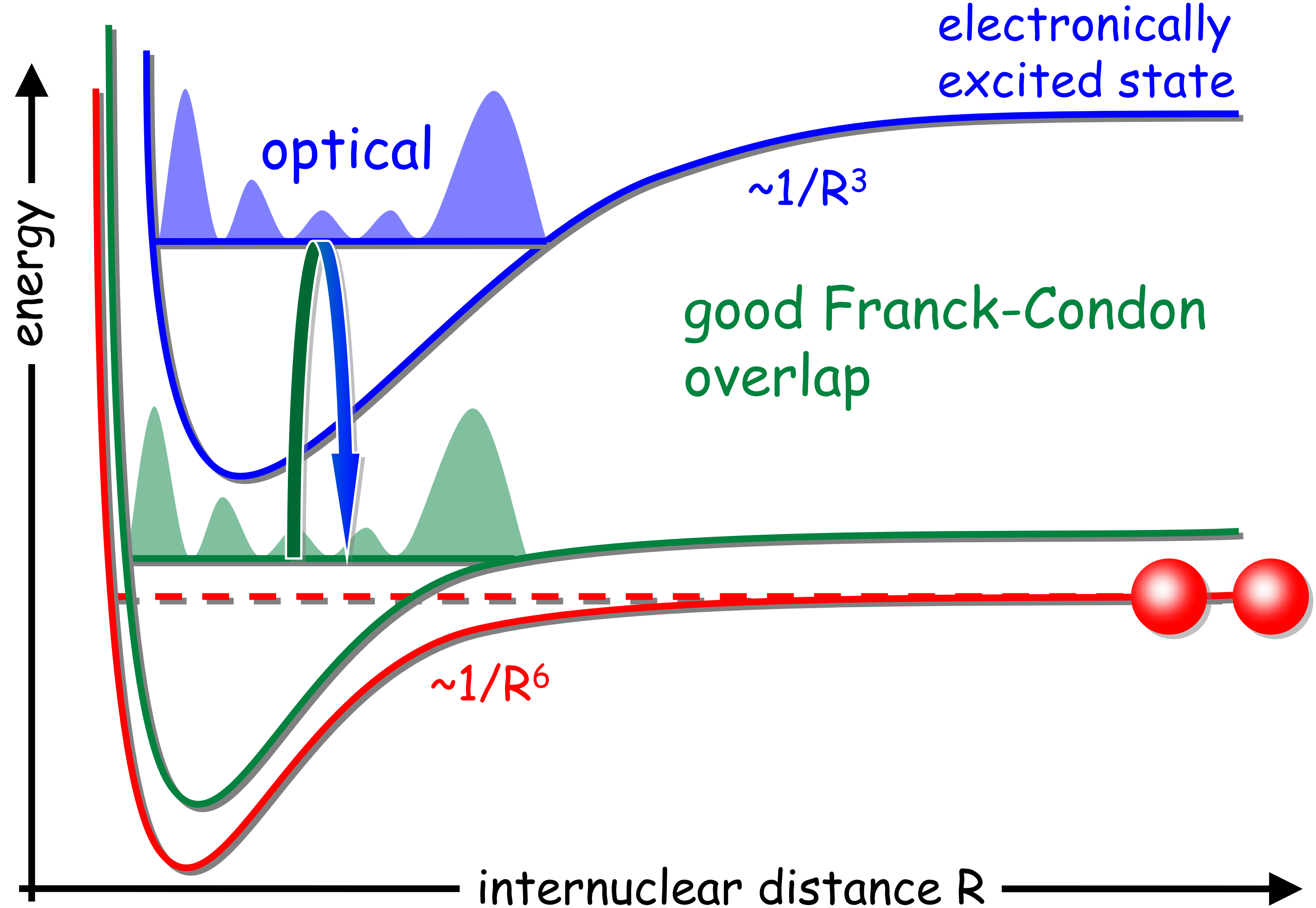
optical Feshbach resonance

Theis et al., Phys. Rev. Lett. **93**, 123001 (2004)



