Yukawas in F-theory GUTs (including E_8 point)

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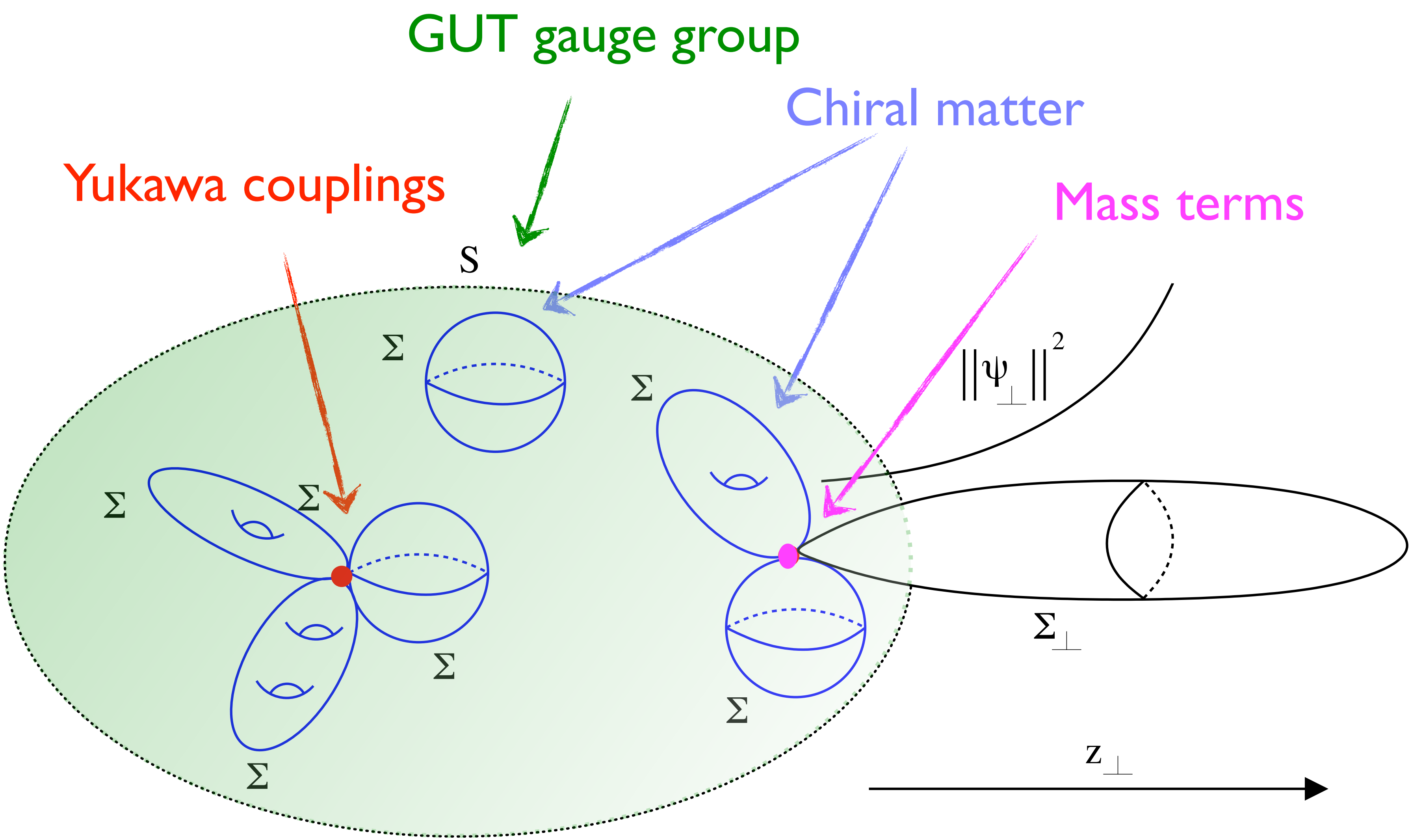
Based on:

Font, Ibáñez, F.M., Regalado [1211.6529]

Font, F.M., Regalado, Zoccarato [1307.8089]

Motivation: GUTs from F-theory

- F-theory GUT models have proven to be a rich and elegant avenue to realize realistic vacua in string theory



Taken from Beasley, Heckman, Vafa '08

Motivation: GUTs from F-theory

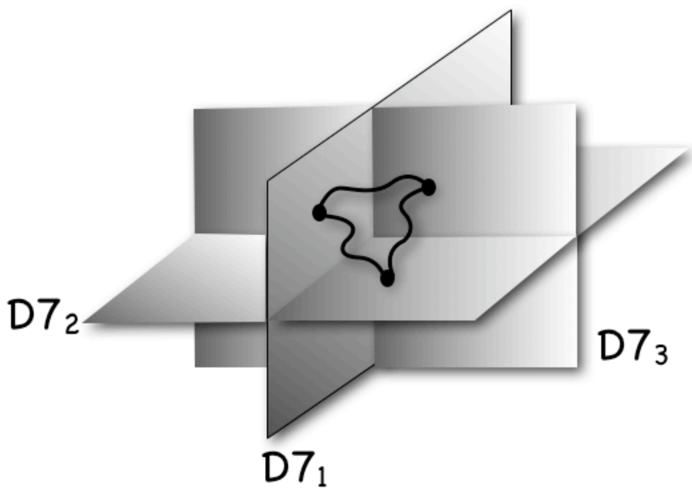
- F-theory GUT models have proven to be a rich and elegant avenue to realize realistic vacua in string theory
- With respect to heterotic strings, they allow to implement a bottom-up approach when constructing 4d vacua, and to analyze several features of the GUT gauge sector at a local level
- With respect to type II strings, they allow for certain couplings and representations that are otherwise forbidden at the perturbative level
  - Example: For type II SU(5) GUTs the Yukawa coupling 5x10x10 is forbidden at the perturbative level and needs to be generated by, e.g., D-instanton effects

F-theory Yukawas

- Despite their differences, one can easily gain intuition in understanding F-theory in terms of their type IIB and heterotic cousins

