A Short Introduction to Game Theory

Heiko Hotz

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Introduction

Game Theory - What is it?

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- A game in the sense of game theory is given by a (mostly finite) number of players, who interact according to given rules.
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Game Theory - What is it?

- Game theory has its historical origin in 1928. By analysing parlour games, John von Neumann realised very quickly the practicability of his approaches for the analysis of economic problems.
- In his book *Theory of Games and Economic Behavior*, which he wrote together with Oskar Morgenstern in 1944, he already applied his mathematical theory to economic applications.
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Introduction

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- When there is competition for a resource to be analysed, game theory can be used either to explain existing behaviour or to improve strategies.
- The first is especially applied by sciences which analyse long-term situations, like biology or sociology.
- In animality, for example, one can find situations, where cooperation has developed for the sake of mutual benefits.
- The latter is a crucial tool in sciences like economics. Companies use game theory to improve their strategical situation in the market.
- Many more sciences like sociology, philosophy, psychology and cultural anthropology use game theory as an appropriate tool.

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DEFINITIONS

Normal Form Games

Normal Form Games

A game in *normal form* consists of:

1. A finite number of players.
2. A strategy set assigned to each player.
3. A payoff function, which assigns a certain payoff to each player depending on his strategy and the strategy of the other players.

Both players choose their strategy simultaneously

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Payoff Matrix

If the number of players is limited to two and if their sets of strategies consist of only a few elements, the outcome of the payoff function can be represented in a matrix, the so-called *payoff matrix*, which shows the two players, their strategies and their payoffs:

Player1\Player2	L	R
U	3 , 3	0 , 5
D	5 , 0	1 , 1

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Extensive Form Games

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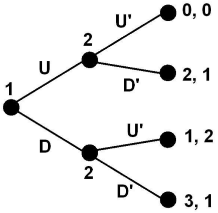
Contrary to the normal form game, the rules of an *extensive form game* are described such that the agents of the game execute their moves consecutively.



Definitions

Nash Equilibrium

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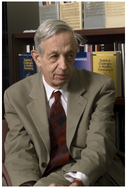
If each player has chosen a strategy and no player can benefit by changing his strategy while the other players keep theirs unchanged, then the current set of strategy choices and the corresponding payoffs constitute a Nash equilibrium.

John Nash showed in 1950, that every game with a finite number of players and finite number of strategies has at least one mixed strategy Nash equilibrium.



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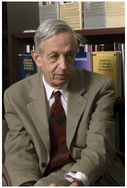

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Best Response

The best response is the strategy (or strategies) which produces the most favorable immediate outcome for the current player, taking other players' strategies as given.

Localizing a Nash Equilibrium in a Payoff Matrix

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Games

Prisoner's Dilemma

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- Two suspects arrested
- Police has insufficient evidence for a conviction
- If both testify:
  - *Both receive 2-year sentence*
- If both stay silent:
  - *Only a six-month-sentence for a minor charge*
- If one testifies and the other remains silent:
  - *Betrayer goes free, accomplice receives 10-year sentence*

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Payoff Matrix of the Prisoner's Dilemma

- Abbreviations: To testify means to betray the other suspect and thus to defect (D), to remain silent means to cooperate (C) with the other suspect.
- We will use positive numbers in the payoff matrix.

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Public Good Game

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- A group of 4 people are given \$ 200 each.
- They can invest an amount of money in a 'project'.
- Every \$1 invested will yield \$ 2, which would be distributed to all group members.
- If everyone invested, each would get \$ 400.
- However, if only one person invested, that "sucker" would take home a mere \$ 100.

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Payoff Function of the Public Good Game

This is a game in normal form and therefore has a payoff function for each player. The payoff function for, let's say, player 1 is given by

$$P_1 = \frac{2 \cdot (s_1 + s_2 + s_3 + s_4)}{4} - s_1$$
$$= \frac{2 \cdot (s_2 + s_3 + s_4)}{4} - 0,5 \cdot s_1$$

- Every investment s_1 of player 1 will diminish his payoff.
- Therefore, a rational player will choose the strategy $s_n = 0$, i.e. he will invest no money at all.

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Rock, Paper, Scissors

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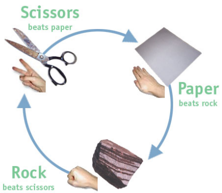
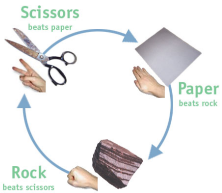


- Two players form a symbol (rock, paper or scissors) with their hands
- *Rock* crushes *Scissors*
- *Scissors* cut *Paper*
- *Paper* covers *Rock*

Rock, Paper, Scissors



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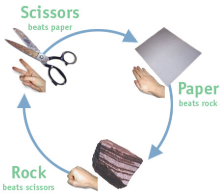

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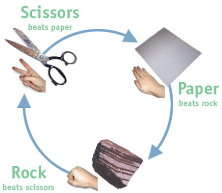

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Evolutionary Game Theory

Why EGT?

