# GUTs without guts

Beatriz Gato-Rivera, Bert Schellekens

**Nucl.Phys. B883 (2014) 529-580**

***Frontiers in String Phenomenology, 28 July 2014***

**Our goal:**

Derive the discrete structure of the Standard Model:
The gauge group and representations.

The standard approach is to use Grand Unification.

But this does not really work.

# Grand Unification

The simplicity is undeniable:

$$SU(3)\times SU(2)\times U(1) \subset SU(5) \subset SO(10)$$

One family matter representation

$$(3,2,\frac{1}{6})+(3^*,1,\frac{1}{3})+(3^*,1,-\frac{2}{3})+(1,2,-\frac{1}{2})+(1,1,1)\ +(1,0,0)$$

Fits beautifully in the $(\mathbf{16})$ of $SO(10)$

And the coupling constants meet each other if there is low energy supersymmetry.

So how could this be wrong?

# Grand Unification

Even if correct GUTs do not lead to a derivation of the SM structure:

- Even the smallest group, $SU(5)$, can break in two ways, to $SU(3)\times SU(2)\times U(1)$ or $SU(4)\times U(1)$.
- The Standard Model Higgs is not determined, and does not fit in an $SU(5)$ multiplet.
- In QFT the representations are determined if one assumes some kind of minimality, but what is the motivation for that?
- No top-down arguments selecting $SU(5)$ or $SO(10)$.

# String Theory

String theory addresses the third point, and to some extent the fourth point, but it really makes the argument far worse.

- Numerous options in addition to GUTs: $E_6$ or $SO(10)$ may have seemed to emerge naturally in heterotic strings in 1984-1986, but this is really just a “lamppost” effect.

- In other contexts (type-II, F-theory, higher level heterotic) GUTs appear by choice, not by necessity.

- Automatic restriction to small representations, but not the right ones:
  - **Heterotic:**
    $(\mathbf{16})$ of $SO(10)$ is automatic, but there are additional fractionally charged representations in $SU(3)\times SU(2)\times U(1)$ which usually appear in the massless spectrum.
  - **Type-II:**
    Undesirable rank-2 tensors

- Coupling convergence always requires human intervention.

# The String Theory Landscape

Our working hypothesis is that the Standard Model is just one of many QFT's that can be realised in the fundamental theory
*(so far string theory is the only candidate for this theory)*.

So then how can we hope to derive the Standard Model?

We still have two clues, that are inevitable in a large landscape:

- Anthropic arguments
- Landscape distributions

We will show that in a certain minimal string setting where GUT realizations are available, anthropic arguments work far better:

- Gauge group determined to be $SU(3)\times SU(2)\times U(1)$.
- Matter determined to be a number of standard families.
- Correct charge quantization without GUTs.
- Standard Model Higgs determined.

Assuming at least one unbroken non-abelian and at least one unbroken electromagnetic interaction

# GUTs, anomalies and Charge Quantization

The observed charged quantization is excellent evidence for BSM physics.

Imagine we end up with a consistent theory of quantum gravity that imposes no constraints on QFT. Then this would allow particles with arbitrary real charges. It is hard to accept that we just happen to live in a universe with quantized charges.

One often hears the arguments that anomaly cancellation imposes charge quantization.

# GUTs, anomalies and Charge Quantization

… but this is not true.

One can add scalars or vector-like particles of arbitrary real charge.

It is true that one Standard Model family with the observed charges is the smallest non-trivial chiral anomaly-free representation of $SU(3)\times SU(2)\times U(1)$.

But the existence of three families and perhaps right-handed neutrinos ruins the argument.

# GUTs, anomalies and Charge Quantization

One can impose one-family charge quantisation on all three families by requiring that they all couple to the same Higgs.

One can also require the see-saw mechanism to work, to restrict the charges of right-handed neutrinos to zero.

But in QFT one can always add other charges, including chiral fermions with irrational charges (in SM units) that get their mass from the SM Higgs

$$(3,2,\frac{1}{6}-\frac{x}{3})+(\bar{3},1,-\frac{2}{3}+\frac{x}{3})+(\bar{3},1,\frac{1}{3}+\frac{x}{3})$$
$$+(1,2,-\frac{1}{2}+x)+(1,1,1-x)+(1,1,-x)$$

# GUTs, anomalies and Charge Quantization

We need some kind of BSM physics to explain charge quantization.

String theory is likely to quantize the charges
(although not necessarily in the right way)

If we already have string theory, do we also need GUTs?

# Towards a derivation of the Standard Model

**Main anthropic assumption:**

We are going to need electromagnetism and a handful of particles with various charges.

