# F-theory: Progress and Prospects

Timo Weigand

ITP, University of Heidelberg

# Why we like F-theory so much

F-theory is perhaps the most general currently controlled framework to think about (non-perturbative) brane configurations

- beyond pert. Type II orientifolds due to [p,q]-branes
- still within (conformal) Calabi-Yau geometry and thus well-controlled

$\Rightarrow$ framework to understand geometric compactifications w/ branes

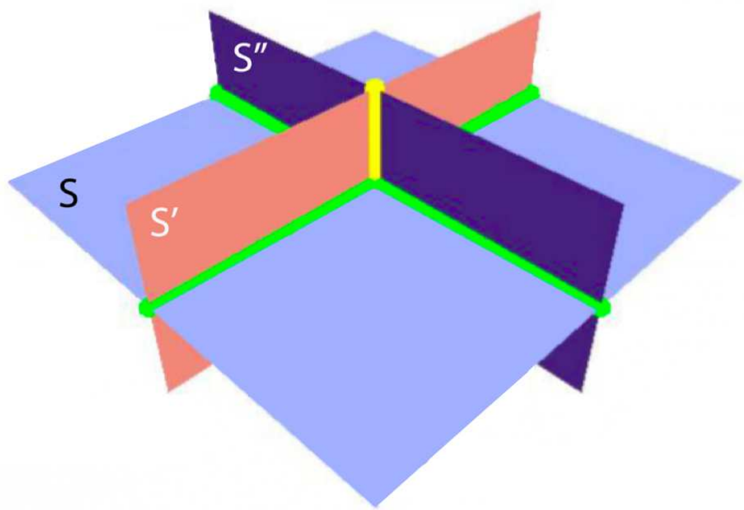
Being general pays off: Application to F-theory GUTs

[Beasley,Heckman,Vafa; Donagi,Wijnholt'08]

Hierarchy of localisation:

- SU(5) $\leftrightarrow$ 4-cycle
- matter $\leftrightarrow$ 2-cycle
- Yukawa $\leftrightarrow$ point

$E_6$-point $\leftrightarrow$ $\mathbf{10}\,\mathbf{10}\,\mathbf{5}$




Pic: Cordova, 0910.2955

# Progress in F-theory

Where were we 5-6 years ago:

✓ local model building via field theory on 7-branes see talk by Marchesano

✓ non-abelian singularities in codimension-one (and two) well understood

✓ . . . (!)

↯ no fully fledged (resolved) F-theory 4-fold suitable e.g. for F-GUTs

↯ not much worked out for codim-2, nothing for codim 3

↯ no (good) understanding of U(1) gauge groups

↯ no understanding of gauge fluxes in global geometries

Today, all these questions have been addressed

- Examples for all such cases worked out
- Ongoing work: systematization and -possibly in future - classification
- better understanding of dualities to M-theory, IIB and heterotic
- Pheno applications have triggered formal progress

# Overview

This review will be biased, incomplete and faulty.

**I) Setting the stage for F-theory compactifications**

**II) Non-abelian gauge symmetry**

**III) Abelian gauge symmetry**

**IV) Gauge fluxes**

**V) Frontiers in Phenomenology**

# I) Setting the stage for F-theory compactifications

# The magic of F-theory

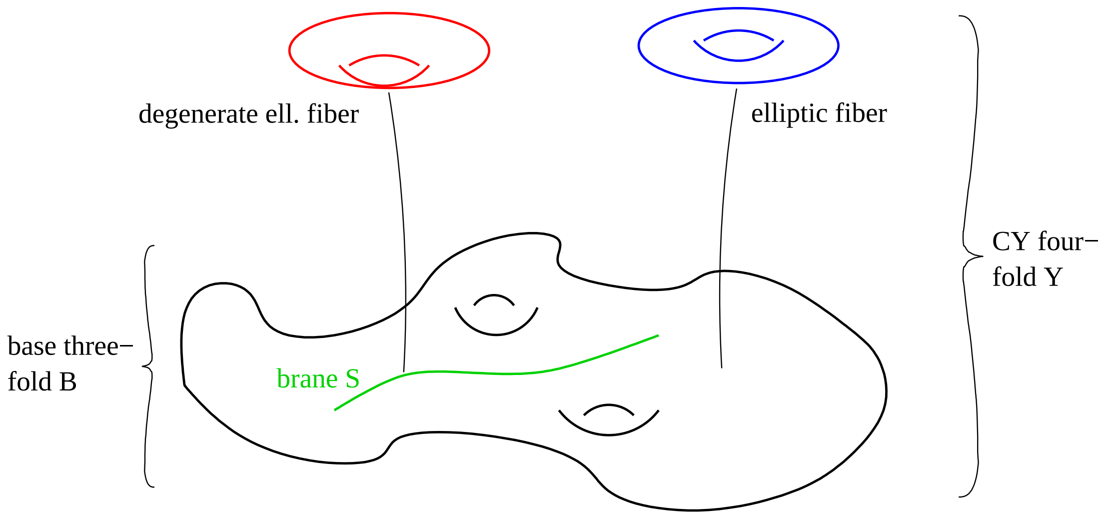
**F-theory epitomises the geometrisation of physics** [Vafa'96]

IIB language:

7-branes wrap 4-cycle $S \in X_6/\sigma$

F-theory language:

S $=$ locus of fiber degeneration




| **IIB picture** | | **F-theory picture** |
|---|---|---|
| compactification space | | base of fibration |
| varying axio-dilaton $\tau(z)$ | $\Longleftrightarrow$ | complex structure of fibre |
| 7-branes | | codim.-one singular fibres |
| D(-1) corrections | | e.g. $\tau(z)$ [Billo et al.'11-'13] |

**But not all physics is geometrised...**

# F-theory via M-theory

F-theory is really defined via **duality with M-theory** [Witten'96]

- M-theory on elliptic 4-fold $\rightarrow \mathcal{N} = 2$ theory in $\mathbb{R}^{1,2}$
- F-theory limit = suitable limit of vanishing fibre volume $v_{T^2} \rightarrow 0$

Effective action by dimensional reduction of **11D sugra coupled to M2/M5-branes** in this very subtle F-theory limit

| | | |
|---|---|---|
| D3-branes on $\mathbb{R}^{1,3}$ | | M2-branes on $\mathbb{R}^{1,2}$ |
| D3-brane instantons | $\Longleftrightarrow$ | vertical M5-brane instantons |
| gauge fluxes | | $G_4$-flux '1 leg along sing. fibres' |
| bulk fluxes | | $G_4$-flux '1 leg along smooth fibres' |

A lot of recent progress in exploring F-theory from 6D and 4D effective action perspective see talk by Grimm