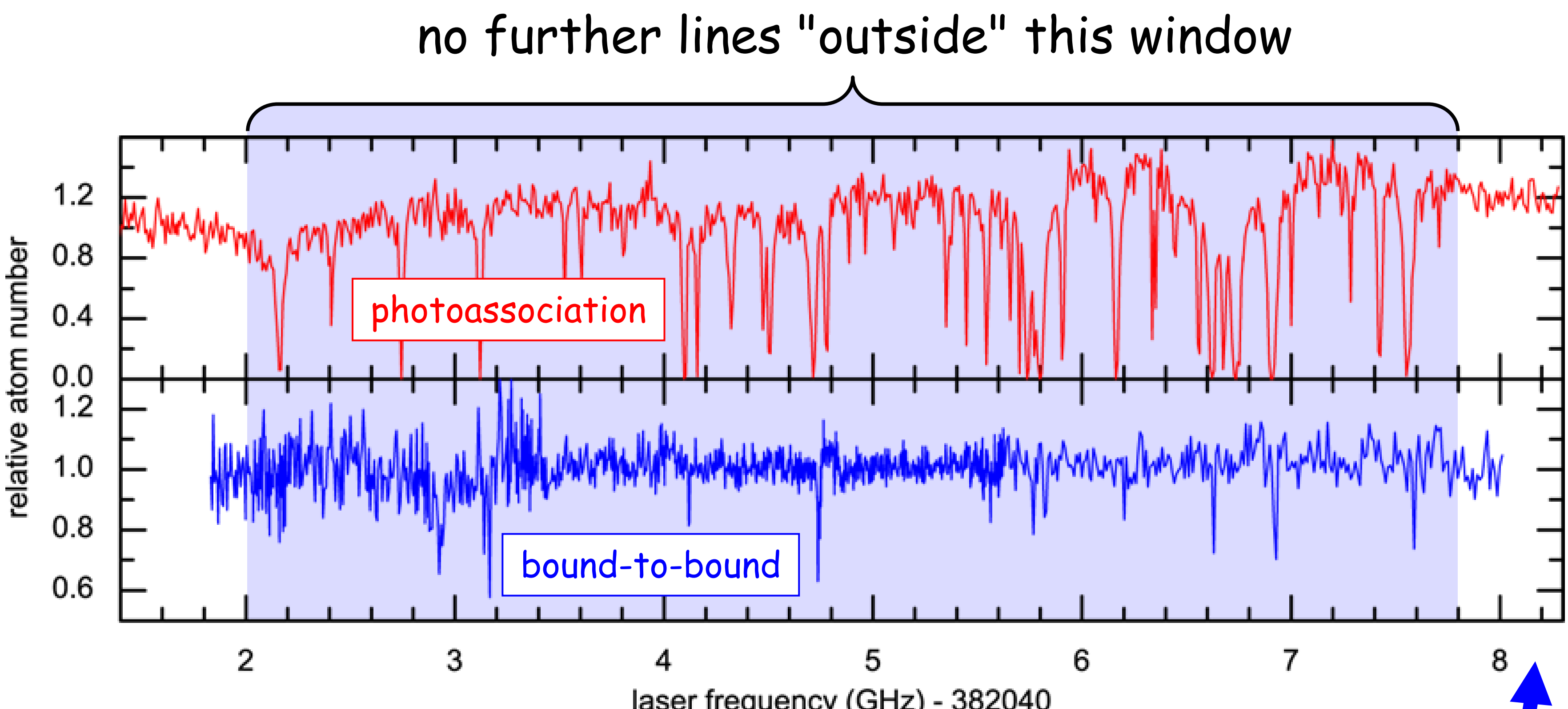
optically controlled magnetic Feshbach resonance

Bauer et al., Nature Phys. **5**, 339 (2009)



molecular lines @ Feshbach resonance (1007 G)

