SU(3) structures and heterotic domain wall solutions

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Motivation and summary

Why string compactifications?

Physics: string models with good phenomenology (particle physics and cosmology)

- Calabi–Yau 3-folds: good 4D physics models, but with moduli.
- Background fluxes can stabilise moduli.
- Fluxes deform geometry $\Longrightarrow$ SU(3) structure instead of SU(3) holonomy.

Math: probe (non-complex, non-Kähler) compact geometry.

This talk: Heterotic compactifications on SU(3) structure manifolds

- Properties of heterotic 4D $\mathcal{N} = 1/2$ domain wall solutions.
- Flow of SU(3) structures and moduli spaces.

Outline

1. Motivation and summary
2. Heterotic supersymmetric vacua
3. SU(3) structure
4. 4D Heterotic $\mathcal{N} = 1$ Minkowski solutions
5. 4D Heterotic $\mathcal{N} = \frac{1}{2}$ domain wall solutions
   - G_2 perspective
   - SU(3) perspective
   - Examples
6. Conclusions

Heterotic supersymmetric vacua

$\mathcal{N} = 1$ Heterotic supergravity

- Bosonic fields: Metric G, B-field B, dilaton ϕ, gauge field A
- Fermionic fields: Gravitino Ψ_M, dilatino λ, gaugino χ
- Bosonic action:

$$S = \frac{1}{2\alpha'} \int \mathrm{d}^{10} x \, e^{-2\phi} \sqrt{|G|} \left(R + 4(\partial\phi)^2 - \frac{1}{12} H^2 + \alpha'(\ldots) \right)$$

where $H = \mathrm{d}B + \alpha'(\ldots)$.

- At lowest order in α': only NSNS fields, Bianchi identity $\mathrm{d}H = 0$

SUSY at $\mathcal{O}(\alpha'^0)$ $\Longleftrightarrow$

$$\left(\nabla_M + \frac{1}{8} H_M \right) \epsilon = 0 \ , \quad \left(\not{\nabla} \hat{\phi} + \frac{1}{12} \not{H} \right) \epsilon = 0$$

where $\not{\nabla} = \Gamma^M \nabla_M$, $\not{H} = \Gamma^{MNP} H_{MNP}$, $H_M = \Gamma^{NP} H_{MNP}$

Heterotic supersymmetric vacua

Compactifications

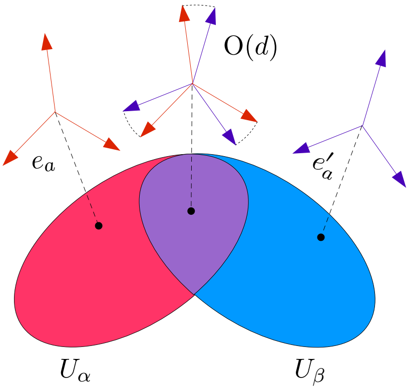
- $\mathcal{M}_{10} = \mathcal{M}_E \times X$
- SUSY

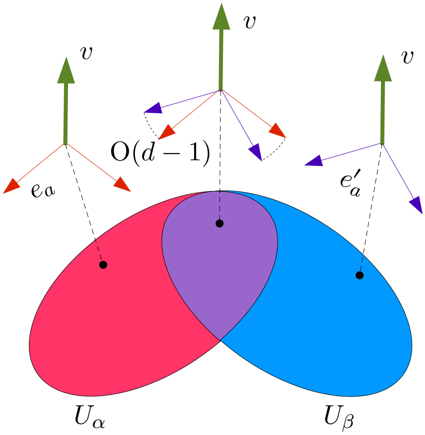
$$\left(\nabla_M + \frac{1}{8} H_M\right)\epsilon = 0 \ , \ \left(\nabla\!\!\!/ \hat{\phi} + \frac{1}{12} H\!\!\!/\right)\epsilon = 0$$

$\iff$ nowhere vanishing spinor η on X: $\epsilon = \rho_E \otimes \eta$
$\iff$ X has reduced structure group

Hitchin:02, Gualtieri:04, Grana et al:05, ...

SU(3) structure





Koerber:10

SU(3) structure

$\mathcal{M}_6$ orientable with metric: $G = \mathrm{SO}(6) \subset \mathrm{GL}(6)$.
$\mathcal{M}_6$ spinnable: $\mathrm{SO}(6)$ lifts to $\mathrm{Spin}(6) \cong \mathrm{SU}(4)$.