We are not asking for a particular quantization, and we are *not* requiring particles of charge 6 (Carbon) to exist, but too simple sets will not do (e.g. charges -1,1,2: just Hydrogen and Helium)

# Towards a derivation of the Standard Model

Pure QED with set of charges has some problems:
No fusion-fueled stars, no stellar nucleosynthesis, baryogenesis difficult, ….

But we focus on another problem, namely that there has to be a hierarchy between the Planck scale and the masses of the building blocks of life.

Maximal number of building blocks with mass $m_p$ of an object that does not collapse into a black hole

$$\left(\frac{m_{\mathrm{Planck}}}{m_p}\right)^3$$

Brain with $10^{27}$ building blocks requires a hierarchy of $10^{-9}$

# Towards a derivation of the Standard Model

So to get a substantial number of light atoms, we have to solve a hierarchy problem for each of the constituents.

In the Standard Model this is solved by getting the particle masses from a single Higgs.

There may be landscape distribution arguments to justify this.

Is having $N$ light fermions statistically more costly than having a single light boson?

Renormalization of scalar masses

$$\mu^2_{\text{phys}} = \mu^2_{\text{bare}} + \sum_i a_i \Lambda^2$$

Computable statistical cost of about $10^{-34}$ for the observed hierarchy. This is the “hierarchy problem”.

Renormalization of fermion masses

$$\lambda_{\text{phys}} = \lambda_{\text{bare}} \left( \sum_i b_i \log(\Lambda/Q) \right)$$

Statistical cost determined by landscape distribution of $\lambda_{\text{bare}}$

It is certainly possible that one fundamental scalar wins against $N$ fermions for moderate $N$ (even for $N \geq 3$).

One would also have to show that one fundamental scalar wins against dynamical Higgs mechanism or low energy supersymmetry.

Not enough is known theoretically to decide this, so we take experiment as our guiding principle.

Currently it seems we have a single Higgs + nothing.

This suggests that in a landscape the Higgs is not the *origin* but the *solution* of the Hierarchy problem: it could be the optimal way to create the anthropically required large hierarchy.

This would immediately imply that there is only a single Higgs.

# No Higgs?

Statistically, no Higgs is better than one.
If there is a credible alternative to the SM with only dynamical symmetry breaking, that would be a serious competitor.

But generically these theories will have a number of problems.

Consider the SM without a Higgs. It is well-known that in that case the QCD chiral condensate will act like a composite Higgs and give mass to the quarks. The photon survives as a massless particle.

But the leptons do not acquire a mass, and the quark masses are not tuneable.

Massless charged leptons turn the entire universe into an opaque particle-antiparticle plasma. *(C. Quigg, R. Shrock, Phys.Rev. D79 (2009) 096002)*

# String Theory Input

We would like to enumerate all QFT’s with a gauge group and chiral matter. Non-chiral matter is assumed to be heavy, with the exception of at most one scalar field, the Higgs. We demand that after the Higgs gets a vev, and all possible dynamical symmetry breakings have been taken into account, at least one massless photon survives, and all charged fermions are massive.

This condition is very restrictive, but still has an infinite number of solutions in QFT.

So at this point we invoke string theory. Its main rôle is to restrict the representations. It also provides a more fundamental rationale for anomaly cancellation.

# Intersecting Brane Models

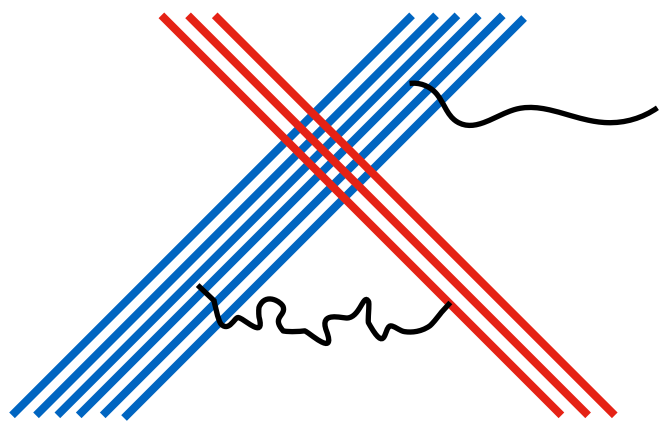
We will assume that all matter and the Higgs bosons are massless particles in intersecting brane models.

The low energy gauge group is assumed to come from $S$ stacks of branes. There can be additional branes that do not give rise to massless gauge bosons: $O(1)$ or $U(1)$ with a massive vector boson due to axion mixing.

We start with $S = 1$, and increase $S$ until we find a solution.

$S = 1$ is easy to rule out. So we go to $S = 2$.