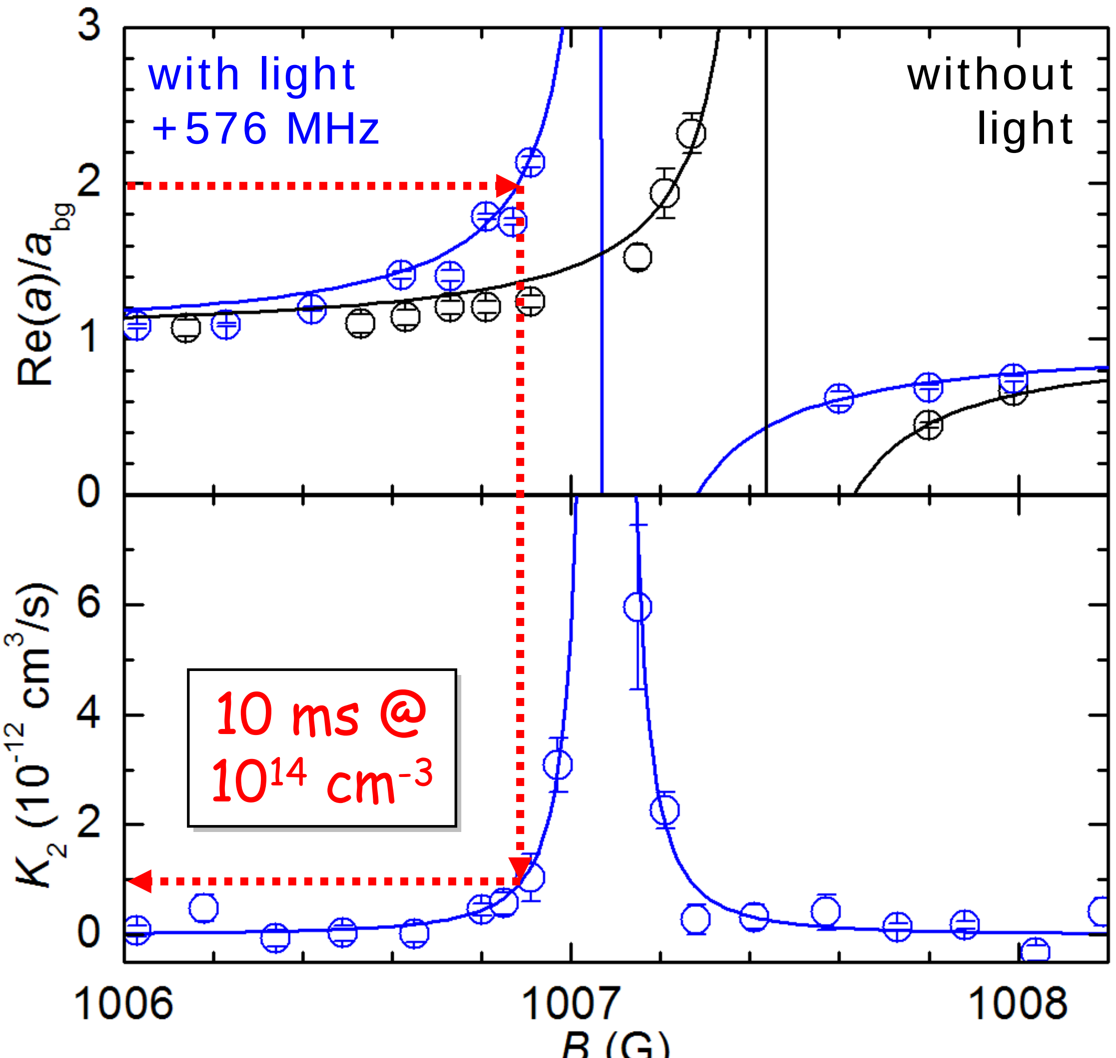
Bauer et al., Phys. Rev. A **79**, 062713 (2009)



laser blue detuned from
all molecular transitions

detuned laser: dynamic Stark shift

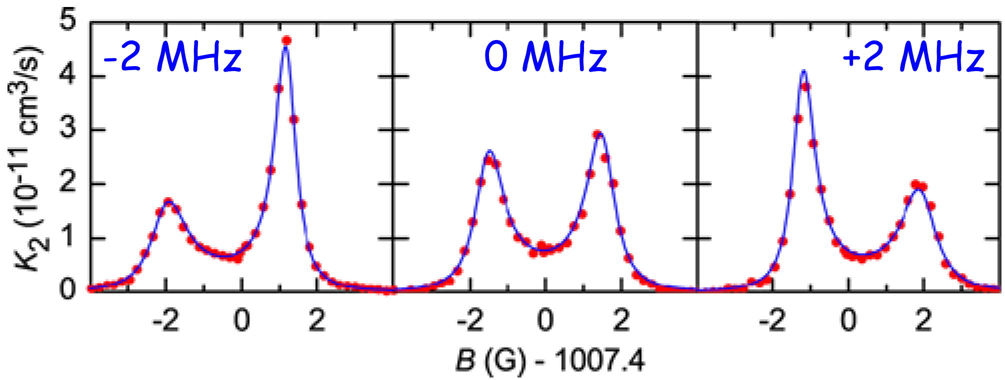
Bauer et al., Nature Phys. **5**, 339 (2009)