[Grimm'10][Grimm,Kerstan,Palti,TW'11][Bonetti,Grimm,(Hohenegger)'11,'12 &13],

including $\alpha'$-corrections: see talk by Weissenbacher

[Hayashi,Garcia-Etxebarria,Savelli,Shiu'12][Grimm,Savelli,Weissenbacher][Grimm,Pugh]'13

# II) Non-abelian gauge symmetry

# Non-abelian gauge symmetry

**Singularity type in co-dim. $1_{\mathbb{C}} \leftrightarrow$ gauge group $G$ along 7-brane**

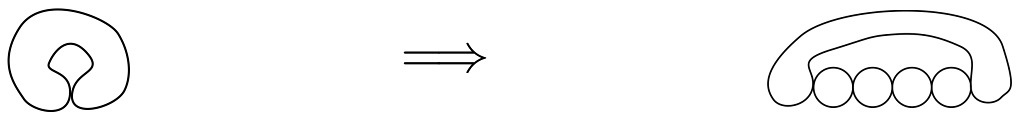
Strategies to study F-theory on singular fibration:

1) Resolve singularity $=$ moving in Coulomb branch of 3D M-theory, or

2) Deform singularity $=$ Higgsing of singularity

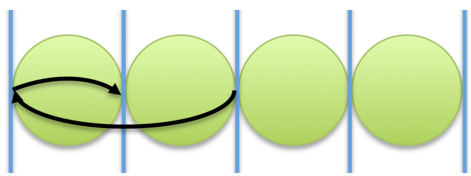
**First consider resolutions:**

- resolve singular point in fibre by tree of $\mathbb{P}^1_i$ $\quad i = 1, \ldots, \mathrm{rk}(G)$



- **Group theory of $G$ $\Longleftrightarrow$ extended Dynkin diagram**

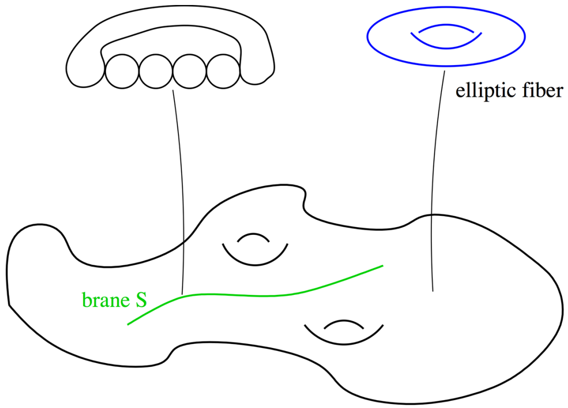
- Each node of Dynkin diagram
  $\leftrightarrow$ stretched open strings
  $\equiv$ $G$-gauge bosons



# Codim 1, 2 and 3

**$G$-gauge bosons:** [Vafa,Morrison'96]

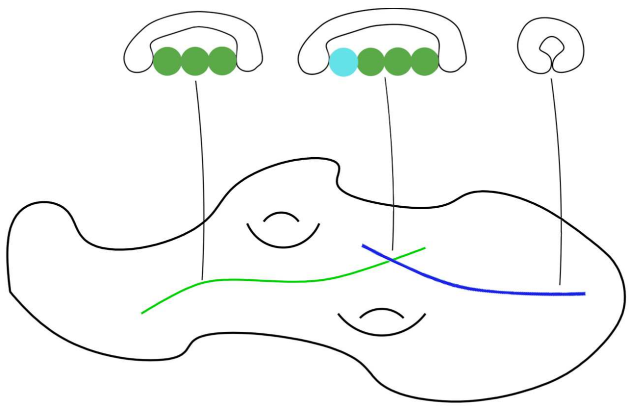
- non-Cartan part from M2-branes along chains of $\mathbb{P}^1_i$
- Cartan part from $C_3 = A_i \wedge [E_i]$




**Enhancement in codimension 2**

extra massless states from wrapped M2-branes

[Katz,Vafa'96][Witten'96]



**Further enhancement in codimension 3**:

Yukawa couplings at intersections of matter curves

[Beasley,Heckman,Vafa; Donagi,Wijnholt'08]

# Fiber splittings in codimension

Splitting of $\mathbb{P}^1_i$ $\rightarrow$ increase of fibre rank in codimension

**Example: SU(5)**

GUT surface

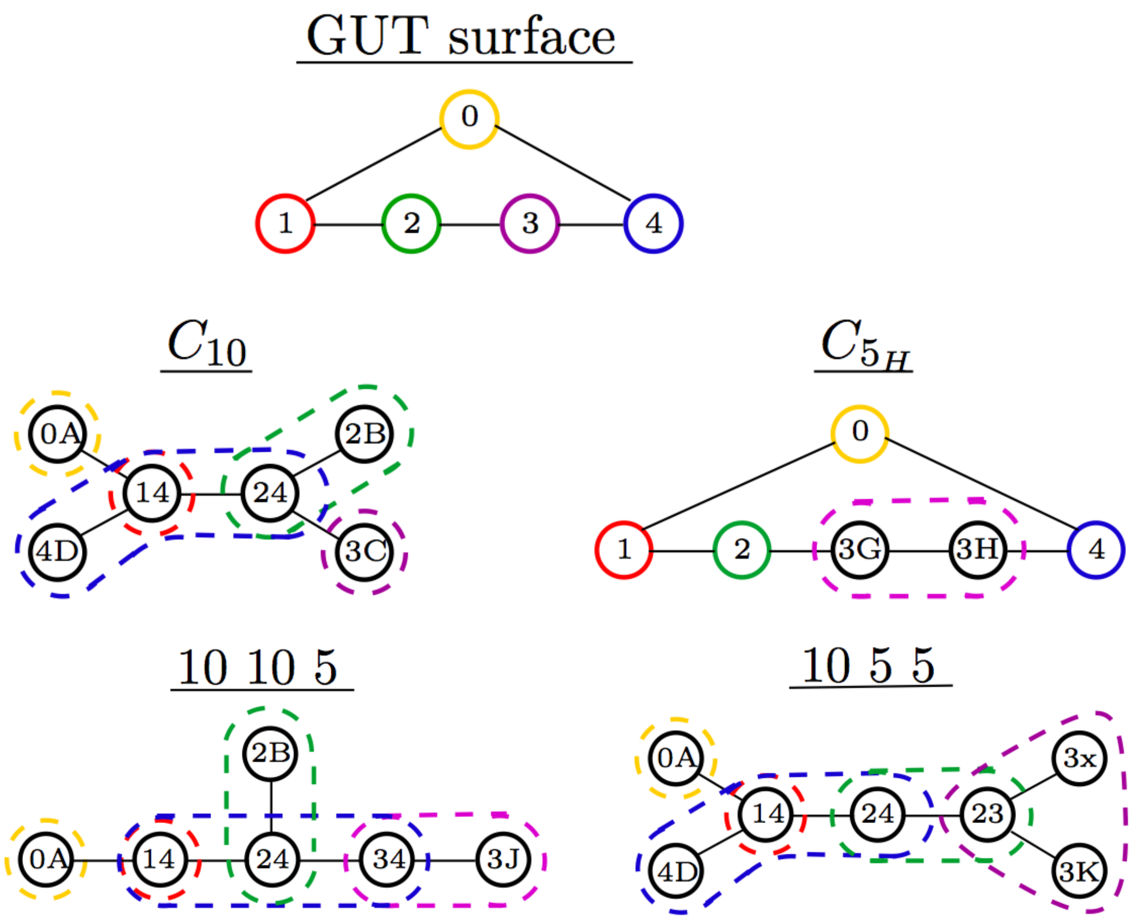
(codimension 1)