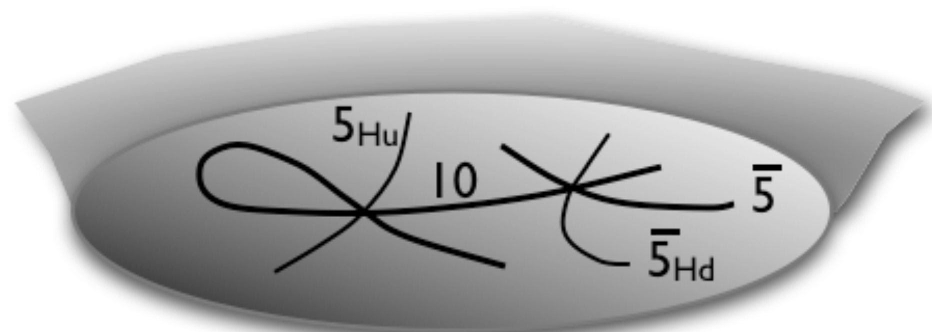
- Just like in type IIB, Yukawa couplings arise from the triple intersection of 4-cycles in a 6d manifold

    - Type IIB:



$$Y = \frac{(S+S^*)^{1/4}}{[(T_1+T_1^*)(T_2+T_2^*)(T_3+T_3^*)]^{1/4}}$$

    - F-theory:



Figures taken from Ibañez & Uranga (2012)

F-theory Yukawas

- Despite their differences, one can easily gain intuition in understanding F-theory in terms of their type IIB and heterotic cousins

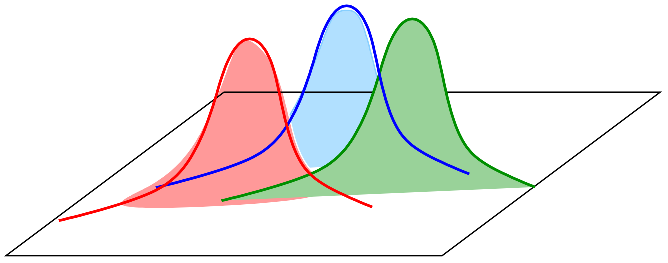
- Like for heterotic strings in CYs, one may compute Yukawas from dim. red. of a higher dimensional field theory

Beasley, Heckman, Vafa '08

Heterotic	F-theory
10d SYM	8d tw. YM
$W = \int_X \Omega \wedge \mathrm{Tr}\,(A \wedge F)$	$W = \int_S \mathrm{Tr}\,(F \wedge \Phi)$
$G_X = E_8 \times E_8,\ SO(32)$	$G_S = SO(2N), E_6, E_7, E_8 \ldots$

- Computation of zero mode wavefunctions in a certain background

- Yukawas = triple overlap of wavefunctions



F-theory Yukawas

- In practice, to compute Yukawa couplings one considers a divisor S and a gauge group G_S = SO(12), E_6, E_7, E_8... on it

  - $\langle \Phi \rangle \neq 0$ describes the intersection pattern near the Yukawa point and breaks $G_S \to G_{GUT} \times U(1)^N$

  - $\langle F \rangle \neq 0$ necessary to generate chirality and family replication at the intersection curves

  - $\langle F_Y \rangle \neq 0$ necessary to break $G_{GUT} \to G_{MSSM}$

Example: SU(5) $\begin{matrix} 5_{H_u} \times 10 \times 10 \\ \bar{5}_{H_d} \times \bar{5} \times 10 \end{matrix} \xrightarrow{F_Y} \begin{matrix} \lambda_u^{ij} Q^i U^j H_u \\ \lambda_d^{ij} Q^i D^j H_d + \lambda_l^{ij} L^i E^j H_d \end{matrix}$

The presence of $\langle F \rangle$ also localizes the wavefunctions and allows for an ultra-local computation of Yukawa couplings

Computing wavefunctions

- The superpotential and D-term encode the 7-brane BPS equations

$$
\begin{aligned}
W &= \int_S \mathrm{Tr}(F \wedge \Phi) \\
D &= \int_S F \wedge \omega + \frac{1}{2}[\Phi, \bar{\Phi}]
\end{aligned}
\longrightarrow
\begin{aligned}
F^{(2,0)} &= 0 \\
\bar{\partial}_A \Phi &= 0 \\
\omega \wedge F &= 0
\end{aligned}
$$

- Which in turn encode the zero mode eom:

$$
\begin{aligned}
\Phi &= \langle \Phi \rangle + \varphi_{xy} dx \wedge dy \\
A &= \langle A \rangle + a_{\bar{x}} d\bar{x} + a_{\bar{y}} d\bar{y}
\end{aligned}
\longrightarrow
D_A \Psi = 0
$$

$$
\mathbf{D_A} = \begin{pmatrix} 0 & D_x & D_y & D_z \\ -D_x & 0 & -D_{\bar{z}} & D_{\bar{y}} \\ -D_y & D_{\bar{z}} & 0 & -D_{\bar{x}} \\ -D_z & -D_{\bar{y}} & D_{\bar{x}} & 0 \end{pmatrix} \qquad \Psi = \begin{pmatrix} 0 \\ a_{\bar{x}} \\ a_{\bar{y}} \\ \varphi_{xy} \end{pmatrix}
$$

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- Which in turn encode the zero mode eom:

$$\Phi = \langle \Phi \rangle + \varphi_{xy} dx \wedge dy$$
$$A = \langle A \rangle + a_{\bar{x}} d\bar{x} + a_{\bar{y}} d\bar{y}$$

$$\longrightarrow \quad D_A \Psi = 0$$

Example: $\langle \Phi \rangle$ and $\langle A \rangle$ linear

Solution: $\Psi_a = J_a \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \psi_a \mathfrak{t}_a, \qquad \psi_a = e^{\lambda_a |x|^2} f_a(y)$