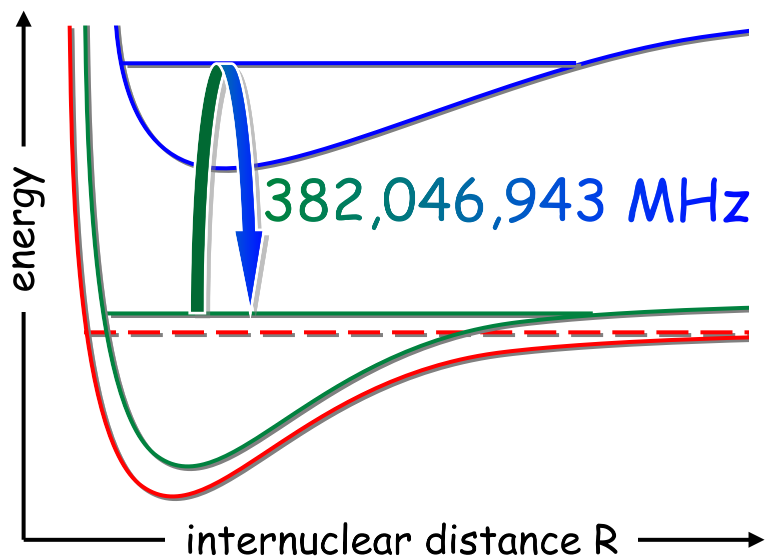


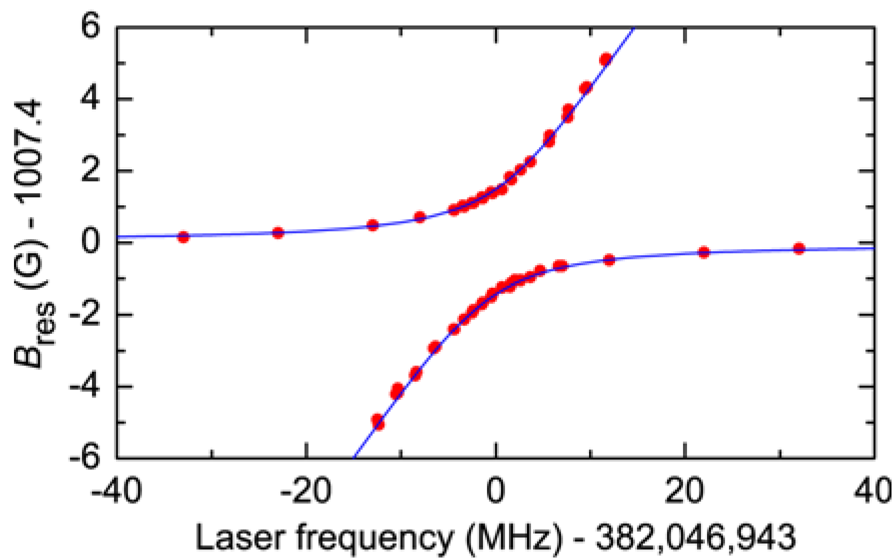
resonant laser: Autler-Townes splitting

Bauer et al., Nature Phys. **5**, 339 (2009)

'weak' magnetic Feshbach resonance as a probe of a 'strong' optical transition





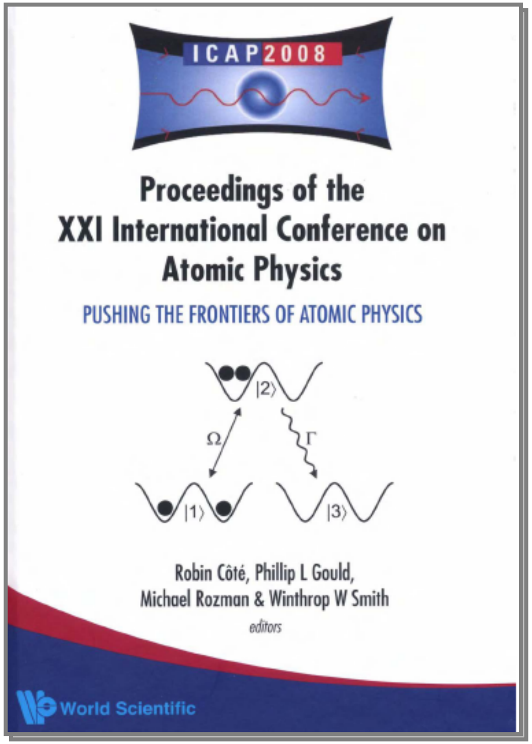


→ all relevant parameters of the molecular transition

quantum optics and many-body physics

quantum optics provides new ideas beyond simulation of 'conventional' many-body systems:

- dissipative Tonks-Girardeau gas
- Zeno effect in an optical lattice
  → suppression of tunneling
- optical control of interactions
  → spatial addressability




thanks to a great team …
Dominik Bauer
Matthias Lettner
Stephan Dürr
Chris Vo
… and you for your attention

thank you for your attention

PRA **63**, 043613 (2001)
PRL **87**, 170404 (2001)
PRL **89**, 283202 (2002)
PRA **68**, 010702(R) (2003)
PRL **92**, 020406 (2004)
PRA **70**, 031601(R) (2004)
PRA **72**, 010704(R) (2005)
PRA **72**, 052707 (2005)
Nature Phys **2**, 692 (2006)
PRA **74**, 062706 (2006)
PRA **75**, 020702(R) (2007)
PRL **99**, 033201 (2007)
Science **320**, 1329 (2008)
NJP **11**, 013053 (2009)
PRA **79**, 023614 (2009)
Nature Phys **5**, 339 (2009)
PRA **79**, 062713 (2009)