matter curves

(codimension 2)

Yukawa points

(codimension 3)



# Weierstrass models

Simplest type of $T^2$-fibration is given by a Weierstrass model:

- $P_T : y^2 = x^3 + fxz^4 + gz^6$ $\qquad (x, y, z) \simeq (\lambda^2 x, \lambda^3 y, \lambda^1 z) \leftrightarrow \mathbb{P}_{2,3,1}$
- $f, g$ depend on coordinates on $B_3$: sections of $\bar{\mathcal{K}}^4$ and $\bar{\mathcal{K}}^6$
- Fibre degenerates over loci where $\Delta := 4f^3 + 27g^2 = 0$

In fact, this describes an elliptic fibration, i.e. a $T^2$-fibration with a section:

- A section = map that assigns to generic point in base a point in the fibre
- A section gives an embedding of $B_3$ into 4-fold, i.e. **identifies $B_3$ with physical spacetime**
- Zero section: $[x : y : z] = [1 : 1 : 0] \Rightarrow \{z = 0\}$ is the base
- Every elliptic fibration is birational to a Weierstrass model.

**Note on recent developments:**

- Many more representations of elliptic/$T^2$ fibrations as hypersurfaces or compl. intersections exist
- For now stick to Weierstrass, but will come back to this soon

# Weierstrass fibers in codim. 1

Codim. one fiber types completely known for Weierstrass models

- on K3: ADE type fibers + a few extra cases - [Kodaira'63],[Néron'64]
- on 3-folds monodromies along 7-brane can 'fold' fibers [Tate'75]

Tate's algorithm: [Tate'75]; [Bershadsky et al.'96]

- $P_T : y^2 = x^3 + fxz^4 + gz^6 \qquad \Delta := 4f^3 + 27g^2$
- $f = f_0 + f_1 w + f_2 w^2 + \ldots, \qquad g = g_0 + g_1 w + g_2 w^2 + \ldots \quad w = 0$ base divisor
- vanishing orders of $(f, g, \Delta)$ determine codim. fibers together with extra monodromies (encoded in extra polynomials)
- **Locally**, in most cases one can achieve Tate form

$$P_{\mathrm{T}} = x^3 - y^2 - x\,y\,z\,a_1 + x^2\,z^2\,a_2 - y\,z^3\,a_3 + x\,z^4\,a_4 + \,z^6\,a_6 \;\; = 0$$

  and $a_i = a_{i,j} w^j$ for $a_{i.j}$ generic
- In few 'outlier' cases in addition $a_{i,j}$ must be non-generic [Katz,Morrison,Schäfer-Nameki,Sully'11]
- **Globally** there may be obstructions - status not clear

# Fibers in codim 2 & 3

Challenge: Fibers in codimension 2 and 3 not fully classified geometrically

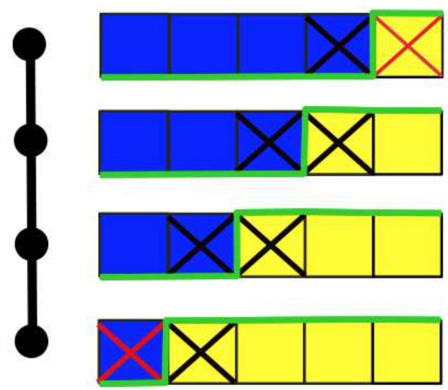
## 1) Codimension-two

- systematic description of 6D matter points for smooth 7-brane curves [Grassi,Morrison,11]
- inclusion of self-intersecting 7-branes in 6D → higher tensor reps. [Morrison,Taylor'12]

## 2) Proposal: 'Box Graphs' [Hayashi,Lawrie,Morrison,Schäfer-Nameki'14]

- sign decorated representation graph based on Kodaira fiber
- blue (yellow) weight curves are effective (anti-effective)

  ↔ phases of classical Coulomb branch of 3D field theory [DeBoer,Hori,Oz][Aharony et al.'97], [Grimm,Hayashi'11][Hayashi,Lawrie,Nameki'13]



- claims to give complete classification of all possible enhanced fiber types in higher codimension - at least for otherwise generic Weierstrass models

# Resolutions of Weierstrass models

## Toric resolutions

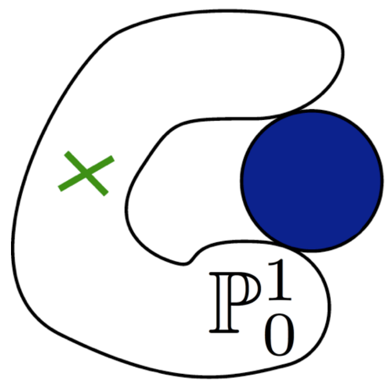
- Foundational work ('tops'): [Candelas,Font,(Rajesh)'96][Perevalov,Skarke'97], ...
  - Toric encoding of Tate vanishing orders and resolution divisors as toric ambient space coordinates
  - Example SU(2):

$$P_T : y^2 + a_1 xyz + a_{3,1} yz^3 e_0 = x^3 e_1 + a_{2,1} x^2 z^2 e_0 e_1 + a_{4,1} xz^4 e_0 + a_{6,2} z^6 e_0^2$$

$E_i : e_i = 0$ is $\mathbb{P}^1_i$-fibration over $S \subset B_3$

$$\mathbb{P}^1_i = \{e_i\} \cap \{P_T|_{e_i=0}\} \quad \cap \quad \{y_a\} \cap \{y_b\}$$

$i = 1$ or $i = 0$




- Explicit construction of toric resolutions of SU(5) GUTs on 4-folds [Blumenhagen,Grimm,Jurke,TW'09]
- and of SO(10) GUTS: [Chen,Knapp,Kreuzer,Mayrhofer,Knapp'10], ...

# Specifics in higher co-dimensions

**Detailed Resolution including analysis of codim- 2 and 3**

- SU(5) fibrations:
  - 'algebraic resolution' [Esole,Yau][Marsano,Schäfer-Nameki]'11
  - 'toric resolution' [Krause,Mayrhofer,TW'11] [Grimm,Hayashi'11]
- Other gauge groups [Collinucci,Savelli'10/12] [Krause,Mayrhofer,TW'12] [Kuntzler,Schäfer-Nameki] [Tatar,Walters][Lawrie,Schäfer-Nameki]'12
- Different types of resolutions/triangulations are different phases of 3D Coulomb branch with same 4D physics [Hayashi,Lawrie,Schäfer-Nameki'13]

**Recent/ongoing developments:**

- Unifying approach behind toric and algebraic and one more type of resolution [Braun,Schafer-Nameki'14][Esole,Yau'14]
- Announced: Extension to all gauge groups - explicit realization of all possible phases/Box graphs as resolved geometries

# Deformations versus resolutions

Resolutions = moving in 3D classical Coulomb branch

Some geometries may not admit a crepant resolution

Alternative way to study physics of singular fibration:
(complex structure) deformation = Higgsing

- Gauge and matter states arise from M2-branes along 2-cycles, which project to string junctions on $B$
- Makes contact with formalism of multi-pronged strings/[p,q]-strings of [Gaberdiel,Zwiebach'97],[DeWolfe,Zwiebach'98]
- Exemplified for K3 and K3$\times T^2/\mathbb{Z}_2$ already in [Braun,Hebecker et al.'08/'09]
- General formalism to determine matter spectrum based on deformations developed in [Grassi,Halverson,Shaneson'13 & 14]

# III) Abelian gauge symmetry

# The quest for U(1)

**Motivation to study non-Cartan U(1)s**:

- desirable for phenomenology as extra **selection rules** (proton decay, flavour structure,...)
- **charged singlets** plays role in phenomenology - e.g. as neutrinos or in SUSY breaking
- precursor to construction of large class of **gauge fluxes**
- **U(1) symmetries and instantons** have rich interplay in Type II and heterotic compactifications
  What's the analogue in F-theory?

General fact from expansion $C_3 = \sum_{i=1} A_i \wedge \mathsf{w}_i$:

non-Cartan U(1)s $\leftrightarrow$ extra resolution divisors not fibered over base 4-cycle

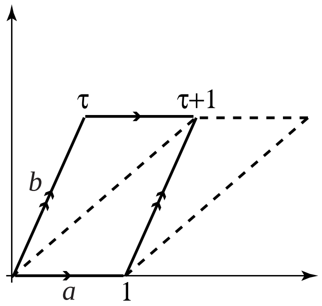
**Claim:** [Morrison,Vafa'96]

These correspond to **extra sections of the fibration**.

# Mordell-Weil group

**1) Elliptic curve**: $\mathcal{E} = \mathbb{C}/\Lambda$

$\leftrightarrow$ addition of points




**Rational points**:

- have $\mathbb{Q}$-rational coordinates $(x, y, z)$ in Weierstrass model

$$y^2 = x^3 + fxz^4 + gz^6, \qquad [x : y : z] \in \mathbb{P}^2_{2,3,1}$$

- form an abelian group under addition = **Mordell-Weil group** $E$

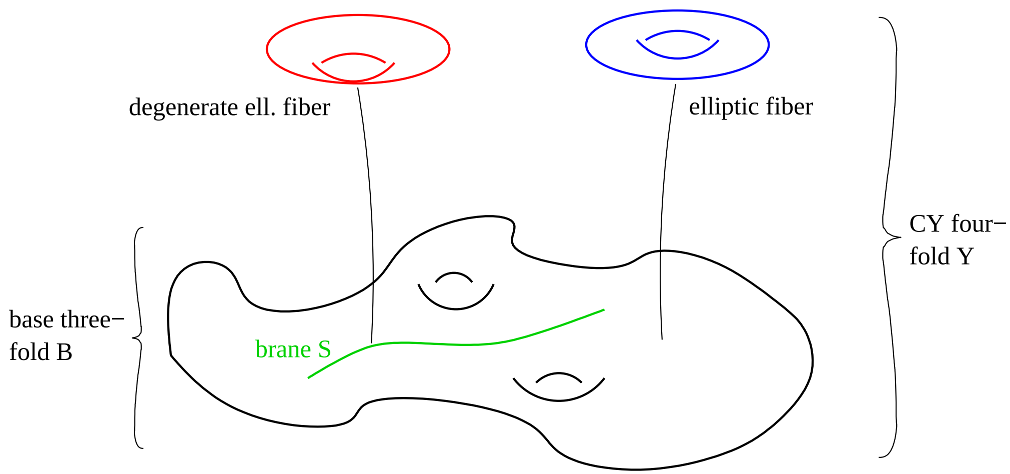
$$E = \mathbb{Z}^r \oplus \mathbb{Z}_{k_1} \oplus \cdots \oplus \mathbb{Z}_{k_n}$$

**2) Elliptic fibration**: $\pi : Y \to \mathcal{B}$

**Rational section** $\sigma$:

$$\mathcal{B} \ni b \mapsto \sigma(b) = [x(b) : y(b) : z(b)]$$

- $\sigma(b)$ is a $K$-rational point in fiber
- degenerations in codimension allowed



# Mordell-Weil group

**Mordell-Weil group $E(K)$= group of rational sections**

- zero-element = zero-section $\sigma_0 : b \to [1 : 1 : 0]$ in $y^2 = x^3 + fxz^4 + gz^6$
- group law = fiberwise addition

$$E(K) = \underbrace{\mathbb{Z}^r}_{\text{free part}} \oplus \underbrace{\mathbb{Z}_{k_1} \oplus \cdots \oplus \mathbb{Z}_{k_n}}_{\text{torsion part}}$$

Physical significance:

- Free part $\leftrightarrow$ $U(1)$ gauge symmetries [Morrison,Vafa'96],[Klemm,Mayr,Vafa'98],. . .
- Torsion part $\leftrightarrow$ Global structure of non-ab. gauge groups ($\pi_1(G)$)

  [Aspinwall,Morrison'98], . . . , [Mayrhofer,Till,Morrison,TW'14] see talk by Mayrhofer

Systematic recent study of U(1)s via rational sections:

✓ extra selection rules, e.g. crucial in F-theory GUTs
✓ window to gauge fluxes and chirality
✓ general interest in any theory with massless U(1)s (landscape studies, . . . )

