Quantum Optics
&
(Mesoscopic) Condensed Matter

UNIVERSITY OF INNSBRUCK

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Innsbruck:
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M. Wallquist
C. Genes
A. Glätzle

AMO - mesoscopic solid state collaborations:
M. Lukin + P. Rabl (Harvard)
E. Polzik (NBI)
Jun Ye (JILA)
H.J. Kimble (Caltech)
F. Marquardt (LMU)
P. Treutlein (LMU)

IQOQI
AUSTRIAN ACADEMY OF SCIENCES

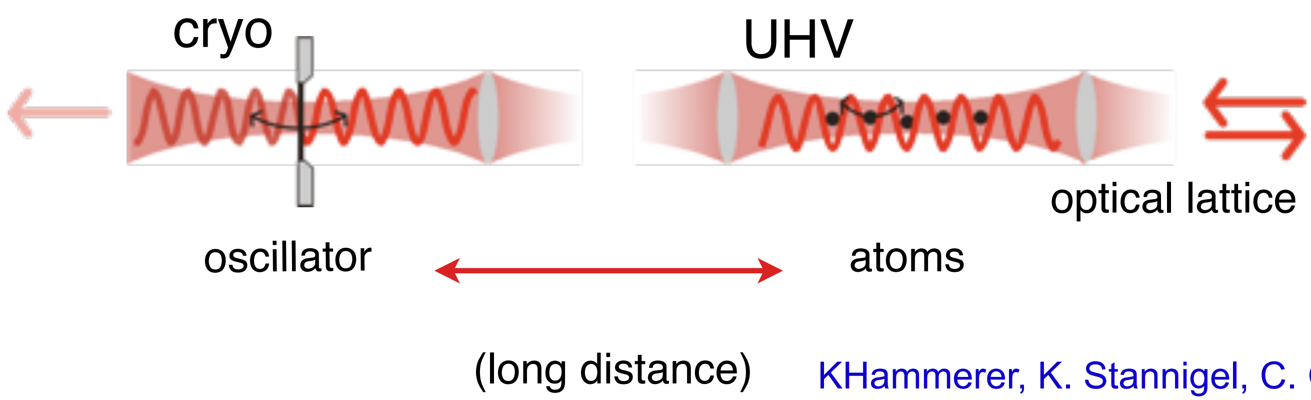
SFB
Coherent Control of Quantum Systems

€U networks

Outline

- **Quantum Noise & Quantum Optics**
  - a mini-tutorial

- **Atoms in Optical Lattices + “Nano-”Mechanical Mirrors / Membranes**



more details: K. Hammerer in his talk on Feb 5

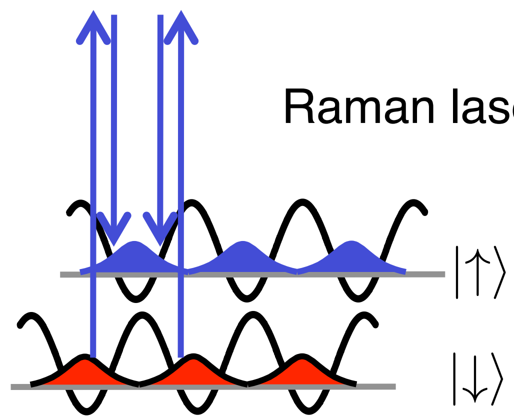
nanomechanics

+ AMO interfaces

KHammerer, K. Stannigel, C. Genes, M. Wallquist, PZ. (Innsbruck)
P. Treutlein, S. Camerer, D. Hunger, T. W. Hänsch (LMU)
in preparation

- **Measurement of Atomic Currents via Light**

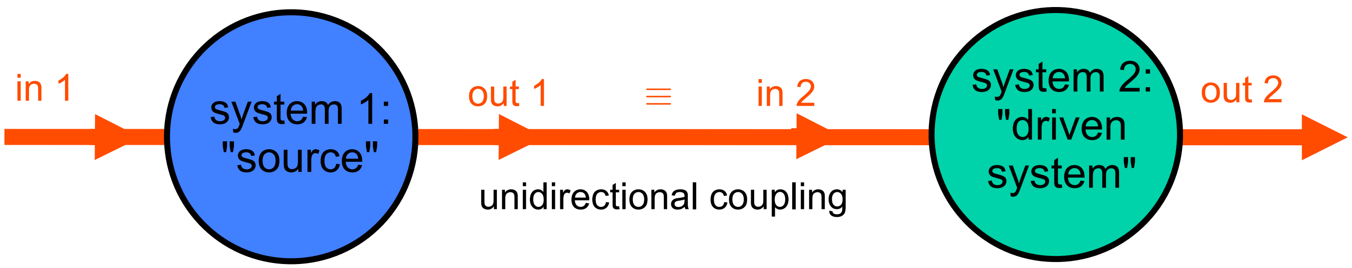
V. Steixner, K Hammerer, A Daley, PZ
in preparation



Mini-Tutorial:

Quantum Optics and Quantum Noise

- **Stochastic Schrödinger equations** (& quantum trajectories)
- **cascaded quantum systems *etc.***



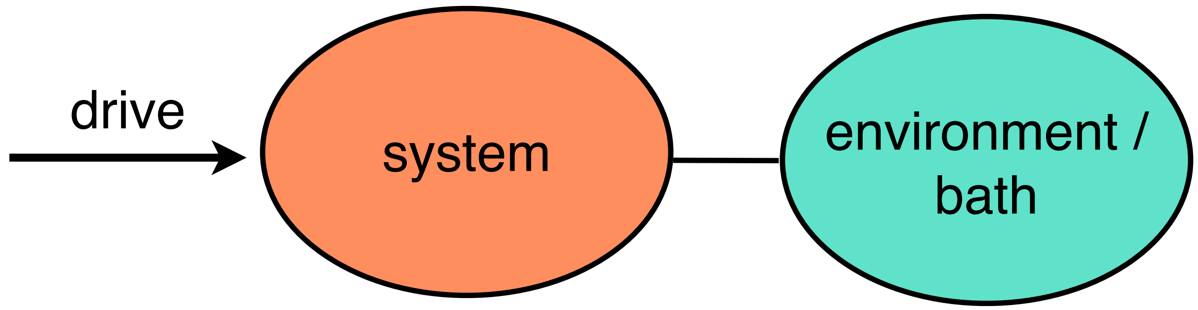
"system 1" drives "system 2"

Stochastic Schrödinger equations with time delays

(in a way not found in Quantum Noise, CW Gardiner & PZ)

Quantum Optics: Open Quantum Systems

- **open quantum system**



role of the environment:

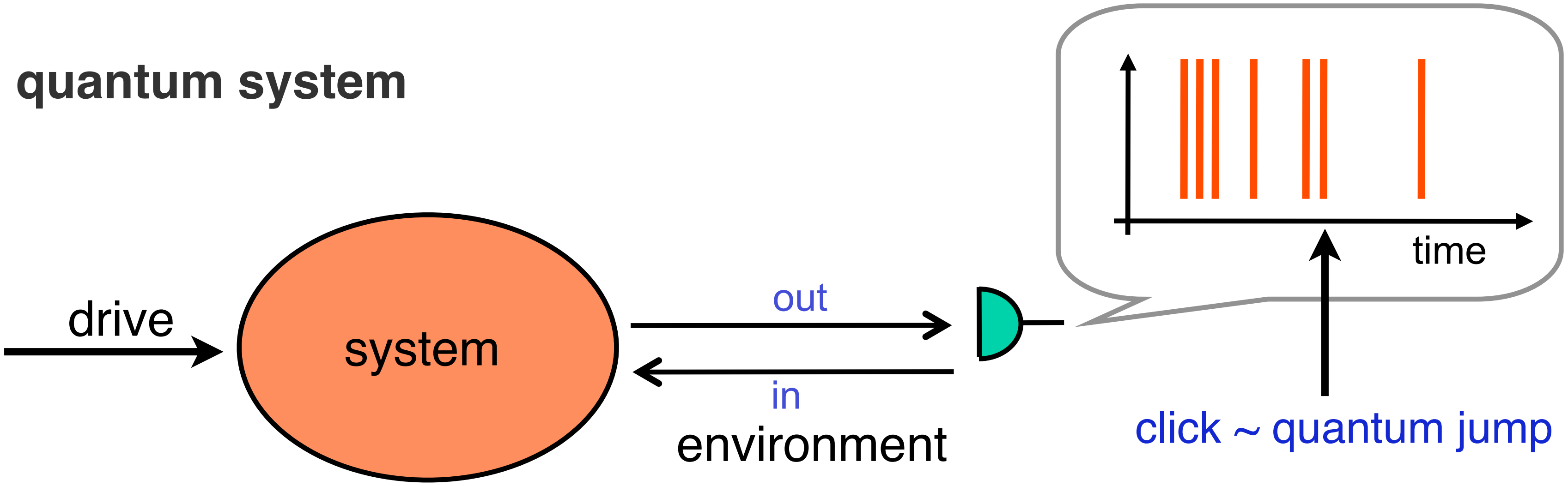
- noise and dissipation (decoherence)
- quantum optics ... tool: state preparation
  - e.g. laser cooling, optical pumping

bath / reservoir: harmonic oscillators

- quantum optics
  - radiation field
  - [Bogoliubov excitation, spin bath]

Quantum Optics: Continuous Measurement

- **open quantum system**



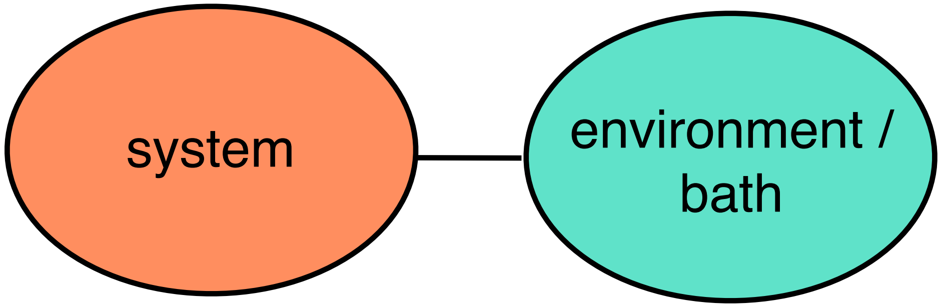
role of the environment:

- continuous observation

quantum optical tools and techniques:

- Quantum Markov processes
- Master Equation
- (Quantum) Stochastic Schrödinger Equation
    - Quantum Trajectories

Generic Quantum Optical Model



$$H = H_{\rm sys} + H_B + H_{\rm int}$$

$$H_B = \int_{\omega_0-\vartheta}^{\omega_0+\vartheta} d\omega\, \omega\, b^\dagger(\omega) b(\omega)$$ bath of oscillators

$$\left[b(\omega), b^\dagger(\omega')\right] = \delta(\omega-\omega')$$

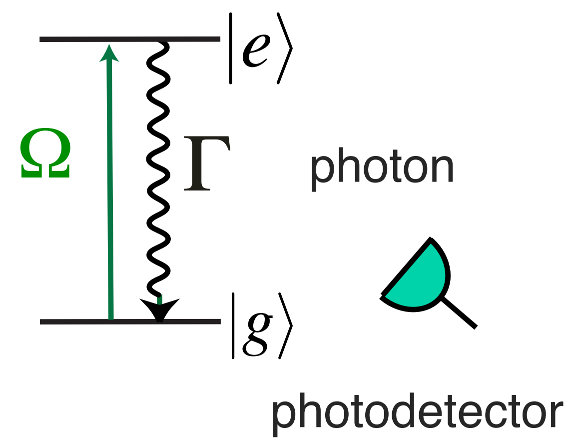
$$H_{\rm int} = i \int_{\omega_0-\vartheta}^{\omega_0+\vartheta} d\omega \kappa(\omega) \left[b^\dagger(\omega) c - c^\dagger b(\omega)\right]$$

system “quantum jump” operator

✓ Rotating wave approximation

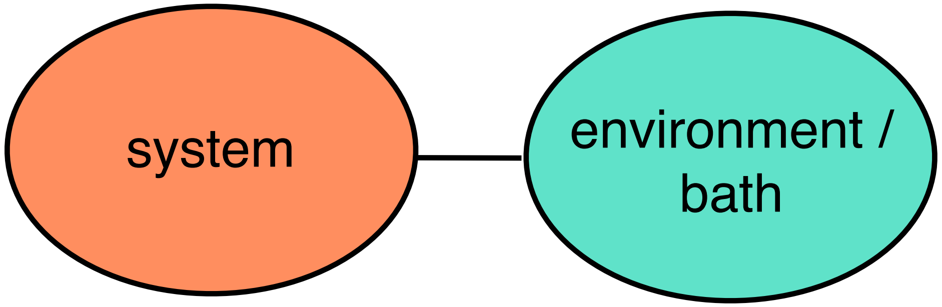
✓ Markov / white noise

Example: spontaneous emission from two level system



$$c = |g\rangle\langle e| \equiv \sigma^-$$

Generic Quantum Optical Model



$$H = H_{\text{sys}} + H_B + H_{\text{int}}$$

$$H_B = \int_{\omega_0-\vartheta}^{\omega_0+\vartheta} d\omega\, \omega\, b^\dagger(\omega) b(\omega)$$ bath of oscillators

