

Review of Double and Exceptional Field Theory

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- O. H., B. Zwiebach 1407.0708, 1407.3803; O. H., W. Siegel, B. Zwiebach 1306.2970
- O. H., H. Samtleben 1308.1673 [PRL], 1312.0614, 1312.4542, 1406.3348
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Overview:

Just reformulation of low-energy supergravity/string theory

Part I: Review of two-derivative DFT

- Efficient reformulation of supergravity ('generalized geometry')
- Beyond supergravity: generalized g.c.t. \Rightarrow globally non-trivial spaces

Generalized Scherk-Schwarz compactifications/Non-geometric fluxes

[see Aldazabal's talk]

Part II: Beyond supergravity: α' corrections

- Higher-derivative corrections \Rightarrow deformed gauge structure
- Geometry for Green-Schwarz mechanism; exact action

Part III: Exceptional Field Theory

- Gauge fields for 'generalized geometry brackets'
- $E_{n(n)}$ covariant form of $D = 11$ supergravity

Part I: Two-derivative Double Field Theory

Reformulation (Extension?) of spacetime action for massless string fields:

$$S_{\text{NS}} = \int d^D x \sqrt{-g} e^{-2\phi} \left[R + 4(\partial\phi)^2 - \frac{1}{12} H^{ijk} H_{ijk} + \frac{1}{4} \alpha' R^{ijkl} R_{ijkl} + \dots \right]$$

generalized metric and doubled coordinates $X^M = (\tilde{x}_i, x^i)$,

$$\mathcal{H}_{MN} = \begin{pmatrix} g^{ij} & -g^{ik} b_{kj} \\ b_{ik} g^{kj} & g_{ij} - b_{ik} g^{kl} b_{lj} \end{pmatrix} \in O(D, D)$$

DFT Action (dilaton density $e^{-2d} = e^{-2\phi} \sqrt{-g}$):

$$S_{\text{DFT}} = \int d^{2D} X e^{-2d} \mathcal{R}(\mathcal{H}, d) \xrightarrow{\tilde{\partial}^i=0} S_{\text{NS}}|_{\alpha'=0}$$

generalized curvature scalar

$$\begin{aligned} \mathcal{R} \equiv & 4 \mathcal{H}^{MN} \partial_M \partial_N d - \partial_M \partial_N \mathcal{H}^{MN} - 4 \mathcal{H}^{MN} \partial_M d \partial_N d + 4 \partial_M \mathcal{H}^{MN} \partial_N d \\ & + \frac{1}{8} \mathcal{H}^{MN} \partial_M \mathcal{H}^{KL} \partial_N \mathcal{H}_{KL} - \frac{1}{2} \mathcal{H}^{MN} \partial_M \mathcal{H}^{KL} \partial_K \mathcal{H}_{NL} \end{aligned}$$

Gauge transformations and generalized Lie derivatives

In DFT gauge invariance governed by generalized Lie derivatives

$$\hat{\mathcal{L}}_{\xi} \mathcal{H}_{MN} = \xi^P \partial_P \mathcal{H}_{MN} + (\partial_M \xi^P - \partial^P \xi_M) \mathcal{H}_{PN} + (\partial_N \xi^P - \partial^P \xi_N) \mathcal{H}_{MP}$$

$$\hat{\mathcal{L}}_{\xi}(e^{-2d}) = \partial_M(\xi^M e^{-2d})$$

Invariance and closure, $[\hat{\mathcal{L}}_{\xi_1}, \hat{\mathcal{L}}_{\xi_2}] = \hat{\mathcal{L}}_{[\xi_1, \xi_2]_C}$,

$$[\xi_1, \xi_2]_C^M = \xi_1^N \partial_N \xi_2^M - \xi_2^N \partial_N \xi_1^M - \frac{1}{2} \xi_{1N} \partial^M \xi_2^N + \frac{1}{2} \xi_{2N} \partial^M \xi_1^N$$

modulo strong constraint

$$\eta^{MN} \partial_M \partial_N = 2\tilde{\partial}^i \partial_i = 0 \quad \eta_{MN} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

solved by

$$\partial_M = \begin{cases} \partial_i & \text{if } M = i \\ 0 & \text{else} \end{cases}$$

$O(D, D)$ covariant, captures IIA/M-theory & IIB simultaneously

Generalized coordinate transformations

Generalized g.c.t. that reproduce this infinitesimally:

$$S'(X') = S(X) \quad A'_M(X') = \mathcal{F}_M^N A_N(X)$$

and analogously on higher tensors, where [O.H., Zwiebach, 1207.4198]

$$\mathcal{F}_M^N \equiv \frac{1}{2} \left(\frac{\partial X^P}{\partial X'^M} \frac{\partial X'_P}{\partial X^N} + \frac{\partial X'_M}{\partial X^P} \frac{\partial X^N}{\partial X'^P} \right)$$

Setting $X'^M = X^M - \xi^M(X)$ we get $\delta_\xi = \hat{\mathcal{L}}_\xi$.