λ_a depends on $\langle \Phi \rangle$ and $\langle A \rangle$

Computing Yukawas

- Inserting these wavefunctions in W we obtain the Yukawa couplings in terms of a triple overlap of wavefunctions

Heckman & Vafa '08
Font & Ibáñez '09
Conlon & Palti '09

$$\int_S \mathrm{Tr}(A \wedge A \wedge \Phi) \longrightarrow Y^{ij} = \mathcal{N}_\lambda f_{abc} \int_S d\mu \, f_a^i \, g_b^j \, h_c$$

$$\mathcal{N}_\lambda = \lambda_a \lambda_b + \lambda_c(\lambda_a + \lambda_b)$$

$$d\mu = d^2x d^2y \, e^{\lambda_a |x|^2 + \lambda_b |y|^2 + \lambda_c |x-y|^2}$$

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U(1) symmetry: $(x, y) \to e^{i\alpha}(x, y)$, only invariant integrands survive:

$f_a^i = x^{3-i} \quad g_b^j = y^{3-j} \quad h_c = 1 \Rightarrow$ only $Y^{33} \neq 0$ ⇒ Yukawas of rank one

Moreover $\int_S d\mu = \pi^2 \mathcal{N}_\lambda^{-1}$ ⇒ Y^{ij} indep. of λ ⇒ indep. of F

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Moreover $\int_S d\mu = \pi^2 \mathcal{N}_\lambda^{-1}$ ⇒ Y^{ij} indep. of λ ⇒ indep. of F

- The same is true for general fluxes ⇒

Rank one Yukawa problem

Cecotti, Cheng, Heckman, Vafa '09

Deforming the superpotential

- A possible way out is to consider a non-commutative deformation of the 7-brane superpotential

Cecotti, Cheng, Heckman, Vafa '09

$$\hat{W}_7 = \int_S \mathrm{Tr}\,(\hat{\Phi} \circledast \hat{F})$$

Non-comm parameter ϵ θ,
θ holomorphic function

Such deformations typically arise for D-branes in β-deformed backgrounds

Kapustin '03
Pestun '06

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Kapustin '03
Pestun '06

- Results:
  - Rank higher than one
  - Holom Y^{ij} can be computed via a residue formula. Depend on coeff. of θ but independent of fluxes
  - Pattern

$$\frac{Y^{\mathrm{hol}}}{Y^{\mathrm{hol}}_{33}} = \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} + \dots$$

Deforming the superpotential

- This nc deformation is however subtle for the groups of interest in F-theory GUTs

- A simple way to realize this is to write down the commutative version of the above deformation

$$\hat{W}_7 = \int_S \mathrm{Tr}\,(\hat{\Phi} \circledast \hat{F})$$

SW map

$$\begin{aligned} \hat{A}_{\bar{m}} &= A_{\bar{m}} - \frac{\epsilon}{2}\theta^{ij}\{A_i, \partial_j A_{\bar{m}} + F_{j\bar{m}}\} + \mathcal{O}(\epsilon^2) \\ \hat{\Phi}_{xy} &= \Phi_{xy} - \frac{\epsilon}{2}\{A_i, (\partial_j + D_j)(\theta^{ij}\Phi_{xy})\} + \mathcal{O}(\epsilon^2) \end{aligned}$$

$$\Big\downarrow$$

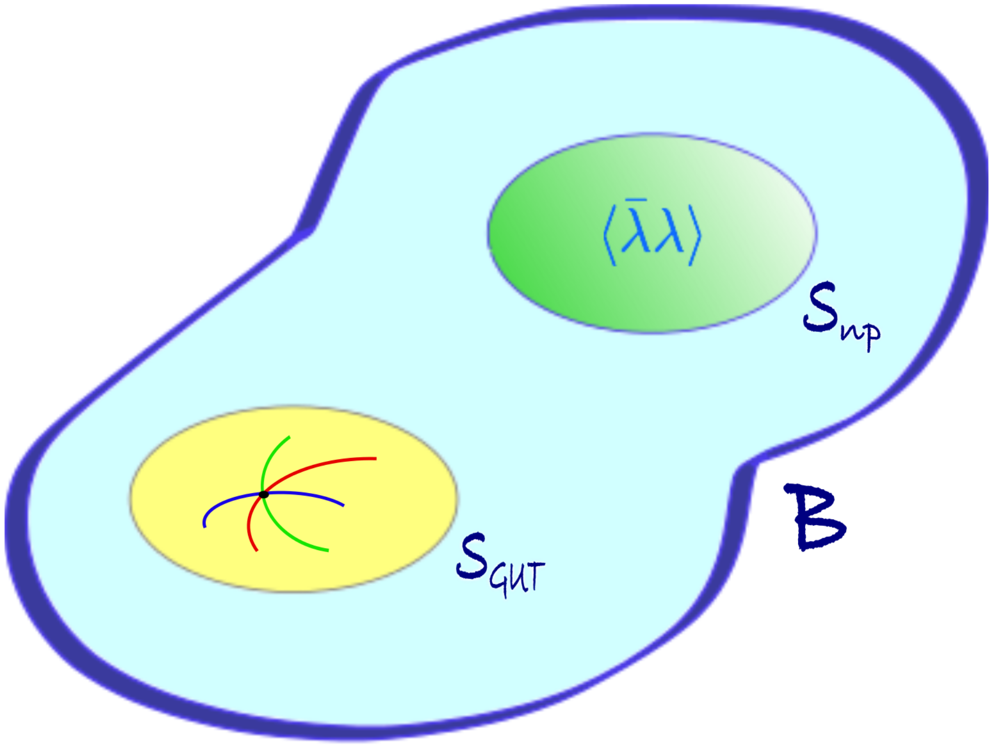
$$W_7 = \int_S \mathrm{Tr}(F \wedge \Phi) + \frac{\epsilon}{2}\int_S \theta\,\mathrm{Tr}\left(\Phi_{xy}F^2\right)$$