Antoniadis,Anderson,Bizet,Borchmann,Braun,Braun,Choi,Collinucci,Cvetič,Etxebarria,Grassi,Grimm,

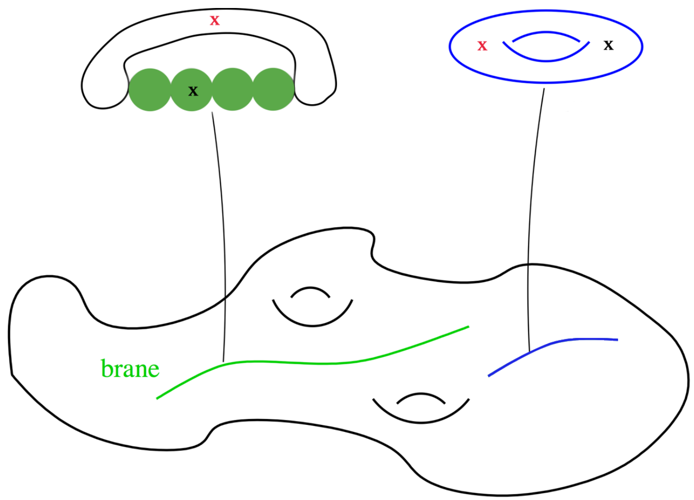
Hayashi, Keitel,Klevers,Küntzler,Krippendorf,Oehlmann,Klemm,Leontaris,Lopes,Mayrhofer, Mayorga,

Morrison, Park,Palti,Piragua,Rühle,S-Nameki,Song,Valandro,Taylor,TW,. . .

# Shioda map

Divisors on elliptic 4-fold $\hat{Y}_4 \to \mathcal{B}$:

- zero-section $Z$
- pullback from base $\pi^{-1}(D_{\rm b})$
- resolution divisors $F_m$, $m = 1, \ldots, {\rm rk}(G)$
- **rational sections** $S_i$




## Shioda map

- homomorphism $\varphi : \underbrace{E(K)}_{\text{group of sections}} \to \underbrace{NS(\hat{Y}_4) \otimes \mathbb{Q}}_{\text{group of divisors}}$ [Shioda'89]

$$\varphi(S - Z) = S - Z - \pi^{-1}(\delta) + \sum l_i F_i, \qquad l_i \in \mathbb{Q}$$

- transversality

$\int_{\hat{Y}_4} [\varphi(S - Z)] \wedge [X] \wedge [\pi^{-1}\omega_4] = 0 \qquad X \in \{Z, F_l, \pi^{-1}(D_{\rm b})\}$

# Abelian gauge symmetries

**F-theory on $Y_4$ $\leftrightarrow$ abelian gauge group $U(1)^{\mathrm{rk}(E(K))}$**

Physics reason:

- section $\sigma$ $\leftrightarrow$ $[\mathcal{S}] \equiv [\varphi(\sigma)]$ is non-trivial in $H^{1,1}(Y_4)$

  Shioda homomorphism $\varphi$ : $\underbrace{E(K)}_{\text{group of sections}} \rightarrow \underbrace{NS(Y_4) \otimes \mathbb{Q}}_{\text{group of divisors}}$ [Shioda'89]

- $[\mathcal{S}]$: 'generator' of $U(1)$ gauge group (by duality with M-theory: [Morrison,Vafa'96])

$$\underbrace{C_3}_{3-\text{form}} = \underbrace{A}_{1-\text{form}} \wedge \underbrace{[\mathcal{S}]}_{2-\text{form}}$$

$A$: $U(1)$ gauge potential

**Arithmetic geometry $\overset{\text{F}-\text{theory}}{\longleftrightarrow}$ Physics of U(1) selection rules**

Detailed field theory analysis: [Cvetic,Grimm,Klevers'12][Grimm,Kapfer,Keitel'13]

# Understanding U(1)s

**Questions:**

1. Which complex structure restrictions lead to extra sections and thus to extra U(1)s?
2. What is the fiber structure in codim 2 and 3, i.e. which charged matter and couplings exist?
3. How does one combine this with non-abelian gauge symmetry?

Elliptic fibrations with non-trivial MW group in principle birational to Weierstass model with non-generic form for $f$ and $g$

Guessing non-generic form of $f$ and $g$ hard, but recent progress via different fiber representations

Before coming to systematics, first consider a simple example

# U(1)s from sections - I

$P_T: y^2 = x^3 + a_1 xyz + a_2 x^2 z^2 + a_3 yz^3 + a_4 xz^4 + \cancel{a_6 z^6}$ [Grimm,TW'10]

Sections: $\mathrm{Sec}_0: [x:y:z] = [1:1:0]$, $\mathrm{Sec}_1: [x:y:z] = [0:0:1]$

Over curve $a_3 = a_4 = 0$: fibre singular at $(x, y) = (0, 0)$

Blow-up: $(x, y) \to (x\, s, y\, s)$

- $P_T: y^2 s = x^3 s^2 + a_1 xyzs + a_2 x^2 z^2 s + a_3 yz^3 + a_4 xz^4$
  with $(x, y) \neq (0, 0)$ and $(z, s) \neq (0, 0)$
- $Z: z = 0 \cap P_T = 0$ is holomorphic zero section as before
- Resolved extra section is $S: s = 0 \cap P_T = 0$:

  ✓ 1 point in fibre, but entire $\mathbb{P}^1$ over curve $a_3 = a_4 = 0$

  ✓ rational section due to degeneracy over curve

  ✓ Holomorphicity of section restored in alternative conifold-type resolution via complete intersection [Braun,Valandro,Collinucci'11]

M2-branes wrapping split fibre over curve $a_3 = 0 \cap a_4 = 0$ matter of charge $q = \int_{\mathbb{P}^1_i} \mathrm{w}$, $\mathrm{w} = S - Z - \bar{K}$

# U(1)s - beyond $\mathbb{P}_{2,3,1}[6]$

✓ Different reps. of fibre, e.g. as hypersurfaces $\mathbb{P}_{1,1,2}[4]$ or $\mathbb{P}_{1,1,1}[3]$

1) Analytic analysis including charged singlets:

- one generic U(1) factor from $\mathrm{Bl}^1\mathbb{P}_{1,1,2}[4]$: [Morrison,Park'12]

  $B\,\mathsf{v}^2\,\mathsf{w} + s\,\mathsf{w}^2 = C_3\,\mathsf{v}^3\,\mathsf{u} + C_2\,s\,\mathsf{v}^2\,\mathsf{u}^2 + C_1\,s^2\,\mathsf{v}\,\mathsf{u}^3 + C_0\,s^3\,\mathsf{u}^4$