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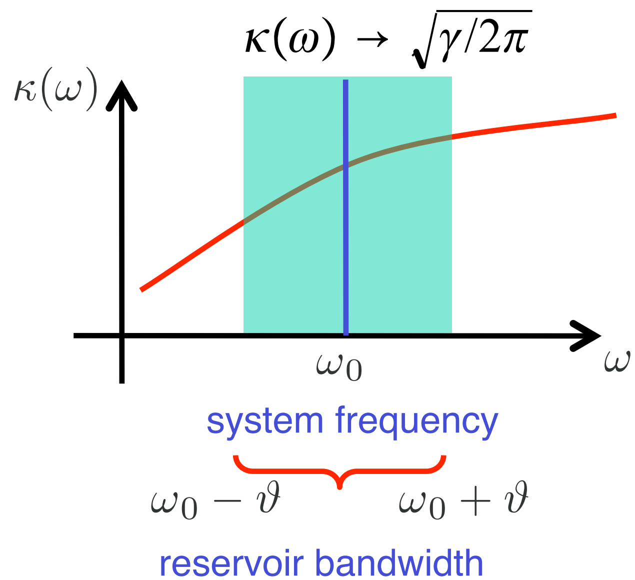
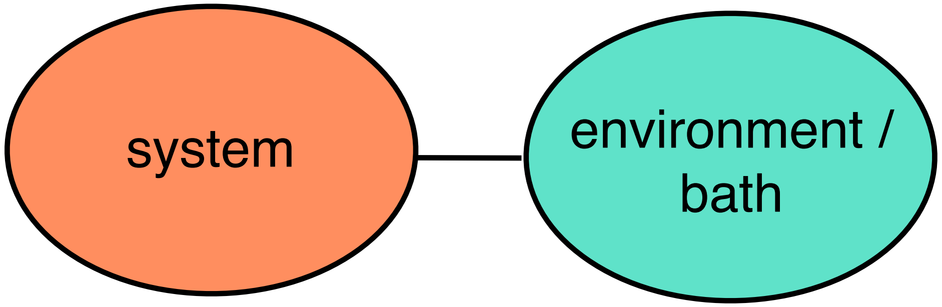
system "quantum jump" operator



✓ Rotating wave approximation

✓ Markov / white noise

Generic Quantum Optical Model



interaction picture

$$H = H_{\text{sys}} + \cancel{H_B} + H_{\text{int}}$$

$$H_B = \int_{\omega_0-\vartheta}^{\omega_0+\vartheta} d\omega\, \omega\, b^\dagger(\omega) b(\omega)$$ bath of oscillators

$$\left[b(\omega), b^\dagger(\omega')\right] = \delta(\omega - \omega')$$

$$H_{\text{int}}(t) = i \int_{\omega_0-\vartheta}^{\omega_0+\vartheta} d\omega \kappa(\omega) \left[b^\dagger(\omega) e^{i\omega t} c - c^\dagger b(\omega) e^{-i\omega t}\right]$$

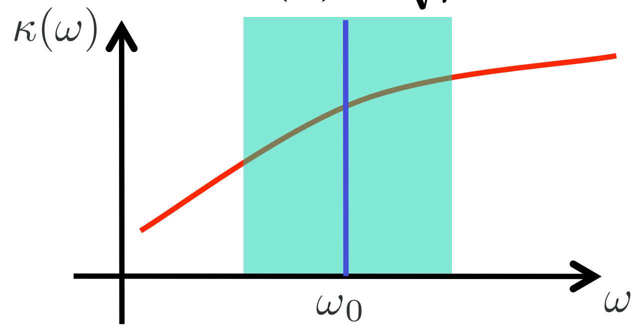
- noise operator: “quantum noiselets”

$$b(t) := \frac{1}{\sqrt{2\pi}} \int_{\omega_0-\vartheta}^{\omega_0+\vartheta} b(\omega) e^{-i(\omega-\omega_0)t} d\omega$$

- “white noise” commutator

$$\left[b(t), b^\dagger(s)\right] = \delta_s(t-s)$$

$\kappa(\omega) \to \sqrt{\gamma/2\pi}$

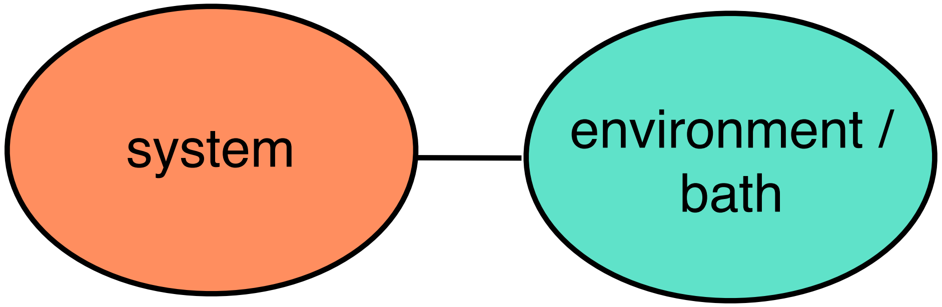


system frequency

$\omega_0 - \vartheta$ $\omega_0 + \vartheta$

reservoir bandwidth

Generic Quantum Optical Model



Stratonovich **Quantum Stochastic Schrödinger Equation (QSSE)**

(S) $$\frac{d}{dt}|\Psi(t)\rangle = \left\{-iH_{\text{sys}} + \sqrt{\gamma}b^\dagger(t)c - \sqrt{\gamma}c^\dagger b(t)\right\}|\Psi(t)\rangle$$

- noise operator: "quantum noiselets"

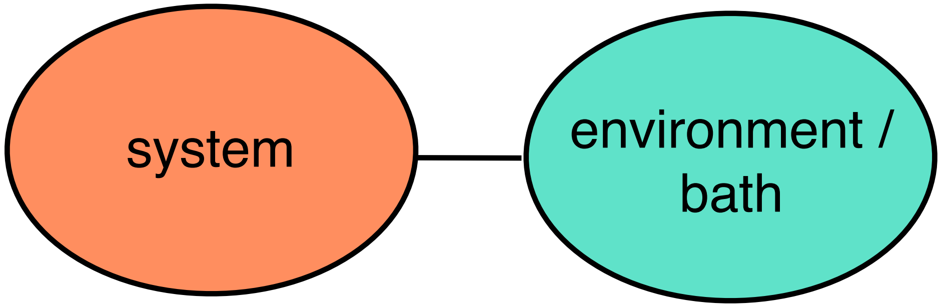
$$b(t) := \frac{1}{\sqrt{2\pi}}\int_{\omega_0-\vartheta}^{\omega_0+\vartheta} b(\omega)e^{-i(\omega-\omega_0)t}d\omega$$

- "white noise" commutator

$$\left[b(t), b^\dagger(s)\right] = \delta_s(t-s)$$

- initial condition $|\Psi(0)\rangle = |\psi_{\text{sys}}\rangle \otimes |\text{vac}\rangle$

Generic Quantum Optical Model



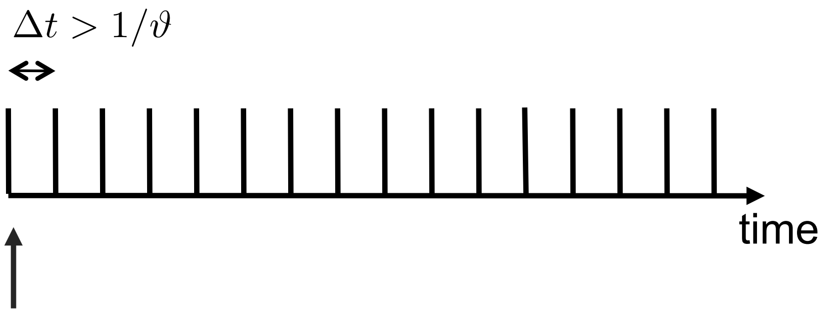
***Stratonovich* Quantum Stochastic Schrödinger Equation (QSSE)**

$$\text{(S)} \quad \frac{d}{dt}|\Psi(t)\rangle = \left\{-iH_{\rm sys} + \sqrt{\gamma}b^\dagger(t)c - \sqrt{\gamma}c^\dagger b(t)\right\}|\Psi(t)\rangle$$

Discussion / Interpretation: derive ...

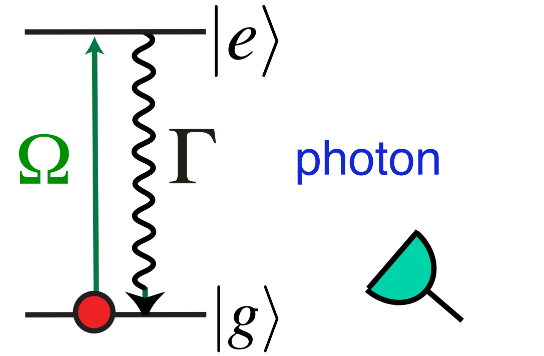
- quantum trajectories ... and conversion to *Ito* QSSE
- master equation

Coarse Grained Integration of the QSSE

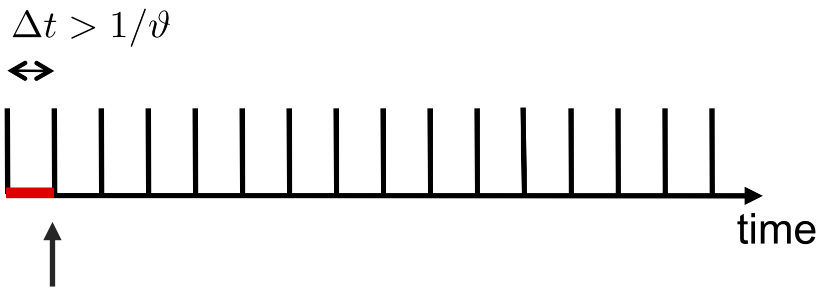


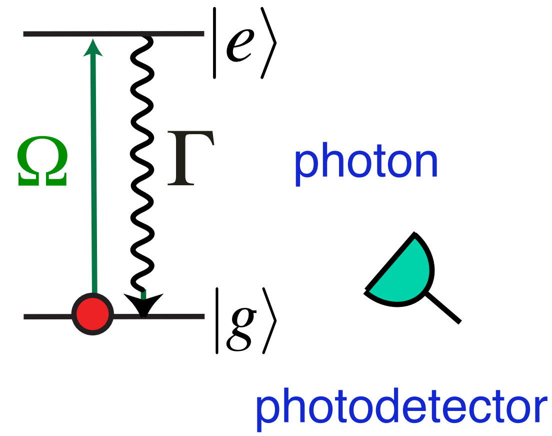
$$|\Psi(0)\rangle = |\psi_{\mathrm{sys}}\rangle \otimes |\mathrm{vac}\rangle$$

system + reservoir



Coarse Grained Integration of the QSSE



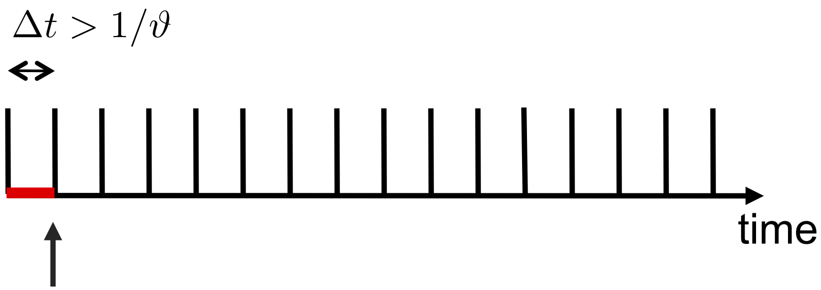


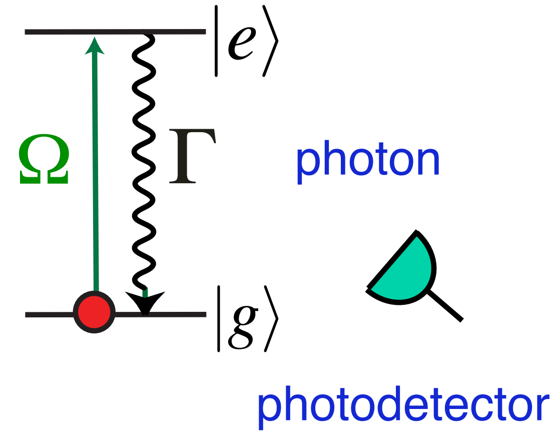
The first time step: up to order $\mathcal{O}(\Delta t)$

$$|\Psi(\Delta t)\rangle = \Big\{ \hat{1} - iH_{\text{sys}}\Delta t + \sqrt{\gamma}\, c \int_0^{\Delta t} b^\dagger(t)\, dt - \sqrt{\gamma} c^\dagger \int_0^{\Delta t} b(t)\, dt$$

$$+ (-i)^2 \gamma c^\dagger c \int_0^{\Delta t} dt \int_0^{t_2} dt'\, b(t) b^\dagger(t') + \ldots + \ldots \Big\} |\Psi(0)\rangle$$

Coarse Grained Integration of the QSSE





The first time step: up to order $\mathcal{O}(\Delta t)$

$$|\Psi(\Delta t)\rangle = \Big\{ \hat{1} - iH_{\rm sys}\Delta t + \sqrt{\gamma}\, c \int_0^{\Delta t} b^\dagger(t)\, dt - \sqrt{\gamma} c^\dagger \int_0^{\Delta t} b(t)\, dt$$

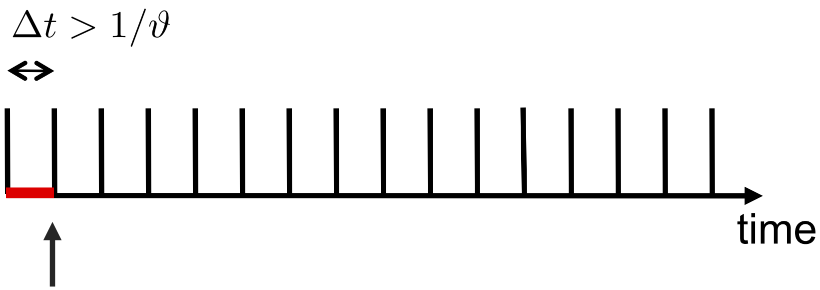
$$+ (-i)^2 \gamma c^\dagger c \int_0^{\Delta t} dt \int_0^{t_2} dt'\, \underline{b(t) b^\dagger(t')} + \ldots |\Psi(0)\rangle$$