Let η Weyl, positive chirality: $\eta \in \mathbf{4}$ of SU(4). Choose basis:

$$\eta = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \eta_0 \end{pmatrix} \text{ invariant under } \begin{pmatrix} U & \mathbf{0_{3\times 1}} \\ \mathbf{0_{1\times 3}} & 1 \end{pmatrix}, \; U \in \mathrm{SU}(3)$$

$$\boxed{\text{Globally defined } \eta \implies \mathrm{G} = \mathrm{SU}(3).}$$

SU(3) structure

Gray–Hervalla:80, Chiossi–Salamon:02

Nowhere vanishing spinor η on 6-manifold $X \iff X$ has SU(3) structure

$\eta \iff$ complex decomposable $(3,0)$-form Ψ and real $(1,1)$ form ω such that

$$\omega \wedge \Psi = 0 \ , \qquad \omega \wedge \omega \wedge \omega \sim \Psi \wedge \bar{\Psi}$$

- ω, Ψ closed $\iff X$ is Calabi–Yau.
- Otherwise: non-zero torsion

$$\mathrm{d}\omega = -\frac{12}{||\Psi||^2} \ \mathrm{Im}(W_0 \, \overline{\Psi}) + W_1^\omega \wedge \omega + W_3 \ ,$$
$$\mathrm{d}\Psi = W_0 \ \omega \wedge \omega + W_2 \wedge \omega + \overline{W}_1^\Psi \wedge \Psi \ .$$

4D Heterotic $\mathcal{N} = 1$ Minkowski solutions

No flux: Calabi–Yau

Candelas, Horowitz, Strominger, Witten:85

SUSY variations, $H = 0$ $\Rightarrow$ covariantly constant spinor η on X: $\nabla\eta = 0$

$\Longleftrightarrow$ holonomy group of X restricted to $SU(3)$.

$\Longleftrightarrow$ X is Calabi–Yau.

With flux: Strominger system

Strominger:86, Hull:86

SUSY variations, $H \neq 0$ * $\Rightarrow$ globally defined spinor η on X: $\nabla_T\eta = 0$

$\Longleftrightarrow$ structure group of X restricted to $SU(3)$.

$\Longleftrightarrow$ X is complex and conformally balanced:

$$\mathrm{d}(e^{-2\phi}\omega \wedge \omega) = \mathrm{d}(e^{-2\phi}\Psi) = 0$$
$$W_0 = W_2 = 0,\ W_1^{\Psi} = 2W_1^{\omega} = 2\mathrm{d}\phi\ .$$

* Need α' corrections to avoid no-go theorem for flux if X is compact without boundary

Ivanov, Papadopoulos:00; Gauntlett, Martelli, Waldram:03

Other SU(3) structure compactifications

SU(3) structure $\iff$ nowhere vanishing spinor η
$\Rightarrow$ compactification to 4D $\mathcal{N} = 1$ effective theory

- 4D $\mathcal{N} = 1$ vacua: fluxless Calabi–Yau or Strominger
- SUSY-breaking vacua: more flux and torsion classes allowed.

Remark: such SU(3) structures are also relevant for type II compactifications

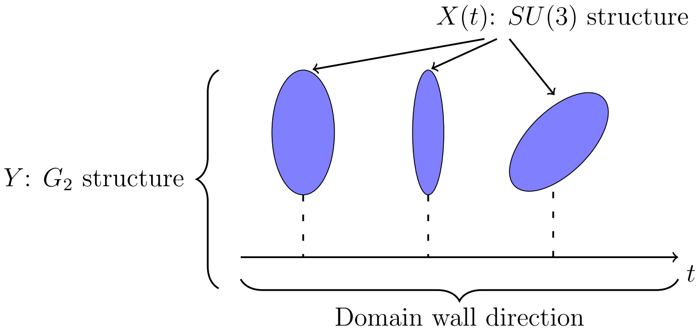
4D domain wall vacuum: $\mathcal{M}_{10} = \mathcal{M}_4 \times_W X(t) \equiv \mathcal{M}_3 \times Y$
$\mathcal{M}_4 = \mathcal{M}_3 \times \mathbb{R}$, $\mathcal{M}_3$ max. symmetric

Will see: several types of solutions, with different torsion classes.

4D Heterotic $\mathcal{N} = \frac{1}{2}$ domain wall solutions

Lukas *et al*:10; Gray, ML, Lüst:12, ...

$\mathcal{M}_{10} = \mathcal{M}_4 \times_W X(t) \equiv \mathcal{M}_3 \times Y$



H-flux allowed by symmetry: $f\epsilon_{\alpha\beta\gamma}$ (along $\mathcal{M}_3$), $\widehat{H}_{tmn}$, $\widehat{H}_{mnp}$ (along Y)

G_2 perspective

G_2 structure: Fernandez–Gray:82, Chiossi–Salamon:02

SUSY $\iff$ Y has G_2 structure determined by 3-form φ ($\psi = *_7\varphi$)

$$\mathrm{d}_7\varphi = \tau_0\,\psi + 3\,\tau_1 \wedge \varphi + *_7\tau_3 \ ,$$
$$\mathrm{d}_7\psi = 4\,\tau_1 \wedge \psi + *_7\tau_2 \ .$$

with torsion

$$\tau_0 = -\tfrac{15}{14}\,f \ \ , \ \ \tau_1 = \tfrac{1}{2}\,d_7\phi \ ,$$
$$\tau_2 = 0 \qquad , \qquad \tau_3 = -\widehat{H} + \tfrac{1}{14}\,f\,\varphi - \tfrac{1}{2}\,d_7\phi \lrcorner \psi$$

This is an integrable G_2 structure.