# Two stack models



$$SU(M) \times SU(N) \times U(1)$$

$$Y = q_a Q_a + q_b Q_b$$

$q_a$, $q_b$ determined by axion couplings

| | |
|---|---|
| $Q$ | $(M, N, q_a + q_b)$ |
| $U$ | $(A, 1, 2q_a)$ |
| $D$ | $(\overline{M}, 1, -q_a)$ |
| $S$ | $(S, 1, 2q_a)$ |
| $X$ | $(M, \overline{N}, q_a - q_b)$ |
| $L$ | $(1, \overline{N}, -q_b)$ |
| $T$ | $(1, S, 2q_b)$ |
| $E$ | $(1, A, 2q_b)$ |

# Anomalies

$SU(M) \times SU(N) \times U(1)$

S W Y

There are six kinds of anomalies:

$$\left.\begin{array}{l}\text{SSS}\\ \text{WWW}\end{array}\right\}$$ From tadpole cancellation: also for $M, N < 3$

YYY

SSY

WWY

GGY Mixed gauge-gravity

At most one linear combination of the U(1)'s is anomaly-free

# Anomalies

$$\begin{aligned}
(S+U)\tilde{q}_a &= C_1 \\
(T+E)\tilde{q}_b &= -C_2 \\
(D+8U)\tilde{q}_a &= (4+M)C_1 + NC_2 \\
L\tilde{q}_b + D\tilde{q}_a &= 0 \\
2E\tilde{q}_b + 2U\tilde{q}_a &= C_1 - C_2
\end{aligned}$$

$$\tilde{q}_a \equiv Mq_a,\ \tilde{q}_b \equiv Nq_b$$

$$C_1 = -(Q-X)\tilde{q}_b$$

$$C_2 = (Q+X)\tilde{q}_a$$

Only five independent ones. In most cases of interest, the stringy $SU(2)^3$ anomaly is not an independent constraint

($q_a = 0$ and/or $q_b = 0$ must be treated separately; see paper)

# Abelian theories

Single $U(1)$: Higgs must break it, no electromagnetism left
$U(1)\times U(1)$: No solution to anomaly cancellation for two stacks

So in two-stack models we need at least one non-abelian factor in the high-energy theory.

# Strong Interactions

It is useful to have a non-abelian factor in the low-energy theory as well, since the elementary particle charge spectrum is otherwise too poor. We need some additional interaction to bind these particles into bound states with larger charges (hadrons and nuclei in our universe).

For this to work there has to be an approximately conserved baryon number.
This means that we need an $SU(M)$ factor with $M \geq 3$, and that this $SU(M)$ factor does not become part of a larger group at the “weak” scale.

Note that $SU(2)$ does not have baryon number, and the weak scale is near the constituent mass scale. We cannot allow baryon number to be broken at that scale.

But let’s just call this an additional assumption.

# Higgs Choice

This implies that at least one non-abelian factor is not broken by the Higgs. We take this factor to be $U(M)$.

Therefore we do not consider bi-fundamental Higgses breaking both $U(M)$ and $U(N)$. We assume that $U(N)$ is the broken gauge factor. Then the only Higgs choices are **L**,**T** and **E**.

# $SU(M)\times U(1)$

Higgs can only break $U(1)$, but then there is no electromagnetism.

Hence there will be a second non-abelian factor, broken by the Higgs.

# $M = 3, N = 2$

## **Higgs = L**

Decompose L, E, T: chiral charged leptons avoided only if

$$L = E, T = 0$$

Substitute in anomaly equation:

$$S\tilde{q}_a = \left(\frac{5 - N - M}{2M}\right) C_1$$

For $M = 3, N = 2$: $S = 0$

Therefore we get standard QCD without symmetric tensors.

# $M = 3,\ N = 2$

Quark sector

$$Q(3, q_a) + Q(3, q_a + 2q_b) + X(3, q_a) + X(3, q_a - 2q_b) - U(3, -2q_a) - D(3, q_a)$$

$Q + X - D = 0$

$Q = U$ **if and only if** $q_a + 2q_b = -2q_a$

*or*

$X = U$ **if and only if** $q_a - 2q_b = -2q_a$

In both cases we get an *SU(5)* type charge relation, and hence standard charge quantization

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# $M = 3, N = 2$

Hence either $Q = 0$ *or* $X = 0$; the choice is irrelevant.

Take $X = 0$.
Then $D = Q = U, T = 0, L = E$
Remaining anomaly conditions: $L = Q$

Hence the only solution is a standard model family, occurring $Q$ times.

The branes **a** and **b** are in principle unrelated, and can generally not be combined to a $U(5)$ stack

# $M = 3,\ N = 2$

## Higgs = T

The symmetric tensor can break $SU(2)\times U(1)$ in two ways, either to $U(1)$, in the same way as **L**, or to $SO(2)$.