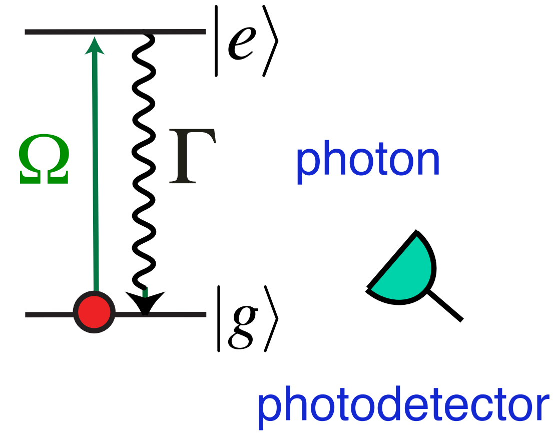
$$\left[b(t), b^\dagger(t') \right] = \delta_s(t - t')$$

$$|\psi_{\rm sys}\rangle \otimes |{\rm vac}\rangle$$

second order term gives $\mathcal{O}(\Delta t)$

Coarse Grained Integration of the QSSE

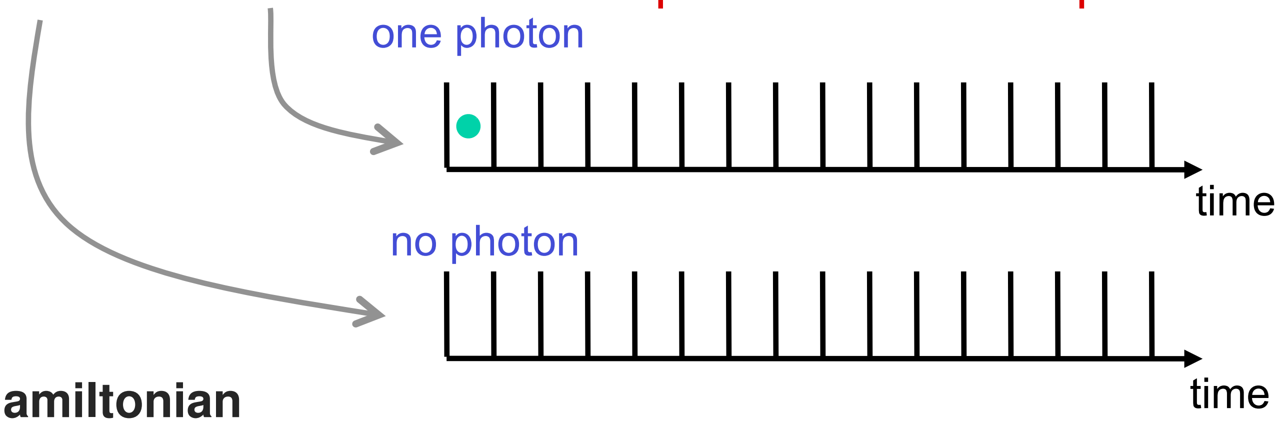




The first time step: up to order $\mathcal{O}(\Delta t)$

$$|\Psi(\Delta t)\rangle = \{\hat{1} - iH_{\text{eff}}\,\Delta t + \sqrt{\gamma}\,c\,\Delta B^{\dagger}(0)\}\,|\Psi(0)\rangle$$

superposition state of "no photon" and "one photon"



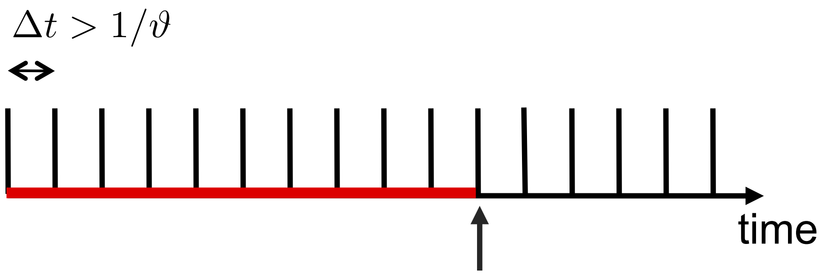
- **effective system Hamiltonian**

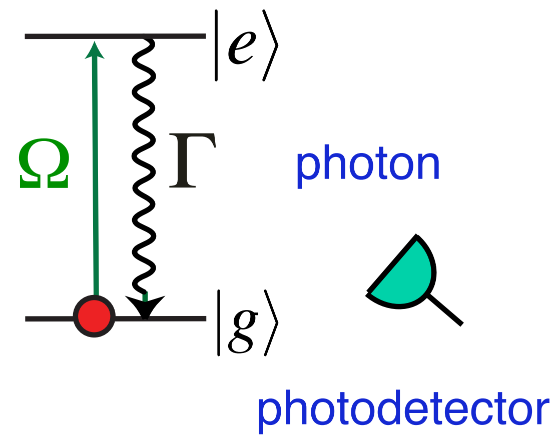
$$H_{\text{eff}} = H_{\text{sys}} - \frac{i}{2}\gamma c^{\dagger} c$$

- **noise increment**

$$\Delta B(t) := \int_{t}^{t+\Delta t} b(s)\,ds$$

Coarse Grained Integration of the QSSE





... and similar for other time steps

- **Ito Quantum Stochastic Schrödinger Equation**

(I) $dt\,|\Psi(t)\rangle = \left\{-iH_{\rm sys}dt + \sqrt{\gamma}dB^\dagger(t)c\right\}|\Psi(t)\rangle \qquad (|\Psi(0)\rangle = |\psi_{\rm sys}\rangle \otimes |{\rm vac}\rangle)$

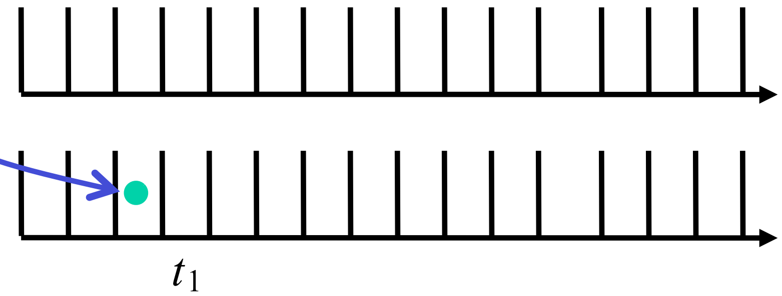
with Ito rules

$$\Delta B(t)\Delta B^\dagger(t)\,|{\rm vac}\rangle = \Delta t\,|{\rm vac}\rangle \quad \longrightarrow \quad dB(t)dB^\dagger(t) = dt$$

Quantum Trajectories

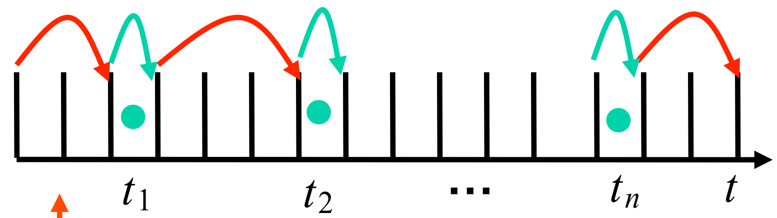
Entangled state of system and bath:

photon emission



$$
\begin{aligned}
|\Psi(t)\rangle = & |\psi_{\rm sys}(t|)\rangle|\text{vac}\rangle \\
& + \sum_{t_1} |\psi_{\rm sys}(t|t_1)\rangle \Delta B^\dagger(t_1)|\text{vac}\rangle \\
& + \ldots \\
& + \sum_{t_n > \ldots > t_1} |\psi_{\rm sys}(t|t_n, \ldots, t_1)\rangle\, \Delta B^\dagger(t_n) \ldots \Delta B^\dagger(t_1)|\text{vac}\rangle \\
& + \ldots
\end{aligned}
$$

...



- ✓ system wave function for count trajectory t_1, t_2, *etc.*
- ✓ gives photon count statistics
- ✓ can be *simulated* as a stochastic c-number Schrödinger equation

click:

"quantum jump" = effect of detecting a photon on system

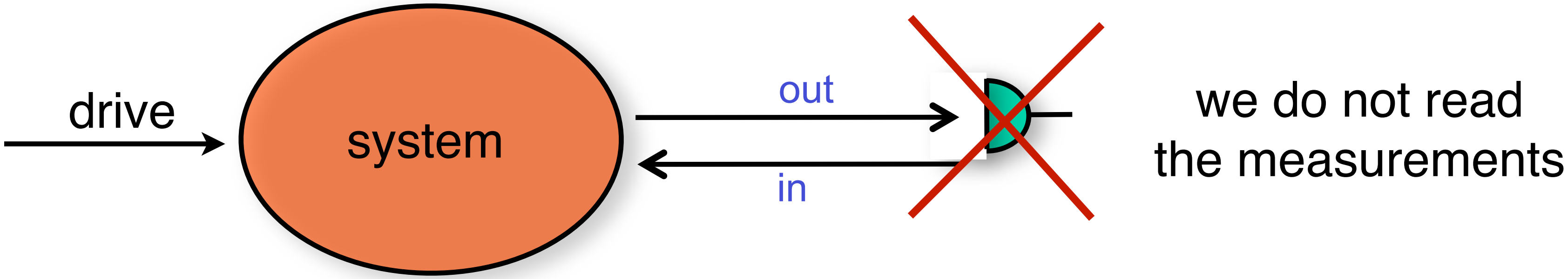
$$|\psi_{\rm sys}(t)\rangle \to \sqrt{\gamma}\,|\psi_{\rm sys}(t)\rangle$$

no click:

$$|\psi_{\rm sys}(0)\rangle \to e^{-iH_{\rm eff}t}|\psi_{\rm sys}(0)\rangle$$

Master Equation

- **open quantum system**

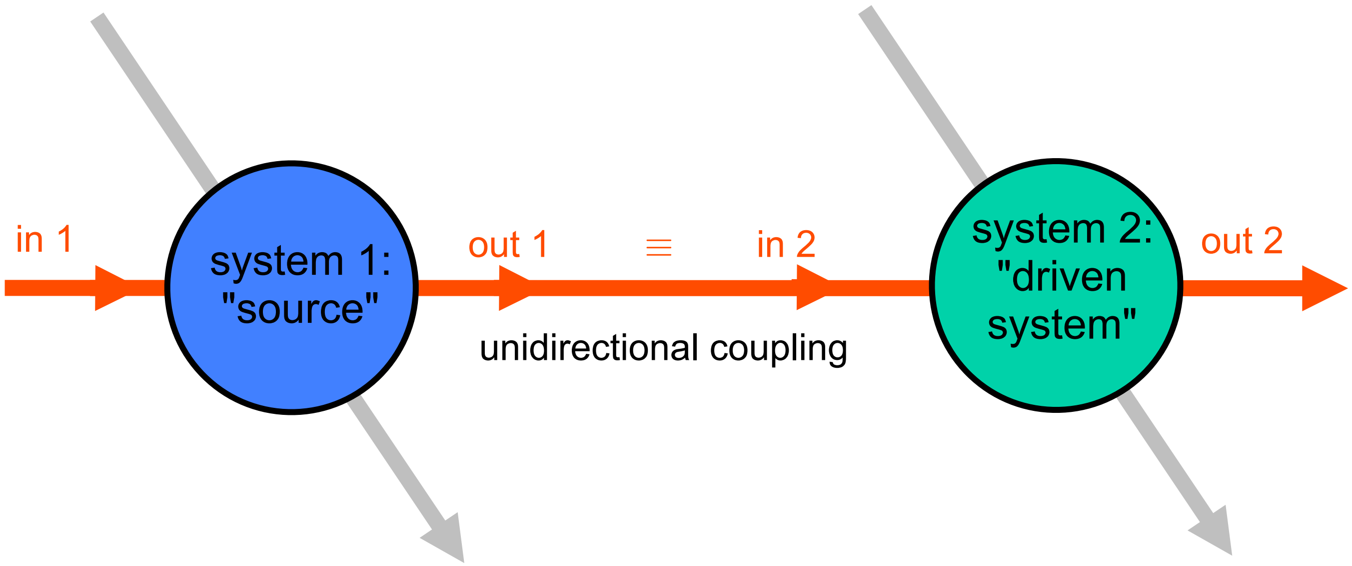


- **Reduced system density operator:** $\rho(t) := \mathrm{Tr}_B |\Psi(t)\rangle\langle\Psi(t)|$
- **Master Equation:** Lindblad form

$$\begin{aligned}\dot{\rho}(t) &= -i\left[H_{\mathrm{sys}}, \rho(t)\right] \\ &\quad + \frac{1}{2}\gamma\left(2c\rho(t)c^\dagger - c^\dagger c\rho(t) - \rho(t)c^\dagger c\right)\end{aligned}$$

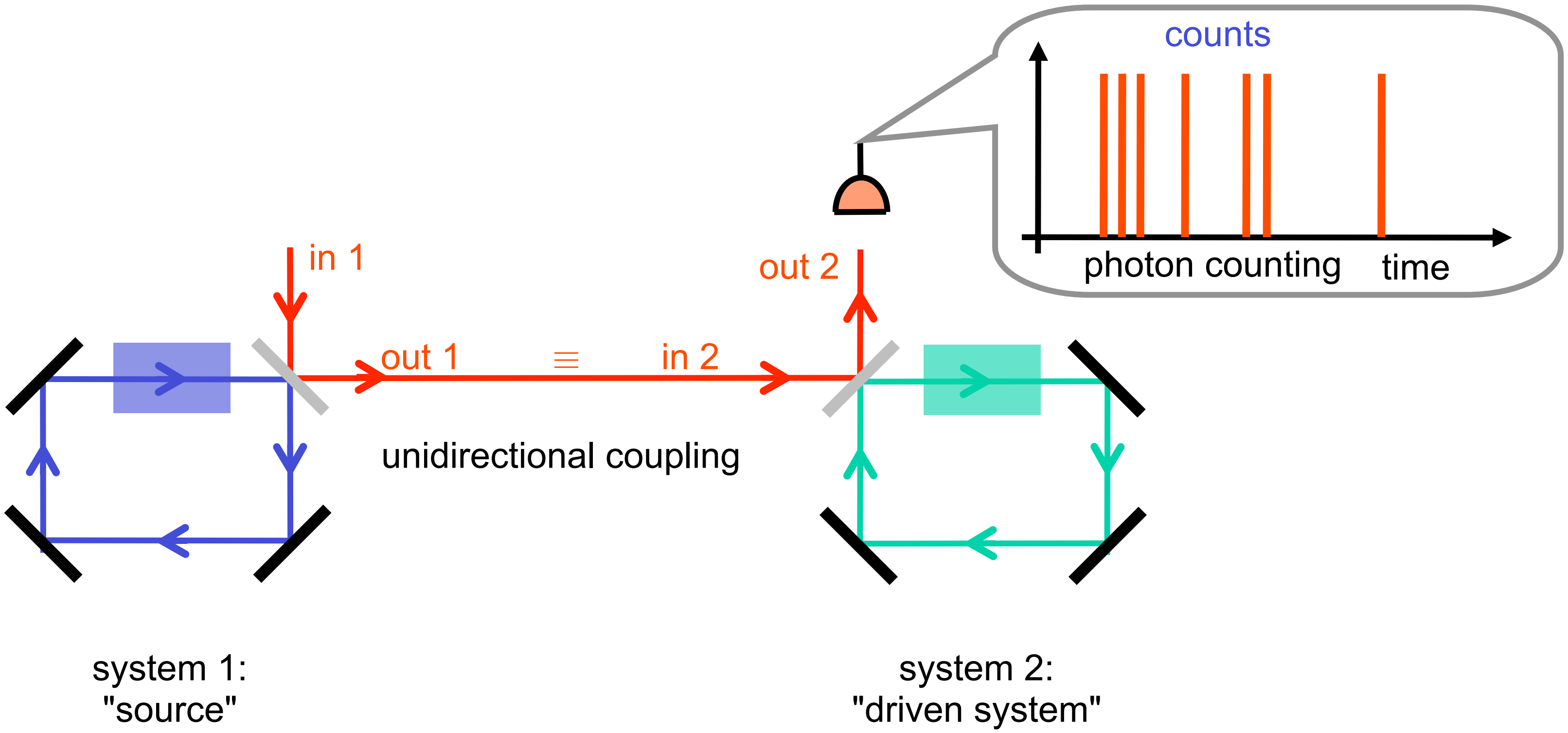
quantum jump operators

Cascaded Quantum Systems



- Quantum Stochastic Schrödinger Equation
- Master Equation

Cascaded Quantum Systems



- Quantum Stochastic Schrödinger Equation
- Master Equation

Cascaded Systems: the Model

Hamiltonian

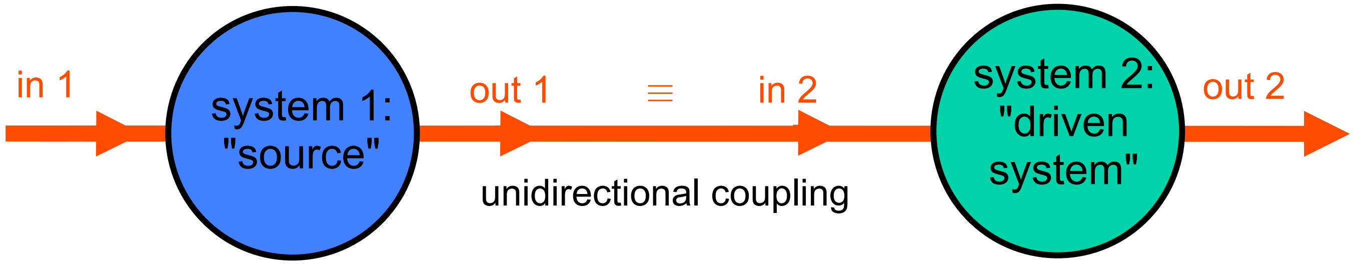
$$H = H_{\text{sys}}(1) + H_{\text{sys}}(2) + H_{\text{B}} + H_{\text{int}}$$

$$H_B = \int_{\omega_0-\vartheta}^{\omega_0+\vartheta} d\omega\, \omega\, b^\dagger(\omega) b(\omega)$$

interaction part

$$\begin{aligned} H_{\text{int}} = \quad & i\hbar \int d\omega \kappa_1(\omega) \left[b^\dagger(\omega) \underline{e^{-i\omega/c x_1}} c_1 - c_1^\dagger b(\omega) e^{+i\omega/c x_1} \right] \\ & + i\hbar \int d\omega \kappa_2(\omega) \left[b^\dagger(\omega) \underline{e^{-i\omega/c x_2}} c_2 - c_2^\dagger b(\omega) e^{+i\omega/c x_2} \right] \qquad (x_2 > x_1) \end{aligned}$$

Cascaded Systems: the Model



Stratonovich **Quantum Stochastic Schrödinger Equation with time delays**

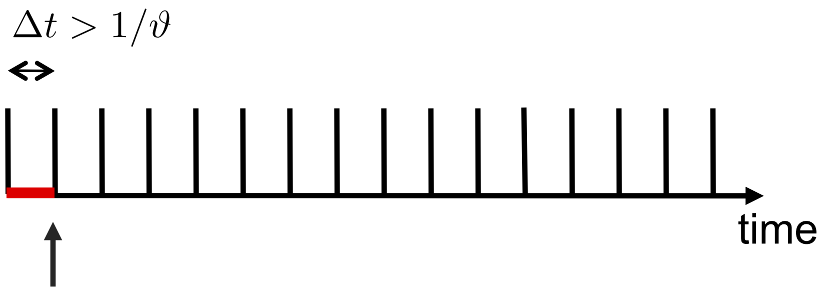
(S) $$\frac{d}{dt}|\Psi(t)\rangle = \{-i\left(H_{\rm sys}(1)+H_{\rm sys}(2)\right) + \sqrt{\gamma_1}\left[b^\dagger(t)c_1 - b(t)c_1^\dagger\right] + \sqrt{\gamma_2}\left[b^\dagger(t-\tau)c_2 - b(t-\tau)c_2^\dagger\right]\}\,|\Psi(t)\rangle$$

time delay

where time ordering / delays reflects causality

Scaling: $\sqrt{\gamma_i}\,c_i \to c_i$

Coarse Grained Integration of the QSSE



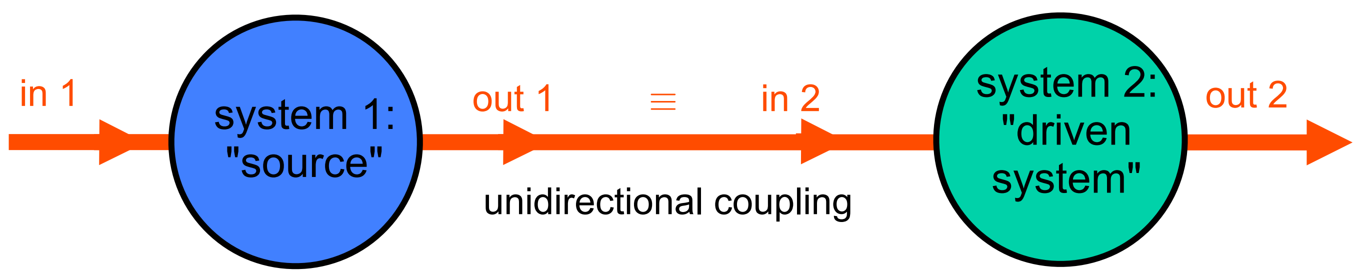
First time step: (for time delay $\tau \to 0^+$)

$$|\Psi(\Delta t)\rangle = \Big\{ \hat{1} - iH_{\text{sys}}\Delta t + \sqrt{\gamma}\, c \int_0^{\Delta t} b^\dagger(t)\, dt - \sqrt{\gamma} c^\dagger \int_0^{\Delta t} b(t)\, dt$$

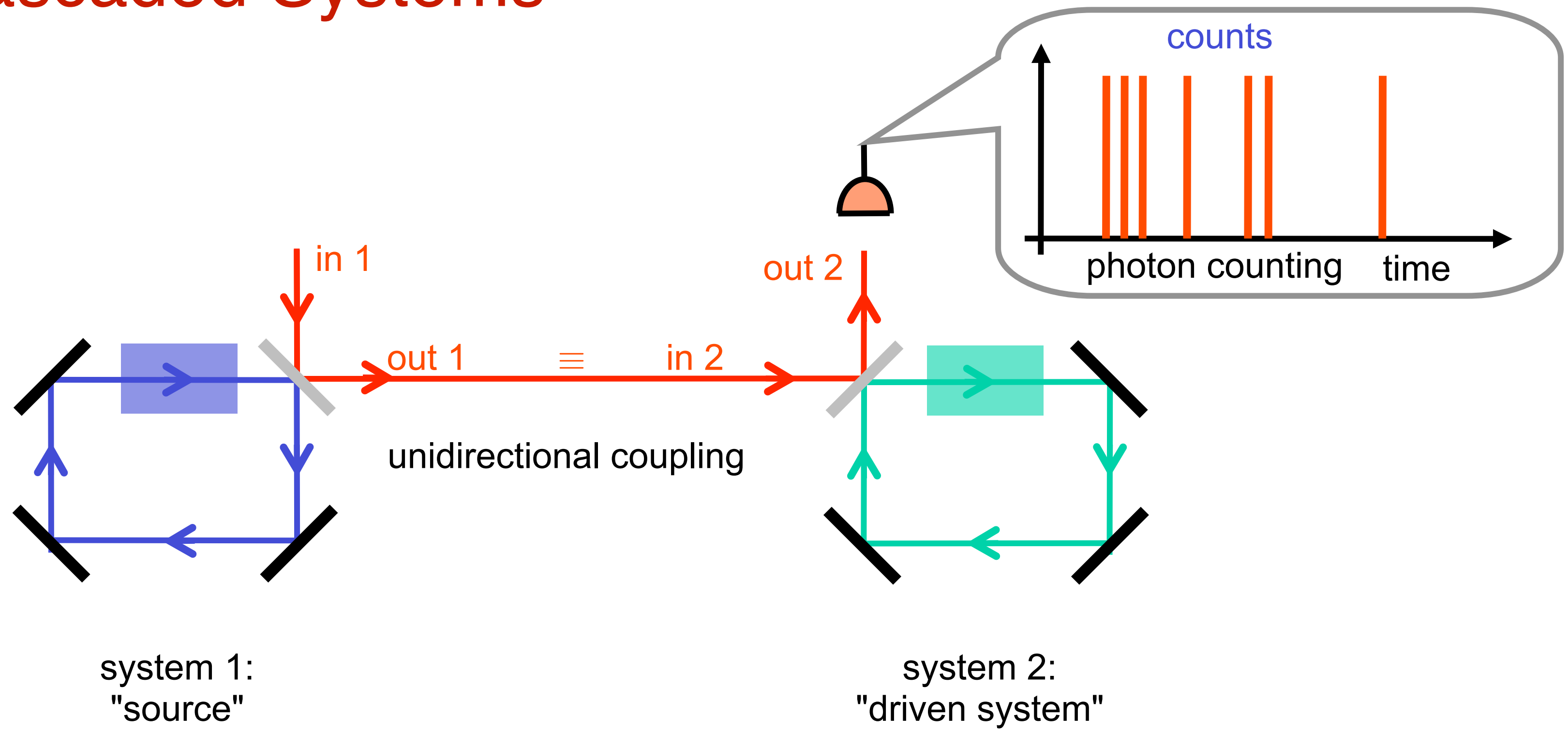
$$+ (-i)^2 \int_0^{\Delta t} dt_1 \int_0^{t_2} dt_2 \left(-b(t_1) c_1^\dagger - b(t_1^-) c_2^\dagger \right) \left(b^\dagger(t_2) c_1 + b^\dagger(t_2^-) c_2 \right) |\Psi(0)\rangle$$

$$\left(-\frac{1}{2} c_1^\dagger c_1 + 0 - c_2^\dagger c_1 - \frac{1}{2} c_2^\dagger c_2 \right) |\text{vac}\rangle \Delta t$$

causality & interaction



Cascaded Systems



Master Equation:

Version 1: Lindblad form

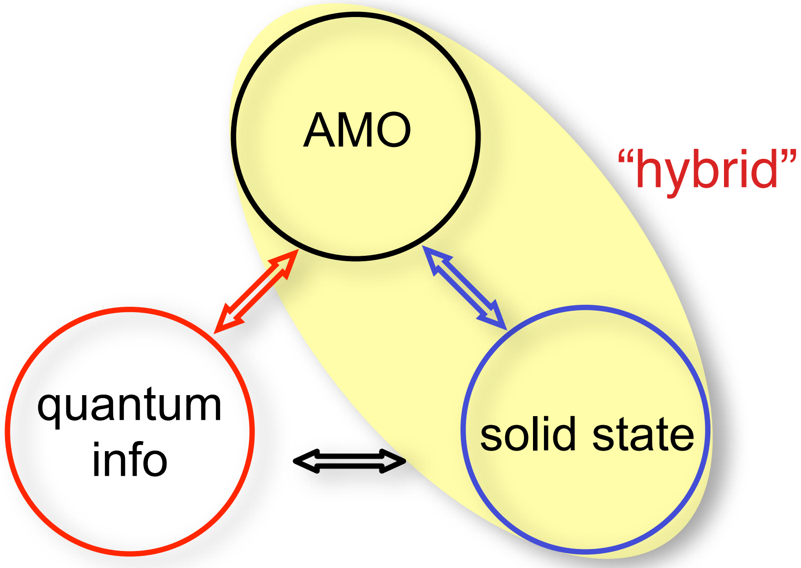
$$\frac{d}{dt}\rho = -i\left(H_{\text{eff}}\,\rho - \rho H_{\text{eff}}^{\dagger}\right) + \frac{1}{2}\left(2c\rho c^{\dagger} - c^{\dagger}c\rho - \rho c^{\dagger}c\right)$$

with jump operator $c \equiv c_1 + c_2$ and

$$H_{\text{eff}} = H_{\text{sys}} + i\frac{1}{2}\left(c_1^{\dagger}c_2 - c_2^{\dagger}c_1\right) - i\frac{1}{2}c^{\dagger}c$$