- $x^{i'} = x^{i'}(x)$, $\tilde{x}'_i = \tilde{x}_i$ leads to usual g.c.t.,
 $\tilde{x}'_i = \tilde{x}_i - \tilde{\xi}_i(x)$, $x^{i'} = x^i$ leads to $b_{ij} \rightarrow b_{ij} + \partial_i \tilde{\xi}_j - \partial_j \tilde{\xi}_i$
- composition according to BCH of C-bracket,
but no 'intrinsic' picture yet
- equivalent to $\exp(\hat{\mathcal{L}}_\xi)$ [Berman, Cederwall, Perry (2014)]

Non-geometric spaces and fluxes

Torus fibration over circle: T^2 with coordinates (x^1, x^2) , and z for circle; globally well-defined iff generalized c.t. in $(x^1, x^2, \tilde{x}_1, \tilde{x}_2)$ so that

$$\mathcal{H}'(g', b')(z = 2\pi) = \mathcal{H}(g, b)(z = 0)$$

1) Non-geometric, but still T-dual to geometric:

Kähler parameter $\rho = -b_{12} + iV$ and complex structure $\tau = \frac{g_{12}}{g_{11}} + i \frac{V}{g_{11}}$

$$\rho(z) = \frac{1}{Hz - iR_1 R_2}$$

Thus:

$$\rho(2\pi) = \frac{1}{2\pi H + \frac{1}{\rho(0)}} = \frac{\rho(0)}{1 + 2\pi H \rho(0)}$$

\Rightarrow genuine T-duality transformation of $\rho \Rightarrow$ T-folds

\Rightarrow implemented by generalized c.t.

$$x^{1'} = x^1 + 2\pi H \tilde{x}_2 \quad x^{2'} = x^2$$

2) Truly non-geometric background: [Condeescu, Florakis, Kounnas & Lüst (2013)]

$$\tau(z) = \frac{\tau_0 \cos(fz) + \sin(fz)}{\cos(fz) - \tau_0 \sin(fz)}, \quad f \in \frac{1}{4} + \mathbb{Z},$$
$$\rho(z) = \frac{\rho_0 \cos(Hz) + \sin(Hz)}{\cos(Hz) - \rho_0 \sin(Hz)}, \quad H \in \frac{1}{4} + \mathbb{Z}.$$

Global structure:

$$\tau(2\pi) = -\frac{1}{\tau(0)}, \quad \rho(2\pi) = -\frac{1}{\rho(0)}.$$

gluing by generalized c.t.

$$\begin{aligned} \tilde{x}'_1 &= -\tilde{x}_2 & \tilde{x}'_2 &= x^1 \\ x'^1 &= -x^2 & x'^2 &= \tilde{x}_1 \end{aligned}$$

⇒ well-defined in DFT, not contained in supergravity

[more non-geometric spaces in: Hassler & Lüst (2014)]

Part II: Higher-derivative α' deformations

Geometrical structures for generalized vector ξ^M in $\alpha' = 0$ DFT:

$$\langle \xi_1 | \xi_2 \rangle = \xi_1^M \xi_2^N \eta_{MN}, \quad [\xi_1, \xi_2]_C^M = 2 \xi_{[1}^N \partial_N \xi_2^M] - \frac{1}{2} \xi_1^K \overleftrightarrow{\partial}^M \xi_{2K}$$

$$\widehat{\mathcal{L}}_\xi V^M = \xi^P \partial_P V^M + (\partial^M \xi_P - \partial_P \xi^M) V^P$$

All receive non-trivial higher-derivative corrections:

$$\langle \xi_1 | \xi_2 \rangle = \xi_1^M \xi_2^N \eta_{MN} - (\partial_N \xi_1^M)(\partial_M \xi_2^N)$$

$$[\xi_1, \xi_2]_C^M = 2 \xi_{[1}^N \partial_N \xi_2^M] - \frac{1}{2} \xi_1^K \overleftrightarrow{\partial}^M \xi_{2K} + \frac{1}{2} (\partial_K \xi_1^L) \overleftrightarrow{\partial}^M (\partial_L \xi_2^K)$$

$$\mathbf{L}_\xi V^M = \xi^P \partial_P V^M + (\partial^M \xi_P - \partial_P \xi^M) V^P - (\partial^M \partial_K \xi^L) \partial_L V^K$$

Closure and gauge invariance exact! ($\mathbf{L}_\xi \langle V, W \rangle = \xi^N \partial_N \langle V, W \rangle$, etc.)