F.M. & Martucci '10

- The deformation is proportional to d_{abc}= STr ($\mathbf{t_a t_b t_c}$), which vanishes for G_S = SO(12), E_6, E_7, E_8

Yukawas from non-perturbative effects

- This commutative version of the deformed superpotential admits a simple physical interpretation in terms of non-perturbative effects



F.M. & Martucci'10

➡ D3-instantons generate non-perturbative superpotentials for D3-branes and magnetized D7-branes

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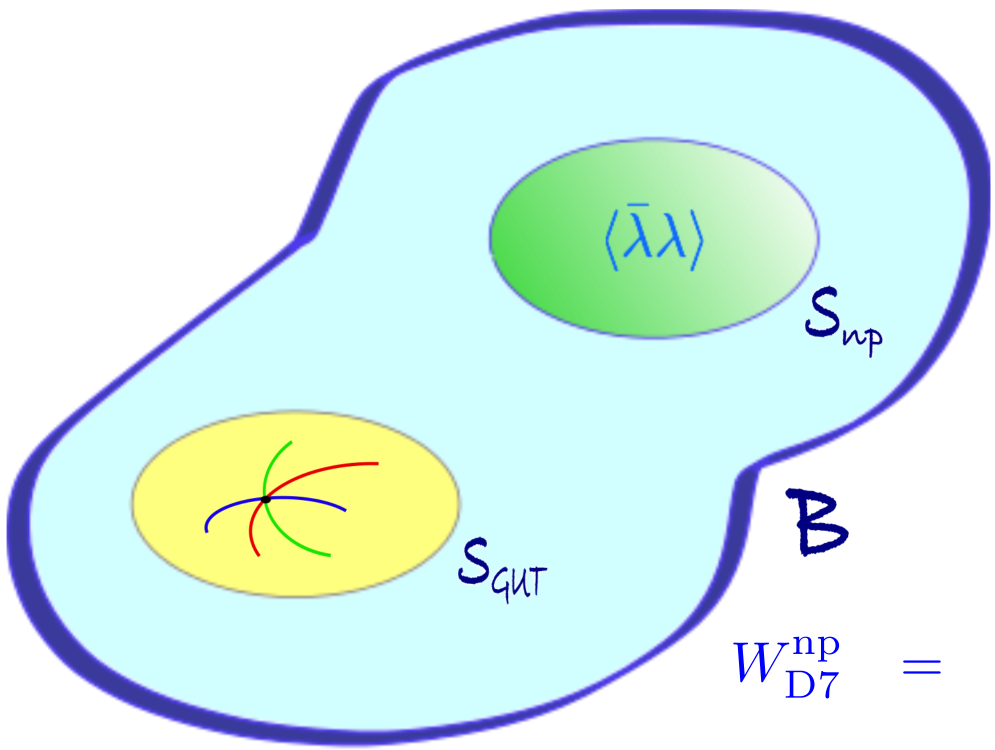
F.M. & Martucci '10



- ➡ D3-instantons generate non-perturbative superpotentials for D3-branes and magnetized D7-branes

$$W_{\rm D7}^{\rm np} \quad = \quad \mu^3 \mathcal{A}\, e^{-T_\Sigma} \exp\left[\frac{1}{8\pi^2}\int_S \mathrm{STr}(\log\, h\, F\wedge F)\right]$$

h = instanton divisor function $\qquad S_{\rm np} = \{h(X) = 0\}$

Yukawas from non-perturbative effects

- h must be Taylor-expanded on the positions field $\Phi_{xy} = z/2\pi\alpha'$, just as in the non-Abelian DBI action

$$W^{\rm np} \quad = \quad m_*^4 \epsilon \left(1 + \int_S {\rm STr}(\log \tilde{h}\, F \wedge F) + \dots \right)$$

$$\epsilon = \mathcal{A}\, e^{-T_{\rm np}} h_0^{N_{\rm D3}} \qquad \tilde{h} = h/h_0$$

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$$\begin{aligned} \log \tilde{h} \quad &= \quad \log \tilde{h}|_S + \Phi_{xy}[\mathcal{L}_z \log \tilde{h}]_S + \Phi_{xy}^2[\mathcal{L}_z^2 \log \tilde{h}]_S + \ldots \\ &= \quad \theta_0 + \theta_1 \Phi_{xy} + \theta_2 \Phi_{xy}^2 + \ldots \end{aligned}$$

$$\downarrow$$

$$W^{\rm np} \quad = \quad m_*^4 \epsilon \left[\int_S \theta_0 {\rm Tr} F^2 + \int_S \theta_1 {\rm Tr}(\Phi_{xy} F^2) + \int_S \theta_2 {\rm STr}(\Phi_{xy}^2 F^2) + \ldots \right]$$

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$h|_S$ const.

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$$W^{\rm np} \quad = \quad m_*^4 \epsilon \left[\int_S \cancel{\theta_0 \mathrm{Tr}} F^2 + \boxed{\int_S \theta_1 \mathrm{Tr}(\Phi_{xy} F^2)} + \int_S \theta_2 \mathrm{STr}(\Phi_{xy}^2 F^2) + \ldots \right]$$

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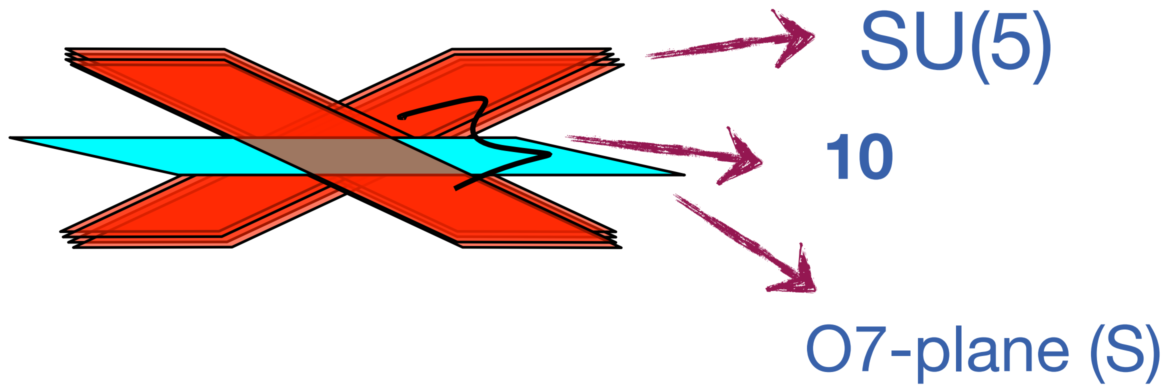
Yukawas in GUTs