- two generic U(1)s from $\mathrm{Bl}^2\mathbb{P}_{1,1,1}[3]$:

  [Borchmann,Palti,Mayrhofer,TW'13] [Cvetič,Grassi,Klevers,Piragua'13]

  $\mathrm{v}\,\mathsf{w}(c_1\,\mathsf{w}\,s_1 + c_2\,\mathrm{v}\,s_0) + \mathrm{u}\,(b_0\,\mathrm{v}^2\,{s_0}^2 + b_1\,\mathrm{v}\,\mathsf{w}\,s_0\,s_1 + b_2\,\mathsf{w}^2\,{s_1}^2) + \mathrm{u}^2(d_0\,\mathrm{v}\,{s_0}^2\,s_1 +$
  $d_1\,\mathsf{w}\,s_0\,{s_1}^2 + d_2\,\mathrm{u}\,{s_0}^2\,{s_1}^2) = 0$

- three generic U(1)s from complete intersection

  [Cvetič,Grassi,Klevers,Piragua'13]

2) Toric analysis: [Braun,Grimm,Keitel'13] (see talk by Keitel)

of 16 reps. of torus as hypersurface of toric spaces (cf. [Grassi,Perduca'12]) )
including examples of 'accidental' non-generic U(1)s

# Engineering $G \times U(1)^n$

**2 types of restrictions:**

1. Generic models:
   restrictions of 'Tate polynomials' of various fiber representations (whenever available) by simple factorization

$$g_m = g_{m,i} w^i, \qquad g_m \text{ otherwise generic}$$

2. Non-generic models
   more general restrictions due to non-trivial relations between $g_m$

**ad 1) Generic models:** 'Toric tops' [Bouchard,Skarke'03]

- $SU(5) \times U(1)$ in $\mathbb{P}_{2,3,1}[6]$ [Mayrhofer,Krause,TW'11][Grimm,Hayashi'11] generalised in [Mayrhofer,Krause,TW'12]
- $\mathbb{P}_{112}[4]$ model: all 4 SU(5) tops in codim1,2,3 [Mayrhofer,Palti,TW'12] [Borchmann,Palti,Mayrhofer,TW'13]
- $\mathbb{P}_{111}[3]$ model: all 5 SU(5) tops in codim1,2,3 [Borchmann,Palti,Mayrhofer,TW'13] (see also [Cvetič,Grassi,Klevers,Piragua'13] for examples)
- list of all 37 SU(5) tops for 16 hypersurfaces + examples [Braun,Grimm,Keitel'13]

# Engineering $G \times U(1)^n$

## 2) Non-generic models

General pattern: describable as 'special generic' models of lower-rank

First exemplified for SU(5) in $\mathbb{P}_{112}[4]$ model in [Mayrhofer,Palti,TW]'12:

- $g_4\, \mathsf{v}^2\, \mathsf{w} + s\, \mathsf{w}^2 = g_3\, \mathsf{v}^3\, \mathsf{u} + g_2\, s\, \mathsf{v}^2\, \mathsf{u}^2 + g_1\, s^2\, \mathsf{v}\, \mathsf{u}^3 + g_0\, s^3\, \mathsf{u}^4$
- Constrain $g_m = g_{m,i} w^i$ such that if $g_{m,i}$ were generic, then $G = SU(4)$
- for $g_{m,i}$ factorising in specific way get instead $G = SU(5)$ with resolution as a complete intersection
- gives rise to new features, e.g. two types of $\mathbf{10}$-curves $\Longrightarrow$ very relevant for pheno!
- no-go of multiple $\mathbf{10}$-curves for hypersurfaces: [Braun,Grimm,Keitel]'13

Systematic classification for $\mathbb{P}_{112}[4]$ recently in [Küntzler,Schäfer-Nameki]'14: 'Tate Trees'

# Fibrations without section

Studied recently: see talks by Grimm and Garcia-Etxebarria

[Braun,Morrison] [Morrison,Taylor] [Anderson,Garcia-Etxebarria,Keitel,Grimm]'14

These have a multi-section = several points in fiber exchanged by monodromies around branch cuts

Several interesting features, including:

1. More general types of monodromies lead to new non-abelian fiber types e.g. type $IV^*$: $E_6$, $F_4$, $G_2$ (new) [Braun,Morrison]
2. Massless matter charged under a $\mathbb{Z}_N$-symmetry remains; understandable via a Higgsing procedure [Morrison,Taylor]
3. Field Theory analysis via fluxed circle reduction in [Anderson,Garcia-Etxebarria,Keitel,Grimm]'14

# IV) Gauge fluxes

# $G_4$-Fluxes - Overview

Gauge fluxes described by $\mathbf{G_4 \in H^{2,2}(Y_4)}$ with **'1 leg along fiber'**

a) $\int_{\hat{Y}_4} G_4 \wedge D_a \wedge D_b = 0$ b) $\int_{\hat{Y}_4} G_4 \wedge D_a \wedge Z = 0$ $\forall D_i \in H^2(B)$, $Z$: fibre

Construction requires detailed knowledge of geometry of 4-fold $Y_4$

$$\mathbf{H^{2,2}(Y_4) = H^{2,2}_{\mathrm{vert}}(Y_4) \oplus H^{2,2}_{\mathrm{hor}}(Y_4) \oplus H^{2,2}_{\mathrm{rest}}(Y_4)}$$

$\mathbf{H^{2,2}_{\mathrm{vert}}(Y_4)}$ generated by elements of $H^{1,1} \wedge H^{1,1}$: factorisable fluxes

$\Longleftrightarrow$ extra 2-forms obtained by **resolution of singularities**

- fluxes associated with massless U(1)s [Grimm,TW '10], [Braun,Collinucci,Valandro '11], [Krause,Mayrhofer,TW'11],[Grimm,Hayashi'11]

  If $C_3 = A \wedge \mathrm{w} \Rightarrow G_4 = F \wedge \mathbf{w}$ $F \in H^{1,1}(B_3)$

- extra 'special' fluxes e.g. 'spectral cover' fluxes [Marsano,Schäfer-Nameki'11]

# $G_4$-Fluxes - Overview

**Systematics of** $H^{2,2}_{\text{vert.}}(\hat{Y}_4)$:

- either find all independent linear combinations of $H^{1,1} \wedge H^{1,1}$
  [Cvetič,Klevers,Grassi,Piragua'13] [Braun,Grimm,Keitel'13]
- or exploit that each matter surface gives rise to one extra vertical flux - modulo relations [Borchmann,Mayrhofer,Palti,TW'13]