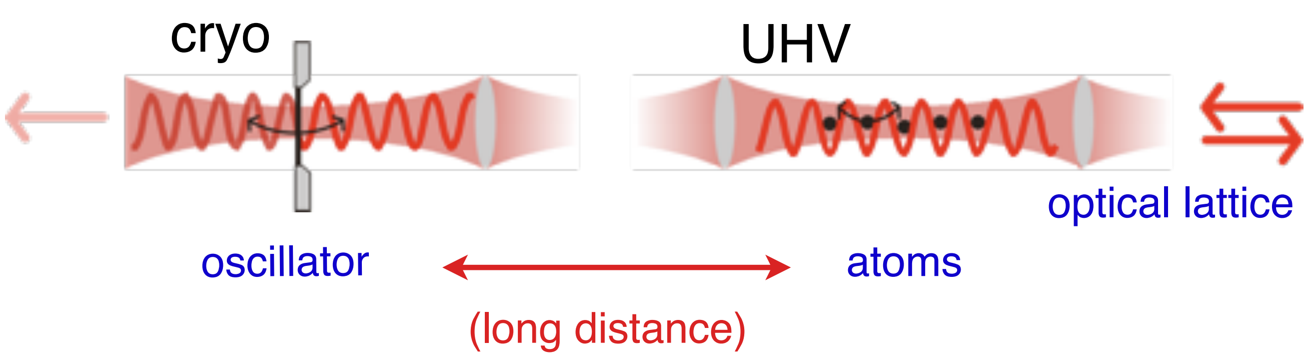
coherent interaction

more details: K. Hammerer
in his talk on Feb 5



AMO - Solid State: Hybrid Systems

- **Free space coupling between nanomechanical mirror + atomic ensemble**



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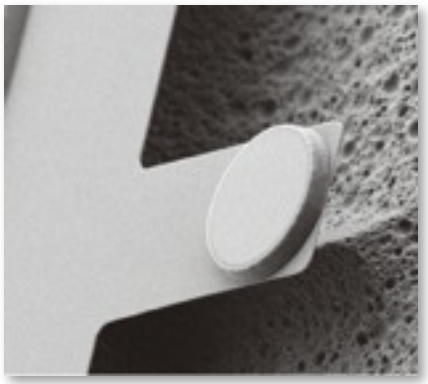
P. Treutlein, S. Camerer, D. Hunger, T. W. Hänsch
in preparation

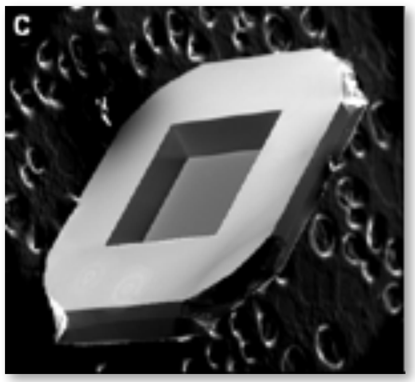
“Opto-nanomechanics”

- **system:** High-quality mechanical oscillators coupled to high-quality, high-finesse optical cavities

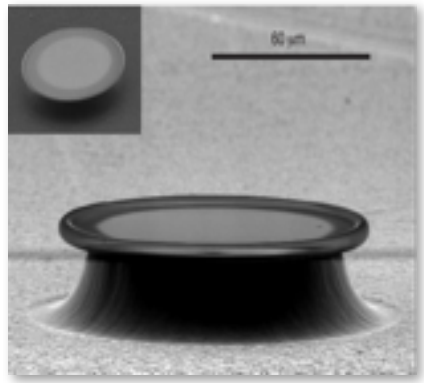
- **goal: see quantum effects & applications in quantum technologies**
  - ground state cooling of the oscillator
  - entanglement …
  - why? … fundamental / applications



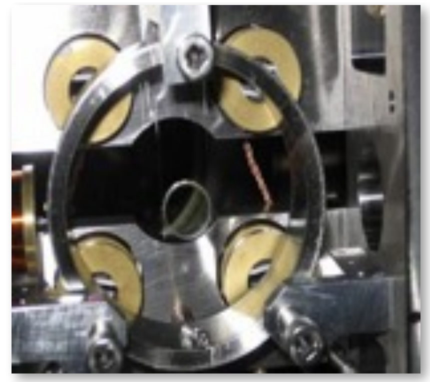
Micromirrors



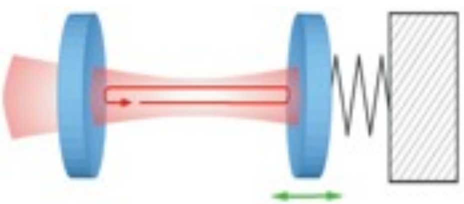
Micromembranes



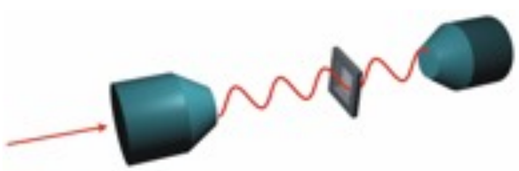
Microtoroids



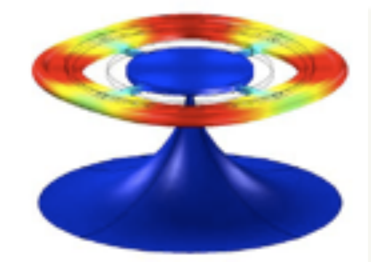
Gravitational Interferometers



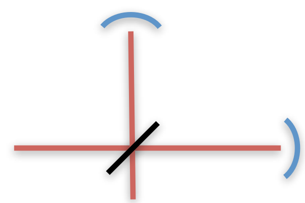
Aspelmeyer (Vienna)
Heidmann (Paris)



Harris (Yale)
Kimble (Caltech)



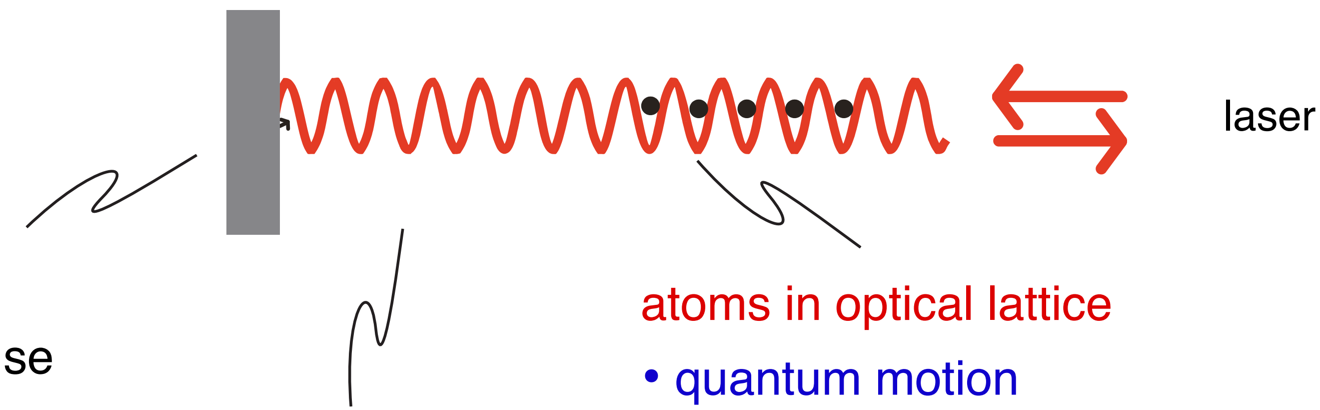
Kippenberg (MPQ)
Weig (LMU)
Vahala (Caletch)
Bowen (UQ)



Danzmann, Schnabel
(MPIG,Hannover)
Mavalvala (LIGO,MIT)

Atoms in Optical Lattices

- **Atoms in optical lattice: standard setup**



fixed mirror

- (thermal) noise

radiation field

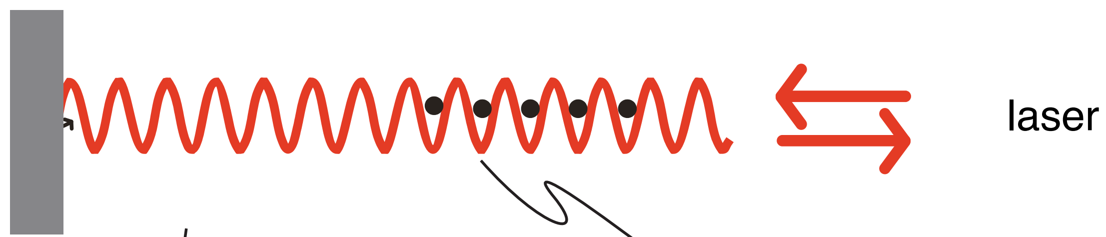
- classical lattice potential: laser
- spontaneous emission:
  laser cooling / decoherence

atoms in optical lattice

- quantum motion
  - near resonant lattice: laser cooling
  - far offresonant lattice: Hubbard models

Atoms in Optical Lattices

- **Atoms in optical lattice: standard setup**



fixed mirror

- (thermal) noise

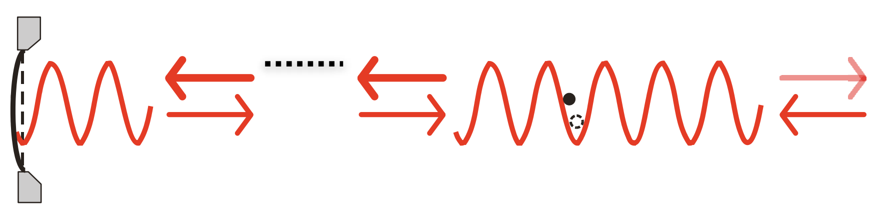
radiation field

- classical lattice potential: laser
- spontaneous emission:
  laser cooling / decoherence

atoms in optical lattice

- quantum motion
  - near resonant lattice: laser cooling
  - far offresonant lattice: Hubbard models

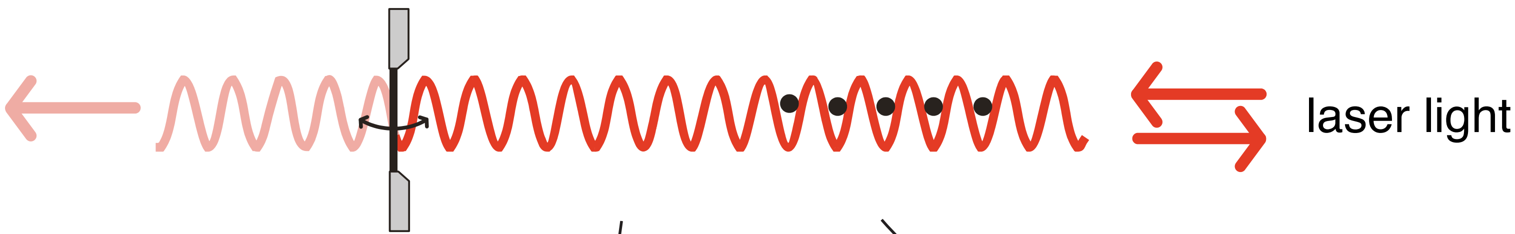
See however:
back action of atoms on lattice beams observed in:
G. Raithel et al. PRL **81** 3615 (1998)
N.V. Morrow et al. PRL **88** 093003 (2002)



here:

Optical Lattices with Micro-Mirrors / Membranes

- **Optical lattice** by *retro-reflection of a single beam* on a partially reflective **oscillating micro–mirror/membrane**



mirror / membrane

- quantum oscillator
- [noise]

atoms in optical lattice

- quantum motion

radiation field

- long distance interaction mediated by quantum fluctuations of the light

---

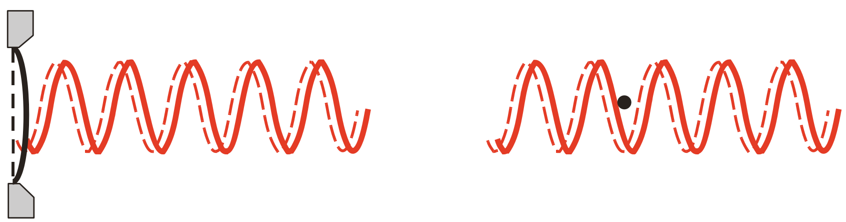
composite quantum dynamics:

mirror + light + atomic motion: coherent coupling vs. dissipation

we can engineer “atomic reservoirs” e.g. laser cooling

1. Naive Semiclassical: Coherent Couplings

- **Classical light / optical potential** [valid for an ideal mirror]
- **Physical picture / expectations:**
  - Membrane vibrations shift phase of field: shift of potential shakes atoms



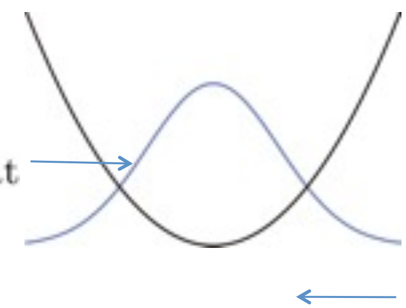
Field modes with boundary condition $E(z) \sim \sin[k(z - z_{\rm mec})]$

Lattice potential $V(z_j) = \frac{m\omega_{\rm at}^2}{2}(z_j - z_{\rm mec})^2 \sim z_j z_{\rm mec}$

Effective coupling

$$H_{\rm int} = \sum_j g_0 \, (a_j + a_j^\dagger)(a_{\rm mec} + a_{\rm mec}^\dagger)$$

$\ell_{\rm mec}, \ell_{\rm at}$



$$g_0 = \frac{\omega_{\rm at}}{2}\frac{\ell_{\rm mec}}{\ell_{\rm at}} \qquad \frac{\ell_{\rm mec}}{\ell_{\rm at}} = \sqrt{\frac{m_{\rm at}\omega_{\rm at}}{m_{\rm mec}\omega_{\rm mec}}} \sim \sqrt{\frac{m_{\rm at}}{m_{\rm mec}}} \sim 10^{-7}$$