- The assumption $\theta_0 = 0$ turns out to be too restrictive

- Example: SO(12) model in type IIB



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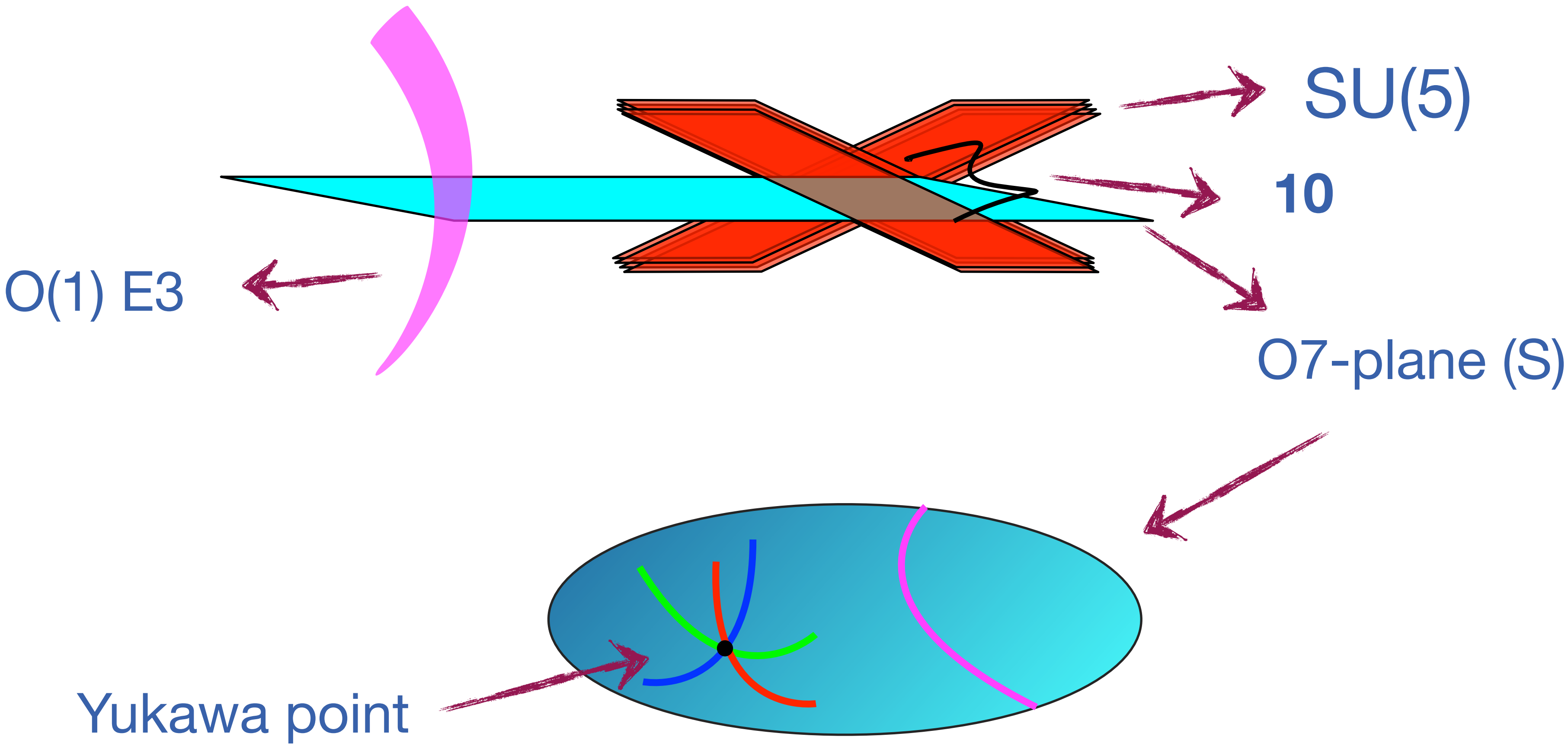