Not removable by $O(D, D)$ covariant redefinitions

DFT relations for $\mathcal{H} \in O(D, D)$

$$(\mathcal{H}^2)_{MN} \equiv \mathcal{H}_{MK}\mathcal{H}^K{}_N = \eta_{MN} \quad \text{Tr } \mathcal{H} \equiv \eta^{MN}\mathcal{H}_{MN} = 0$$

get deformed \Rightarrow dynamical equations!

$$(\mathcal{M} \star \mathcal{M})_{MN} \equiv 2(\mathcal{M}^2)_{MN} - \frac{1}{2}\partial_M\mathcal{M}^{PQ}\partial_N\mathcal{M}_{PQ} + \dots = 2\eta_{MN}$$

$$\text{tr } \mathcal{M} \equiv \eta^{MN}\mathcal{M}_{MN} - 3\partial_M\partial_N\mathcal{M}^{MN} + \dots = 0$$

In derivative expansion:

$$\mathcal{M}_{MN} = \mathcal{H}_{MN} + \frac{1}{2}\{\mathcal{H}, \mathcal{V}^{(2)}\}_{MN} + \dots, \quad \mathcal{H}^2 = \eta$$

\Rightarrow dilaton eq. & gravity eq. plus α' corrections!

Exact gauge invariant action (with deformed products)

$$S = \int e^\phi \left(\text{Tr } \mathcal{M} - \frac{1}{3}\mathcal{M}^3 + \dots \right)$$

Interpretation on physical subspace?

(Perturbative) analysis shows that b -field transforms as

$$\delta_{\xi+\tilde{\xi}} b = d\tilde{\xi} + \mathcal{L}_{\xi} b + \frac{1}{2} \text{tr}(d(\partial\xi) \wedge \Gamma)$$

with (Christoffel) connection 1-form $(\Gamma)^k_l \equiv \Gamma^k_{il} dx^i$

deformed gauge invariant 3-form curvature

$$\hat{H}(b, \Gamma) = db + \frac{1}{2} \Omega(\Gamma), \quad \Omega(\Gamma) = \text{tr}(\Gamma \wedge d\Gamma + \frac{2}{3} \Gamma \wedge \Gamma \wedge \Gamma)$$

\Rightarrow Green-Schwarz anomaly cancellation mechanism of heterotic string
but with deformed diffeomorphisms rather than deformed Lorentz

Green-Schwarz terms of heterotic string theory subsector to $\mathcal{O}(\alpha')$
but: in terms of g, b arbitrary orders in $\mathcal{O}(\alpha')$

[work in progress with B. Zwiebach and A. Sen]

Deformation of Courant bracket

Deformed gauge transformations close according to bracket

$$\begin{aligned} \left[\xi_1 + \tilde{\xi}_1, \xi_2 + \tilde{\xi}_2 \right]' &= \left[\xi_1, \xi_2 \right] + \mathcal{L}_{\xi_1} \tilde{\xi}_2 - \mathcal{L}_{\xi_2} \tilde{\xi}_1 - \frac{1}{2} d(i_{\xi_1} \tilde{\xi}_2 - i_{\xi_2} \tilde{\xi}_1) \\ &\quad - \frac{1}{2} (\tilde{\varphi}(\xi_1, \xi_2) - \tilde{\varphi}(\xi_2, \xi_1)) \end{aligned}$$

with the map $\tilde{\varphi}$ that produces a 'one-form' from 2 vectors

$$\tilde{\varphi}(V, W) \equiv \text{tr}(d(\partial V) \partial W) \equiv \partial_i \partial_k V^l \partial_l W^k dx^i$$

not genuine 1-form \Rightarrow anomalous transformation under diffeomorphisms

Bracket covariant under *deformed* diffeomorphisms

$$\delta_{\xi + \tilde{\xi}} \tilde{V} \equiv \mathcal{L}_{\xi} \tilde{V} - i_V d\tilde{\xi} - \tilde{\varphi}(\xi, V)$$

α' