100 Years of General Relativity

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Twenty-sixth Arnold Sommerfeld Lecture Series
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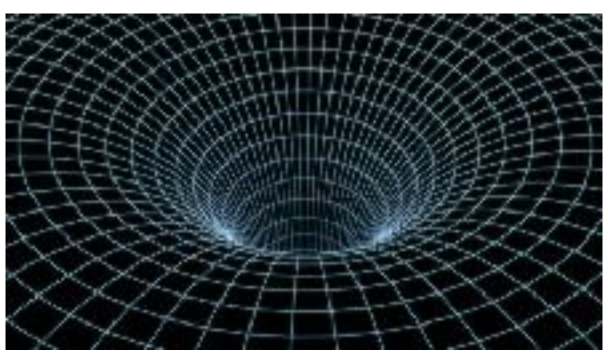
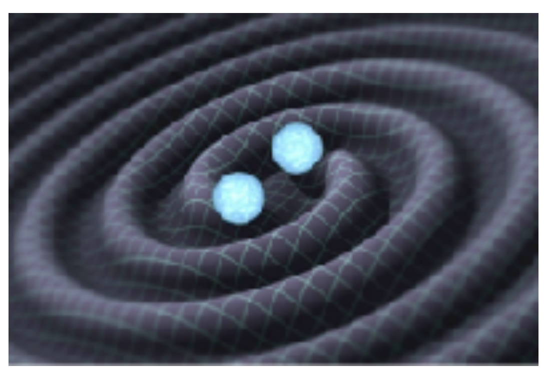
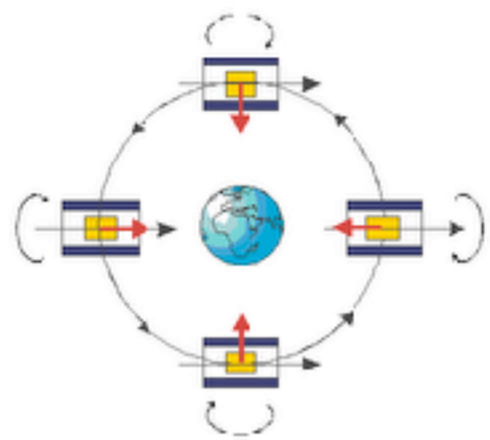
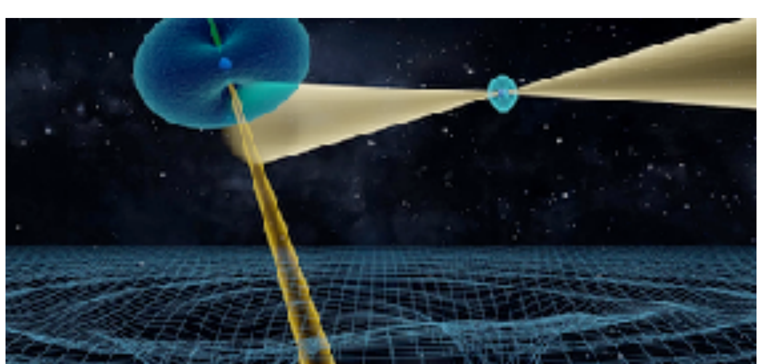
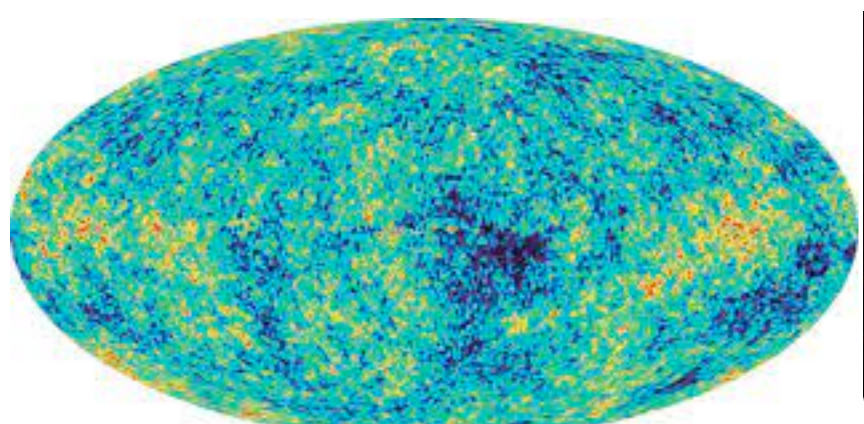
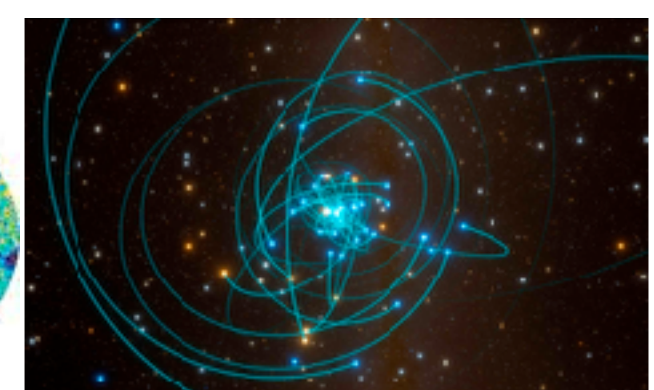
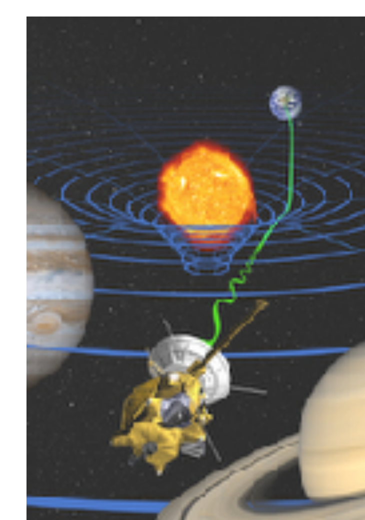
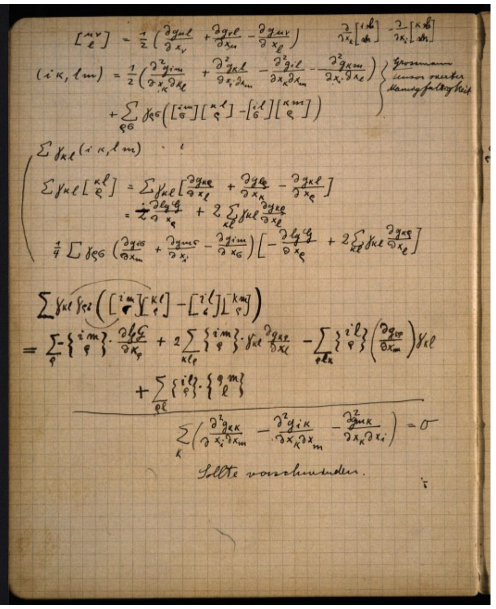
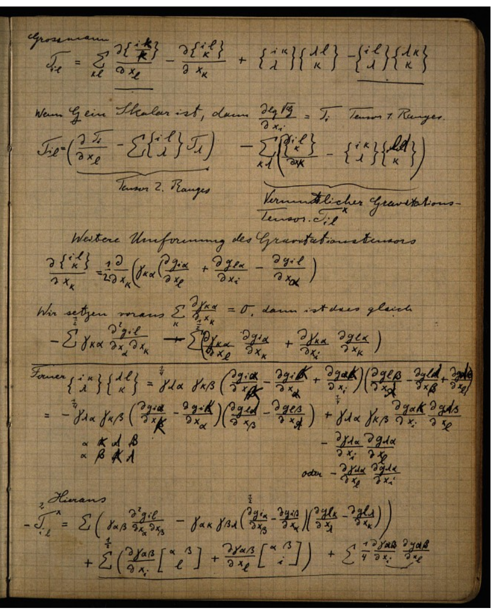
Ludwig Maximilians Universität, Munich

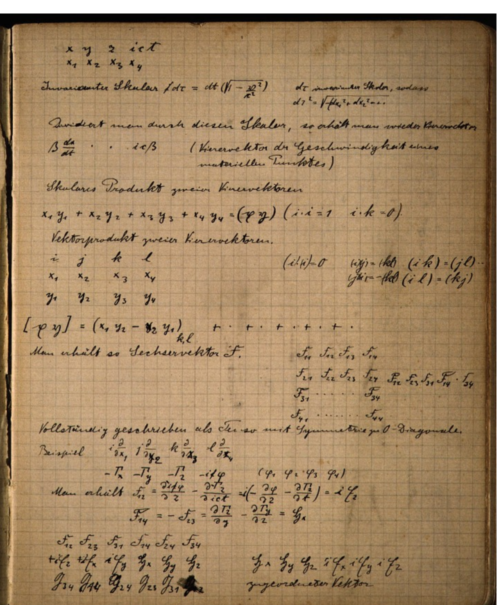




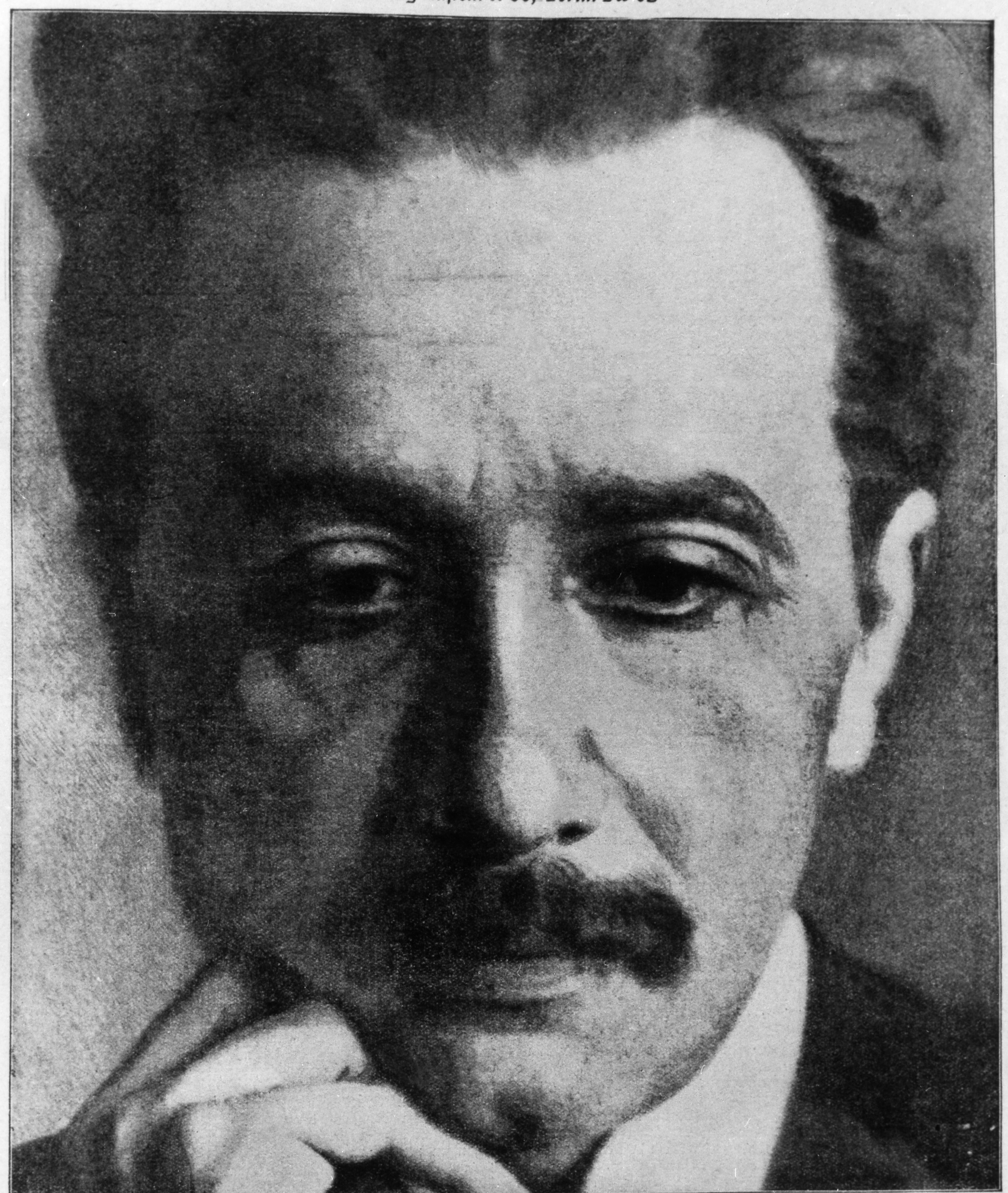






Novembre 1915









14. Dezember 1919
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Phot. Suse Byk.

Eine neue Größe der Weltgeschichte: Albert Einstein, dessen Forschungen eine völlige Umwälzung unserer Naturbetrachtung bedeuten und den Erkenntnissen eines Kopernikus, Kepler und Newton gleichwertig sind.

DOC. 25 FIELD EQUATIONS OF GRAVITATION 245

844 Sitzung der physikalisch-mathematischen Klasse vom 25. November 1915

Die Feldgleichungen der Gravitation.

Von A. Einstein.

In zwei vor kurzem erschienenen Mitteilungen[1] habe ich gezeigt, wie man zu Feldgleichungen der Gravitation gelangen kann, die dem Postulat allgemeiner Relativität entsprechen, d. h. die in ihrer allgemeinen Fassung beliebigen Substitutionen der Raumzeitvariabeln gegenüber kovariant sind.

Der Entwicklungsgang war dabei folgender. Zunächst fand ich Gleichungen, welche die Newtonsche Theorie als Näherung enthalten und beliebigen Substitutionen von der Determinante 1 gegenüber kovariant waren. Hierauf fand ich, daß diesen Gleichungen allgemein kovariante entsprechen, falls der Skalar des Energietensors der »Materie« verschwindet. Das Koordinatensystem war dann nach der einfachen Regel zu spezialisieren, daß $\sqrt{-g}$ zu 1 gemacht wird, wodurch die Gleichungen der Theorie eine eminente Vereinfachung erfahren. Dabei mußte aber, wie erwähnt, die Hypothese eingeführt werden, daß der Skalar des Energietensors der Materie verschwinde.

Neuerdings finde ich nun, daß man ohne Hypothese über den Energietensor der Materie auskommen kann, wenn man den Energietensor der Materie in etwas anderer Weise in die Feldgleichungen einsetzt, als dies in meinen beiden früheren Mitteilungen geschehen ist. Die Feldgleichungen für das Vakuum, auf welche ich die Erklärung der Perihelbewegung des Merkur gegründet habe, bleiben von dieser Modifikation unberührt. Ich gebe hier nochmals die ganze Betrachtung, damit der Leser nicht genötigt ist, die früheren Mitteilungen [2]

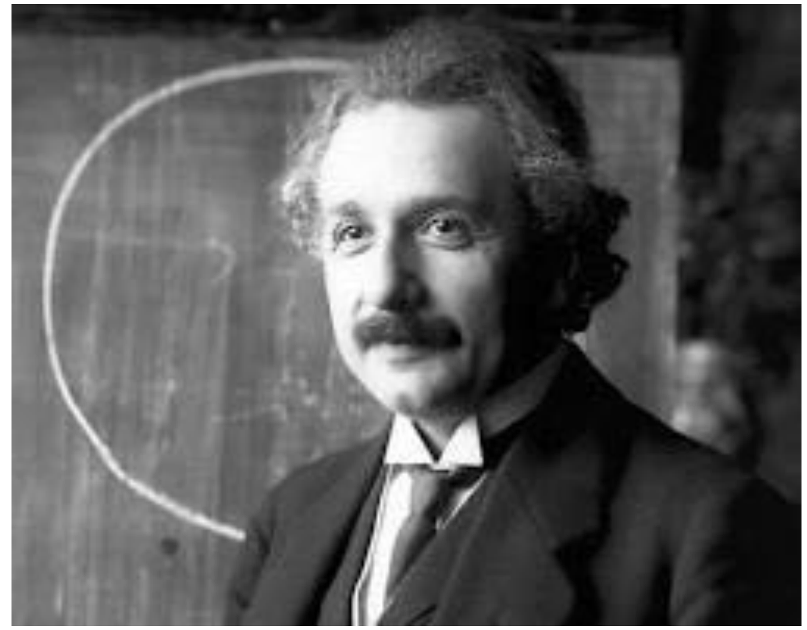
$$G_{im} = R_{im} + S_{im}$$

$$R_{im} = -\sum_l \frac{\partial \begin{Bmatrix} im \\ l \end{Bmatrix}}{\partial x_l} + \sum_{l\rho} \begin{Bmatrix} il \\ \rho \end{Bmatrix} \begin{Bmatrix} m\rho \\ l \end{Bmatrix}$$

$$S_{im} = \sum_l \frac{\partial \begin{Bmatrix} il \\ l \end{Bmatrix}}{\partial x_m} - \sum_{l\rho} \begin{Bmatrix} im \\ \rho \end{Bmatrix} \begin{Bmatrix} \rho l \\ l \end{Bmatrix}$$