- **“naive” approach**

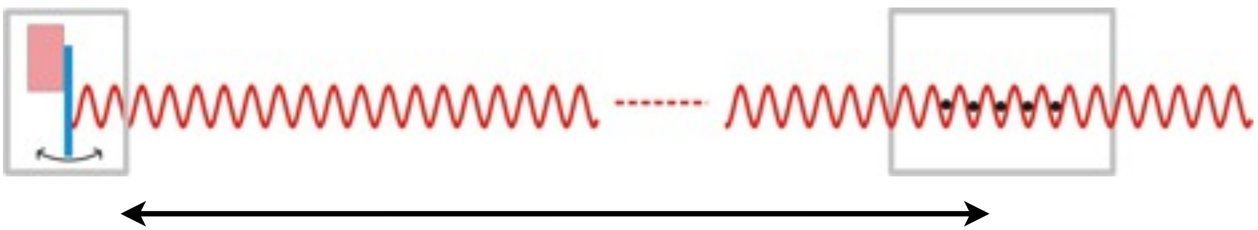
$$H_{\rm int} = \sum_j g_0 \ (a_j + a_j^\dagger)(a_{\rm mec} + a_{\rm mec}^\dagger)$$

Collectively enhanced coupling to com mode

$$a_{\rm com} = \frac{1}{\sqrt{N}} \sum_j a_j$$

$$H_{\rm int} = g(a_{\rm com} + a_{\rm com}^\dagger)(a_{\rm mec} + a_{\rm mec}^\dagger) \qquad g = g_0 \sqrt{N_{\rm at}}$$

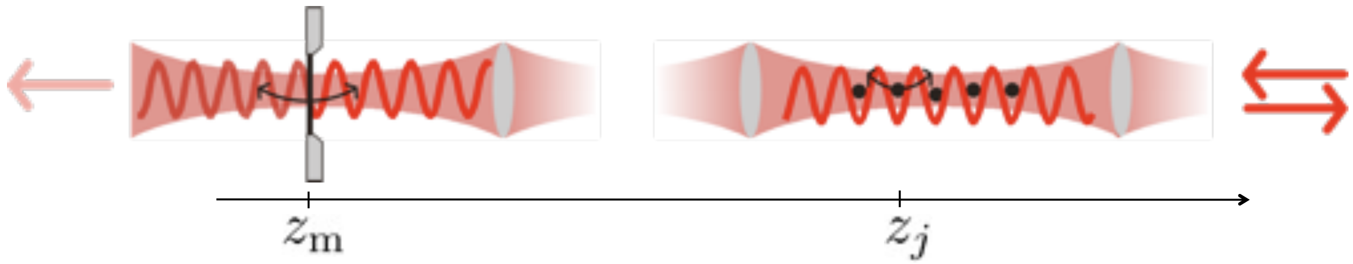
- **retardation / causality (?)**



how to the atoms and
mirror talk to each other? … by exchange of photons

2. Quantum Treatment

- **Hamiltonian:** including membrane, atoms *and* electro-magnetic field as degree of freedom



$$H = \left(\frac{\hat{p}_{\rm m}^2}{2m_{\rm m}} + \frac{m\omega_{\rm m}^2}{2}\hat{z}_{\rm m}^2\right) + \sum_j \frac{\hat{p}_j^2}{2m_{\rm at}}$$

… membrane + atoms

$$- \sum_j \frac{\mu^2}{\hbar\delta}\hat{E}^-(\hat{z}_j,t)\hat{E}^+(\hat{z}_j,t)$$

…optical potential for atoms

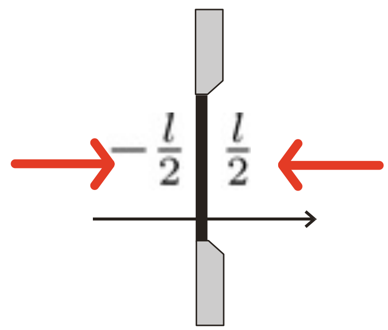
$$- \frac{\epsilon_0 A(n^2-1)}{2}\left[\hat{E}^-(-\tfrac{l}{2})\hat{E}^+(-\tfrac{l}{2}) - \hat{E}^-(\tfrac{l}{2})\hat{E}^+(\tfrac{l}{2})\right]\hat{z}_{\rm m}$$

…radiation pressure potential for membrane

~ intensity on left – intensity on right side

$$|\Psi\rangle = |\psi_{\rm m,at}\rangle \otimes |\alpha_{\rm laser}\rangle \otimes |{\rm vac}\rangle$$

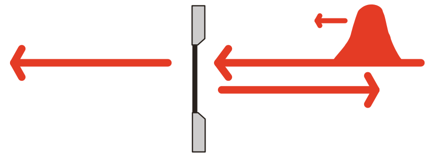
$=$ anything $\otimes$ coherent laser field $\otimes$ vacuum



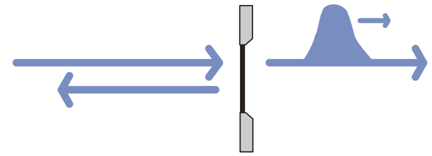
$$i\hbar\frac{d}{dt}|\Psi(t)\rangle = H|\Psi(t)\rangle$$

- here: 1D model (actually 3D ...)

- **Electric Field modes:**



R-field



L-field

$$E(z) = E_R(z) + E_L(z)$$

$$E_\alpha^+(z,t) = \mathcal{E} \int d\omega A_\alpha(k,z) b_{\omega,\alpha} e^{-i\omega t} \quad (\alpha = L, R)$$

mode function

- **laser as classical driving field:** displacement

$$b_{\omega,R} \to \alpha\delta(\omega - \omega_l) + b_{\omega,R}$$ (laser driving R mode)

$$\sum_{j=1}^{N} \frac{\mu^2}{\hbar\delta} E^-(z_j,t)E^+(z_j,t) = V_0 \sum_{j=1}^{N} \sin^2(kz_j) + \text{quantum noise}$$

$$V_0 = \frac{\mu^2 \mathcal{E}^2 \alpha^2 \sqrt{\mathfrak{r}}}{\hbar\delta}$$

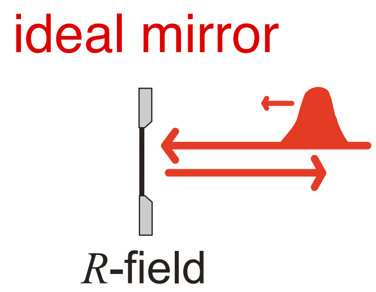
- **lowest order quantum fluctuations ...**

Rem.: for an ideal mirror only the R mode appears

Quantum Stochastic Schrödinger Equation

- **Linearization** around laser amplitude: keep terms linear and quadratic in α_{laser}
- Interpretation as **Stratonovich QSSE** *with time delays.*

$$
\begin{aligned}
i\hbar\tfrac{d}{dt}|\Psi\rangle &= H(t,t^-,t^+)|\Psi\rangle \\
&= \Big\{H_{\mathrm{m}} + H_{\mathrm{at}} \\
&\quad - i g_{\mathrm{at},R}\, z_{\mathrm{at}} \left[b_R(t^+) - b_R^\dagger(t^+)\right] \\
&\quad + g_{\mathrm{m},R}\, z_{\mathrm{m}} \left[b_R(t) + b_R^\dagger(t)\right] + \qquad] \\
&\quad + i g_{\mathrm{at},R}\, z_{\mathrm{at}} \left[b_R(t^-) - b_R^\dagger(t^-)\right] + \qquad \Big\}|\Psi\rangle
\end{aligned}
$$



atomic motion unbalance laser beams

mirror motion: phase modulation

membrane & atomic motion: sidebands

time delays: retardation & causality

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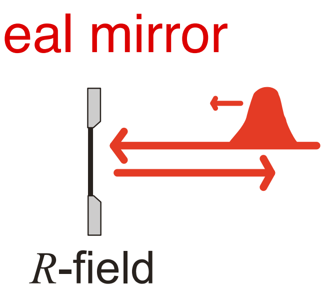

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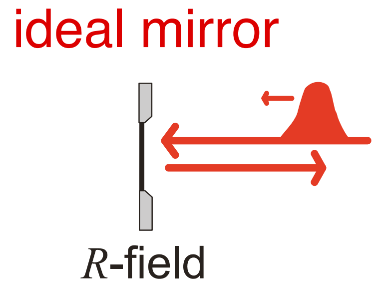

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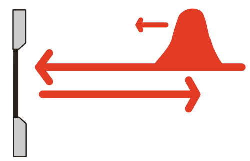
ideal mirror



R-field

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mirror motion: phase modulation

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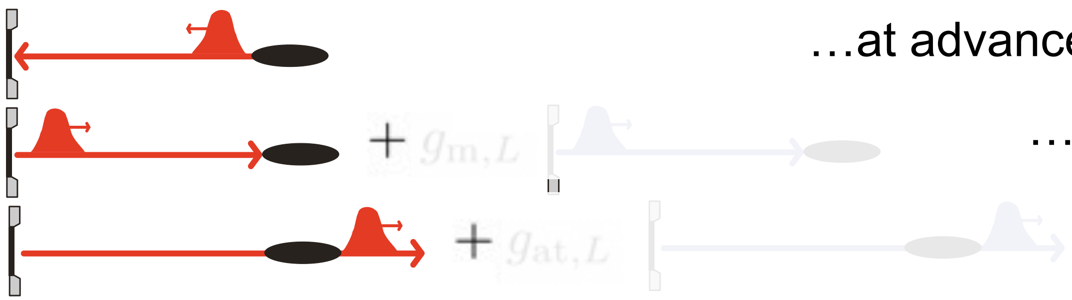
&\quad + i g_{\mathrm{at},R}\, z_{\mathrm{at}} \left[b_R(t^-) - b_R^\dagger(t^-) \right] + g_{\mathrm{at},L}\, z_{\mathrm{at}} \left[b_L(t^-) + b_L^\dagger(t^-) \right] \Big\} |\Psi\rangle
\end{aligned}
$$

… unbalance laser beams

… phase modulation

$$
= \Big\{ H_{\mathrm{m}} + H_{\mathrm{at}} \; - i g_{\mathrm{at},R} \;\; + g_{\mathrm{m},R} \;\; + g_{\mathrm{m},L} \;\; + i g_{\mathrm{at},R} \;\; + g_{\mathrm{at},L} \Big\} |\Psi\rangle
$$



…at advanced time $t^+ = t + d/c$

…at time t

…at retarded time $t^- = t + d/c$

- **Convert to Ito QSSE & master equation**

Monday, February 1, 2010

Markovian Master Equation

- Equivalent Markovian Master Equation

$$\dot{\rho} = -i[H_{\mathrm{m}} + H_{\mathrm{at}} + g z_{\mathrm{at}} z_{\mathrm{m}}, \rho] + L_{\mathrm{m}}\rho + L_{\mathrm{at}}\rho + C\rho$$

Hamiltonian term for coherent atom-membrane interaction at strength

$$g = \omega_{at}\sqrt{\frac{N m_{\mathrm{at}}}{m_{\mathrm{m}}}}$$

$$= 2\pi \cdot 10^6 \sqrt{\frac{10^7\, 10^{-25}}{10^{-13}}} \simeq 10\,\mathrm{kHZ}$$

optical "spring" between membrane and atomic COM motion

Lindblad terms describing radiation pressure induced momentum diffusion of membrane, eg

$$L_{\mathrm{m}} = \gamma_{\mathrm{m}}^{\mathrm{diff}}(2 z_{\mathrm{m}} \rho z_{\mathrm{m}} - z_{\mathrm{m}}^2 \rho - \rho z_{\mathrm{m}}^2)$$

and atoms at rates

$$\gamma_{\mathrm{m}}^{\mathrm{diff}}, \gamma_{\mathrm{at}}^{\mathrm{diff}} \ll g$$ ☺

agrees with

K. Karrai PRL **100**, 240801 (2008)
Gordon, Ashkin, Cohen Tannoudji….

Application: Sympathetic Cooling of a Mirror via Atoms

- **Master Equation** including thermal bath for membrane, laser cooling of atoms

$$\dot{\rho} = -i[H_{\rm m} + H_{\rm at} + g z_{\rm at} z_{\rm m}, \rho] + L_{\rm m}\rho + L_{\rm at}\rho + C\rho + L_{\rm m}^{\rm heat}\rho + L_{\rm at}^{\rm cool}\rho$$

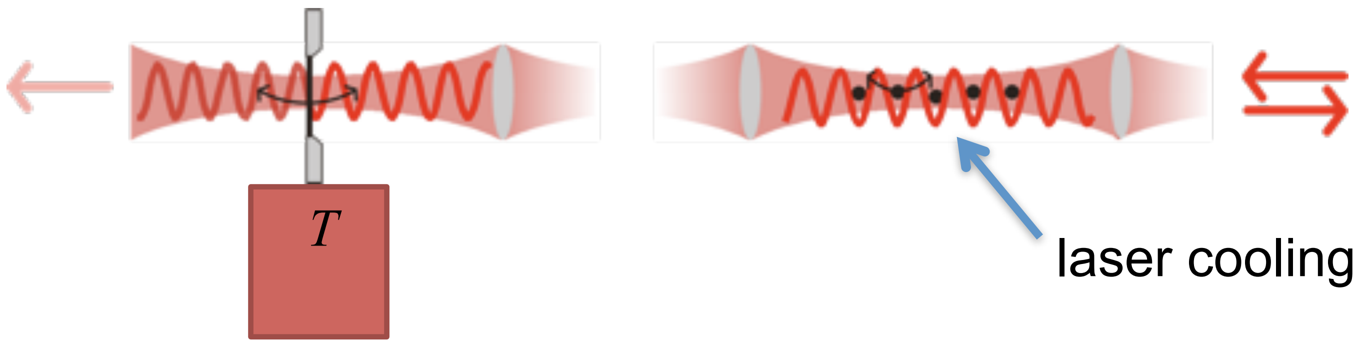
heating of membrane mode
due to coupling to thermal reservoir

laser cooling of atoms
to motional ground state

rate $\gamma_{\rm m}^{\rm heat}$

$\gamma_{\rm at}^{\rm cool}$ rate

equilibrium thermal occupation $\bar{n}_{\rm initial} \simeq \frac{k_B T}{\hbar \omega_{\rm m}}$