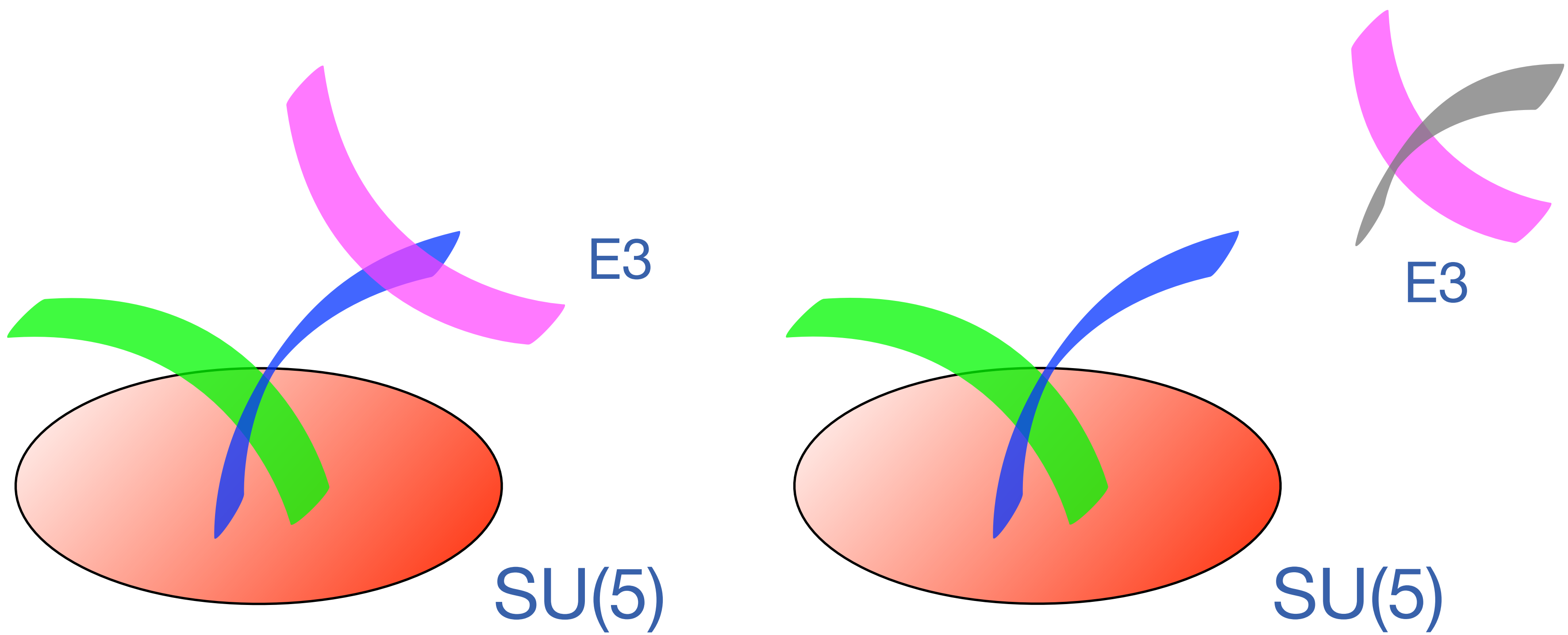
Yukawas in GUTs

- The assumption $\theta_0 = 0$ turns out to be too restrictive
- F-theory perspective: an E3-instanton with the right number of zero modes must intersect one 7-brane

Bianchi, Collinucci, Martucci'11

Cvetic, Garcia-Etxebarria, Halverson'11

Two possible scenarios:



Yukawas in SO(12)

- In the first scenario $\theta_0 \neq 0$, and the full superpotential is

$$W_{\rm total} = m_*^4 \left[\int_S \mathrm{Tr}(\Phi_{xy} F) \wedge dx \wedge dy + \frac{\epsilon}{2} \int_S \theta_0 \, \mathrm{Tr}\left(F \wedge F\right) + \theta_2 \mathrm{STr}\left(\Phi_{xy}^2 F \wedge F\right) \right]$$

  - No obvious non-commutative interpretation
  - We can still solve for the wavefunctions and compute the Yukawas, using a residue formula to identify the holomorphic part

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  - No obvious non-commutative interpretation
  - We can still solve for the wavefunctions and compute the Yukawas, using a residue formula to identify the holomorphic part
  - Result for SO(12) point, with $\theta_0 = i(\theta_{00} + x\,\theta_{0x} + y\,\theta_{0y})$, θ_2 const.

$$\frac{Y^{\text{hol}}}{Y^{\text{hol}}_{33}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \epsilon \begin{pmatrix} 0 & 0 & \theta_{0x} \\ 0 & \theta_{0x} + \theta_{0y} & \theta_2 \\ \theta_{0y} & -\theta_2 & 0 \end{pmatrix} + \mathcal{O}(\epsilon^2)$$

hierarchy $(1, \epsilon, \epsilon^2)$ of eigenvalues, still independent of worldvolume fluxes

Yukawas in SO(12)

- The hypercharge flux F_Y is the only GUT → MSSM gauge group breaking effect. This means that at the holomorphic level

$$Y_L^{ij} = Y_{D_R}^{ij}$$

- If that was the final answer it would imply

$$\frac{m_\mu}{m_\tau} = \frac{m_s}{m_b}\,, \quad \frac{m_e}{m_\tau} = \frac{m_d}{m_b} \qquad \text{vs.} \qquad \frac{m_\mu}{m_\tau} \simeq 3\,\frac{m_s}{m_b}\,, \quad \frac{m_e}{m_\tau} \simeq \frac{1}{3}\frac{m_d}{m_b}$$

Georgi & Jarlskog '79

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$$Y_L^{ij} \;=\; Y_{D_R}^{ij}$$

- However, the physical Yukawas depend on F_Y via wavefunction normalization

$$Y_{phys}^{ij} \;=\; K_i^{-1/2} K_j^{-1/2} K_H^{-1/2}\, Y_{hol}^{ij}$$

$$K_i \;=\; \int |\psi|^2 \;\propto\; \int_0^\infty dy\, e^{-\pi |M||y|^2}\; |f^i(y)|^2$$

- These normalization factors depend on the family and on the flux M

$$K_i^{-1/2} \;\propto\; \left(\frac{\pi}{\sqrt{2}}|M|, \sqrt{\pi}|M|^{1/2}, 1\right) \qquad\qquad M \;=\; N + q_Y N_Y$$

- For higher hypercharge we have thinner wavefunctions and larger quotients. One can then accommodate a realistic GUT scale mass ratio

$$\frac{m_\mu}{m_\tau} \simeq 3\,\frac{m_s}{m_b} \qquad \text{for} \qquad \frac{N_Y}{N} \simeq 1.8$$

Yukawas in SO(12)