The Space-Time Block

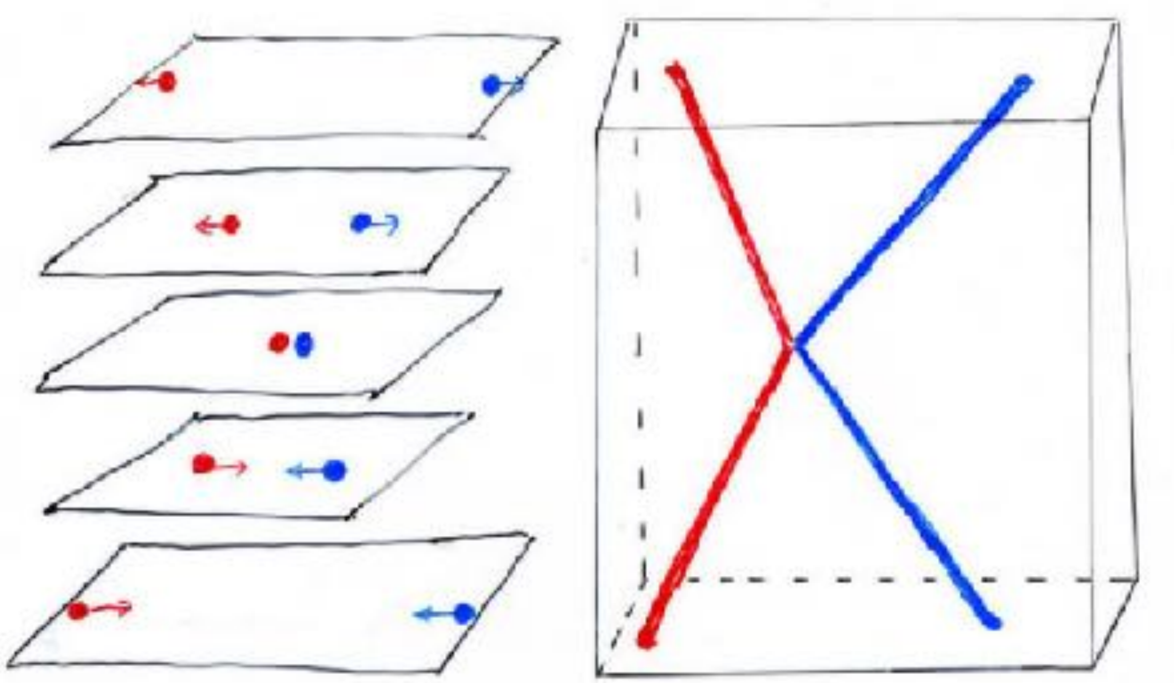
H. Poincaré

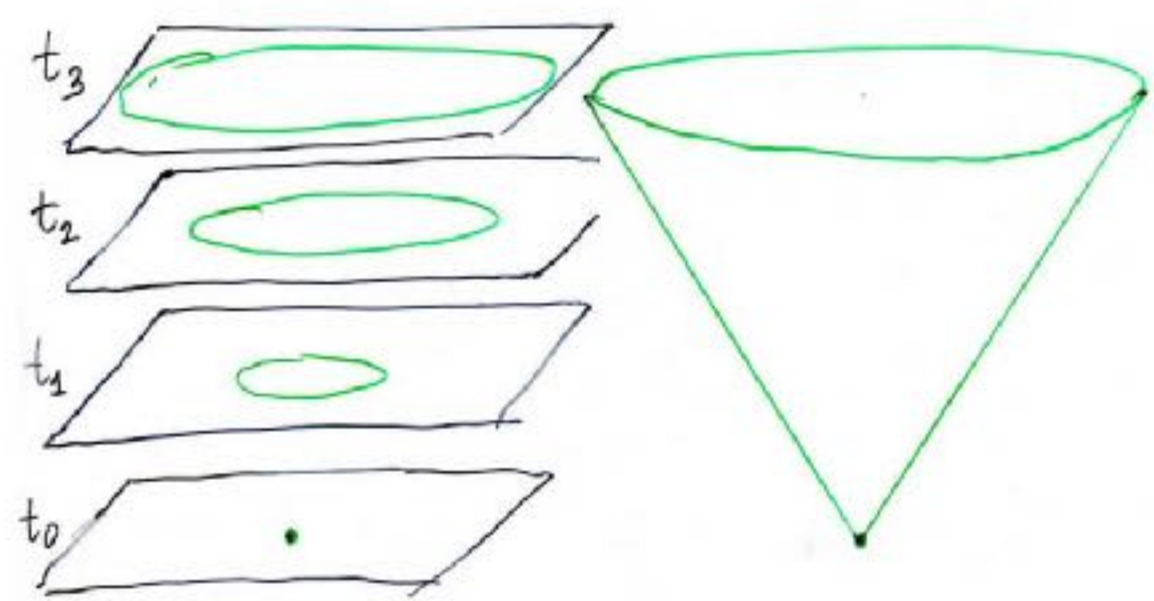


A. Einstein

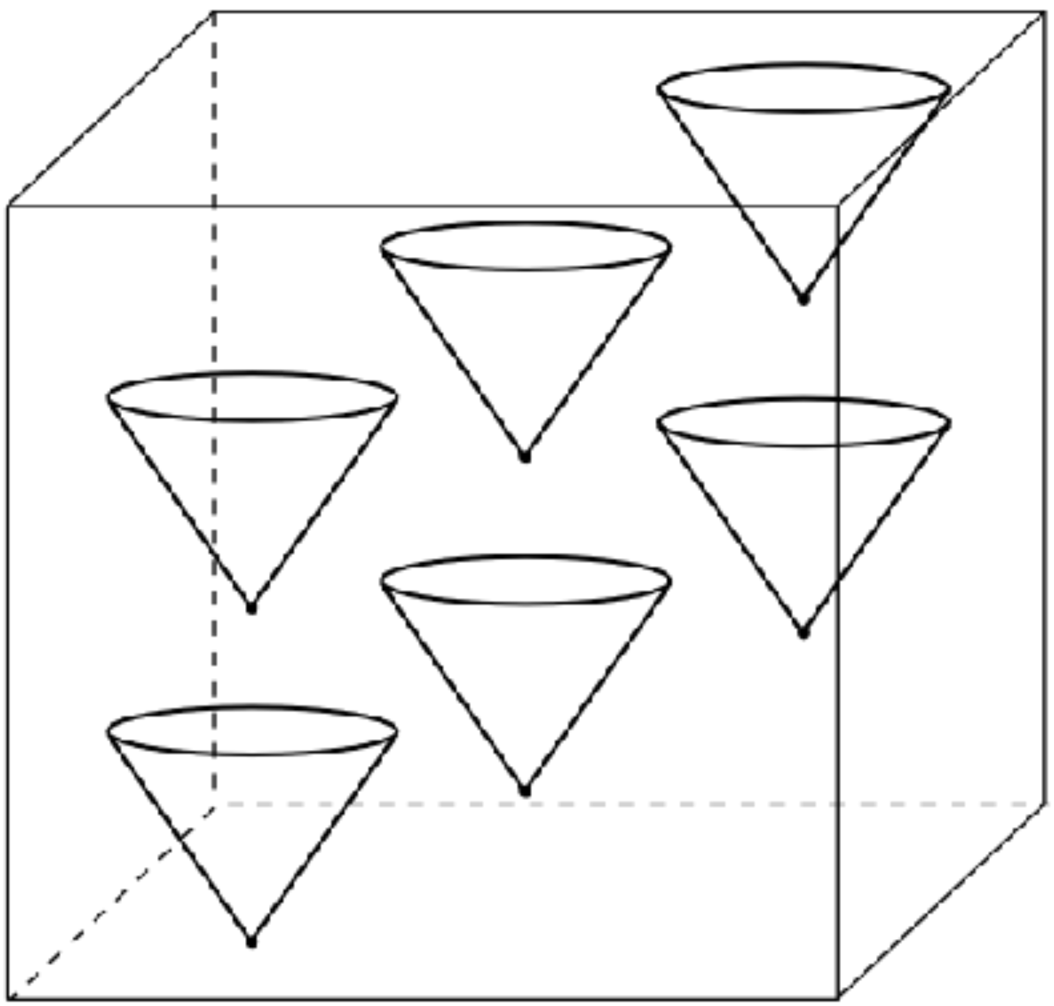
H. Minkowski

SPACE + TIME → SPACETIME

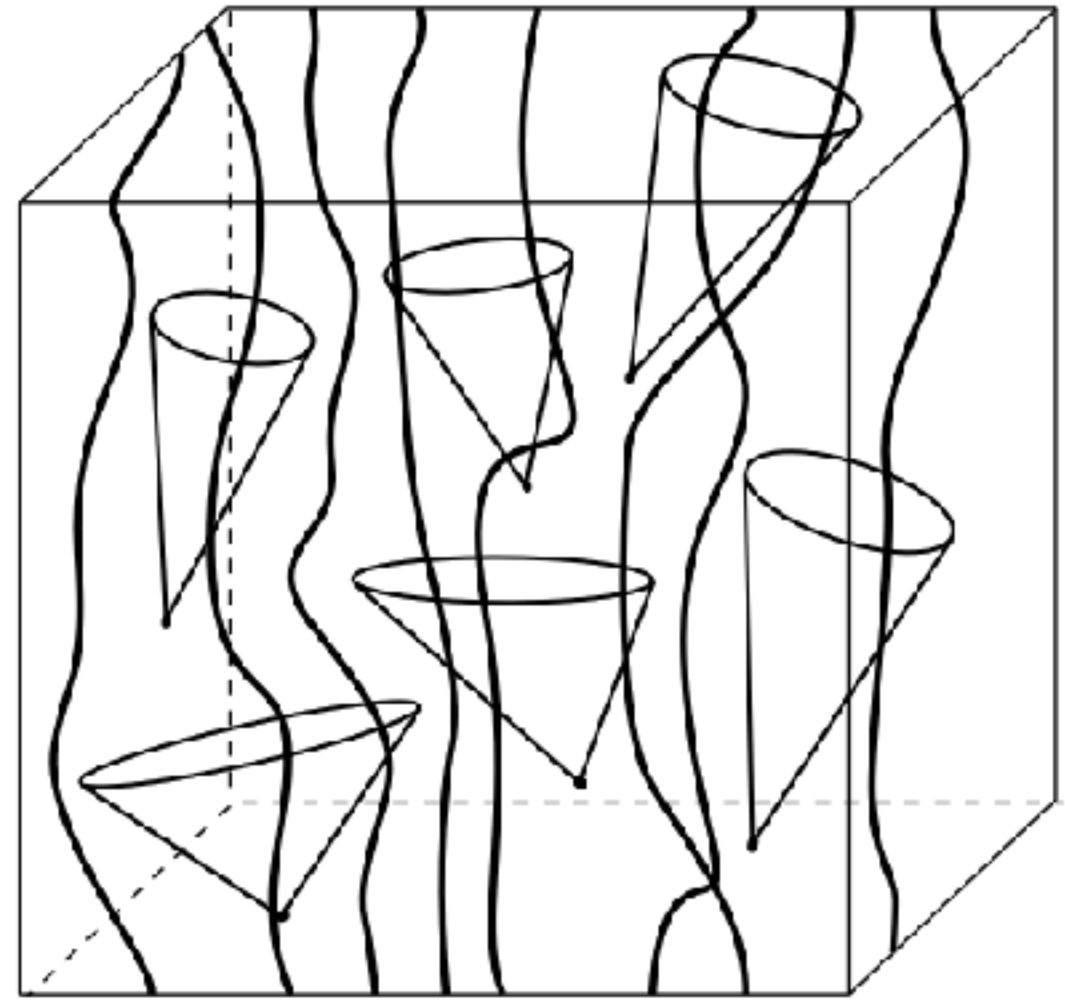




Special Relativity



General Relativity



$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$$
$$= \eta_{\mu\nu}dx^\mu dx^\nu$$

$$ds^2 = g_{\mu\nu}(x^\lambda)\,dx^\mu dx^\nu$$

$$R_{\mu\nu} - \frac{1}{2}R\,g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

Explicit form of Einstein's equations

(in harmonic coordinates)

$$R_{\mu\nu} = \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{D-2} T\, g_{\mu\nu}\right)$$

$$-g^{\mu\nu} g_{\alpha\beta,\,\mu\nu} + g^{\mu\nu} g^{\rho\sigma} (g_{\alpha\mu,\,\rho} g_{\beta\nu,\,\sigma} - g_{\alpha\mu,\,\rho} g_{\beta\sigma,\,\nu}$$

$$+ g_{\alpha\mu,\,\rho} g_{\nu\sigma,\,\beta} + g_{\beta\mu,\,\rho} g_{\nu\sigma,\,\alpha} - \frac{1}{2} g_{\mu\rho,\,\alpha} g_{\nu\sigma,\,\beta})$$

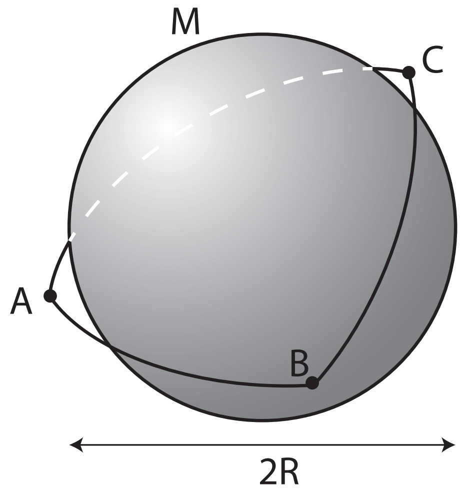
$$= \frac{16\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{D-2} T\, g_{\mu\nu}\right)$$

$$ds^2 = g_{\mu\nu}(x^\lambda)\, dx^\mu dx^\nu \qquad R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Einstein's Theory in one sentence:

Space-Time is an elastic structure which is deformed by the presence, within it, of Mass-Energy

Space = Jelly



$$\widehat{A} + \widehat{B} + \widehat{C} \approx \pi(1 + \frac{2GM}{c^2 R})$$

Brief history of General Relativity

1915-1930: Blossoming

Perihelion of Mercury'15, « Schwarzschild Black Hole»'16, Solar system (De Sitter'16), Relativistic Cosmology (Einstein'17,Friedmann'22, Lemaître'27,Hubble'29), Deflectionion of light rays'19, Generalisations (Weyl, Eddington,Cartan, Einstein), Exact solutions (Weyl, Levi-Civita),Kaluza-Klein,…

1930-1960: Eclipse of GR (Quantum M.+ lack of observational results)

but: dark matter (Zwicky), black hole (Oppenheimer-Snyder'39), Hot Big Bang (Gamow, Alpher, Herman), Y. Choquet-Bruhat

1960-1974: First golden era (Pound-Rebka, Quasars, Pulsars,Penzias-Wilson, Dicke, Nordtvedt, Shapiro, Bondi,Trautman, Weber, Kerr, Regge-Wheeler, Zeldovich, Penrose,…)

1974-now: Second golden era (theoretical et experimental):

Gravitation and string theory (Scherk-Schwarz'74), Quantum Evaporation of black holes (Hawking74), Supergravity (76 Freedman-vanNieuwenhuizen-Ferrara; Deser-Zumino), Superstrings (Green-Schwarz82), inflationary cosmologies '80

Binary Pulsar (Hulse-Taylor74), Binary X ray Sources, Gamma Ray Bursts, Dark matter(Rubin-Ford'80), COBE-Planck, Cosmic Acceleration, Lunar-Laser-Ranging, Cassini, gravit. lenses, SgrA*, LIGO-Virgo, Microscope, Event Horizon Telescope,….

22 décembre 1915

Karl Schwarzschild
decembre 1915-
january 1916

Schwarzschild: Über das Gravitationsfeld eines Massenpunktes 189

Sitzber. Deut. Akad. Wiss. (1916)

Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie.

Von K. Schwarzschild.

(Vorgelegt am 13. Januar 1916 [s. oben S. 42].)

§ 1. Hr. Einstein hat in seiner Arbeit über die Perihelbewegung des Merkur (s. Sitzungsberichte vom 18. November 1915) folgendes Problem gestellt:

Ein Punkt bewege sich gemäß der Forderung

$$\delta \int ds = 0,$$

wobei

$$ds = \sqrt{\sum g_{\mu\nu} dx_\mu dx_\nu} \quad \mu, \nu = 1, 2, 3, 4 \qquad (1)$$

ist, $g_{\mu\nu}$ Funktionen der Variabeln x bedeuten und bei der Variation am Anfang und Ende des Integrationswegs die Variablen x festzuhalten sind. Der Punkt bewege sich also, kurz gesagt, auf einer geodätischen Linie in der durch das Linienelement ds charakterisierten Mannigfaltigkeit.

Die Ausführung der Variation ergibt die Bewegungsgleichungen des Punktes

$$\frac{d^2 x_\alpha}{ds^2} = \sum_{\mu,\nu} \Gamma^\alpha_{\mu\nu} \frac{dx_\mu}{ds} \frac{dx_\nu}{ds}, \quad \alpha, \beta = 1, 2, 3, 4 \qquad (2)$$

wobei

$$\Gamma^\alpha_{\mu\nu} = -\frac{1}{2} \sum_\beta g^{\alpha\beta} \left(\frac{\partial g_{\mu\beta}}{\partial x_\nu} + \frac{\partial g_{\nu\beta}}{\partial x_\mu} - \frac{\partial g_{\mu\nu}}{\partial x_\beta} \right) \qquad (3)$$

ist und $g^{\alpha\beta}$ die zu $g_{\alpha\beta}$ koordinierte und normierte Subdeterminante in der Determinante $|g_{\mu\nu}|$ bedeutet.

Dies ist nun nach der Einsteinschen Theorie dann die Bewegung eines masselosen Punktes in dem Gravitationsfeld einer im Punkt $x_1 = x_2 = x_3 = 0$ befindlichen Masse, wenn die »Komponenten des Gravitationsfeldes« Γ überall, mit Ausnahme des Punktes $x_1 = x_2 = x_3 = 0$, den »Feldgleichungen«

$$R = (3x_1 + \rho)^{\frac{1}{3}} = \left(r^3 + \alpha^3\right)^{\frac{1}{3}}$$

eingeführt ist.

Setzt man diese Werte der Funktionen *f* im Ausdruck (9) des Linienelements ein und kehrt zugleich zu gewöhnlichen Polarkoordinaten zurück, so ergibt sich das Linienelement, welches die strenge Lösung des Einsteinschen Problems bildet:

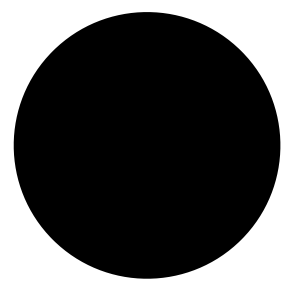
$$ds^2 = (1 - \alpha/R)dt^2 - \frac{dR^2}{1 - \alpha/R} - R^2 \left(d\vartheta^2 + \sin^2\vartheta d\phi^2\right), \quad R = \left(r^3 + \alpha^3\right)^{\frac{1}{3}}. \qquad (14)$$

Dasselbe enthält die eine Konstante *α*, welche von der Größe der im Nullpunkt befindlichen Masse abhängt.

Strange Structure of the Schwarzschild Solution

$$ds^2 = -\left(1 - \frac{R_g}{r}\right) c^2 dt^2 + \frac{dr^2}{1 - R_g/r} + r^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)$$

$$R_g = \frac{2G\,M}{c^2}$$



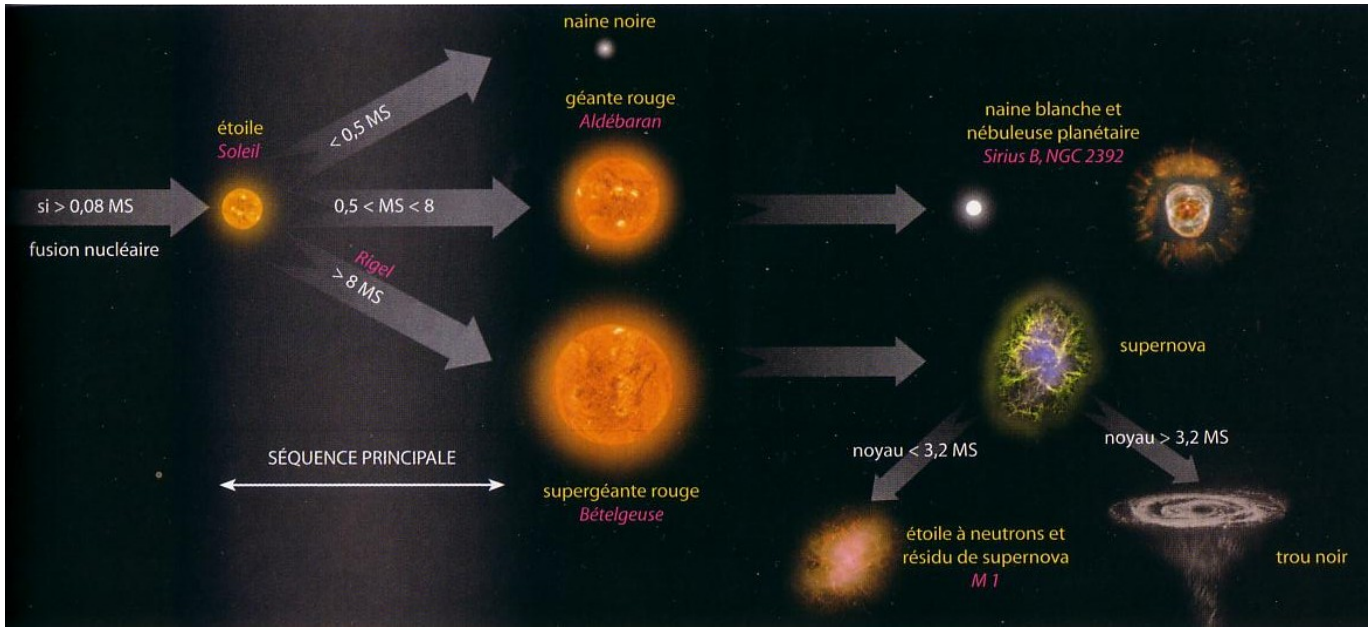
« Schwarzschild Singularity» : when r=R_g ??

$$R_g = \frac{2GM}{c^2} = 3\,\mathrm{km}\frac{M}{M_\odot}$$

Theoretical developpements and observational discoveries that allowed the emergence of the BH concept (1)

- 1920-: evolution of stars (Eddington)
- 1926 Special Relativity + Quantum M. + Exclusion Principle

theory of cold relativistic fermionic matter (Fowler 1926)



- 1929-35 **maximum mass** of White Dwarf
- (Chandrasekhar, Landau)

- 1934 Concept of neutron star ([Landau], Baade-Zwicky) density of neutron star:

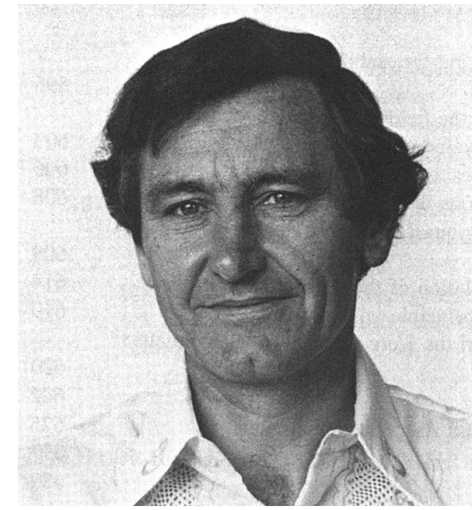
$$\sim M_\odot \text{ dans } 10\text{km} \rightarrow \rho \sim 10^{14}\text{g}/\text{cm}^3 \sim \rho_{\text{nucleaire}}$$

- 1938: energy source =nuclear fusion (Bethe)

- 1939 Oppenheimer-Volkoff : neutron stars in General Relativity : **maximum mass** neutron stars

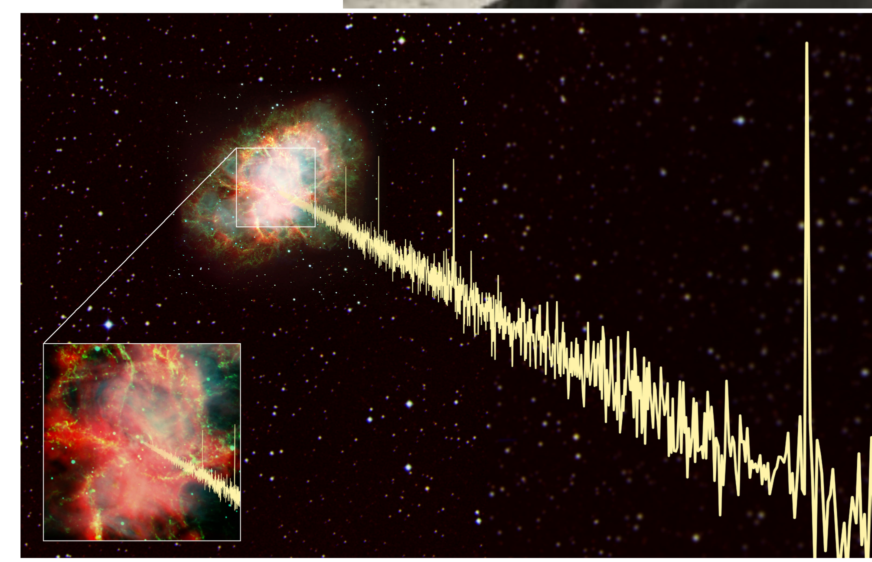
- **July 1939 Oppenheimer-Snyder** : stars which are too massive to « end their lives » as neutron stars, undergo gravitational collapse, until the light emitted by the star cannot escape to infinity

Theoretical developpements and observational discoveries that allowed the emergence of the BH concept (2)



- 1963 Quasars
  Existence of supermassive black holes?
- 1963 exact solution of Roy Kerr
  (generalizing Schwarzschild)

1965 Doroshkevich-Zel'dovich-Novikov: gravitational collapse of a rotating star -> « frozen star »



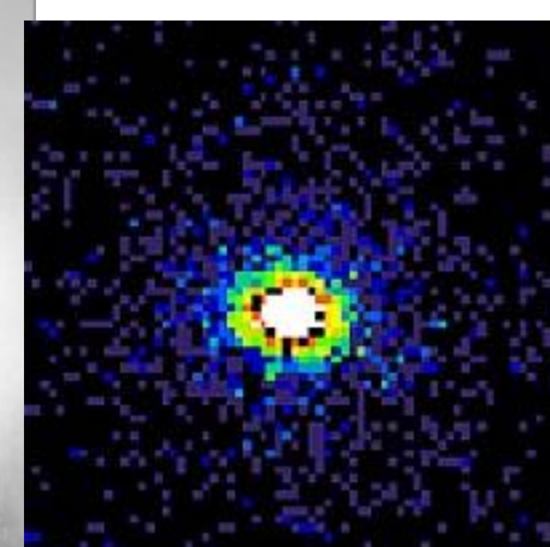
- 1967 Pulsars
- 1968 Crab Pulsar : existence of neutron stars
- 1960-68 the name « black hole » is invented
- (Dicke+Journalists+Wheeler)
- 1969 global space-time picture
  of a black hole (Penrose)
- 1970 binary X-ray sources and existence of
  black holes ~ 10 M_sun
- 1973 [1963] gamma-ray bursts

………………………………

- 2015 gravitational waves from

the coalescence of two black holes
~30 M_sun (LIGO)

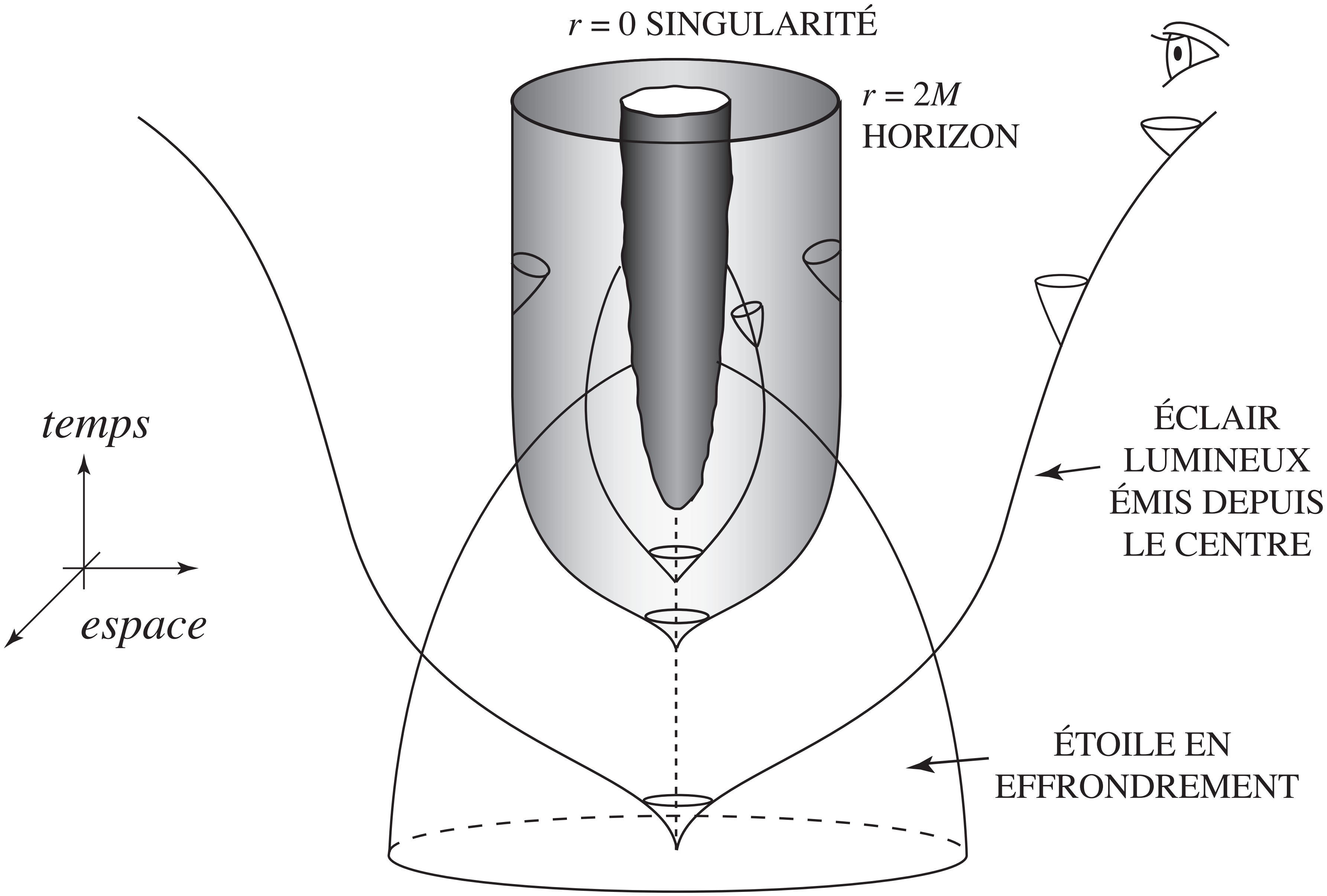
- 2019 image of M87 (EHT)



Collapse of a star and formation of a black hole

(Oppenheimer-Snyder 1939, Doroshkevich-Zel'dovich-Novikov 1965, Penrose 1969)



Black hole surface

(or « horizon »):

a bubble of light

that « stays put ».

one-way

membrane

Black Hole interior:

the spacetime

jelly

is torn in a

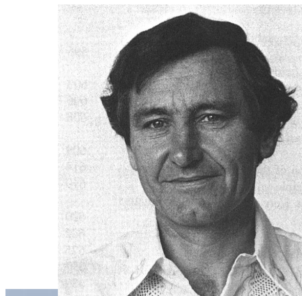
« big crunch »


Black Holes as Physical Objects

It is shown that a Schwarzschild singularity, spherically symmetrical and endowed with mass, will undergo small vibrations about the spherical form and will therefore remain stable if subjected to a small nonspherical perturbation.

Regge-Wheeler1957

$$ds^2 = -\left(1 - \frac{r_s r}{\Sigma}\right) c^2 dt^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \left(r^2 + a^2 + \frac{r_s r a^2}{\Sigma} \sin^2\theta\right) \sin^2\theta \, d\phi^2 - \frac{2 r_s r a \sin^2\theta}{\Sigma} c \, dt \, d\phi$$




Kerr (1963):
two parameters:
mass M and
angular momentum
J= GM a/cˆ2

Vishveswara 1970

Scattering of gravitational waves on a black hole, and discovery of « damped vibration modes » (Quasi-Normal Modes) of a BH


Energetics of black holes

(Penrose '69; Christodoulou-Ruffini '70, Hawking'71)

$$M^2 = \left(M_{\text{irr}} + \frac{Q^2}{4M_{\text{irr}}}\right)^2 + \frac{J^2}{4M_{\text{irr}}^2}$$

$$\delta M_{\text{irr}} \geq 0$$

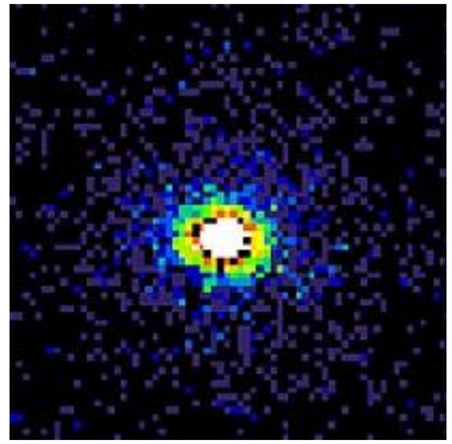
Thermodynamic, electric and hydrodynamic properties of black holes (Bardeen-Carter-Hawking, Hartle-Hawking, Damour)

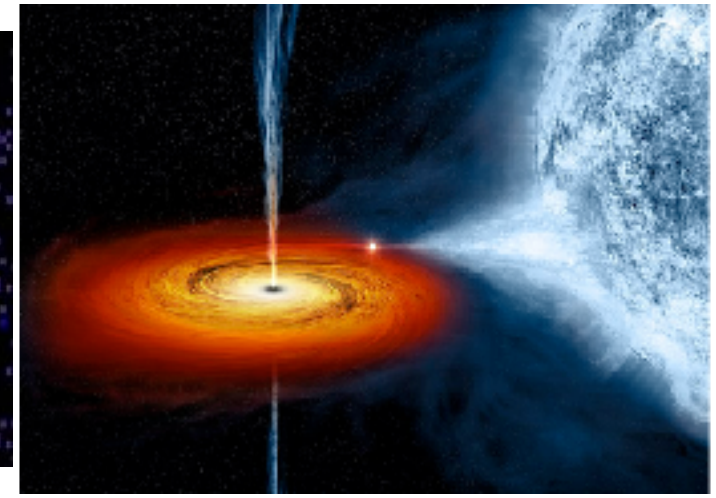
surface resistivity=377 Ohm

Navier-Stokes equation with a viscosity equal to 1/(16 pi)

Black Holes in Astrophysics

X-ray binaries: Cygnus X1:

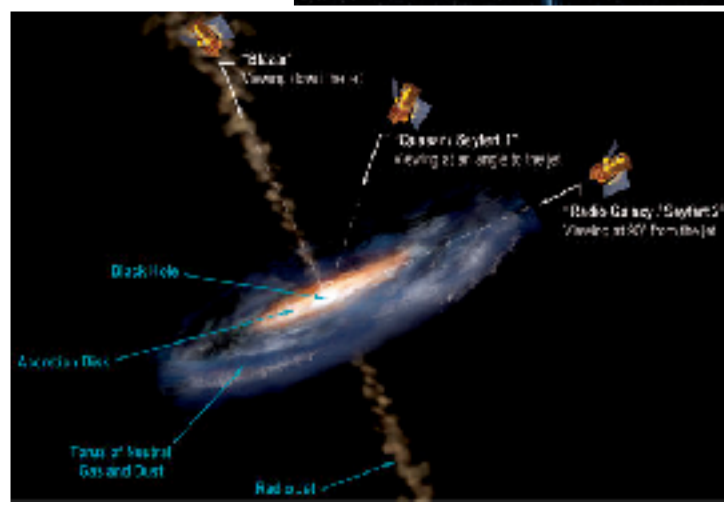




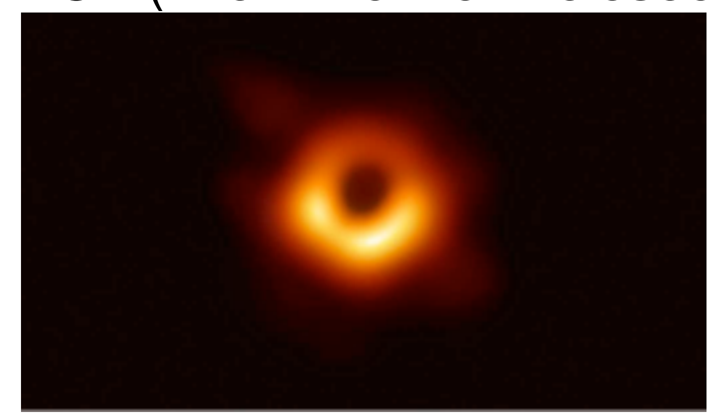
Giacconi

Gursky

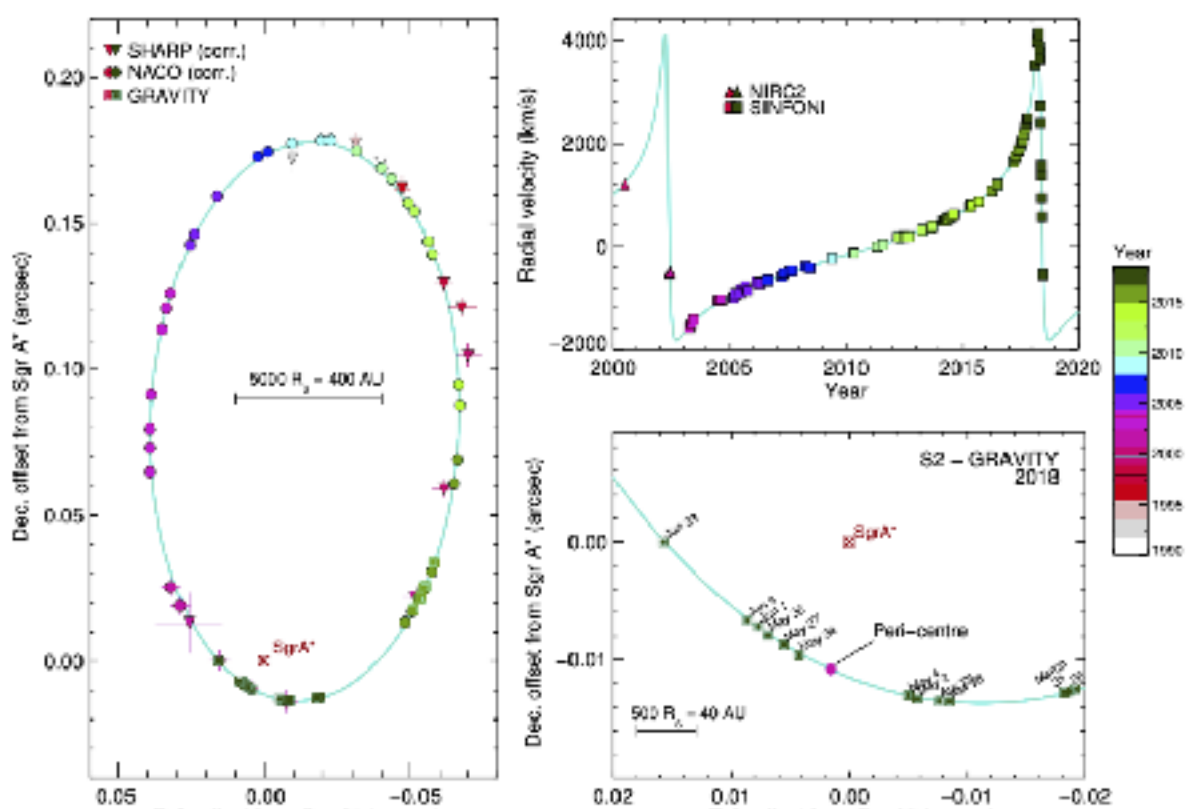
active galacti nuclei



M87 (Event Horizon Telescope)



centre of our Galaxy SgrA*: notably S2



Penrose Genzel Ghez

Gravitational Waves

linearized waves: Einstein 1916,1918

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\Box h_{\mu\nu} + \partial_\mu H_\nu + \partial_\nu H_\mu = -16\pi G\left(T_{\mu\nu} - \frac{1}{D-2}\eta_{\mu\nu}T\right)$$

$$H_\mu = \frac{1}{2}\partial_\mu h - \partial^\nu h_{\mu\nu} \qquad \partial^\nu T_{\mu\nu} = 0$$

$$ds^2 = -c^2 dt^2 + (\delta_{ij} + h_{ij})dx^i dx^j$$

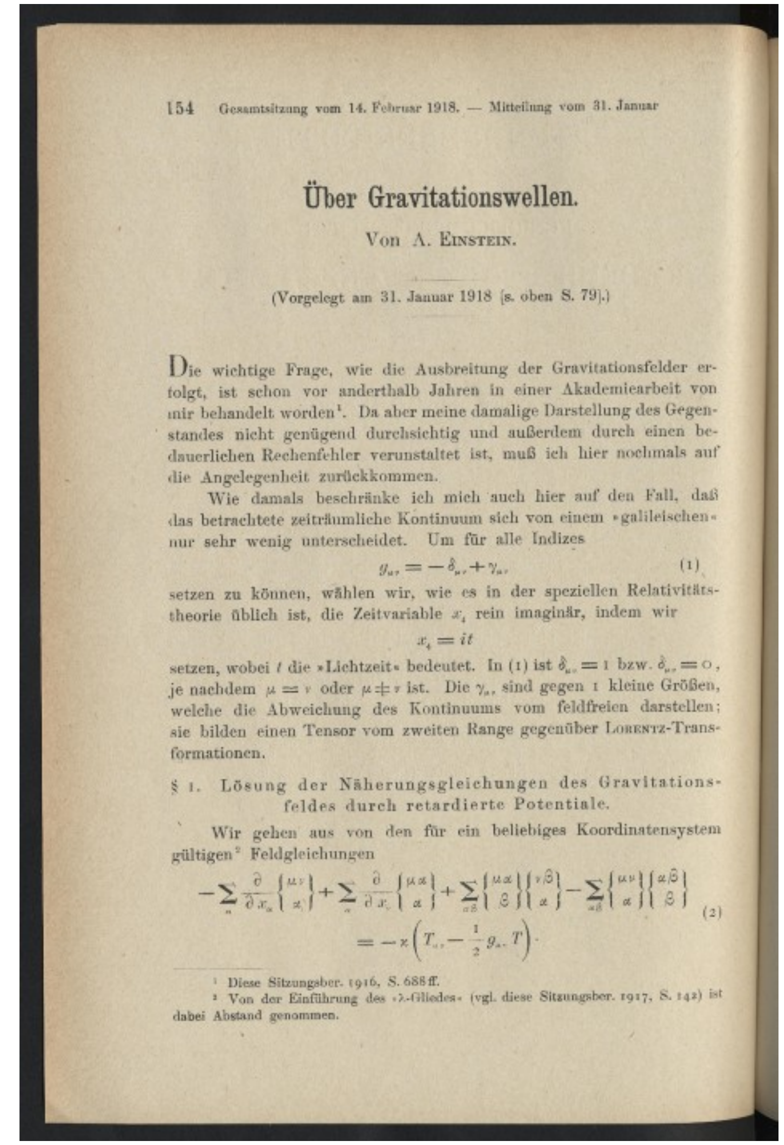
Two tensor polarisations transverse-traceless (TT)
propagating at v=c

$$g_{ij} = \delta_{ij} + h_{ij} \qquad h_{ij} = h_+(x_i x_j - y_i y_j) + h_\times(x_i y_j + y_i x_j)$$

$$\frac{\delta L}{L} = \frac{1}{2}h_{ij}n^i n^j$$

slow-motion approximation

$$h_{ij} \simeq \frac{2G}{c^4 r}\ddot{Q}_{ij}^{TT}(t - r/c)$$



154 Gesamtsitzung vom 14. Februar 1918. — Mitteilung vom 31. Januar

Über Gravitationswellen.

Von A. Einstein.

(Vorgelegt am 31. Januar 1918 [s. oben S. 79].)

Die wichtige Frage, wie die Ausbreitung der Gravitationsfelder erfolgt, ist schon vor anderthalb Jahren in einer Akademiearbeit von mir behandelt worden[1]. Da aber meine damalige Darstellung des Gegenstandes nicht genügend durchsichtig und außerdem durch einen bedauerlichen Rechenfehler verunstaltet ist, muß ich hier nochmals auf die Angelegenheit zurückkommen.

Wie damals beschränke ich mich auch hier auf den Fall, daß das betrachtete zeiträumliche Kontinuum sich von einem »galileischen« nur sehr wenig unterscheidet. Um für alle Indizes

$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu} \qquad (1)$$

setzen zu können, wählen wir, wie es in der speziellen Relativitätstheorie üblich ist, die Zeitvariable x_4 rein imaginär, indem wir

$$x_4 = it$$

setzen, wobei t die »Lichtzeit« bedeutet. In (1) ist $\delta_{\mu\nu} = 1$ bzw. $\delta_{\mu\nu} = 0$, je nachdem $\mu = \nu$ oder $\mu \neq \nu$ ist. Die $\gamma_{\mu\nu}$ sind gegen 1 kleine Größen, welche die Abweichung des Kontinuums vom feldfreien darstellen; sie bilden einen Tensor vom zweiten Range gegenüber Lorentz-Transformationen.

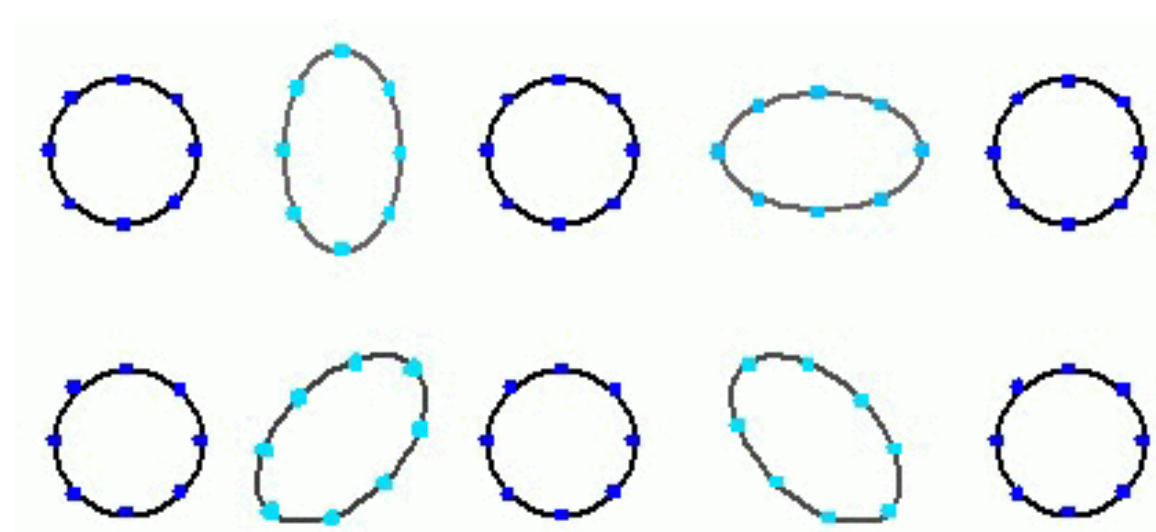
§ 1. Lösung der Näherungsgleichungen des Gravitationsfeldes durch retardierte Potentiale.

Wir gehen aus von den für ein beliebiges Koordinatensystem gültigen[2] Feldgleichungen

$$-\sum_\alpha \frac{\partial}{\partial x_\alpha}\begin{Bmatrix}\mu\nu\\ \alpha\end{Bmatrix} + \sum_\alpha \frac{\partial}{\partial x_\nu}\begin{Bmatrix}\mu\alpha\\ \alpha\end{Bmatrix} + \sum_{\alpha\beta}\begin{Bmatrix}\mu\alpha\\ \beta\end{Bmatrix}\begin{Bmatrix}\nu\beta\\ \alpha\end{Bmatrix} - \sum_{\alpha\beta}\begin{Bmatrix}\mu\nu\\ \alpha\end{Bmatrix}\begin{Bmatrix}\alpha\beta\\ \beta\end{Bmatrix} = -\varkappa\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right) \qquad (2)$$

[1] Diese Sitzungsber. 1916, S. 688 ff.

[2] Von der Einführung des »λ-Gliedes« (vgl. diese Sitzungsber. 1917, S. 142) ist dabei Abstand genommen.



A few pioneers of gravitational waves

Yvonne Choquet-Bruhat

1952 Acta Math. 88, 141-225

At a time where many physicists were still confused about the existence and reality of GWs!

THÉORÈME D'EXISTENCE POUR CERTAINS SYSTÈMES D'ÉQUATIONS AUX DÉRIVÉES PARTIELLES NON LINÉAIRES.

Par

Y. FOURÈS-BRUHAT.

Introduction.

Je me suis posé le problème de Cauchy pour les équations aux dérivées partielles hyperboliques non linéaires à propos des équations de la gravitation d'Einstein. Ces équations se présentent en effet comme un système de dix équations du second ordre, linéaires, à quatre variables (espace et temps) et dix fonctions inconnues, les potentiels de gravitation. Ces équations sont du types hyperbolique normal dans un système de coordonnées spatio-temporelles régulier. Le problème du déterminisme se pose, dans la théorie d'Einstein, sous forme du problème de Cauchy, les données étant portées par une variété orientée dans l'espace, relativement à ce système d'équations. L'étude de ce problème, en supposant les données de Cauchy analytiques[1], avait montré que, moyennant quatre conditions vérifiées par ces données, il correspondait à des données initiales, portées par une surface S non caractéristique, un espace-temps einsteinien au voisinage de S. L'étude des surfaces caractéristiques, définies par le fait que les données de Cauchy, portées par une telle surface, ne déterminait pas au voisinage un espace-temps, avait montré que ces surfaces étaient tangentes en l'un quelconque de leurs points M au «conoïde caractéristique» de sommet M, ce conoïde étant engendré par les rayons lumineux, géodésiques de longueur nulle. On voyait ainsi apparaître des ondes et rayons gravifiques, donnant au champ de gravitation le caractère d'un phénomène de propagation et on constatait l'identité entre les lois de propagation de la lumière et du champ de gravitation. Il apparaissait alors comme très important d'étendre ces résultats au cas de données de Cauchy non analytiques, d'une part parce qu'une telle hypothèse d'analyticité n'a pas de sens

[1] G. Darmois [1]. A. Lichnerowicz [2].

Cauchy problem

Einstein's gravitation equations

characteristic conoid

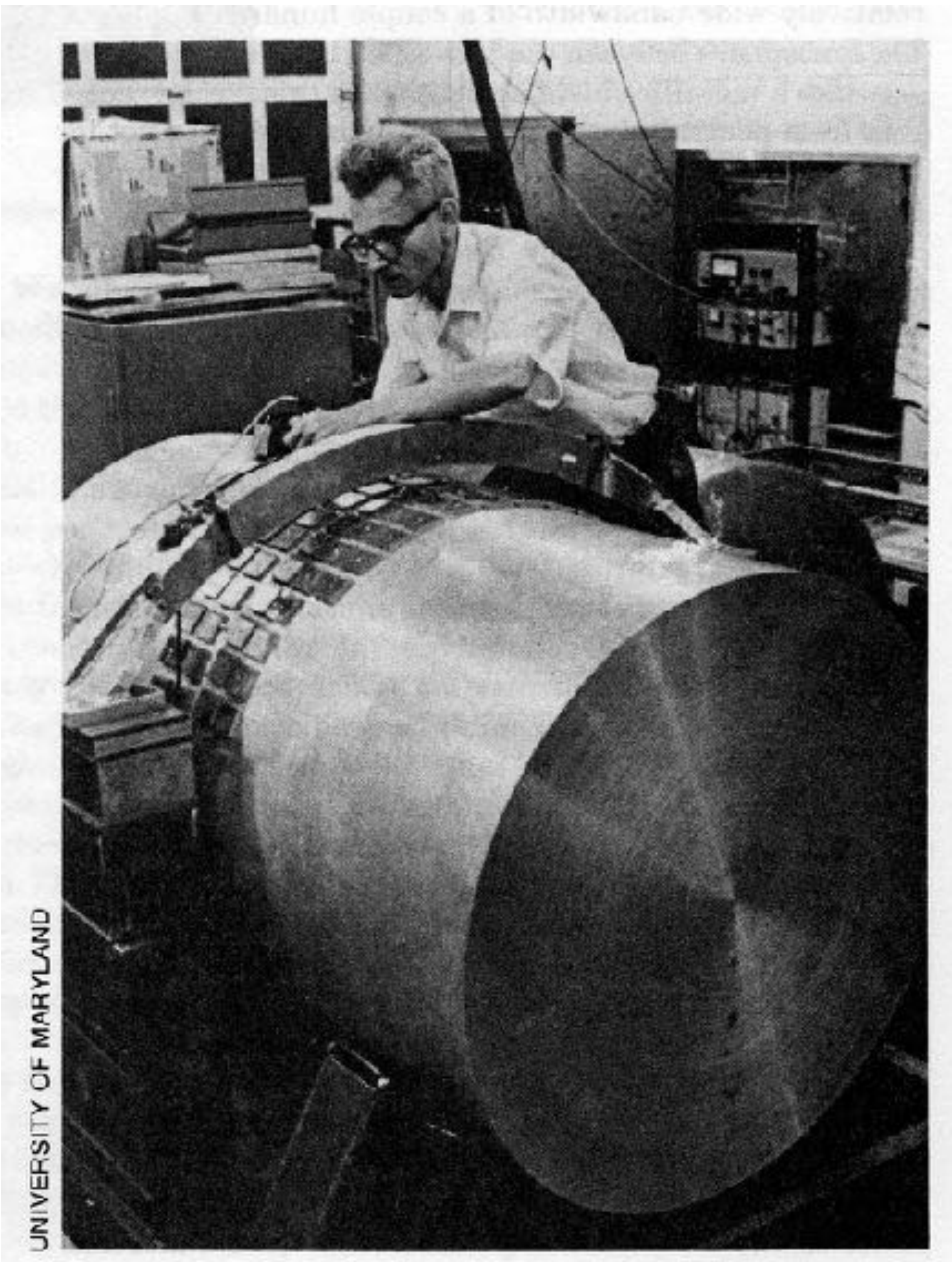
gravitational waves and rays

propagation phenomenon

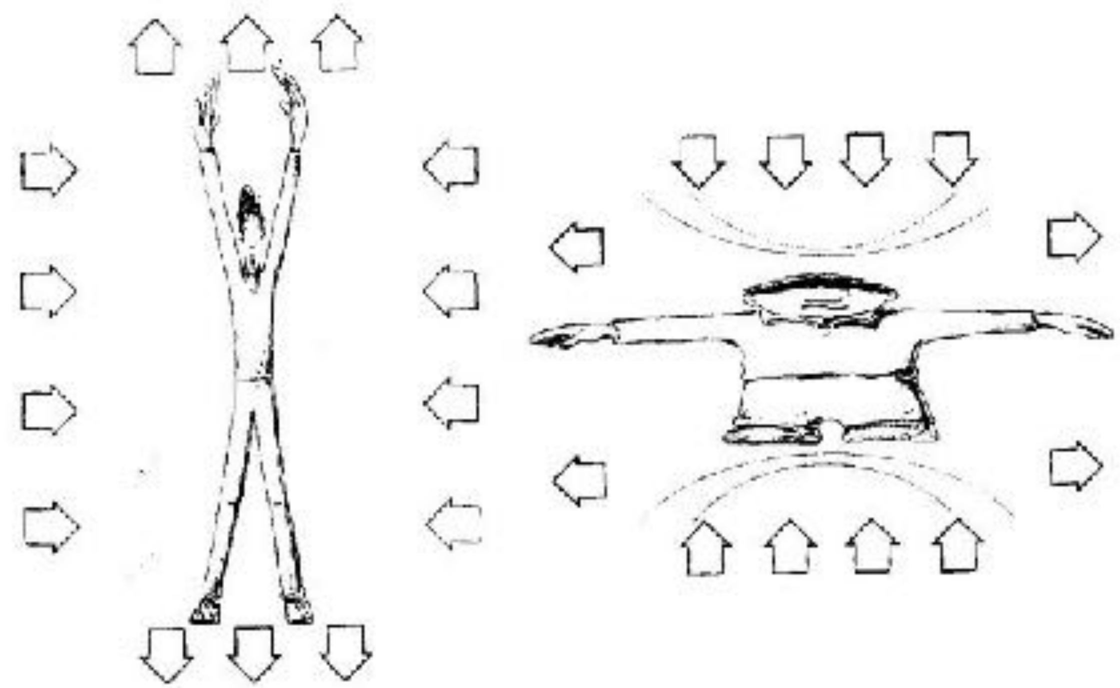
non analytic

Lichnerowicz's versus Leray reactions

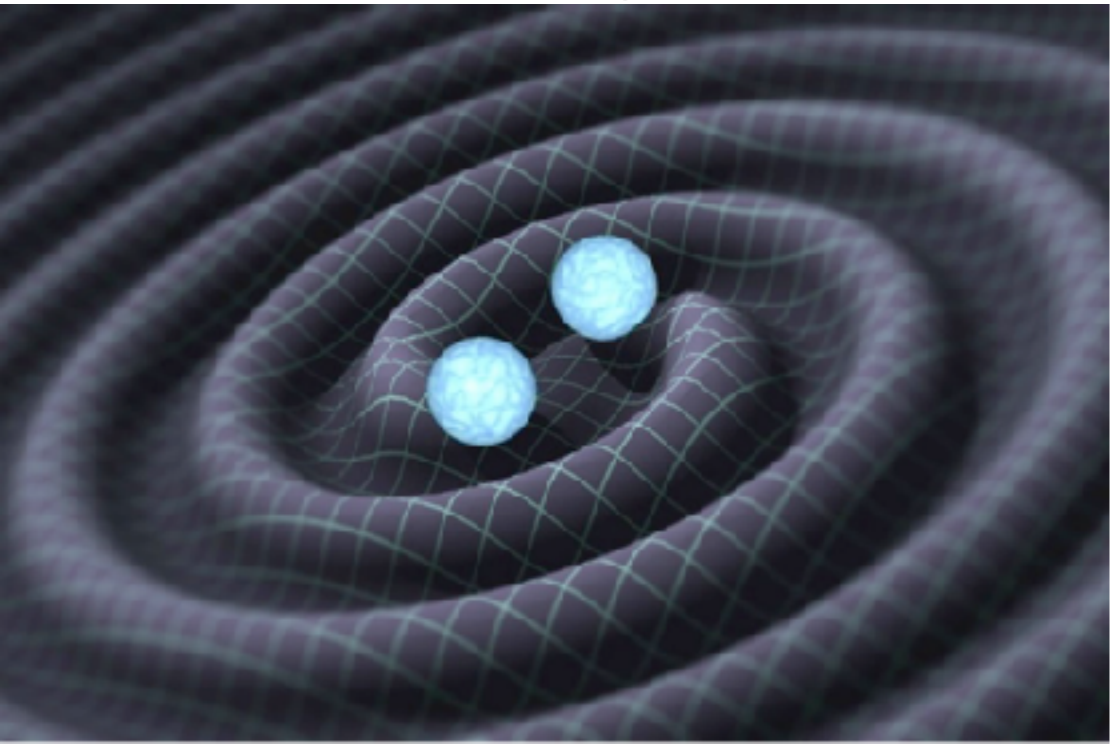
Joseph Weber
(starting in 1955, invited by Dyson at the IAS)

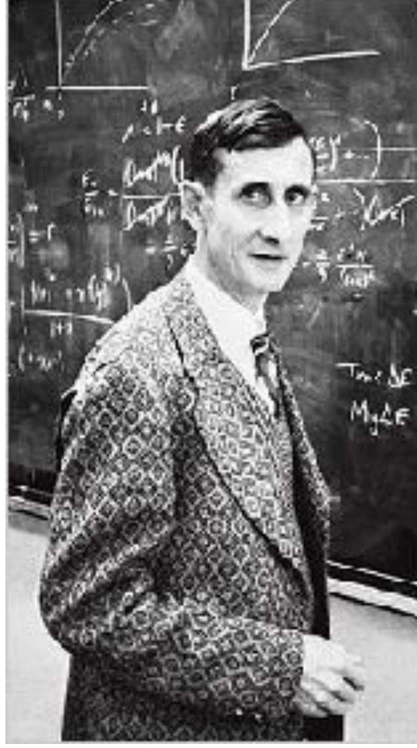


$$\frac{\delta L}{L} = \frac{1}{2} h_{ij} n^i n^j$$



Freeman Dyson (1963) GWs from compact binaries



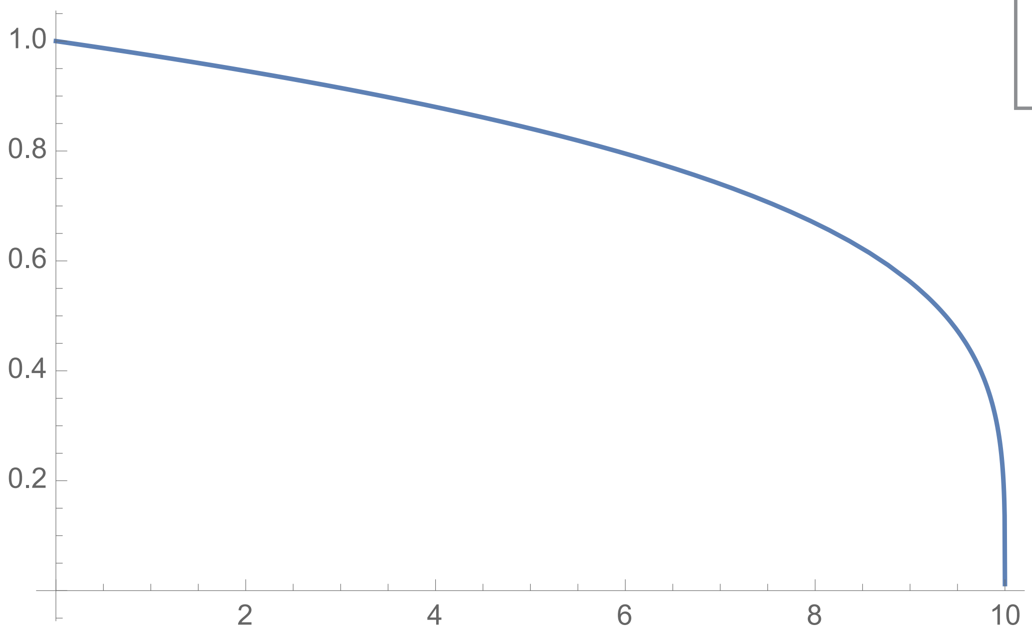


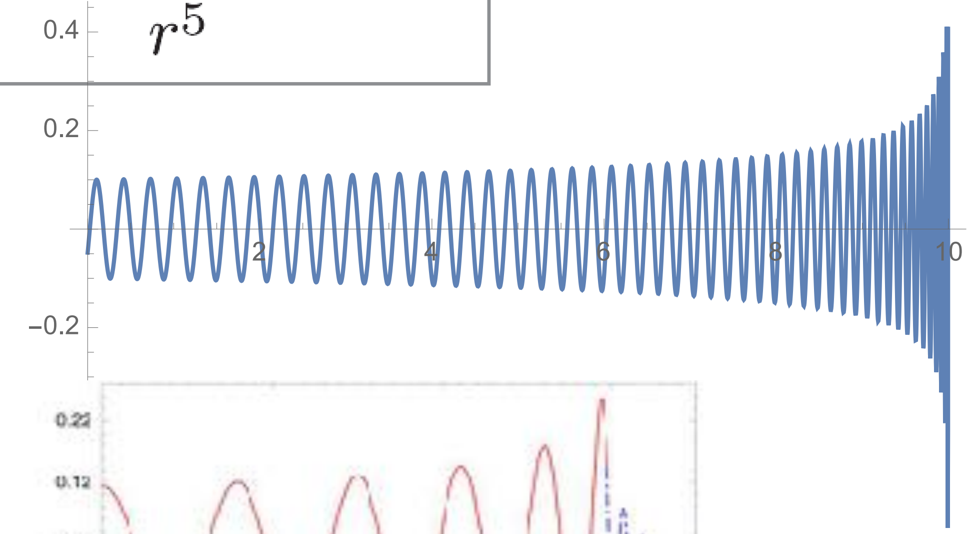
$$E = -\frac{G\, m_1\, m_2}{2r}$$

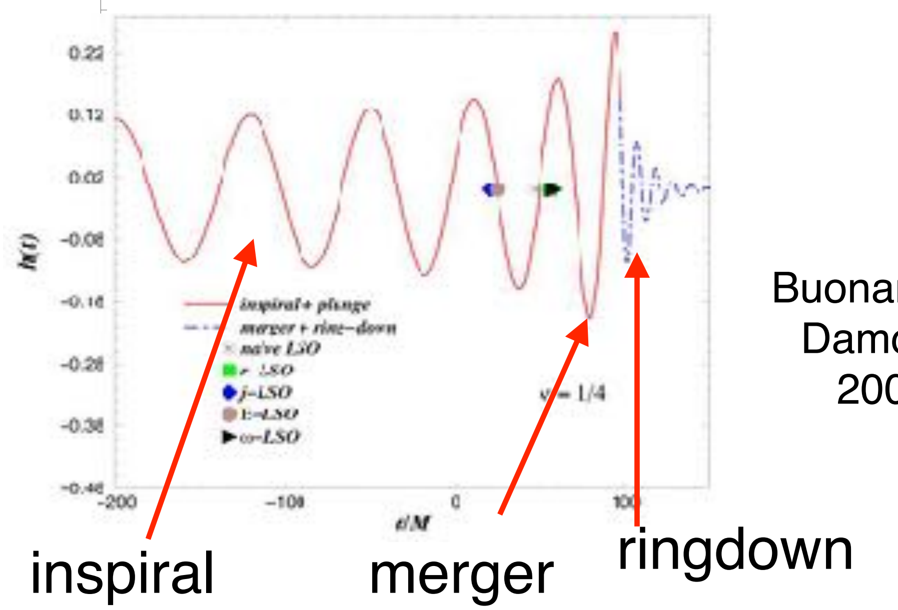
$$\frac{d}{dt} E = -F$$

Einstein 1918 + Landau-Lifshitz 1941

$$F = \frac{32\, G^4}{5\, c^5}\, \frac{m_1^2\, m_2^2 (m_1 + m_2)}{r^5}$$







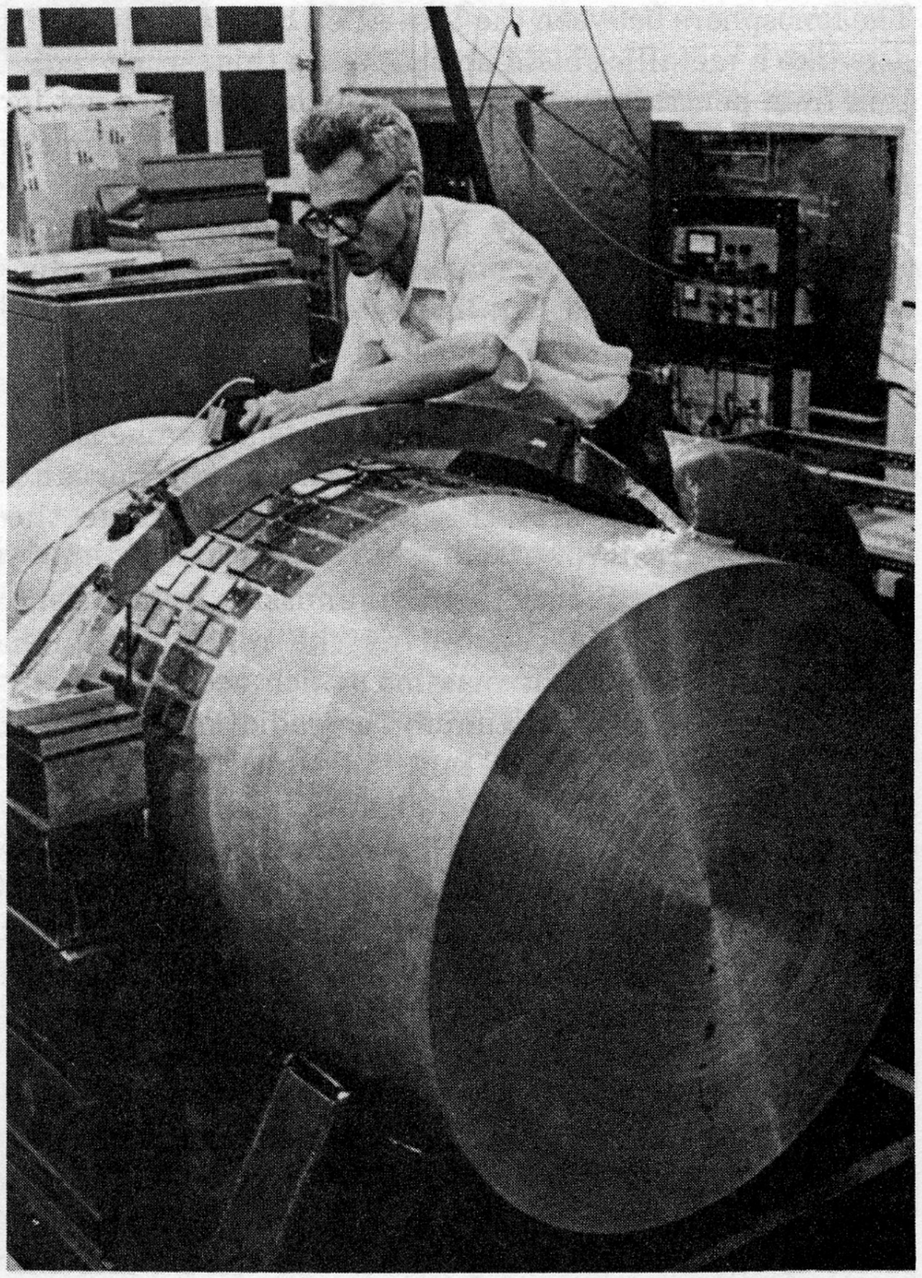
Buonanno-Damour 2000

Freeman Dyson's challenge: describe the intense flash of GWs emitted by the last orbits and the merger of a binary BH, when v~c and r~GM/c^2

Gravitational Wave Detectors

Joseph Weber (1919-2000)

Understands (helped by Marcel Riesz)
the reality of GWs
Develops the theory of GW detectors
(during a sabbatical at the IAS, invited
by Freeman Dyson in 1955)
Constructs resonant GW detectors ~ 1960
Conceives interferometric detectors


UNIVERSITY OF MARYLAND

Laser Interferometric GW detectors

Initial idea: [Weber]   Gertsenshtein-Pustovoit 1962
First detailed noise analysis: Rainer Weiss 1967-1972
First prototype: Forward ~ 1970
First search of GW signals: Levine-Hall, Levine-Stebbins 1972
Development of sensitive interferometric detectors ~ 1980's
Garching: Billing H, Maischberger K, Ruediger A, Schilling R, Schnupp L, Winkler, W 1979;
Shoemaker D et al 1988
Glasgow: Drever, Hough, Pugh, Edelstein, Ward, Ford, Robertson 1979
Caltech: Drever et al 1981
France: Brillet, Man, Vinet 1982
Italy: Giazzoto
Founders/Deciders/Managers: Thorne, Isaacson, Barish, …

Rainer Weiss (1972)

importance of broad-band antenna

(V. GRAVITATION RESEARCH)

B. ELECTROMAGNETICALLY COUPLED BROADBAND GRAVITATIONAL ANTENNA

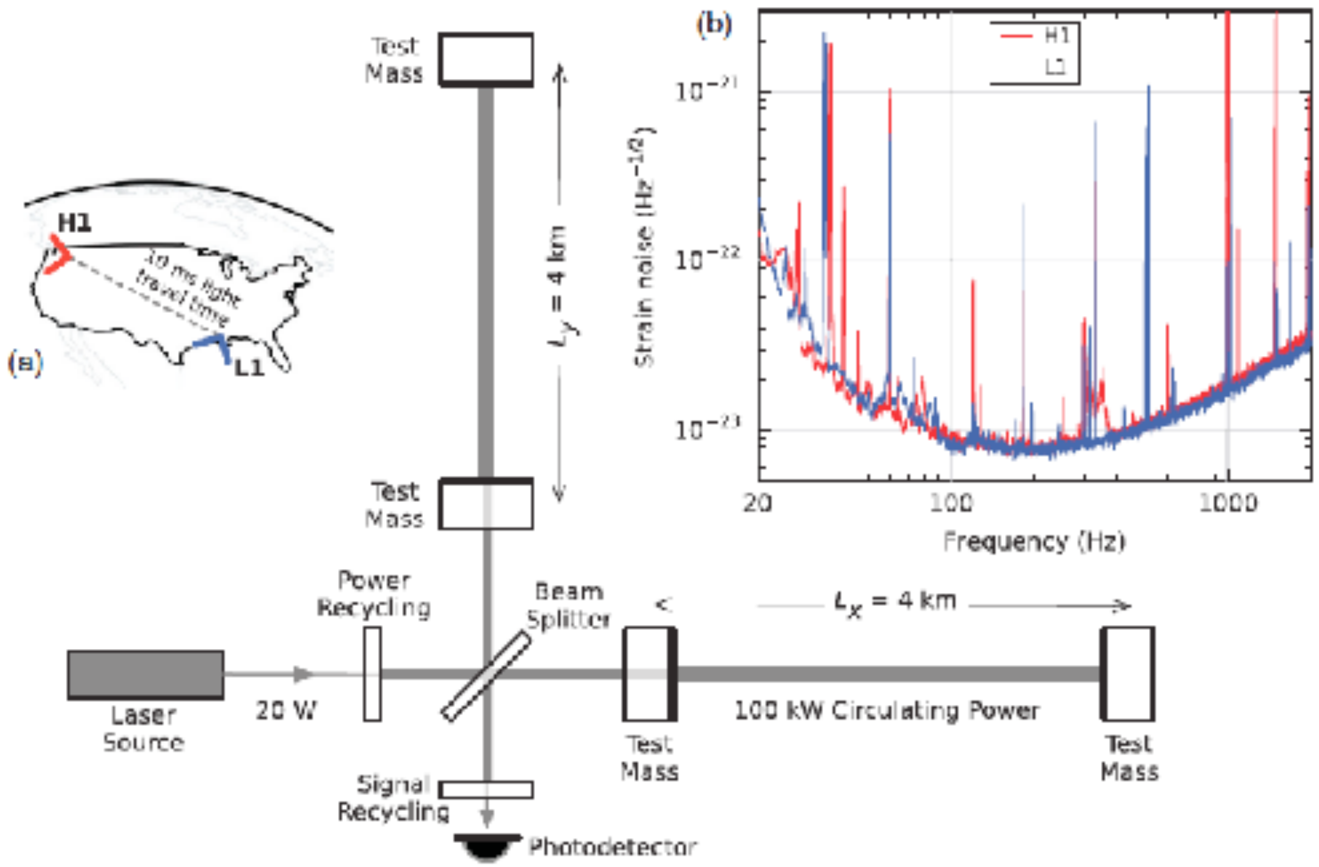
1. Introduction

The prediction of gravitational radiation that travels at the speed of light has been an essential part of every gravitational theory since the discovery of special relativity. In 1918, Einstein,[1] using a weak-field approximation in his very successful geometrical theory of gravity (the general theory of relativity), indicated the form that gravitational waves would take in this theory and demonstrated that systems with time-variant mass quadrupole moments would lose energy by gravitational radiation. It was evident to Einstein that since gravitational radiation is extremely weak, the most likely measurable radiation would come from astronomical sources. For many years the subject of gravitational radiation remained the province of a few dedicated theorists; however, the recent discovery of the pulsars and the pioneering and controversial experiments of Weber[2,3] at the University of Maryland have engendered a new interest in the field.

Weber has reported coincident excitations in two gravitational antennas separated 1000 km. These antennas are high-Q resonant bars tuned to 1.6 kHz. He attributes these excitations to pulses of gravitational radiation emitted by broadband sources concentrated near the center of our galaxy. If Weber's interpretation of these events is correct, there is an enormous flux of gravitational radiation incident on the Earth.

Several research groups throughout the world are attempting to confirm these results with resonant structure gravitational antennas similar to those of Weber. A broadband antenna of the type proposed in this report would give independent confirmation of the existence of these events, as well as furnish new information about the pulse shapes.

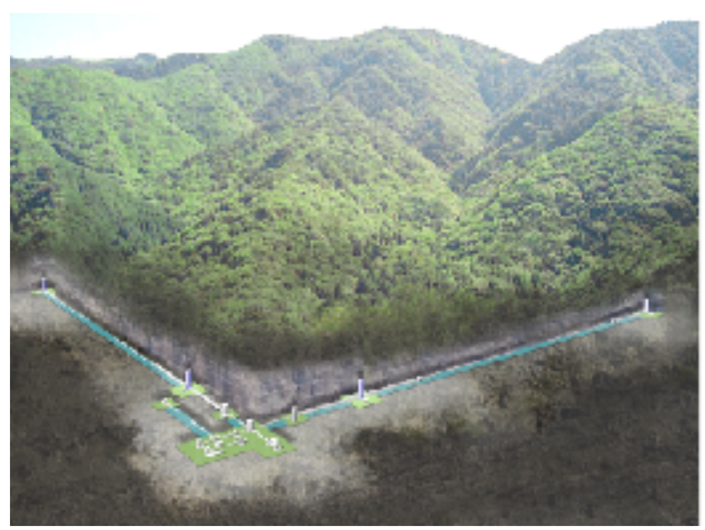
STARTING FROM 14 SEPT 2015: GRAVITATIONAL WAVE (GW) DETECTIONS BY TWO LIGO (+ VIRGO+KAGRA+…) GW DETECTORS



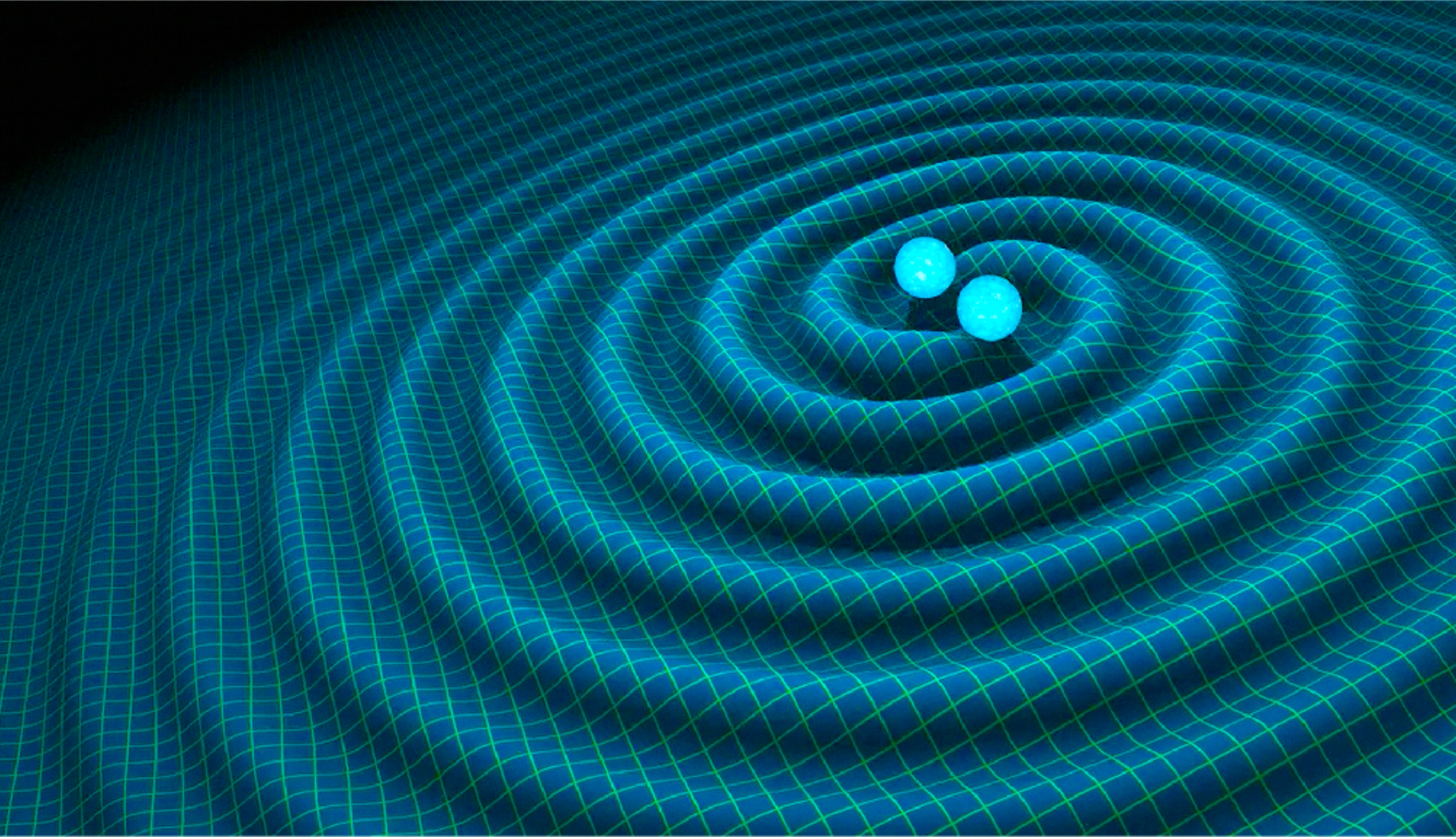
LIGO Hanford

LIGO Livingston

KAGRA



Virgo (IT)



$$m_1 = 36^{+5}_{-4} M_\odot$$

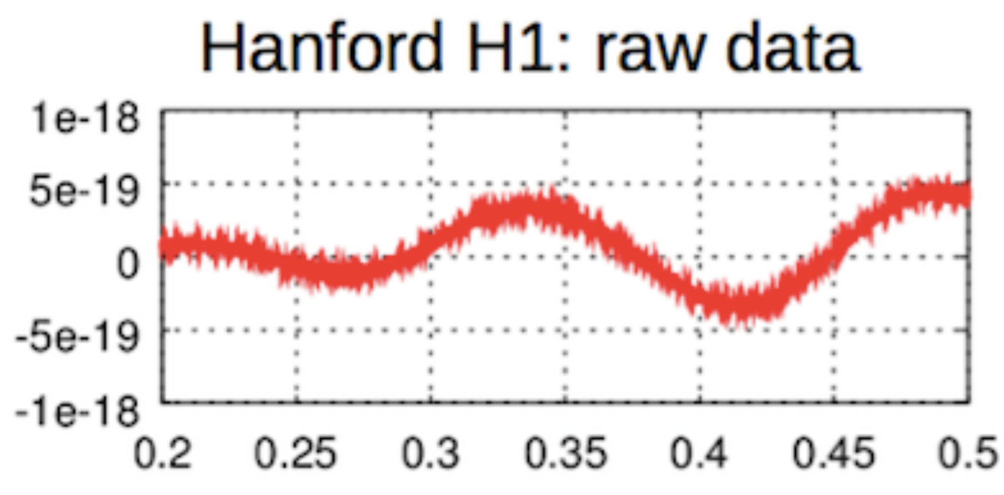
$$m_2 = 29^{+4}_{-4} M_\odot$$

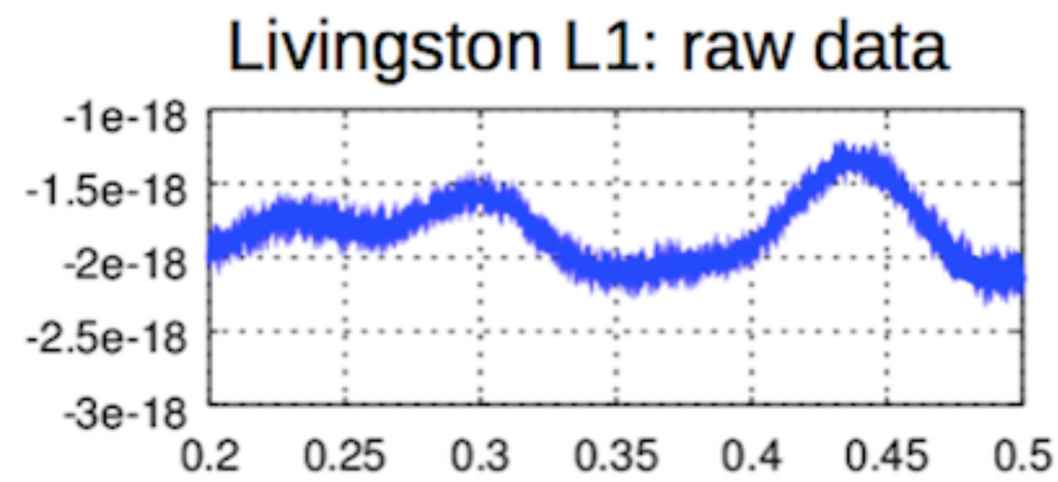
$$\chi_{\mathrm{eff}} = -0.06^{+0.17}_{-0.18}$$

$$D_{\mathrm{L}} = 410^{+160}_{-180} \mathrm{Mpc}$$

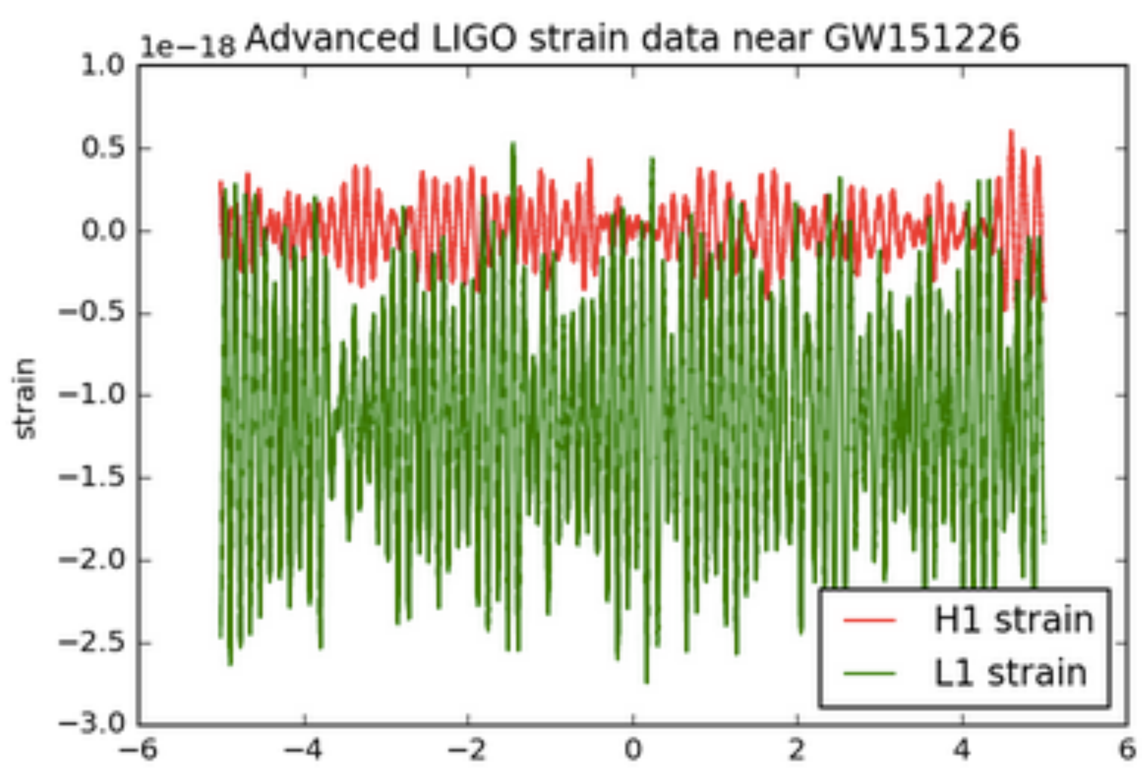
LIGO Raw Data for First Binary Black Hole Events

GW150914 from LIGO open data



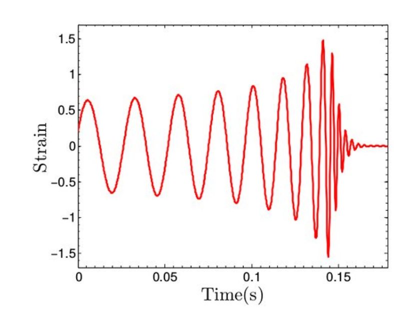


GW151226 from LIGO open data

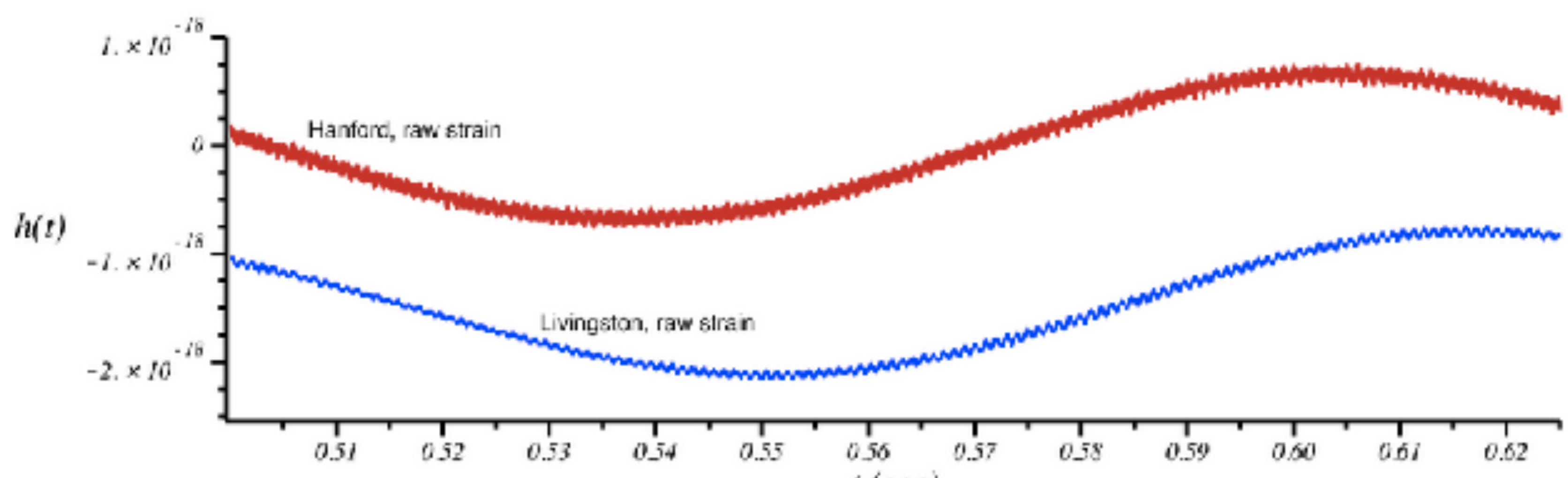


$$\left[\frac{\delta L}{L}\right]_{\rm obs} \sim 10^{-18}$$

$$\gg h_{GW} \lesssim 10^{-21}$$



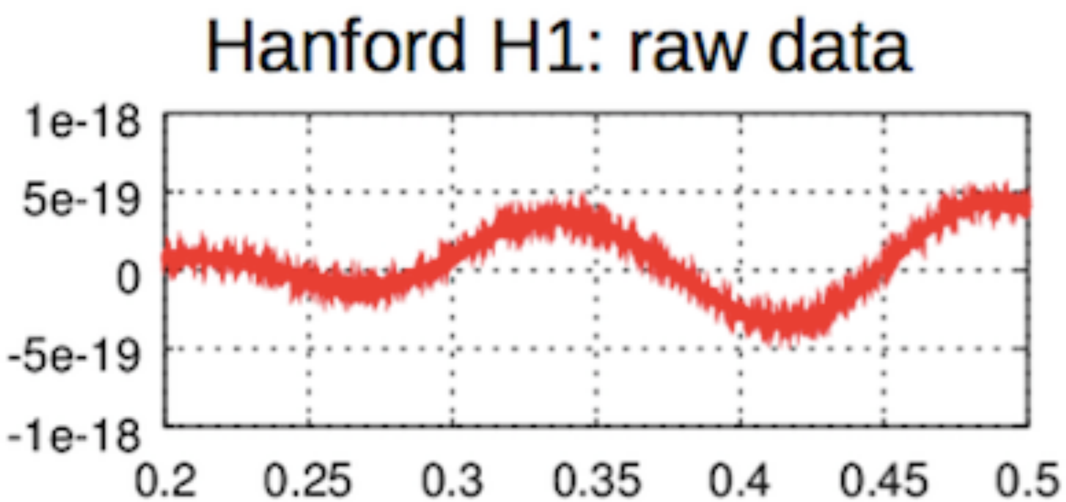
GW170104 from LIGO open data

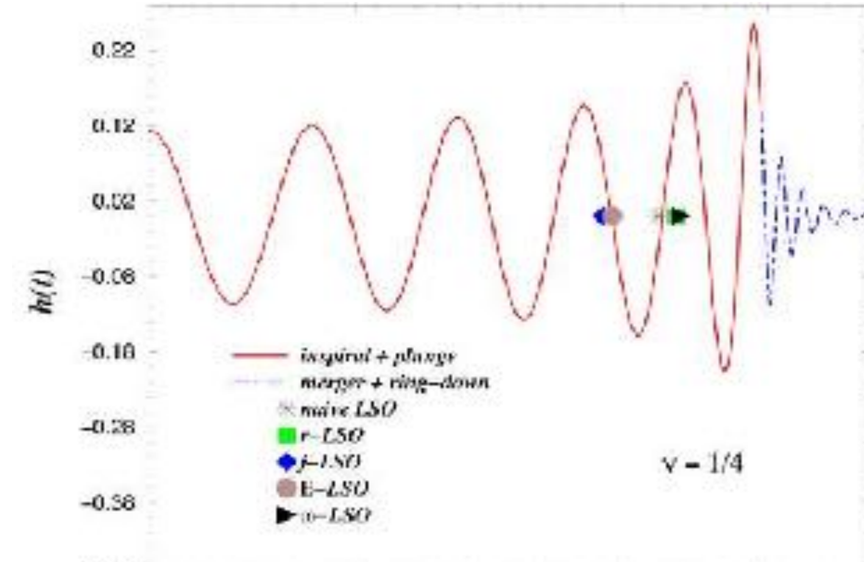


GW150914 with matched-filter SNR=24

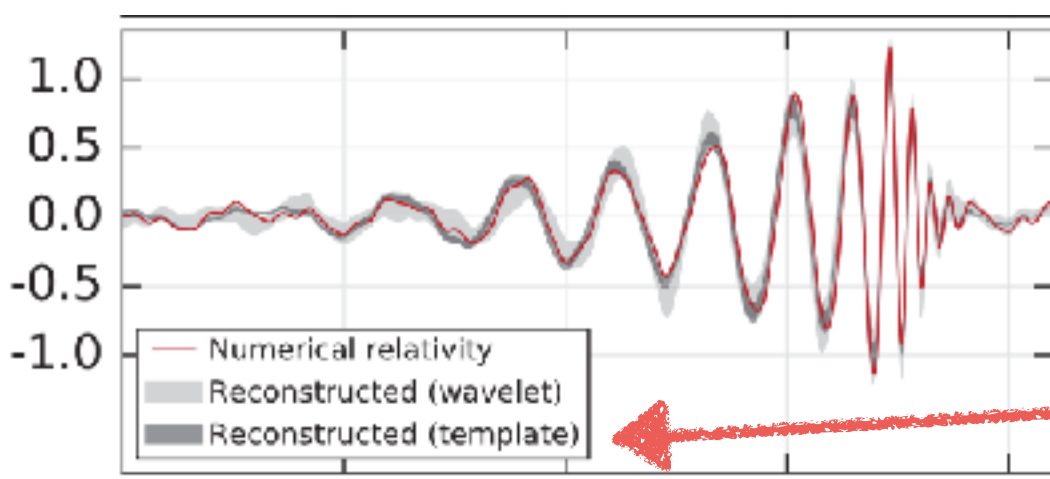
Buonanno-Damour 2000 + NR (Pretorius 2005,…)

10^-18

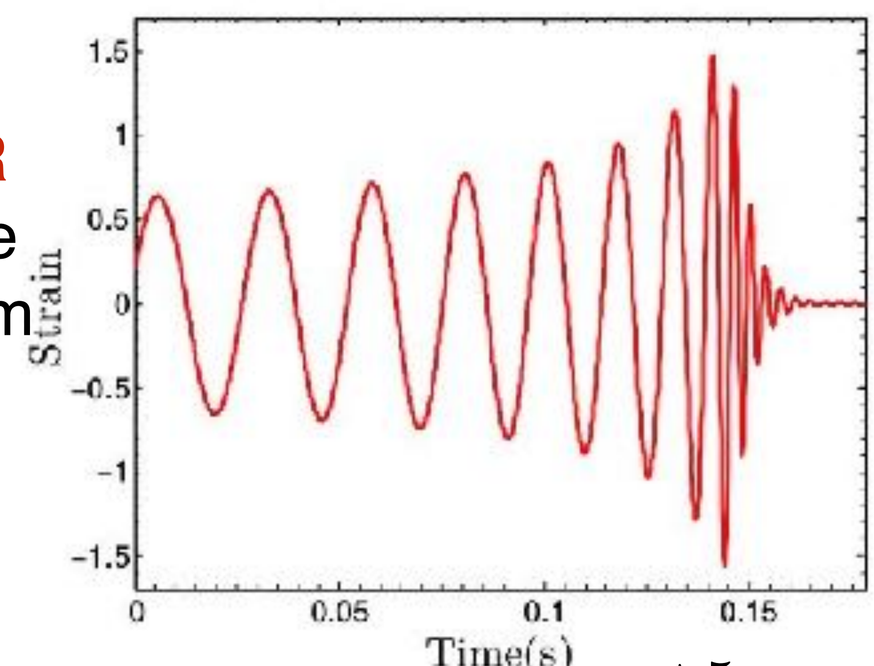


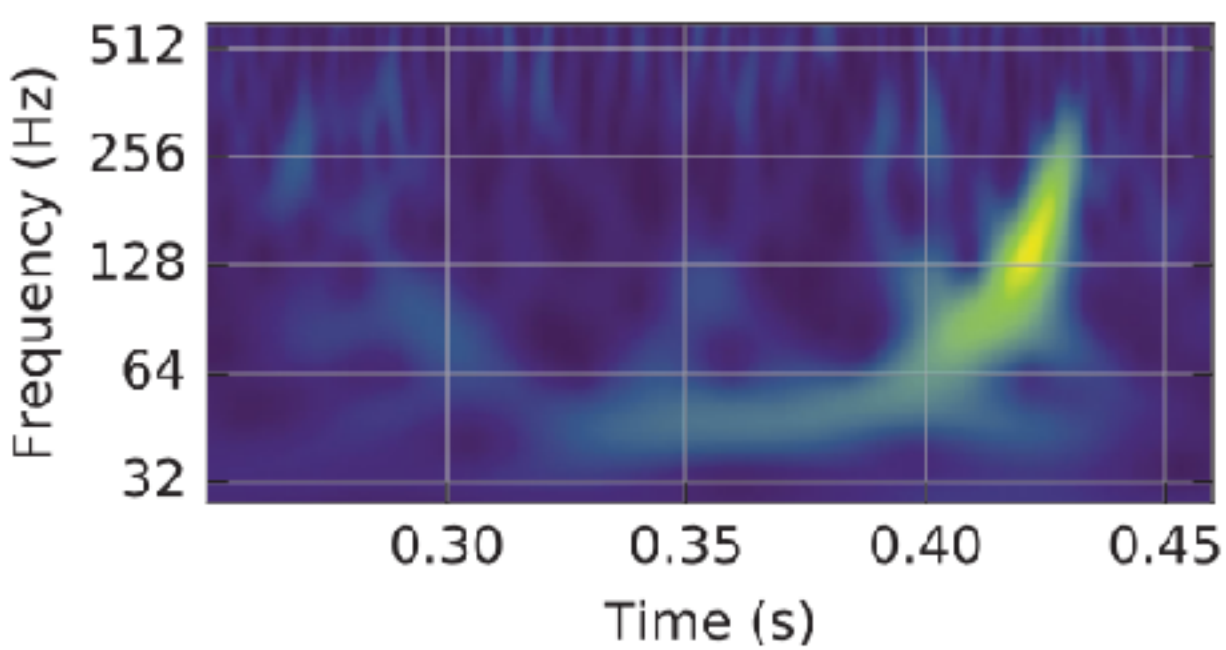


10^-21



EOBNR
template
waveform





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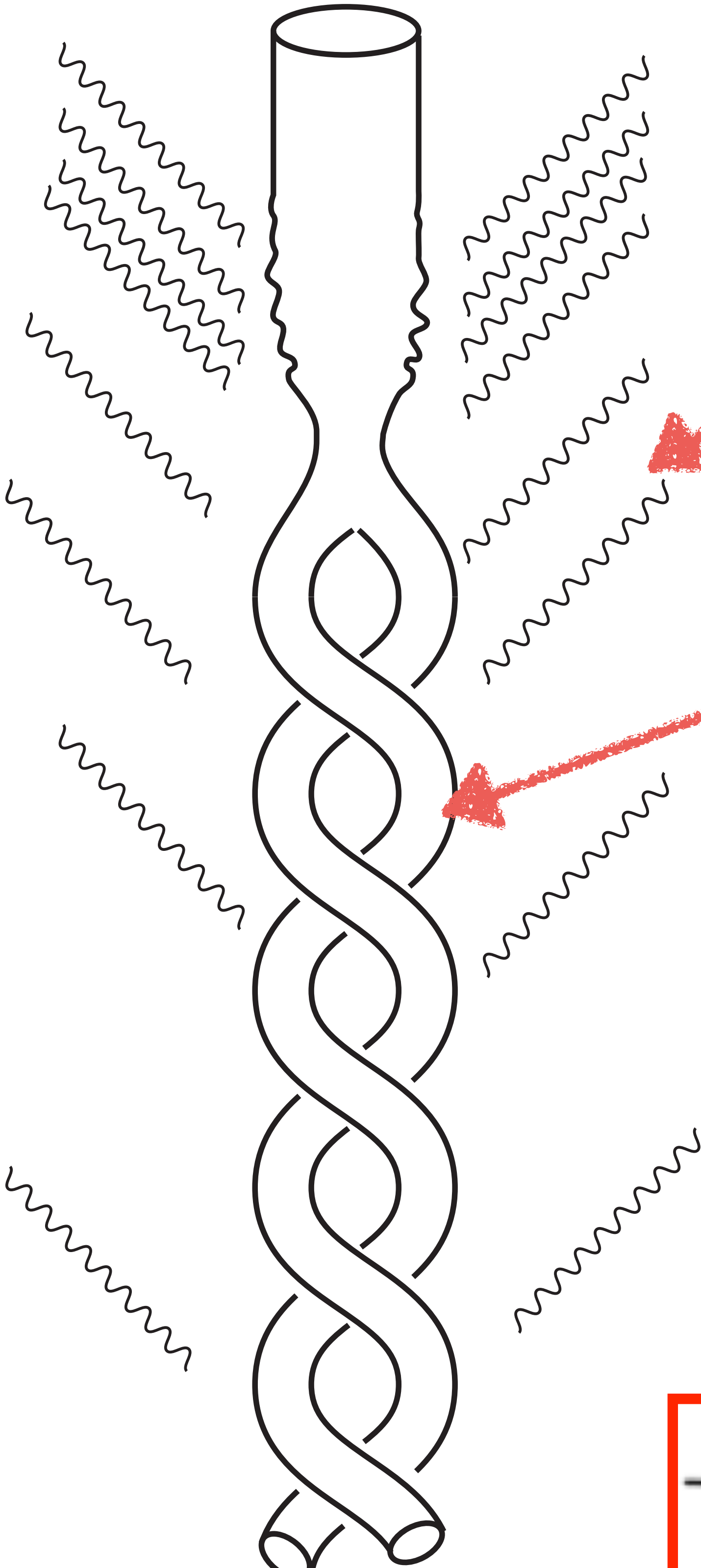
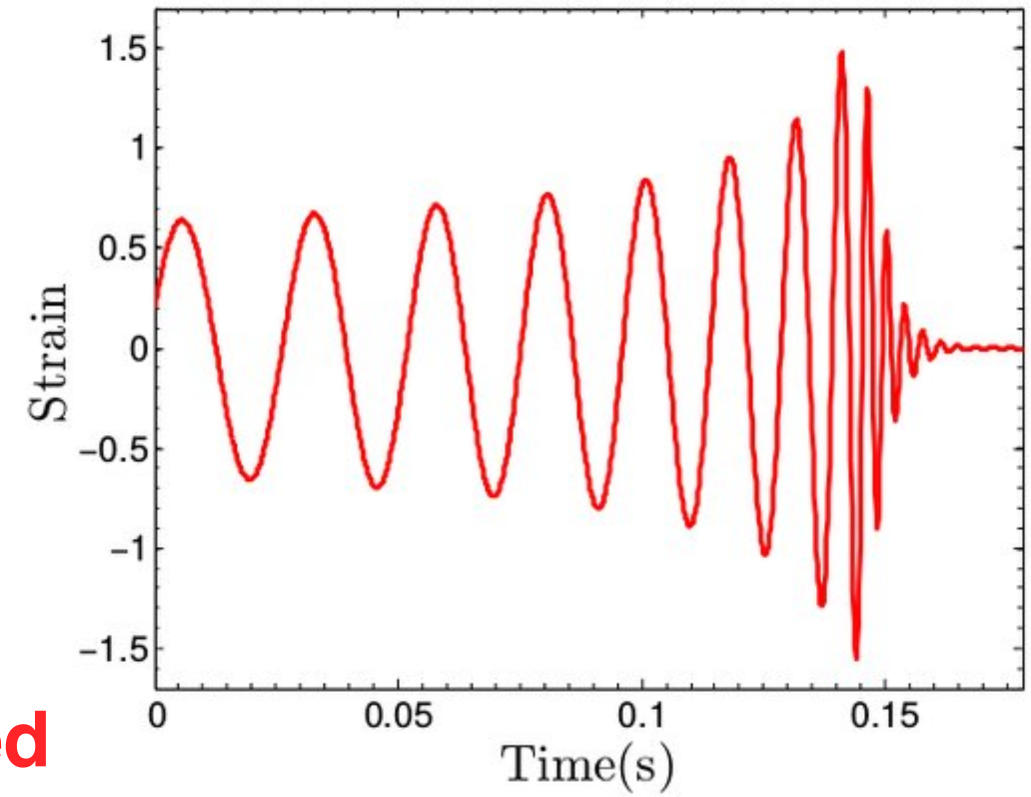


GW emission

from **radiation-reacted** binary dynamics



$$ds^2 = g_{\mu\nu}(x^\lambda)\, dx^\mu dx^\nu$$

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$R_{\mu\nu} = 0$$

$$-g^{\mu\nu} g_{\alpha\beta,\,\mu\nu} + g^{\mu\nu} g^{\rho\sigma} \big(g_{\alpha\mu,\,\rho} g_{\beta\nu,\,\sigma} - g_{\alpha\mu,\,\rho} g_{\beta\sigma,\,\nu} + g_{\alpha\mu,\,\rho} g_{\nu\sigma,\,\beta} + g_{\beta\mu,\,\rho} g_{\nu\sigma,\,\alpha} - \frac{1}{2} g_{\mu\rho,\,\alpha} g_{\nu\sigma,\,\beta}\big) = 0$$

TWO-BODY/EOB "CORRESPONDENCE":

THINK QUANTUM-MECHANICALLY (J.A. WHEELER)

Real 2-body system
(in the c.o.m. frame)
(m_1, m_2)

1:1 map

An effective particle
in some effective metric

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$g_{\mu\nu}^{\text{eff}}$$

$$\mathcal{E} = f(E)$$

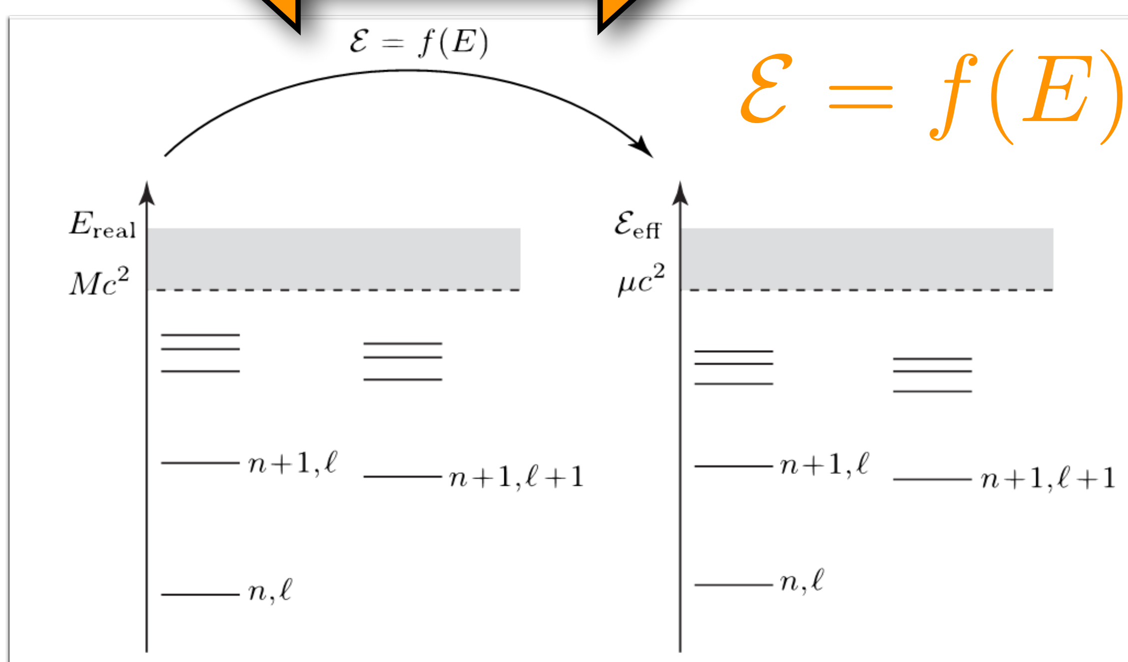


Figure 1: Sketch of the correspondence between the quantized energy levels of the real and effective conservative dynamics. n denotes the 'principal quantum

$$\mu^2 + g_{\text{eff}}^{\mu\nu} \frac{\partial S_{\text{eff}}}{\partial x^\mu} \frac{\partial S_{\text{eff}}}{\partial x^\nu} + \mathcal{O}(p^4) = 0$$

Bohr-Sommerfeld's Quantization Conditions

(action-angle variables & Delaunay Hamiltonian)

$$J = \ell\hbar = \frac{1}{2\pi} \oint p_\varphi d\varphi$$

$$N = n\hbar = I_r + J$$

$$I_r = \frac{1}{2\pi} \oint p_r dr$$

$$\mathcal{E}_{\text{eff}} = \frac{(\mathcal{E}_{\text{real}})^2 - m_1^2 - m_2^2}{2(m_1 + m_2)}$$

$$H^{\text{classical}}(q,p) \rightarrow H^{\text{classical}}(I_a) \rightarrow E^{\text{quantum}}(I_a = n_a h) = f^{-1}[\mathcal{E}_{\text{eff}}^{\text{quantum}}(I_a^{\text{eff}} = n_a h)]$$

EOB

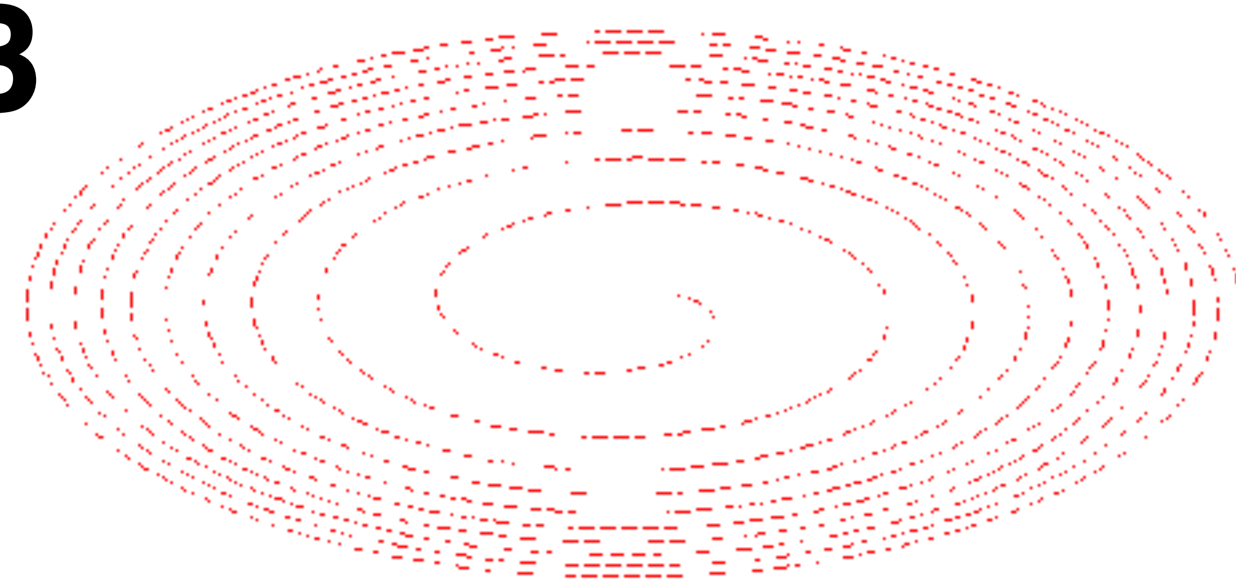
$$\frac{dr}{dt} = \left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial p_{r_*}} ,$$

$$\frac{dp_{r_*}}{dt} = -\left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial r} ,$$

$$\Omega \equiv \frac{d\varphi}{dt} = \frac{\partial \hat{H}_{\rm EOB}}{\partial p_\varphi} ,$$