G_2 perspective

- Bianchi identity constrains the G_2 structure further:

$$\tau_0 = \text{constant} \; ,$$
$$0 = \mathrm{d}_7 \left(\tau_3 + \tau_1 \lrcorner \psi + \tfrac{1}{15} \, \tau_0 \, \varphi \right)$$

- To zeroth order in α', can show Martelli, Sparks:10

SUSY + BI $\implies$ Einstein equation + dilaton EOM + flux EOM

SU(3) perspective

SU(3) structure and embedding

SU(3) structure: $(3,0)$-form Ψ and real $(1,1)$ form ω

Embed in G_2 using 1-form $N = N_t(t,x)\,\mathrm{d}t$, cpl function α (=1 for this talk):

$$\varphi = N \wedge \omega + \mathrm{Re}(\alpha\Psi) \ .$$

SUSY and BI

Restricts torsion and t-flow of the SU(3) structure, and the flux.

SU(3) torsion: $X(t)$ conformally balanced, but otherwise generic

$$W_1^\omega = \mathrm{d}\phi;\ W_0, W_2, W_1^\Psi, W_3 \neq 0.$$

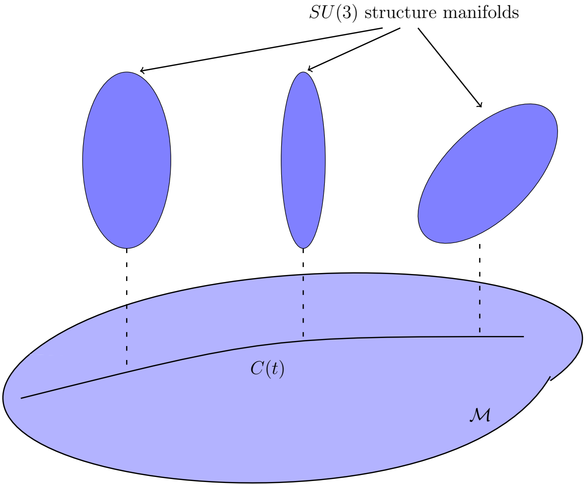
SU(3) flow: $\partial_t\omega$ fixed in terms of N_t, ϕ and SU(3) torsion

$\partial_t\Psi$ fixed up to primitive (2,1)+(1,2)-form γ.

Flux $\widehat{H}$: fixed by SUSY in terms of ϕ and SU(3) torsion up to γ

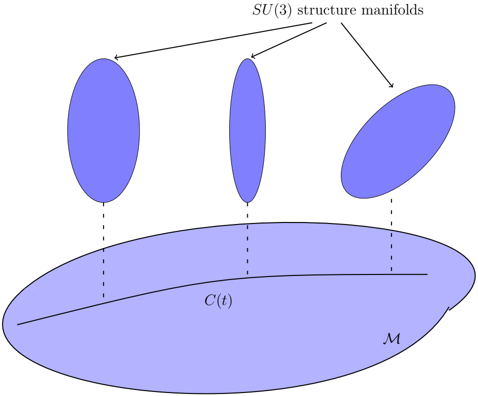
$\Rightarrow$ can check Bianchi idenitity.

Flow of SU(3) structures



t parametrizes a curve in the moduli space of SU(3) structures

Flow of SU(3) structures



Two options:

- Fix torsion classes of SU(3) structure.
- Flow between different types of SU(3) structure.

Examples: Hitchin flow

Hitchin:00

Assume G_2 holonomy: $\tau_a = 0$, $a = 0, .., 4$ and embed $\varphi = \mathrm{d}t \wedge \omega + \mathrm{Re}(\Psi)$

SUSY $\Longrightarrow$ No flux and constant dilaton

$\Longrightarrow$ Half-flat SU(3) structure

$$\begin{aligned} \mathrm{d}(\omega \wedge \omega) &= 0 \ , \\ \mathrm{dRe}(\Psi) &= 0 \ , \\ \mathrm{dIm}(\Psi) &= \mathrm{Im}(W_0)\,\omega \wedge \omega + \mathrm{Im}(W_2) \wedge \omega \ . \end{aligned}$$

Hitchin flow:

$$\begin{aligned} \partial_t(\omega \wedge \omega) &= 2\mathrm{dIm}(\Psi) \\ \partial_t \mathrm{Re}(\Psi) &= \mathrm{d}\omega \ . \end{aligned}$$

The presence of flux/G_2 torsion allows to find generalisations of Hitchin flow.