Numbers

- **SiN membrane:** 100μm$\times$100μm$\times$50nm, $\omega_m = 2\pi \times 1.3$MHz, $m_m = 4 \times 10^{-13}$kg, $Q = 10^7$ at $T \lesssim 2$K, $r = 0.31$ at $\lambda = 780$nm (^{87}Rb)
- **Lattice beam** with power $P = 4$mW and a waist 100μm, detuning $\delta = 2\pi \times 1$GHz, so that $\omega_{at} \simeq \omega_m$. Thus for $N \simeq 10^7$ atoms we have a coherent coupling

$$g = \omega_{at}\sqrt{\frac{m_{at}N}{m_m}} \simeq 10 \text{ kHz}$$

- **Decoherence**
  - radiation pressure noise: $\gamma_m^{diff} = 10 \text{ Hz}$
  - atomic momentum diffusion rate in the lattice $\gamma_{at}^{diff} = 35 \text{ Hz}$
  - membrane thermal decoherence at rate $\gamma_m^{th} = 4$MHz at room temperature, or $\gamma_m^{th} = 4$KHz at $T = 300$mK
- **Raman sideband cooling of atoms** at a (fast) rate $\gamma_{at}^{\text{cool}} = 10$ kHz
- **Coherent coupling regime accessible:** $\omega_m = \omega_{at} \gg g \simeq \gamma_{at}^{\text{cool}} \gg \gamma_{m(at)}^{\text{diff}}$

Application: Sympathetic Cooling of a Mirror via Atoms

- **Cooling efficiency:** Consider $\gamma_{at}^{cool} \gg g$, then one finds a rate equation after adiabatic elimination of atoms

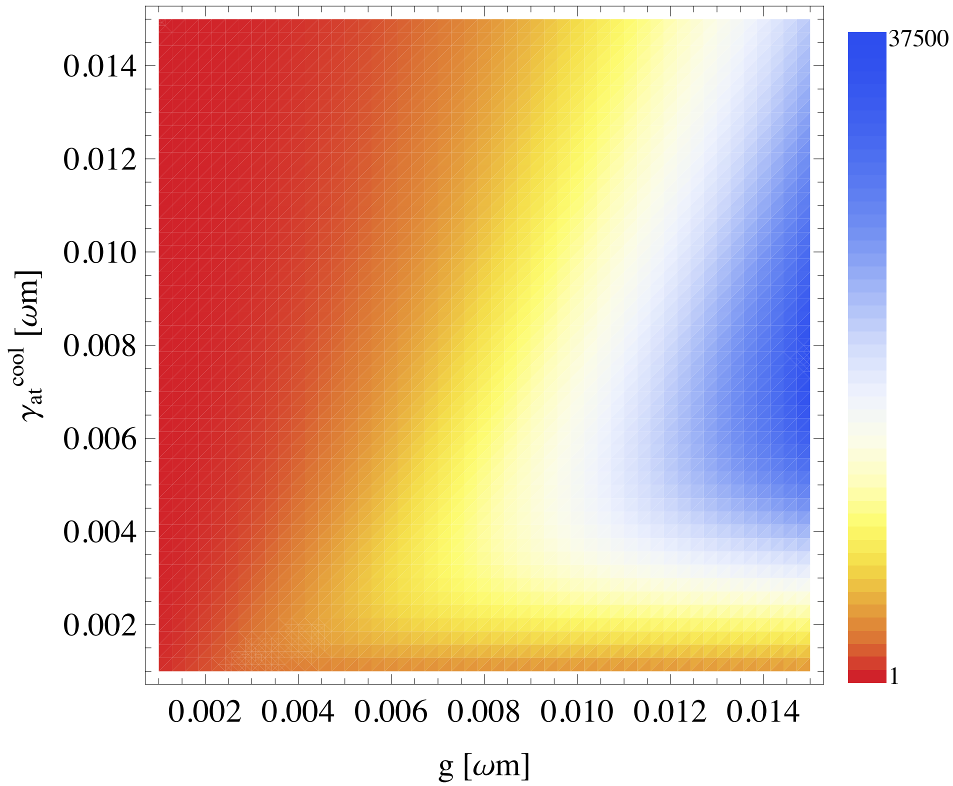
$$\frac{d}{dt}\langle a_m^\dagger a_m\rangle = -\Gamma_m(\langle a_m^\dagger a_m\rangle - \bar{n}_{ss})$$

analogous to optomechanical laser cooling
I. Wilson-Rae et al., PRL **99**, 093901 (2007)
F. Marquardt et al., PRL **99**, 093902 (2007)

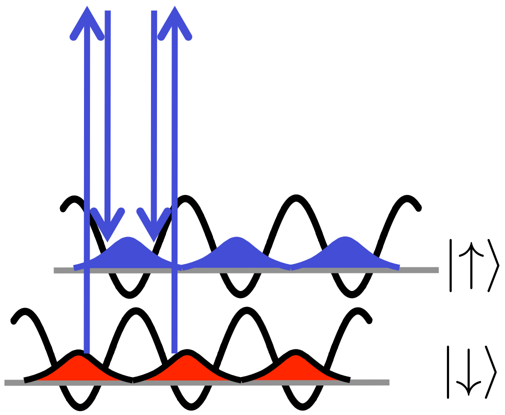
with an effective cooling rate $\Gamma_m = \gamma_m + \mathfrak{r}g^2/4\gamma_{at}^{cool}$ and a final occupation

$$\bar{n}_{ss} \equiv \langle a_m^\dagger a_m\rangle_{ss} \simeq \frac{\gamma_m}{\Gamma_m}\bar{n} + \frac{\gamma_{at}^{\mathrm{cool}}}{4\omega_m^2}$$

- **Cooling factor** $f = \bar{n}/\bar{n}_{ss}$ vs. effective coupling g and Raman sideband laser cooling γ_{at}^{cool} ($\omega_m = \omega_{at}$) for $\omega_m = 2\pi \times 1.3$MHz and $Q_m = 10^7$, momentum diffusion $\gamma_{m(at)}^{diff} = 10^{-5}\omega_m$.
- For $g \simeq \gamma_{at}^{cool} \simeq 10$kHz we find $f = 2 \times 10^4$, and $\bar{n}_{ss} < 1$ for $T = 1$K.



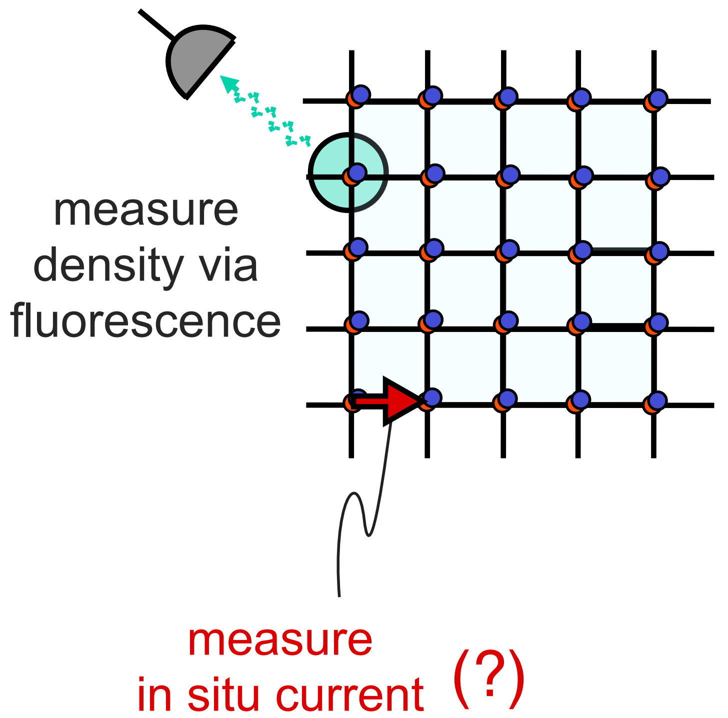
Continuous Measurement of Atomic Currents



with: V. Steixner, A. Daley and K Hammerer

Single Shot / Continuous Measurement of Atoms

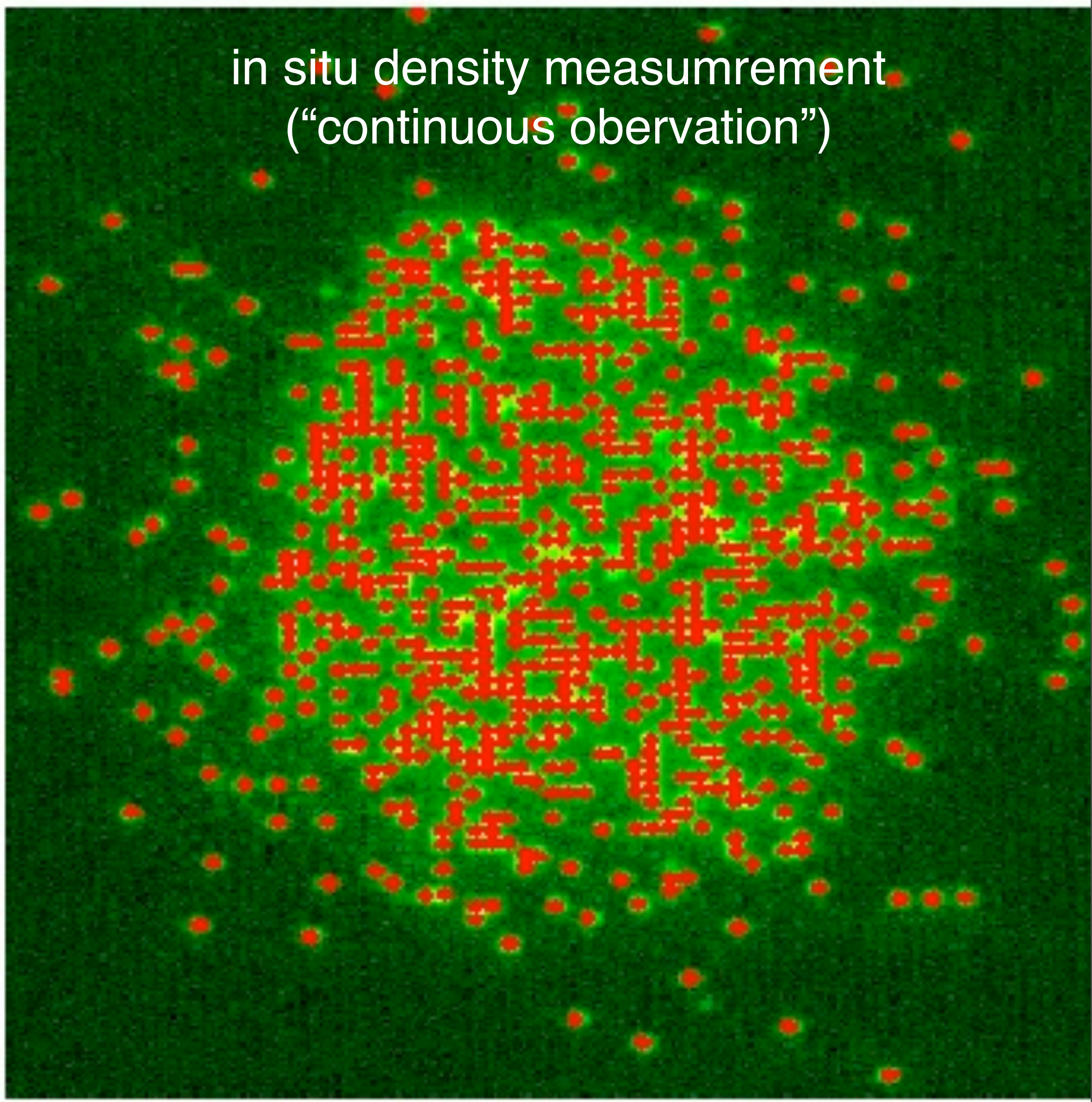
- **optical lattice**



- single atom / single site (?)
- many atoms / site (JJ array)

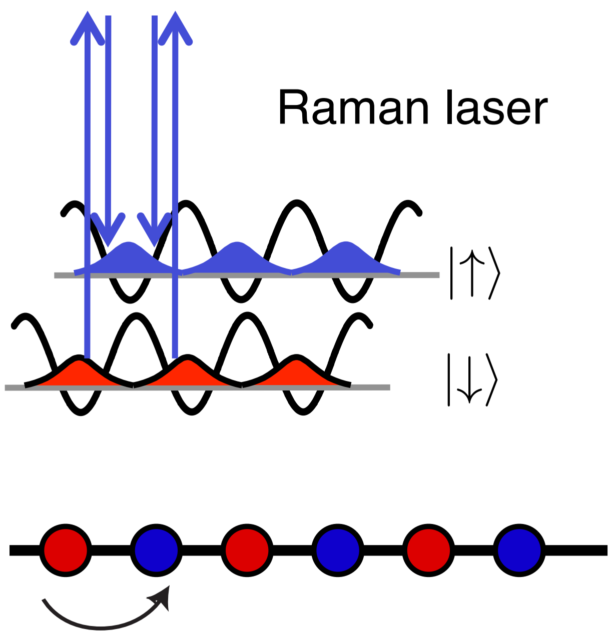
idea: via homodyne measurement

- Microscope: Greiner (Harvard), [LMU, ...]