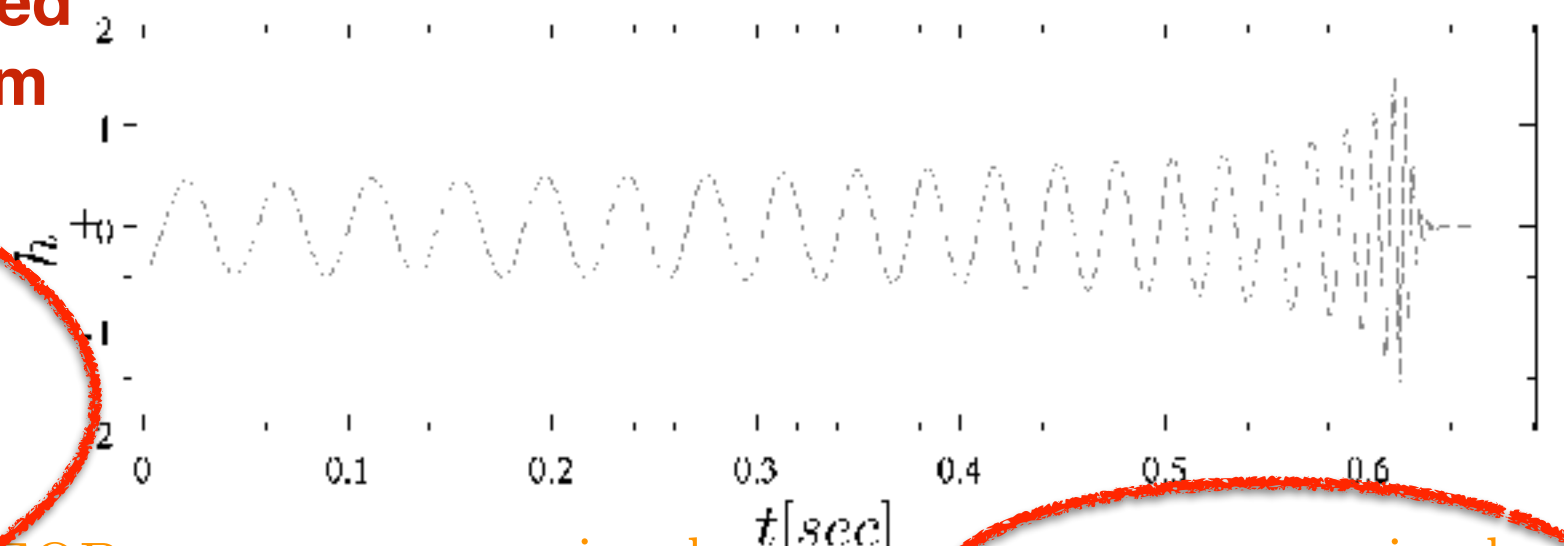
$$\frac{dp_\varphi}{dt} = \hat{\mathcal{F}}_\varphi .$$

Hamiltonian: conservative dynamics



Rad Reac Force

Resummed waveform



$$h_{\ell m} \equiv h^{(N,\epsilon)}_{\ell m} \hat{h}^{(\epsilon)}_{\ell m} \hat{h}^{\rm NQC}_{\ell m}$$

$$\hat{h}^{(\epsilon)}_{\ell m} = \hat{S}^{(\epsilon)}_{\rm eff} T_{\ell m} e^{i\delta_{\ell m}} \rho^{\ell}_{\ell m}$$

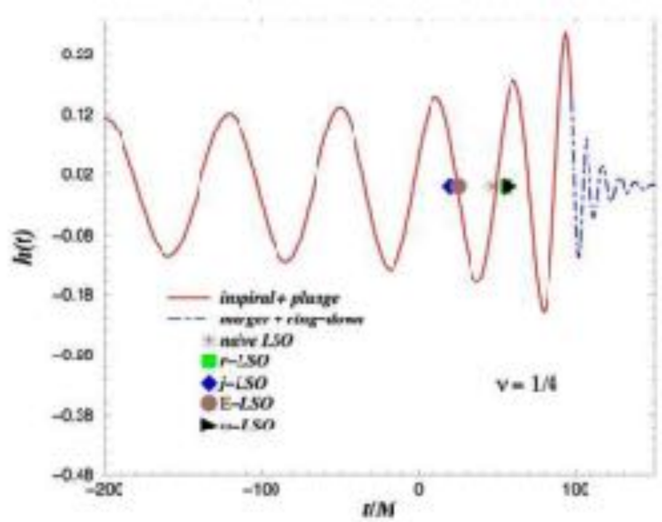
$$\mathcal{F}_\varphi \equiv -\frac{1}{8\pi\Omega} \sum_{\ell=2}^{\ell_{\rm max}} \sum_{m=1}^{\ell} (m\Omega)^2 |R h^{(\epsilon)}_{\ell m}|^2$$

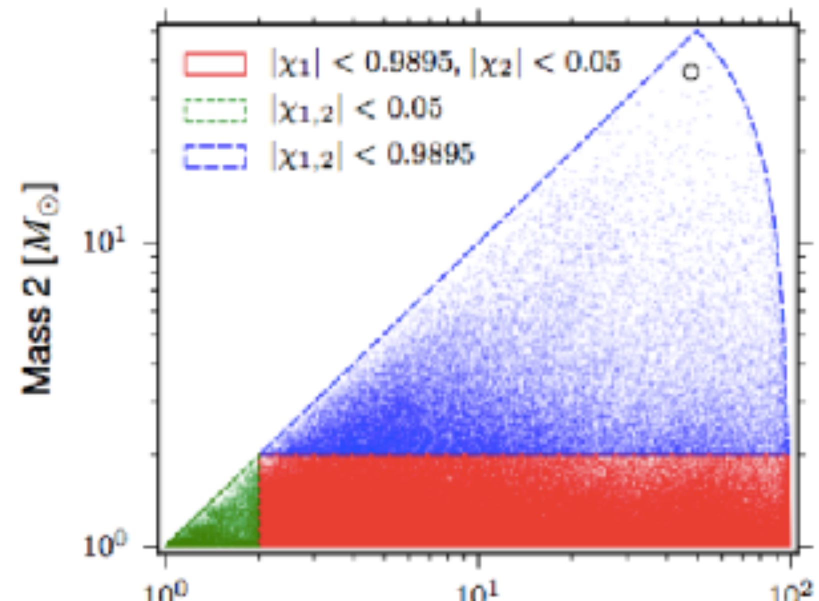
$$h^{\rm EOB}_{\ell m} = \theta(t_m - t) h^{\rm insplunge}_{\ell m}(t) + \theta(t - t_m) h^{\rm ringdow}_{\ell m}$$

$$h^{\rm ringdown}_{\ell m}(t) = \sum_N C^+_N e^{-\sigma^+_N (t - t_m)}$$

$$T_{\ell m} = \frac{\Gamma(\ell + 1 - 2i\hat{k})}{\Gamma(\ell + 1)} e^{\pi \hat{k}} e^{2i\hat{k}\log(2kr_0)},$$

Complete waveforms for BBH coalescenceS:





General Relativity and Experiment

(review: particle data group)

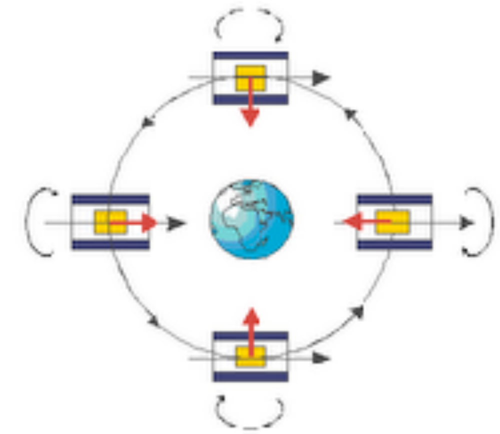
Tests of the Equivalence Principle

Sommerfeld's « constant » 1916

$$\alpha \equiv \frac{e^2}{\hbar c} \approx \frac{1}{137.036}$$

$$d\ln(\alpha_{\rm em})/dt = (-2.5 \pm 2.6) \times 10^{-17}{\rm yr}^{-1},$$
$$d\ln(\mu)/dt = (-1.5 \pm 3.0) \times 10^{-16}{\rm yr}^{-1},$$
$$d\ln(m_q/\Lambda_{\rm QCD})/dt = (7.1 \pm 4.4) \times 10^{-15}{\rm yr}^{-1}.$$

Microscope (Touboul et al.)



$$(\Delta a/a)_{\rm BeTi} = (0.3 \pm 1.8) \times 10^{-13};$$
$$(\Delta a/a)_{\rm BeAl} = (-0.7 \pm 1.3) \times 10^{-13};$$
$$(\Delta a/a)_{\rm TiPt} = (-1 \pm 9({\rm stat}) \pm 9({\rm syst})) \times 10^{-15}$$

Lunar-Laser ranging

$$(\Delta a/a)_{\rm EarthMoon} = (-3 \pm 5) \times 10^{-14}.$$

Tests of post-Newtonian gravity (solar system)

Two post-Newtonian parameters

$$g_{00} = -1 + \frac{2}{c^2}V - \frac{2\beta}{c^4}V^2 + O\left(\frac{1}{c^6}\right)$$

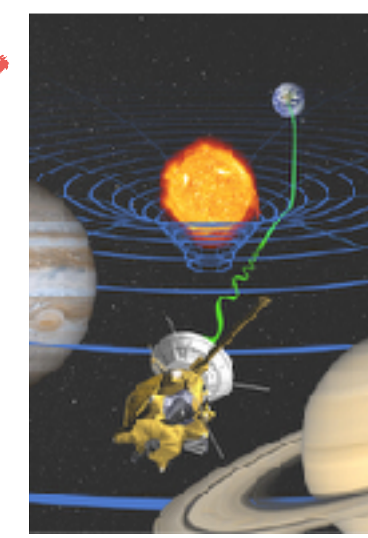
$$g_{0i} = -\frac{2(\gamma+1)}{c^3}V_i + O\left(\frac{1}{c^5}\right),$$

$$g_{ij} = \delta_{ij}\left[1 + \frac{2\gamma}{c^2}V\right] + O\left(\frac{1}{c^4}\right),$$

$$\bar{\gamma} = \gamma - 1$$

$$\bar{\beta} = \beta - 1$$

Cassini Mission



$$\overline{\gamma} = (2.1 \pm 2.3) \times 10^{-5}$$

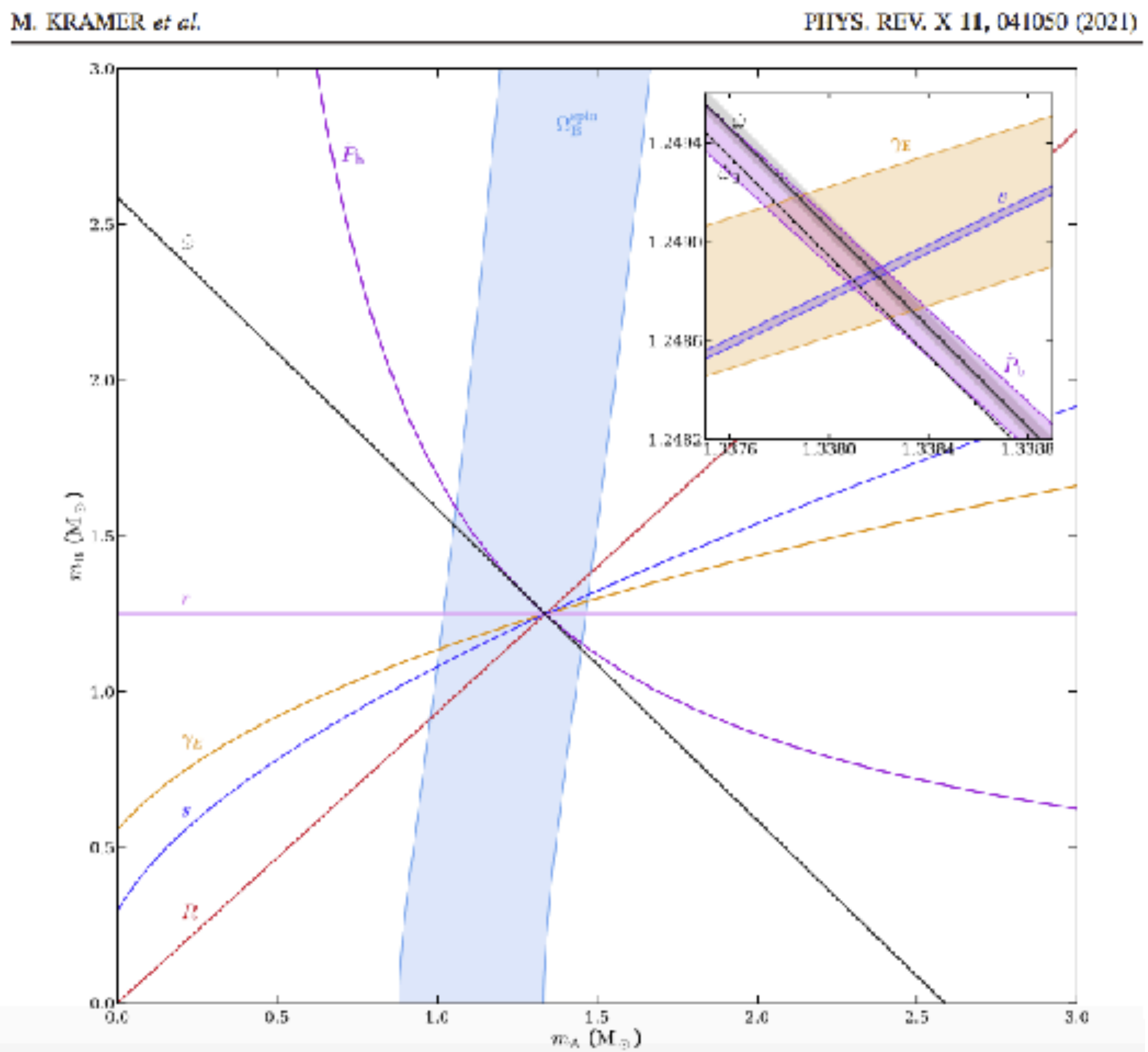
$$|\overline{\beta}| < 7 \times 10^{-5}$$

Binary Pulsar Tests: strong and radiative fields

PSR1913+16

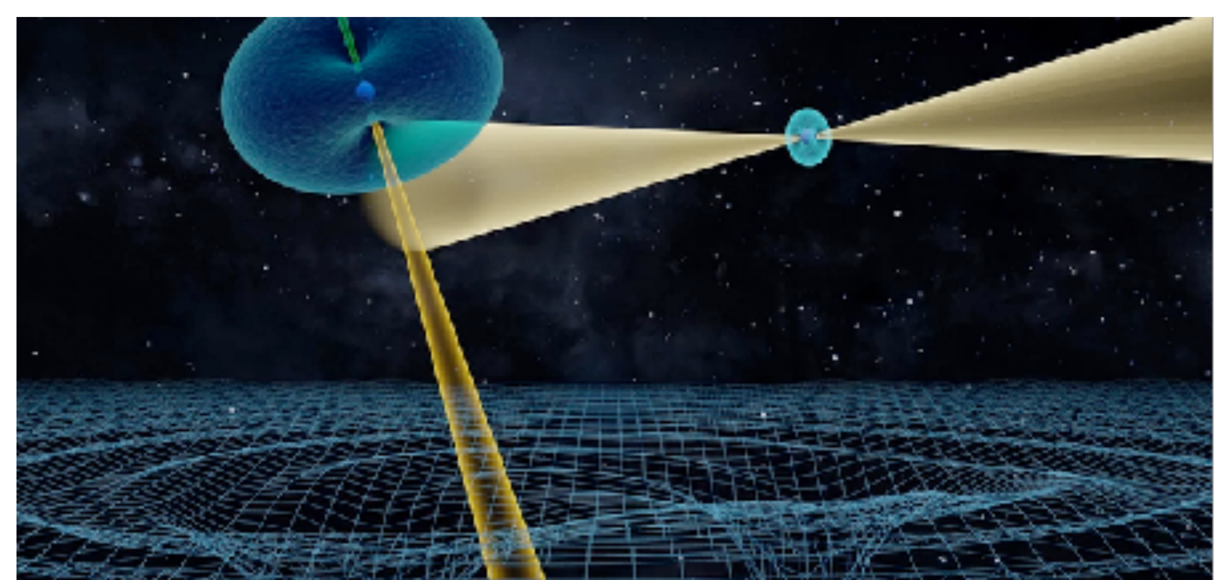
$$\left[\frac{\dot{P}_b^{\mathrm{obs}} - \dot{P}_b^{\mathrm{gal}}}{\dot{P}_b^{\mathrm{GR}}[k^{\mathrm{obs}}, \gamma_{\mathrm{timing}}^{\mathrm{obs}}]}\right]_{1913+16} = 0.9983 \pm 0.0016$$

Double Pulsar(Kramer et al)



Taylor Hulse

5 precision tests of GR

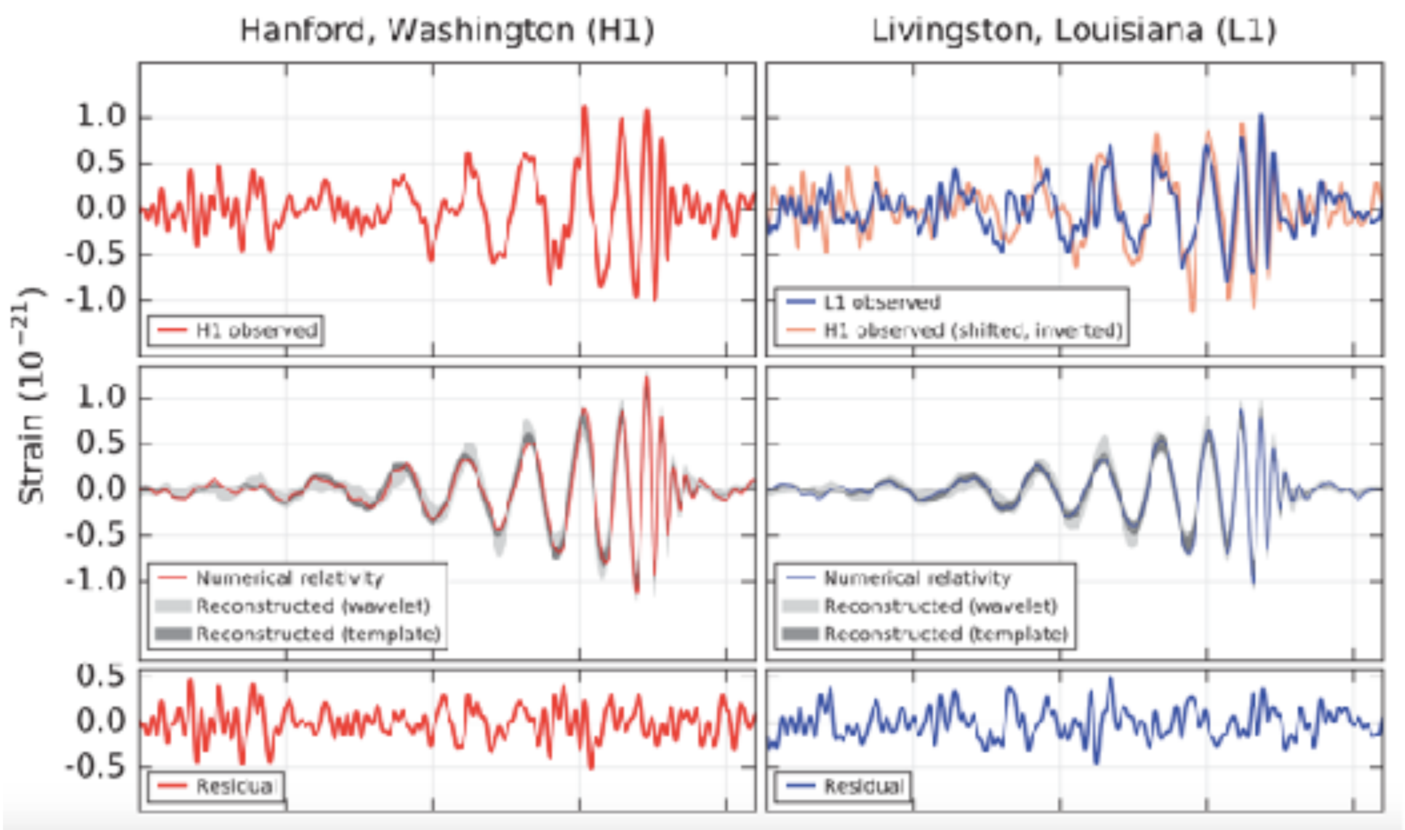


GR tests from LIGO-Virgo

Correlation between the observed GW signal from the coalescence of two black holes and the GR prediction:

> 96%

The most direct evidence that the BHs predicted by GR exist and have the expected structure, notably the final damped vibration modes.