### Breaking to $U(1)$ (same subgroup as **L**)

No allowed Higgs couplings to give mass to the charged components of L, E and T, so we must require $E = L = T = 0$. Then there is no solution.

### Breaking to $SO(2)$

Then $SO(2)$ must be electromagnetism. Y-charges forbid cubic T couplings, so $T = 0$ to avoid massless charged leptons. Quark charge pairing (to avoid chiral QED, broken by QCD) requires $Q = -X$. **If we also require $S = 0$, everything vanishes.**

Note: stronger dynamical assumption: $S = 0$

# $M > 3$ and/or $N > 2$

Unless $Q = -X$, we get quarks and anti-quarks coupling to $SU(N)$ representations that are not mutually conjugate. Hence dynamical symmetry breaking breaks $SU(N)$ completely.

If we also use the fields $D$ and $U$ (for $M = 3$) then $SU(N) \times U(1)$ contains a current that is non-chiral. It must be a linear combination

$$Q_{\mathrm{em}} = \Lambda + \Upsilon$$

Where $\Lambda \in SU(N)$. There can be at most one such $U(1)$ factor.

($Q = -X$: see paper)

# $M > 3$ and/or $N > 2$

Lepton charge pairing: $$-L + (N-1)E + (N+1)T = 0$$

Combined with the five anomaly constraints this gives the following solution

$$\begin{aligned}
U\tilde{q}_a &= \tfrac{3+M}{6}C_1 \\
S\tilde{q}_a &= \tfrac{3-M}{6}C_1 \\
D\tilde{q}_a &= NC_2 - \tfrac{M}{3}C_1 \\
L\tilde{q}_b &= -NC_2 + \tfrac{M}{3}C_1 \\
E\tilde{q}_b &= -\tfrac{1}{2}C_2 + \tfrac{M}{6}C_1 \\
T\tilde{q}_b &= -\tfrac{1}{2}C_2 - \tfrac{M}{6}C_1
\end{aligned}$$

$$C_1 = -(Q-X)\tilde{q}_b$$

$$C_2 = (Q+X)\tilde{q}_a$$

For $M = 3$, $S = 0$ automatically!

# $M > 3$ and/or $N > 2$

$$\Lambda = \mathrm{diag}(\lambda_1, \ldots, \lambda_N)$$

Charges of $Q$: $q_a + q_b + \lambda_i$

Charges of $X$: $q_a - q_b - \lambda_i$

Charges of $D$: $-q_a$

Charges of $U$,$S$: $2q_a$

Lepton Charges: $q_b + \lambda_i;\ 2q_b + \lambda_i + \lambda_j$

Define $q_b + \lambda_i = \alpha q_a$

Quark charge pairing is possible only for $\alpha = 0, \pm 3$

**All solutions satisfy Standard Model charge quantization!**

# $M > 3$ and/or $N > 2$

We can obtain a solution for any $Q$ and $X$

$$\Lambda : n \times \{-q_b\} + n_+ \times \{-q_b + 3q_a\} + n_- \times \{-q_b - 3q_a\}$$

$$n_+ = \frac{Q}{R}$$

$$n_- = -\frac{X}{R}$$

$$N = n + n_+ + n_-$$

$$R = -(Q + X)\frac{\tilde{q}_a}{\tilde{q}_b} \in \mathbb{Z}$$

The trace of $\Lambda$ must vanish

$$\mathrm{Tr}\ \Lambda = \tilde{q}_b \left( \frac{3}{M} - 1 \right)$$

Hence $M = 3$!

# $M > 3$ and/or $N > 2$

The spectrum can be computed

$$D = n(Q + X)$$

$$U = (N - n)(Q + X)$$

$$L = nR$$

$$E = \frac{1}{2}(N - n + 1)\,R$$

$$T = -\frac{1}{2}(N - n - 1)\,R$$

Absence of massless charged leptons only for $N = 2$!

# Conclusions

- The Standard Model is the only anthropic solution within the set of two-stack models.
- Family structure, charge quantization, the weak interactions and the Higgs choice are all derived.
- Standard Model charge quantization works the same way, for any value of N, even if $N+3 \neq 5$.
- The GUT extension offers no advantages, only problems (doublet-triplet splitting)
- Only if all couplings converge (requires susy), GUTs offer an advantage.
- The general class is like a GUT with its intestines removed, keeping only the good parts: **GUTs without guts**.

# Couplings

$$\frac{1}{\alpha_Y} = \frac{2}{3}\frac{1}{\alpha_s} + \frac{1}{\alpha_w}$$

see also:
*Ibañez, Munos, Rigolin, 1998;*
*Blumenhagen, Kors, Lüst, Stieberger, 2007*

This is satisfied at $5.7\times10^{13}$ GeV  ($1.4\times10^{16}$ GeV for susy)