Examples: Flow of Calabi–Yau with flux

Flow that preserves CY

Assume that $W_i = 0$ for all $t \iff X$ is CY for all t.

Embed $\varphi = N \wedge \omega + \mathrm{Re}(\Psi)$.

Flux (determined by SUSY):
$\widehat{H} = \frac{14}{15}\,\tau_0 N \wedge \omega - \frac{49}{60}\,\tau_0 \mathrm{Re}(\alpha\Psi) + N_t^{-1}\,\tau_{1\,t}\,\mathrm{Im}(\alpha\Psi) + J\gamma$.

Analysis of SUSY and BI gives

$$\begin{aligned} \mathrm{d}\partial_t\omega = 0 \quad &\iff \quad \mathrm{d}\mathrm{d}^\dagger(N_t\,\Psi) = 0 \quad \iff \quad \mathrm{d}N_t = 0\ , \\ \mathrm{d}\partial_t\Psi = 0 \quad &\iff \quad \mathrm{d}\gamma = 0\ , \\ \mathrm{d}_7\widehat{H} = 0 \quad &\iff \quad \mathrm{d}^\dagger\gamma = 0 \end{aligned}$$

Conclusion:
Flow preserves CY $\iff$ N_t is constant and the primitive form γ is harmonic

Examples: Flow of Calabi–Yau with flux

Flow away from CY

Assume X has $W_i = 0$ for $t = 0$.

What does X flow to if N_t is non-constant and γ is not harmonic?

- Taylor expand all forms in the equations $\beta(t) = \beta_0 + \delta_1\beta\, t + \mathcal{O}(t^2)$
- Solve for W_i order by order

First order result:

$$\begin{aligned}
\delta_1 W_0 &= -\tfrac{i}{3}\, \mathrm{d}^{\dagger_0}\mathrm{d}\, N_0 \ , \\
\delta_1 W_1^{\omega} &= 0 \ , \\
\delta_1 \overline{W}_1^{\Psi} &= -N_0^{-1}\, (\partial_m N_0)\, \Delta_0^m + \tfrac{1}{2}\, \left(\lambda_0\, N_0^{-1} + \tfrac{7}{2}\, i\, \tau_0\right)\, \bar{\partial} N_0 \ , \\
\delta_1 W_2 &= -2\, \omega_0 \lrcorner \bar{\partial}(N_0\, \gamma_0)^{(2,1)} + i\, \left(\tfrac{1}{3}\, (\mathrm{d}^{\dagger_0}\mathrm{d}\, N_0)\, \omega_0 - \mathrm{d}(J(\mathrm{d}N_0))\right) \ , \\
\delta_1 W_3 &= \tfrac{1}{2}\, \left(\bar{\partial}\partial^{\dagger_0}(N_0\, \Psi) + \partial\bar{\partial}^{\dagger_0}(N_0\, \overline{\Psi})\right) \ .
\end{aligned}$$

where $\Delta_0^m = \frac{1}{8}\, \overline{\Psi}_0^{mpq}\, \left(2\, (\partial_p N_0)\, \omega_{0\, qn} - N_0\, \gamma_{0\, pqn}^{(2,1)}\right) \mathrm{d}x^n$

Examples: Flow of Calabi–Yau with flux

Flow away from CY: 1st order results

Remarks:

- No flow from a CY to a complex non-CY manifold
- Integrability of non-CY flow: under study
- Simplified case with $\mathrm{d}N_0 = 0$:
  flow from CY to SU(3) structure with only $\mathrm{Re}W_2 \neq 0$.

Flow of symplectic half-flat SU(3) structure

Assume $W_i = 0$, $i \neq 2$
constant embedding $\varphi = \mathrm{d}t \wedge \omega + \mathrm{Re}(\Psi)$

Flow of W_2: $(\partial_t \mathrm{Re}W_2 - \frac{1}{2}\lambda_t \mathrm{Re}W_2) \wedge \omega = \mathrm{d}\gamma$

Examples: Flow of Calabi–Yau with flux

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Conclusions

Conclusions

General 4D heterotic $\mathcal{N} = 1/2$ domain wall solutions

- Y Non-compact with integrable G_2 structure
- $X(t)$ Conformally balanced (non-complex) SU(3) structure
- Flow equations generalize Hitchin flow
- Flow can change the torsion of the SU(3) structure

Work in progress and outlook

- Integrability of flow
- Study moduli space of SU(3) structure manifolds
- Higher order in α': gauge sector, BI
- Non-perturbative "uplift" to 4D AdS.

Lukas *et al*:11, 12, 13 (CY and Nearly-Kähler cosets)

Thank You