$\mathbf{H^{2,2}_{\text{hor}}(Y_4)}$: fluxes for specific compl. structure [Braun,Collinucci,Valandro '11]

- specific algebraic 4-cycle, e.g. given by complete intersection on ambient space which lies on $\hat{Y}_4$ for special complex structure

$\mathbf{H^{2,2}_{\text{rest}}(Y_4)}$: the rest

- e.g. Cartan fluxes over non-abelian brane which are trivial in ambient space [Mayrhofer,Palti,TW'13][Braun,Collinucci,Valandro '14]

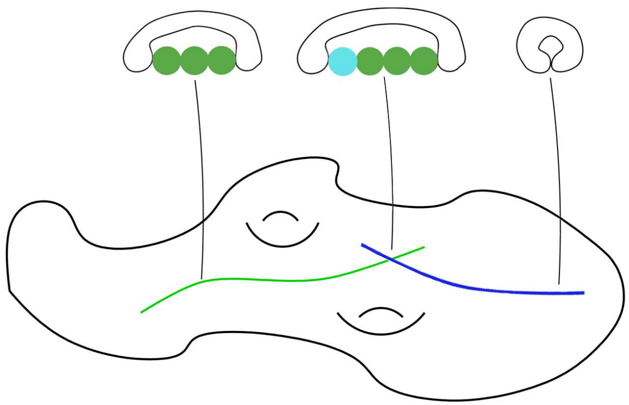
# Matter multiplicities in F-theory

# of charged zero modes $\leftrightarrow$ background gauge field $C_3$ with $G_4 = dC_3$

- chiral index:

$$\nu_+ - \nu_- = \int_{\mathcal{C}_4} G_4$$

[Donagi,Wijnholt'09], [Braun,Collinucci,Valandro] [Marsano,S-Nameki], [Krause,Mayrhofer,TW], [Grimm,Hayashi]'11 ...



- What is the spectrum of states beyond the chiral index?

$\Longrightarrow$ need $C_3$ beyond its field strength [(Curio),Donagi'98], ...

$$0 \longrightarrow \underbrace{J^2(\hat{Y}_4)}_{\oint C_3 \text{ 'Wilson lines'}} \longrightarrow \underbrace{H^4_D(\hat{Y}_4, \mathbb{Z}(2))}_{\textbf{Deligne cohomology}} \xrightarrow{\hat{c}_2} \underbrace{H^{2,2}_{\mathbb{Z}}(\hat{Y}_4)}_{\text{field strength } G_4} \longrightarrow 0$$

Framework for computation of non-chiral states: [Bies,Mayrhofer,Pehle,TW'14]

[cf. talk by Christoph Mayrhofer]

# Flux consistency conditions

- Quantisation: $G_4 + \frac{1}{2}c_2(\hat{Y}_4) \in H^4(\hat{Y}_4, \mathbb{Z})$ [Collinucci,Savelli '10 &'12]
- D3/M2 tadpole: $N_{M_2} + \frac{1}{2}\int_{\hat{Y}_4} G_4 \wedge G_4 = \frac{1}{24}\chi(\hat{Y}_4)$
- F-term condition: $G_4 \in H^{2,2}(\hat{Y}_4)$:
  ✓for $H^{2,2}_{\text{vert}}(\hat{Y}_4)$ for $H^{2,2}_{\text{hor}}(\hat{Y}_4)$ fixes compl. structure
- D-term condition: from detailed analysis of F/M- theory effective action [Grimm '10] [Grimm,Kerstan,Palti,TW '11]

$$D_X = -\frac{2}{\mathcal{V}_B}\int_{\hat{Y}_4} J \wedge G_4 \wedge \mathsf{w}_X$$

Chiral examples of SU(5) GUTs on resolved 4-folds:

✓explicit example of 3-generation $SU(5) \times U(1)_X$ model

[Krause,Mayrhofer,TW '11], [Marsano,Clemens,Pantev,Raby,Tseng'12]

✓explicit examples of chiral $SU(5) \times U(1)_i$ models

[Cvetic,Klevers,Grassi,Piragua][Grimm,Braun,Keitel] [Borchmann,Mayrhofer,Palti,TW]

# Gluing data/T-branes

So far have assumed zero VEV for charged massless fields $\Phi_R$, $\tilde{\Phi}_{\bar{R}}$

Simple 4D field theory model: $V_D \simeq |\phi|^2 - |\tilde{\phi}|^2 - \xi, \qquad \xi \simeq \int_S F \wedge J$

2 different types of VEVs possible:

1. $\langle \phi \tilde{\phi} \rangle \neq 0$ in D-flat manner
   $=$ brane recombination $\leftrightarrow$ complex structure deformation (Higgsing)
2. 'Chiral' VEV $\langle \phi \rangle \neq 0$ , $\langle \tilde{\phi} \rangle = 0$
   D-flatness $\leftrightarrow$ $F \neq 0$
   $=$ 'Gluing data'[Donagi,Wijnholt'11]/'T-branes' [Cecotti,Cordova,Heckman,Vafa'10]

Phenomenological relevance of 'gluing':

Degrees of freedom affect matter spectrum and couplings

Conceptual interest in 'gluing':

Extra data not present in pure geometry as it does involve 'gauge flux'

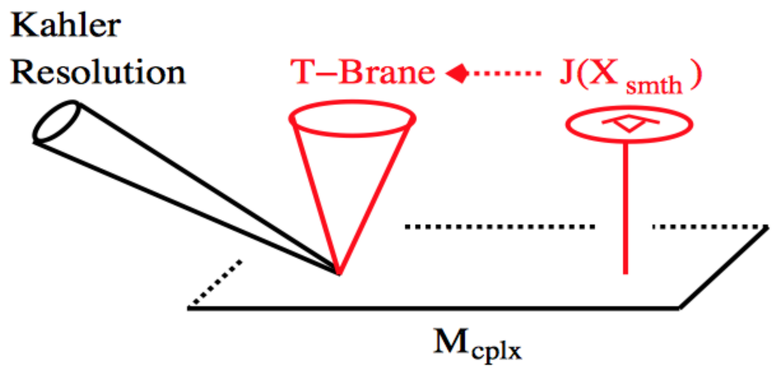
In particular, not accessible via Coulomb branch of resolution

# Gluing data/T-branes

How is gluing data captured in compactifications?