- One can in general accommodate the GUT scale masses

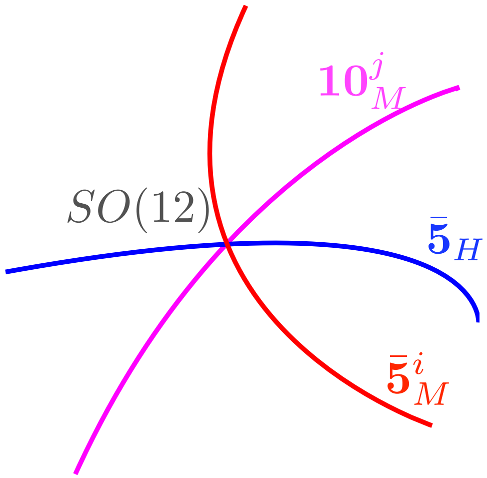
$\tan\beta$	10	38	50
m_d/m_s	$5.1 \pm 0.7 \times 10^{-2}$	$5.1 \pm 0.7 \times 10^{-2}$	$5.1 \pm 0.7 \times 10^{-2}$
m_s/m_b	$1.9 \pm 0.2 \times 10^{-2}$	$1.7 \pm 0.2 \times 10^{-2}$	$1.6 \pm 0.2 \times 10^{-2}$
m_e/m_μ	$4.8 \pm 0.2 \times 10^{-3}$	$4.8 \pm 0.2 \times 10^{-3}$	$4.8 \pm 0.2 \times 10^{-3}$
m_μ/m_τ	$5.9 \pm 0.2 \times 10^{-2}$	$5.4 \pm 0.2 \times 10^{-2}$	$5.0 \pm 0.2 \times 10^{-2}$
m_b/m_τ	0.73 ± 0.03	0.73 ± 0.03	0.73 ± 0.04
Y_τ	0.070 ± 0.003	0.32 ± 0.02	0.51 ± 0.04
Y_b	0.051 ± 0.002	0.23 ± 0.01	0.37 ± 0.02
Y_t	0.48 ± 0.02	0.49 ± 0.02	0.51 ± 0.04

for large tan β and $\epsilon \sim 10^{-3} - 10^{-4}$

T-branes and Up-type Yukawas

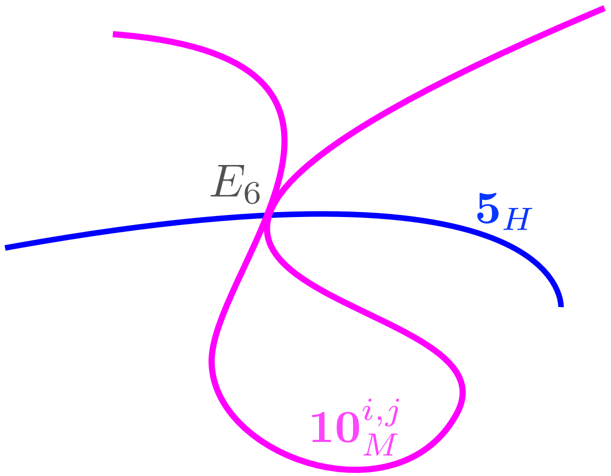
Down-type

$$Y_D^{ij}: \bar{\mathbf{5}}_H\ \bar{\mathbf{5}}_M^i\ \mathbf{10}_M^j$$



Up-type

$$Y_U^{ij}: \mathbf{5}_H\ \mathbf{10}_M^i\ \mathbf{10}_M^j$$



T-branes and Up-type Yukawas

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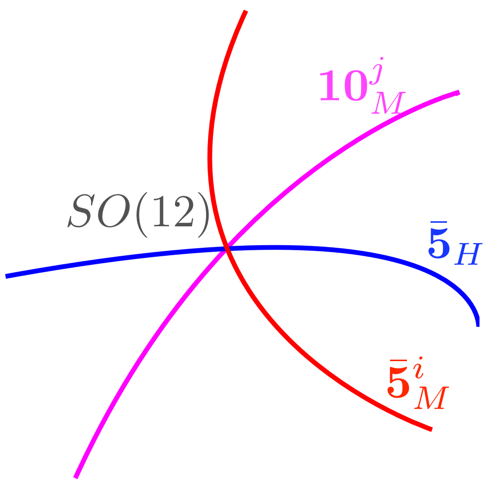


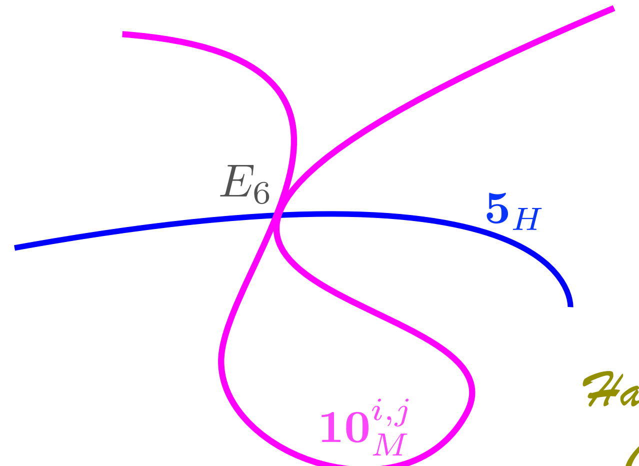
Intersecting branes, $[\langle\Phi\rangle, \langle\bar{\Phi}\rangle] = 0$

$$\langle\Phi\rangle \sim \begin{pmatrix} -x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & x-y \end{pmatrix} dx \wedge dy$$

$$\omega \wedge F = 0$$

Up-type

$$Y_U^{ij} : \mathbf{5}_H \, \mathbf{10}_M^i \, \mathbf{10}_M^j$$



Hayashi et al. '09

Cecotti et al. '10

T-branes, $[\langle\Phi\rangle, \langle\bar{\Phi}\rangle] \neq 0$

$$\langle\Phi\rangle \sim \begin{pmatrix} 0 & 1 & 0 \\ x & 0 & 0 \\ 0 & 0 & y \end{pmatrix} dx \wedge dy$$

$$\omega \wedge F + \frac{1}{2}[\Phi, \bar{\Phi}] = 0$$

Yukawas in E_6

- We now have the breaking

$$E_6 \xrightarrow{\langle \Phi \rangle} SU(5) \xrightarrow{\langle F_Y \rangle} SU(3) \times SU(2) \times U(1)$$

where Φ lives in $\mathfrak{su}(5) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1) \subset \mathfrak{e}_6$

$$78 \to (24,1)_0 \oplus (1,3)_0 \oplus (1,1)_0 \oplus (10,2)_{-1} \oplus (\overline{10},2)_1 \oplus (5,1)_2 \oplus (\overline{5},1)_{-2}$$