Measurement of Atomic Currents

- **laser induced tunneling**



- **Raman transition**

Hamiltonian

$$H \sim \mu^2 \frac{E_2^-(x)E_1^+(x)}{\delta} a_2^\dagger a_1 + \text{h.c.}$$

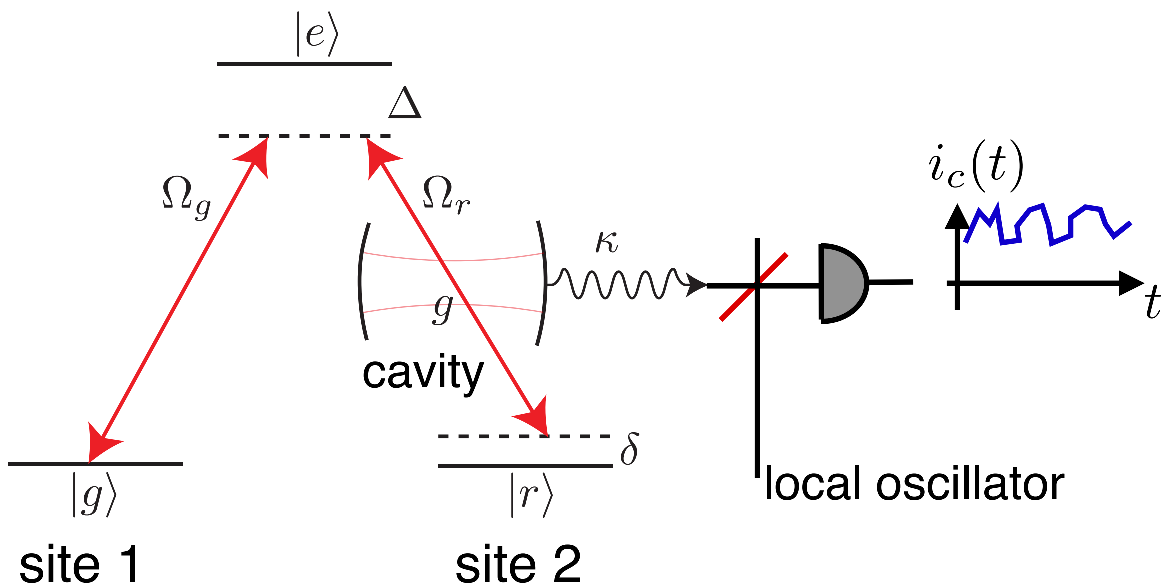
$$= \frac{\Omega_2\Omega_1}{\delta}\left(a_2^\dagger a_1 + \text{h.c.}\right) + \frac{g\Omega_1}{\delta} b^\dagger(t) a_2^\dagger a_1 + \text{h.c.}$$

tunneling — back action

homodyne current

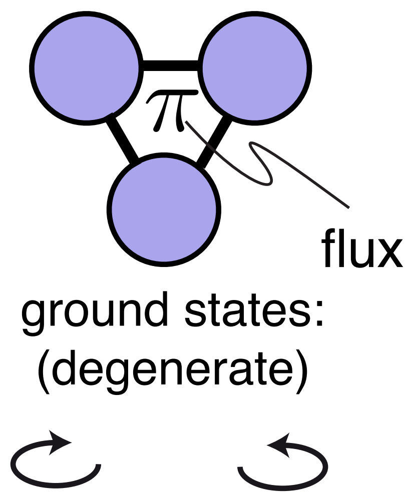
$$i_c(t) \sim \gamma_c i \langle a_2^\dagger a_1 - a_i^\dagger a_2 \rangle + \sqrt{\gamma_c}\,\xi(t)$$

atomic current — shot noise

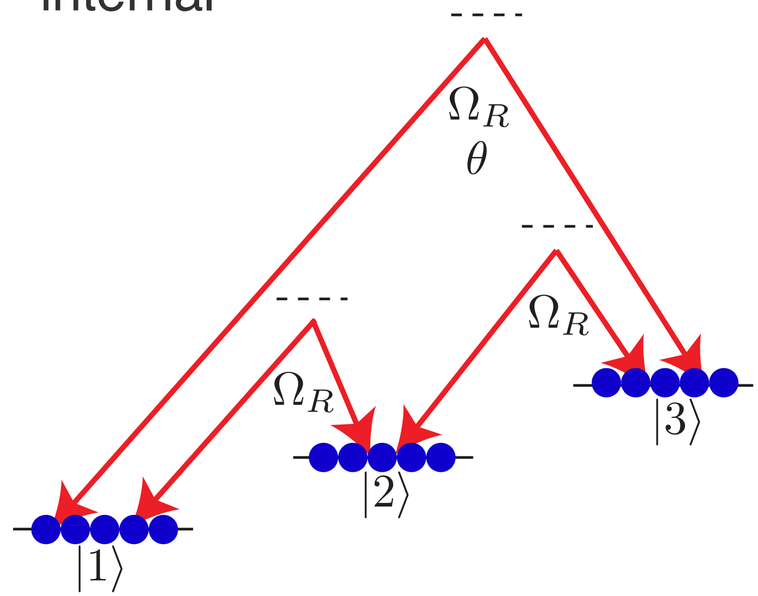


Toy Model: “3 Site JJ”

- external: 3 BECs



- internal



closed Raman cycle with π phase

Bose Hubbard

$$H_{\mathrm{BH}} = -\sum_{i,j} J_{ij} e^{-i\theta_{ij}} a_i^\dagger a_j + \frac{U}{2} \sum_j a_j^{\dagger 2} a_j^2$$

Phase Model

$$H_{BH} = -\frac{U}{2} \sum_i \frac{\partial^2}{\partial \phi_i^2} + 2JN \sum_{\langle i,j \rangle} \cos(\phi_i - \phi_j + \theta_{ij})$$

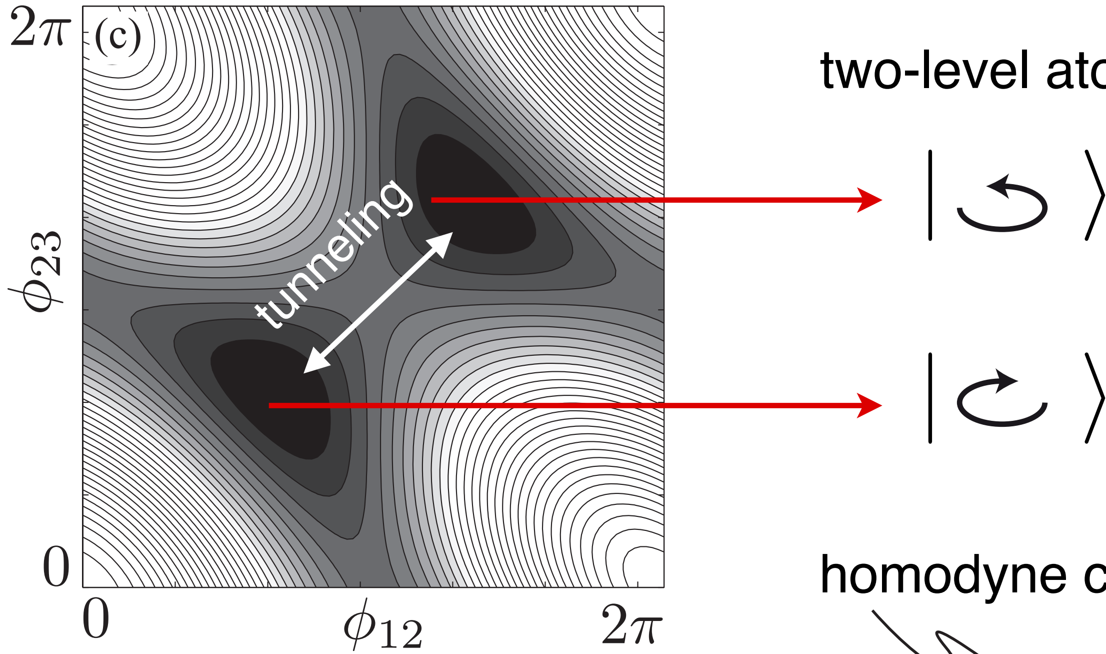
number conservation: $\sum_i \hat{N}_i = \hat{N} \to N$

Monday, February 1, 2010

Toy Model: “3 Site JJ”

- motion of ficitious particle in potential

two-level atom: $H = \omega\sigma_x$

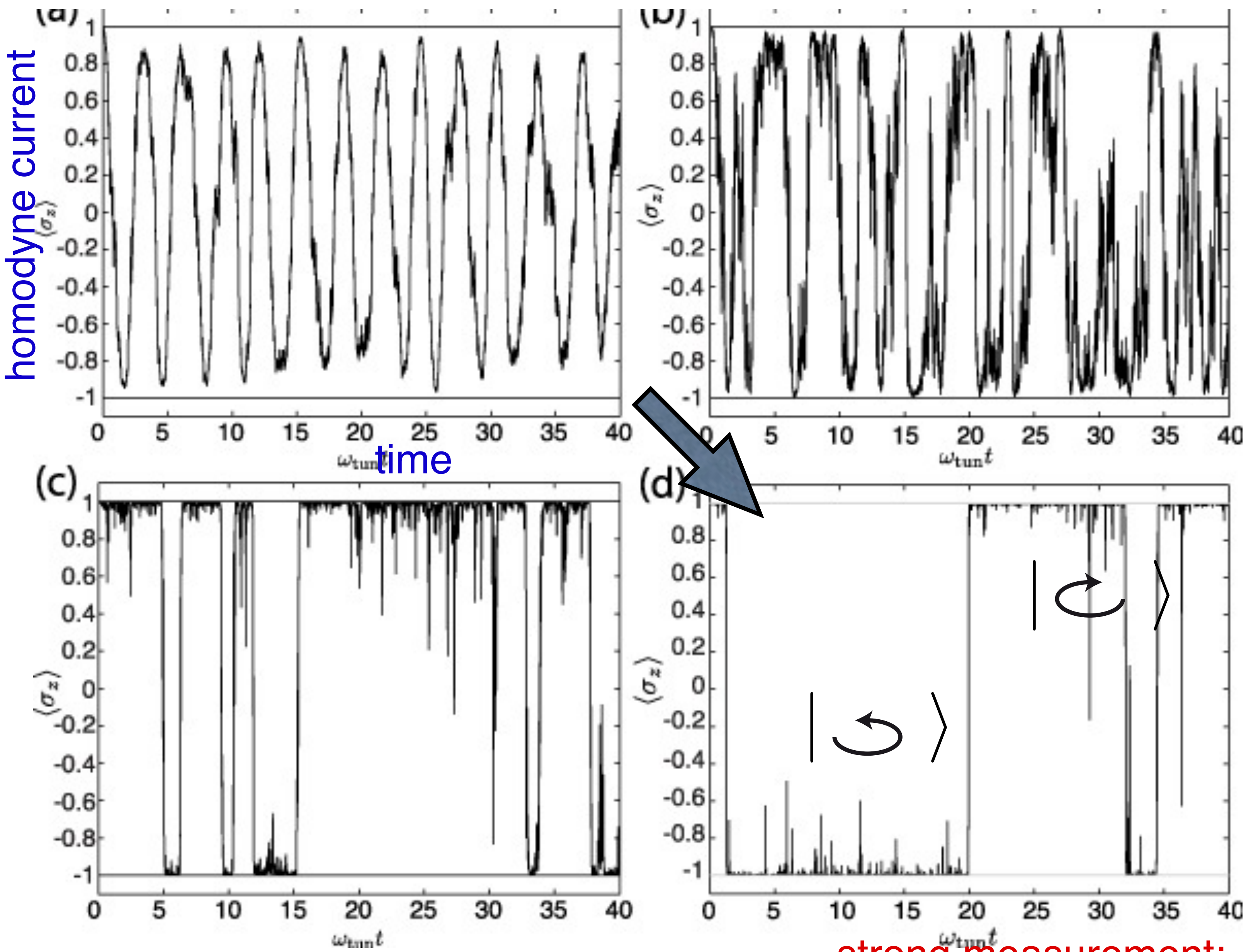


homodyne current:

$$i_c(t) \sim \langle\sigma_z\rangle_c(t) + \text{noise}$$

atomic current

Rabi oscillations between wells:
weak measurement

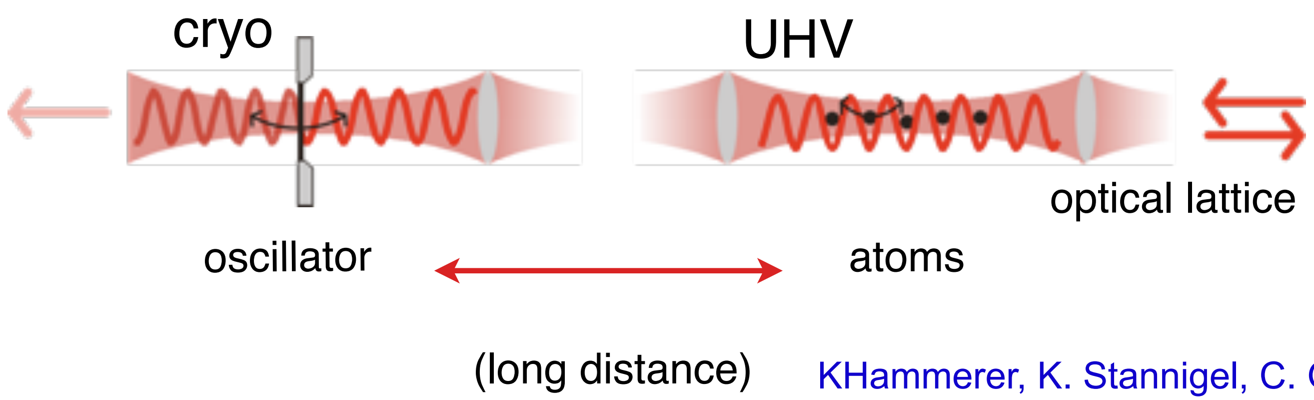


strong measurement:
projective measurement
Zeno effect

Summary

- **Quantum Noise & Quantum Optics**
  - a mini-tutorial

- **Atoms in Optical Lattices + "Nano-"Mechanical Mirrors / Membranes**



nanomechanics

+ AMO interfaces

KHammerer, K. Stannigel, C. Genes, M. Wallquist, PZ. (Innsbruck)
P. Treutlein, S. Camerer, D. Hunger, T. W. Hänsch (LMU)
in preparation

- **Measurement of Atomic Currents via Light**

V. Steixner, K Hammerer, A Daley, PZ
in preparation