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- Games can be played repeatedly, and also with a population of players.
- Which populations/strategies are stable?
- Is it possible for mutants to invade a given population?
- How does cooperation arise and evolove?
- How does biodeversity emerge?

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- Let $E(\mu, \sigma)$ be the payoff for a player playing μ against a player playing σ.
- For σ to be an Nash equilibrium it is demanded: $E(\sigma, \sigma) \geq E(\mu, \sigma)$
- For σ to be an ESS it is demanded:
  1. $E(\sigma, \sigma) > E(\mu, \sigma)$ (equlibrium condition), or
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- Two individuals compete for a resource V.
- Each individual follows exactly one of two strategies described below:
  - **Hawk**: Initiate aggressive behaviour, not stopping until injured or until one's opponent backs down.
  - **Dove**: Retreat immediately if one's opponent initiates aggressive behaviour.

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- whenever two individuals both initiate aggressive behaviour, conflict eventually results and the two individuals are equally likely to be injured,
- the cost of the conflict reduces individual fitness by some constant value C,
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- the cost of the conflict reduces individual fitness by some constant value C,
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This leads to the following payoff matrix:

	Hawk	Dove
Hawk	$\frac{1}{2}(V-C), \frac{1}{2}(V-C)$	$V, 0$
Dove	$0, V$	$V/2, V/2$

- *Dove* is no stable strategy, since $\frac{V}{2} = E(D,D) < E(H,D) = V$.
- Because of $E(H,H) = \frac{1}{2}(V-C)$ and $E(D,H) = 0$, H is an ESS if $V \geq C$.
- But what if $V < C$?
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Evolutionary Game Theory

The Replicator Dynamics

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- Let us consider now a population consisting of n types, and let $x_i(t)$ be the frequency of type i at some time t.
- The state of the population is given by the vector $\mathbf{x}(t) = x_1(t), \ldots, x_n(t)$.
- The state of the population will change, since some species will get higher payoffs and will reproduce better than others, or others adopt the strategy of the species with the better fitness.
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- If individuals meet randomly and then engage in a symmetric game with payoff matrix A, then $(A\mathbf{x})_i$ is the expected payoff for an individual of type i
- $\mathbf{x}^T A\mathbf{x}$ is the average payoff in the population state $\mathbf{x}$.
- The evolution of $\mathbf{x}$ over time is described by the replicator equation:

$$\dot{x}_i = x_i[(A\mathbf{x})_i - \mathbf{x}^T A\mathbf{x}]$$

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- By setting $\dot{x}_i = 0$, we obtain the *evolutionary stable states* of a population.
- A population is said to be in an *evolutionarily stable state* if its genetic composition is restored by selection after a disturbance, provided the disturbance is not too large.
- For $V > C$, the only evolutionary stable state is a population consisting of hawks.
- For $V < C$, a mixed population with a fraction V/C of hawks and a fraction $1 - V/C$ of doves is evolutionary stable.

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Applications

Evolution of Cooperation

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- Consider spatially structured populations
- Limited local interactions enable cooperators to form clusters
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- Theoretical models have shown that competing species can coexist if ecological processes such as dispersal, movement, and interaction occur over small spatial scales.
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Biodiversity

- There exist three strains of Escherichia coli bacteria.
- Type A releases toxic colicin and produces, for its own protection, an immunity protein.
- Type B produces the immunity protein only.
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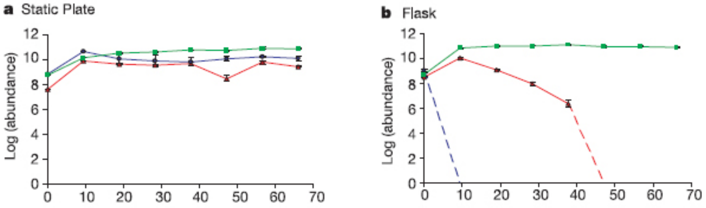

green: resistant strain
red: colicin producing strain
blue: sensitive strain

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- Most evolutionary game dynamics have been studied in the context of infinitely large populations. We expect that finite population size effects will lead to surprising outcomes and might question the importance of traditional evolutionary stability.
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