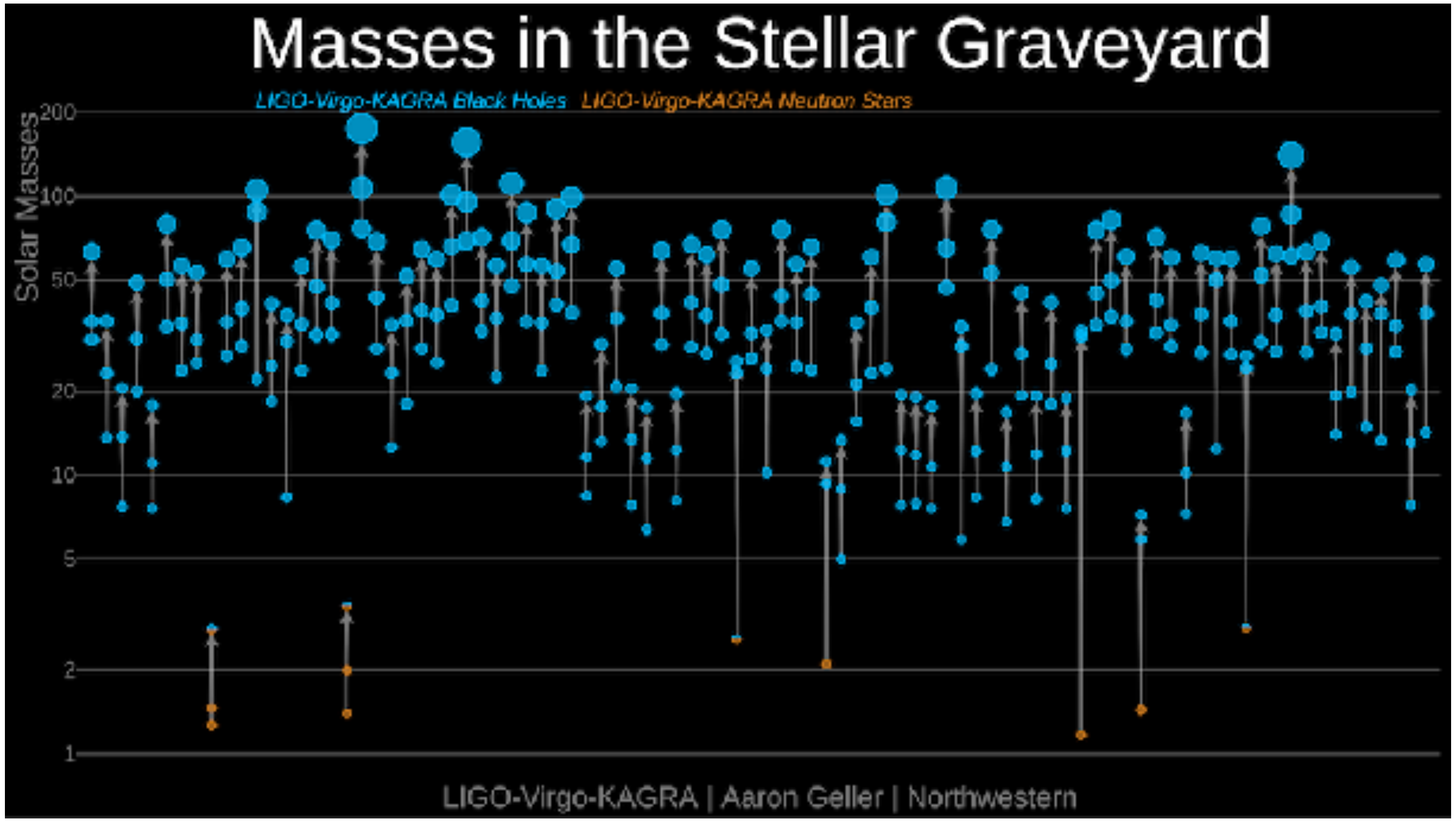


90 events, incl.:

2 NS-NS; 3 NS-BH; 85 BH-BH

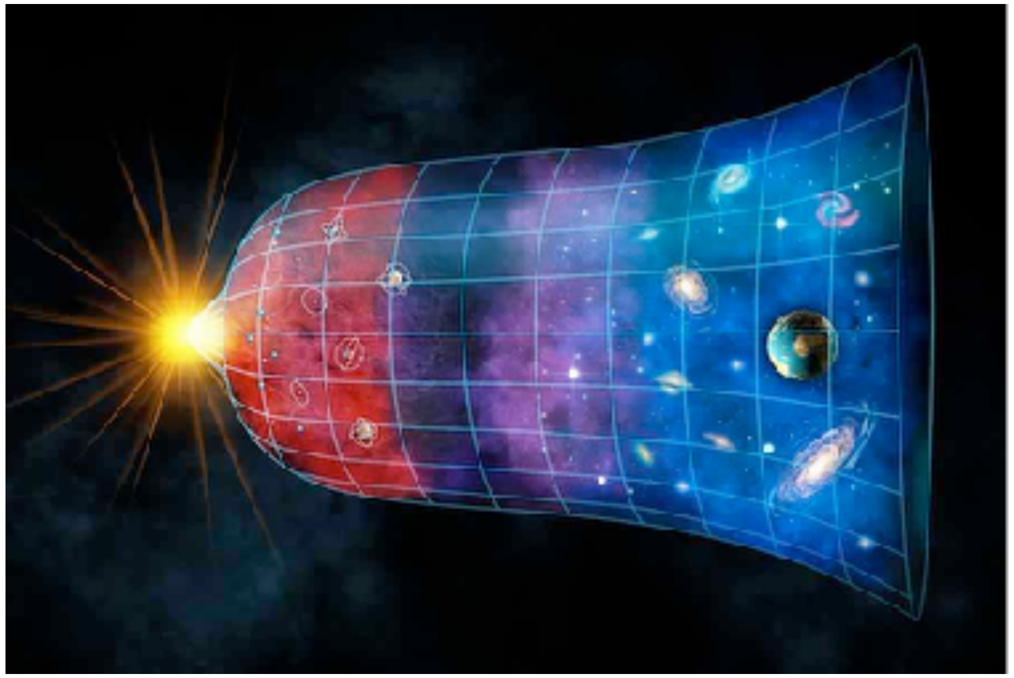
GW170817

$$-3 \times 10^{-15} < \frac{c_{GW} - c}{c} < +7 \times 10^{-16}.$$

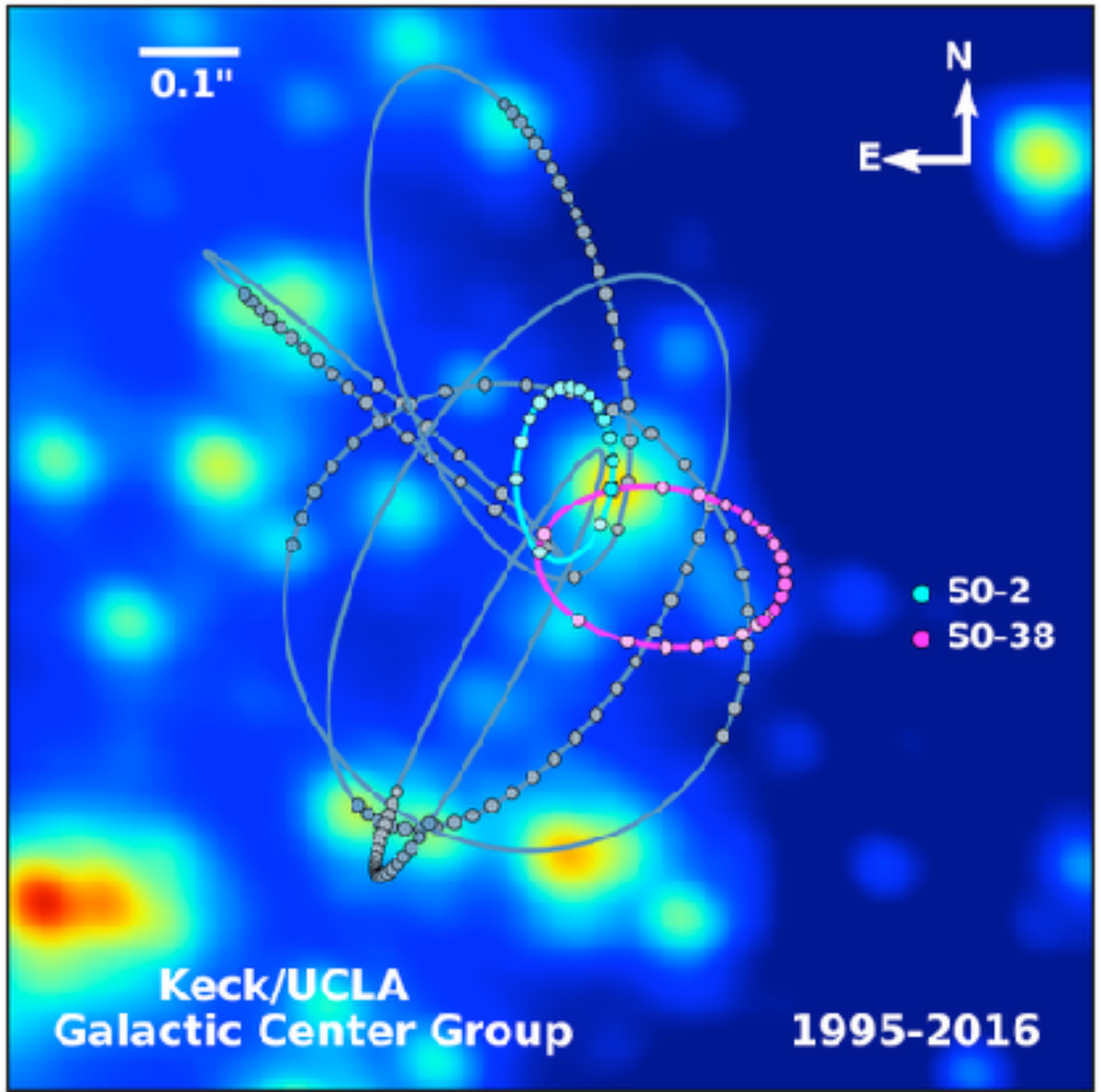


Other tests

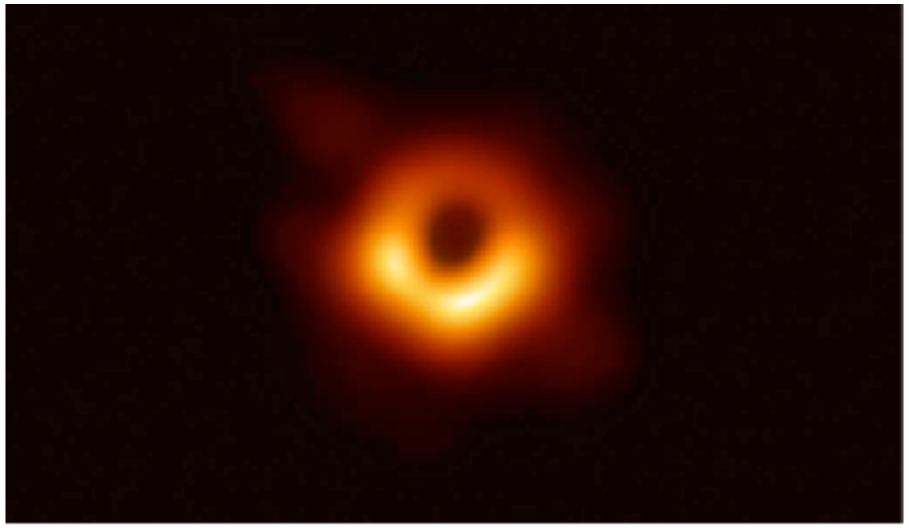
cosmology



centre of our Galaxy SgrA*: notably S2

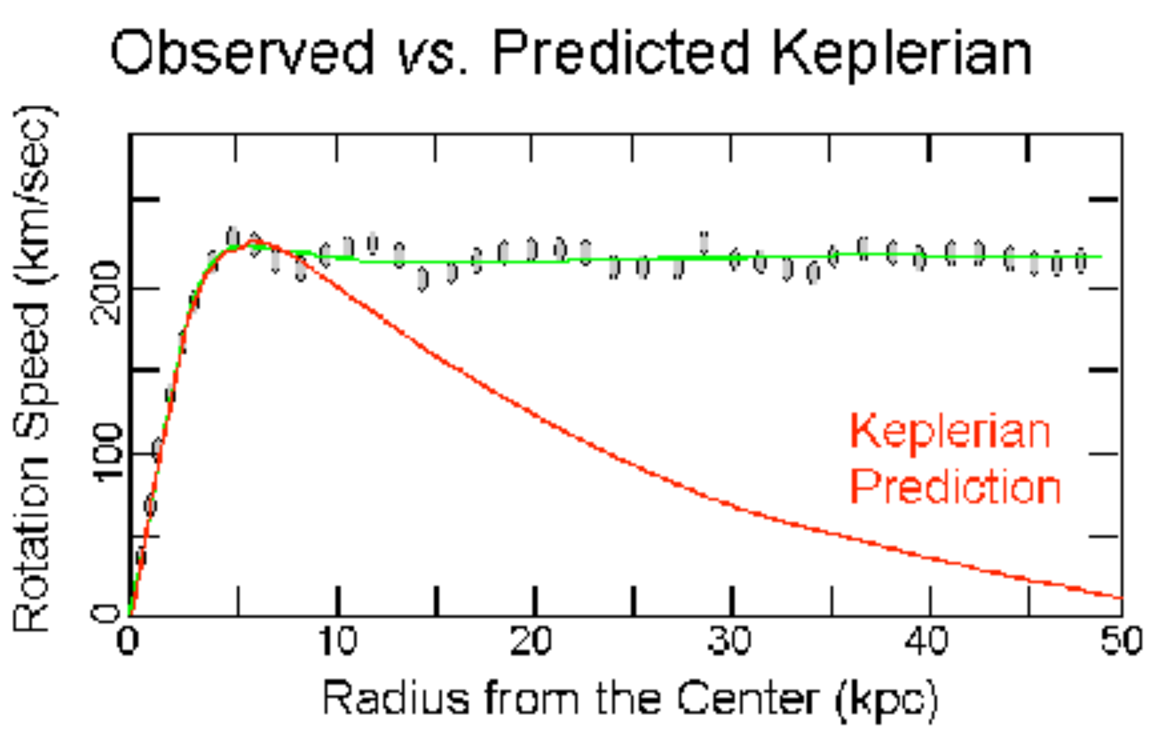


EHT

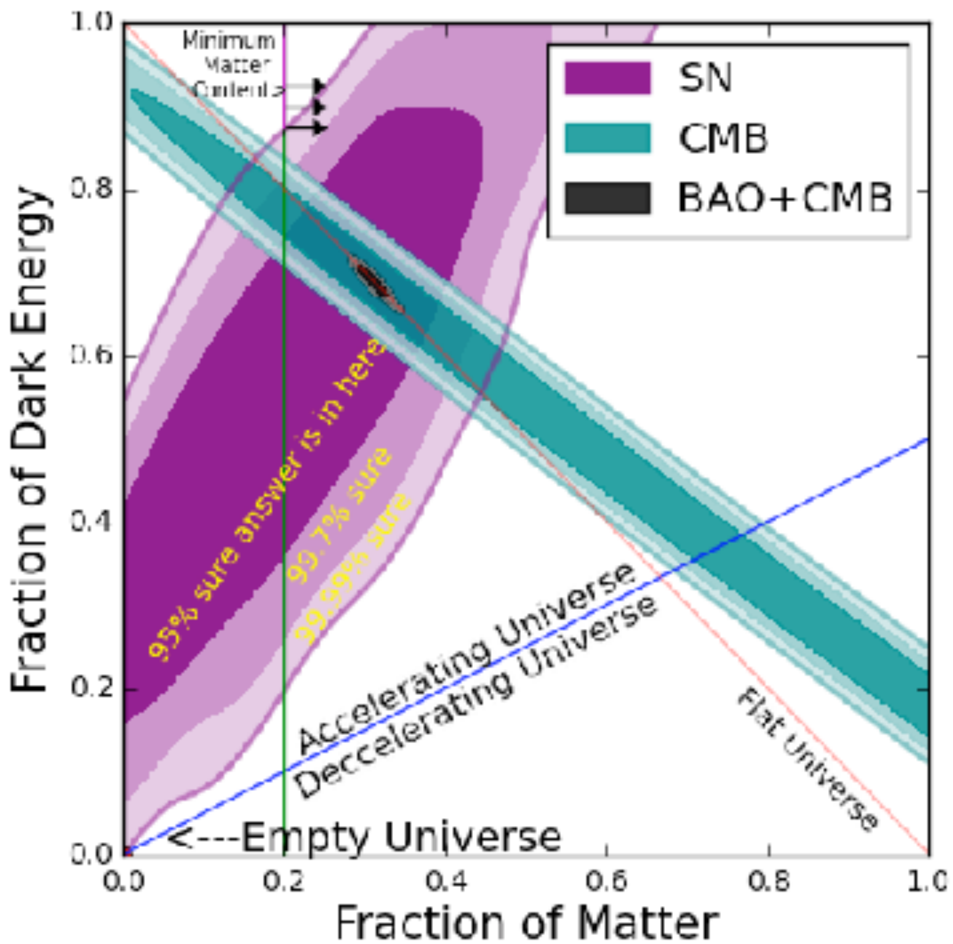


Two « black clouds » ??

dark matter



dark energy



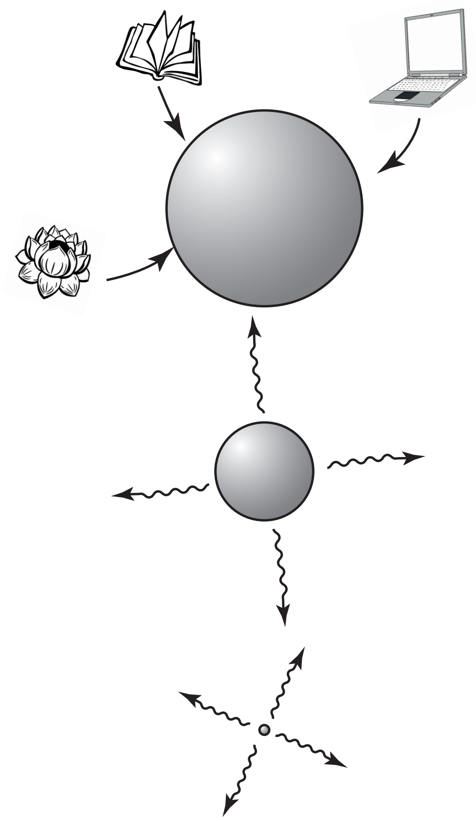
Theoretical Puzzles

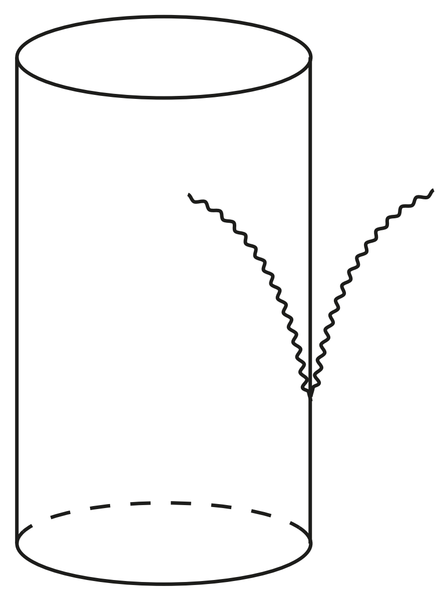
perturbative non renormalisability

$$l_{\mathrm{P}} = \sqrt{\frac{\hbar G}{c^3}}$$

$$l_P \approx 1.66 \times 10^{-33} \mathrm{cm}$$

quantum mysteries of black holes

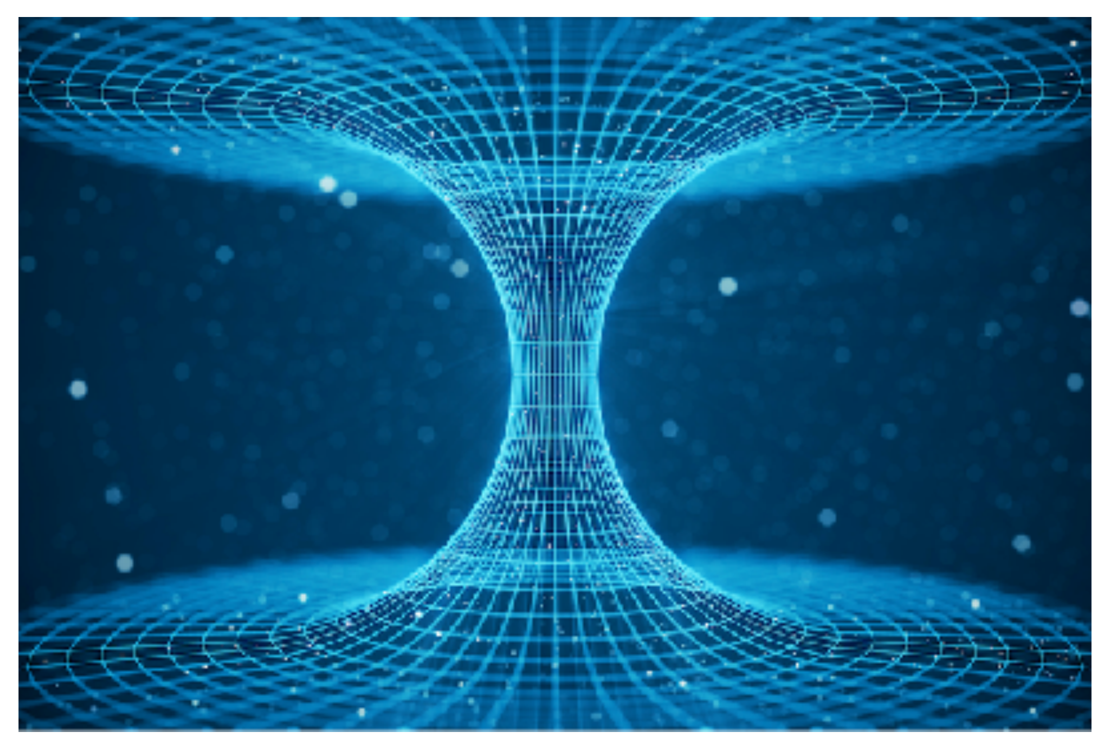




Bekenstein

Hawking

Quantum Evaporation: information ?



EPR= ER ?? (Maldacena-Susskind 2013**)**

Quantum Cosmos (Mukhanov-Chibisov 1981)
(compatible with CMB observations)