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$$\langle\Phi_{xy}\rangle = m(e^f E^+ + mxe^{-f}E^-) + \mu^2(ax+by)Q, \qquad \langle A_{0,1}\rangle = -\frac{i}{2}\bar{\partial}f P$$

$$f = \log c + m^2c^2r^2 + \dots$$

Matter curves:

$$\Sigma_5 = \{ax+by=0\}$$

$$\Sigma_{10} = \{m^3x - \mu^4(ax+by)^2 = 0\}$$

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- Using the superpotential

$$W_{\text{total}} = m_*^4 \left[\int_S \mathrm{Tr}(\Phi_{xy} F) \wedge dx \wedge dy + \frac{\epsilon}{2} \int_S \theta_0 \, \mathrm{Tr}\,(F \wedge F) \right]$$

with $\theta_0 = i(\theta_{00} + x\,\theta_{0x} + y\,\theta_{0y})$ we obtain

$$\frac{Y^{\text{hol}}}{Y^{\text{hol}}_{33}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \tilde{\epsilon} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} + \mathcal{O}(\tilde{\epsilon}^2)$$

$$\tilde{\epsilon} = \epsilon\,(a\theta_y - b\theta_x)$$

Yukawas in E_6

- The physical Yukawas read

$$Y^{ij} = \frac{\pi^2\gamma_5}{4\rho_\mu\rho_m}\begin{pmatrix} 0 & 0 & \tilde{\epsilon}\rho_\mu^{-1}\gamma_{10}^1\gamma_{10}^3 \\ 0 & \tilde{\epsilon}\rho_\mu^{-1}\gamma_{10}^2\gamma_{10}^2 & 0 \\ \tilde{\epsilon}\rho_\mu^{-1}\gamma_{10}^1\gamma_{10}^3 & 0 & -2\gamma_{10}^3\gamma_{10}^3 \end{pmatrix} + \mathcal{O}(\epsilon^2)$$

where

$$\tilde{\epsilon} = \epsilon\,(a\theta_y - b\theta_x) \qquad \rho_m = \frac{m^2}{m_*^2} \qquad \rho_\mu = \frac{\mu^2}{m_*^2}$$

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- For instance for the values

$$M = 0.3\,, \quad N = 0.03\,, \quad \tilde{N}_Y = 0.6\,, \quad N_Y = -0.18\,, \quad m = 0.5\,, \quad \mu = 0.1$$

we obtain that $Y_t \sim 0.5$. A realistic value for Y_c is obtained by taking

$$\tilde{\epsilon} \sim 10^{-4}$$

Beyond E_6

- Realistic values for up and down-type Yukawas are obtained with similar flux densities and n.p. parameter ϵ
- One may then consider models where both type of Yukawas are generated at the same point, an scenario that is independently motivated by a hierarchical CKM matrix
- Possible enhancements:
  - E_7
  - E_8

Beyond E_6

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- One may then consider models where both type of Yukawas are generated at the same point, an scenario that is independently motivated by a hierarchical CKM matrix

- Possible enhancements:

  - E_7 $\longrightarrow$ Vanishing Yukawas in SU(5) T-brane models

    Chiou, Faraggi, Tatar, Walters '11

  - E_8 $\longrightarrow$ Motivated by neutrino sector and local computability

    Heckman, Tavanfar, Vafa '09

    Palti '12

Yukawas in E_8: an example

- The details of the computation will depend on the spectral cover splitting, which gives rise to different kinds of models

$$E_8 \to SU(5)_{GUT} \times SU(5)_\perp$$

- Let us for instance take the E_8 T-brane model of Cecotti et al.

$$\Phi \sim \begin{pmatrix} \lambda_1 & 1 & 0 & 0 & 0 \\ x & \lambda_1 & 0 & 0 & 0 \\ 0 & 0 & -2\lambda_1 - \lambda_2 & 1 & 0 \\ 0 & 0 & y & -2\lambda_1 - \lambda_2 & 0 \\ 0 & 0 & 0 & 0 & 2(\lambda_1 + \lambda_2) \end{pmatrix}$$

Cecotti, Cordova, Heckman, Vafa '10

Yukawas in E8: an example

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  - Up-type sector: same like in E6, same conditions for large top Yukawa

$$\lambda_{ij}^{(u)} \sim \begin{pmatrix} 0 & 0 & \mathcal{O}(\epsilon) \\ 0 & \mathcal{O}(\epsilon) & 0 \\ \mathcal{O}(\epsilon) & 0 & 1 \end{pmatrix} + \mathcal{O}(\epsilon^2)$$

  - Down-like sector: more complicated than SO(12), but same hierarchy

$$\lambda_{ij}^{(d)} \sim \begin{pmatrix} 0 & 0 & \mathcal{O}(\epsilon) \\ 0 & \mathcal{O}(\epsilon) & \mathcal{O}(\epsilon) \\ \mathcal{O}(\epsilon) & \mathcal{O}(\epsilon) & 1 \end{pmatrix} + \mathcal{O}(\epsilon^2)$$

Conclusions

- Simplest F-theory GUTs have rank one Yukawas at tree-level
- Non-perturbative effects change this result, in the sense that they correct the superpotential of seven-branes
- We can have a explicit and simple expression for this correction, which allows to compute its effects at a local level
- In simple cases one may express the new superpotential as a non-commutative deformation of the previous superpotential. However, this is not true for the cases of interest in F-theory GUTs.
- The np effect provides rank 3, flux-indep holomorphic Yukawas. The hierarchy of eigenvalues is $\mathcal{O}(1), \mathcal{O}(\epsilon), \mathcal{O}(\epsilon^2)$
- The flux dependence comes from wavefunction normalization. This in principle allows to accommodate a large top Yukawa and realistic MSSM mass ratios via F_Y GUT breaking, more naturally than in 4d GUTs