1) Approach of [Anderson,Heckman,Katz'14]

- Gluing = 3-form moduli $\int C_3$ on $X_{smth}$ in singular limit
- Compensating flux captured in $G_4$ on resolved space




- Encoded in element in 'singular limit' of Deligne cohomology

$$0 \longrightarrow \underbrace{J^2(X_{smth})}_{\oint C_3 \ '\text{Wilson lines}'} \longrightarrow \underbrace{H^4_D(X_{smth}, \mathbb{Z}(2))}_{\textbf{Deligne cohomology}} \xrightarrow{\hat{c}_2} \underbrace{H^{2,2}_{\mathbb{Z}}(X_{smth})}_{\text{field strength } G_4} \longrightarrow 0$$

2) Announcement of [Collinucci,Savelli, to appear]

Can be understood via certain matrix factorizations directly in singular limit

# V) Frontiers in F-Phenomenology

(small selection)

# F-theory GUT Phenomenology

initiated by [Beasley,Heckman,Vafa; Donagi,Wijnholt'08]

- **GUT breaking** $SU(5) \to SU(3) \times SU(2) \times U(1)_Y$:
  hypercharge flux due to localisation of GUT brane in codimension
- **Doublet-triplet (3-2) splitting** (and $\mu$-problem):
  localisation of $\bar{\mathbf{5}}_m$ , $\mathbf{5}_{H^u}$, $\mathbf{5}_{H^d}$ on separate curves
- **Proton stability**: U(1) symmetries and localisation

All of these are now understood in global context, with one exception:

How can one achieve doublet-triplet splitting by hyperfluxes with $H_u$ and $H_d$ on different curves?

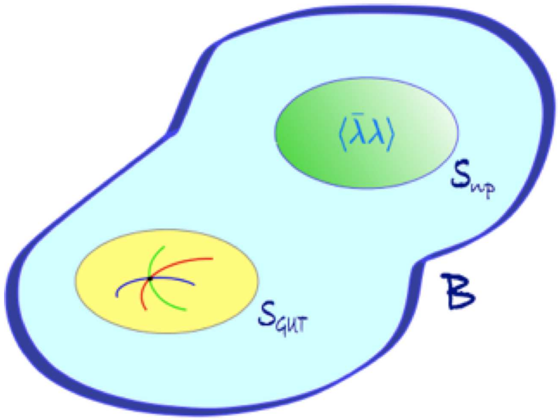
- Need fluxes and curves such that
  $\chi_{(\mathbf{3},1)_{-2Y}} = \int_{C_{\mathbf{5_H}}} (F - 2F_Y) = 0, \qquad \chi_{(1,\mathbf{2})_{3Y}} = \int_{C_{\mathbf{5_H}}} (F + 3F_Y) = \pm 1$
- Proof of principle in IIB limit in [Mayrhofer,Palti,TW'13]
- More on hypercharge flux: [Braun,Collinucci,Valandro'14]

# Yukawa textures

Yukawas $\leftrightarrow$ overlap of matter wavefunction at curve intersection point

**Approach 1:** All families from the same curve

- For single Yukawa point, mass matrix of rank 1 [Heckman,Vafa'08]
- Subleading non-pert corrections from D3/M5-instantons see talk by Marchesano [Marchesano,Martucci'09] [Font,Ibanez,Marchesano13] [Font,Marchesano,Regalado,Zoccarato'13]
- instantons are global data!



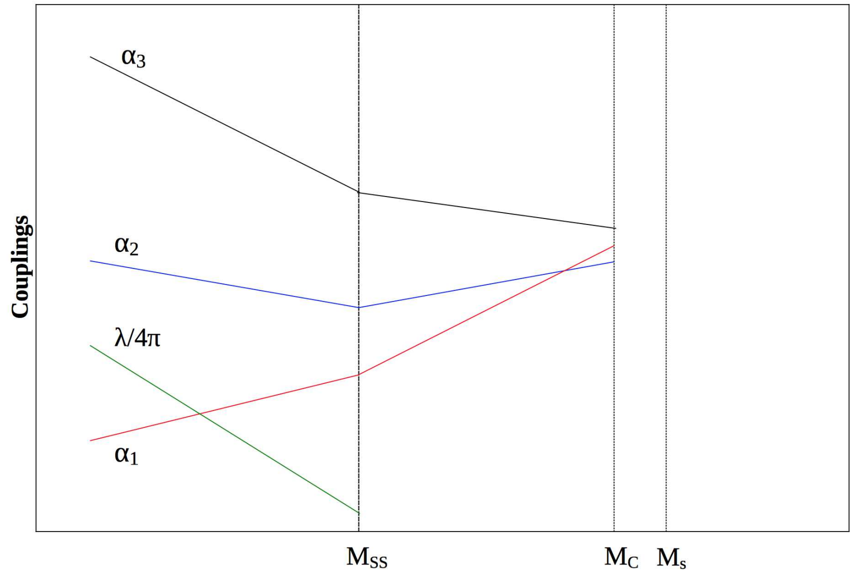

Font et al.,1307.8089

**Approach 2:** Different families from different curves

- U(1) selection rule allows for top coupling, but forbids others
- subleading correction either from instantons or due to Froggatt-Nielson mechanism after giving VEV to singlets [Palti'09], ...

# Beyond SUSY GUTs

- Push $M_{\mathrm{SUSY}}$ up to $10^{11}$ GeV, where quartic Higgs coupl. $\lambda = 0$.
- Standard gauge coupling unification is destroyed.
- Effect can be cancelled in principle against hypercharge-flux correction of [Blumenhagen'08]




**Scenario**: see talk by Hebecker

$M_{\mathrm{SUSY}} = 10^{11}$GeV $\quad M_{\mathrm{GUT}} = \; 10^{14}$GeV [Ibanez,Marchesano,Regalado,Valenzuela'12]

Dimension 6 proton decay from $X - Y$ boson exchange might favour lower SUSY scale $M_{\mathrm{SUSY}} = 100$ TeV [Hebecker,Unwin'14]

Without TeV-SUSY may be more natural to give up GUT idea direct $SU(3) \times SU(2) \times U(1)_Y$ phenomenology within F-theory [Lin,TW'14]

# Closing remarks

F-theory phenomenology has triggered great progress in F-theory compactifications in past 5-6 years

Many more topics should have been covered in this talk, including

- better understanding of duality to IIB [Donagi,Katz,Wijnholt'12] [Grimm,Kerstan,Palti,TW'11] [Braun,Collinucci,Valandro'14]
- connection to heterotic theory [Heckman,Lin,Yau'13]
- M5-instantons [Blumenhagen,Collinucci,Jurke'10] [Cvetič,Garcia-Etxebarria,Richter'09/Halverson'10] [Kerstan,TW'12],...
- connections to 'formal field theory' [Heckman,Morrison,Vafa13], ...

GUTs are a beautiful application, but F-theory is much broader

Elephant in the room:
Moduli stabilization and SUSY breaking:
in principle as in IIB, but more details will be worked out in future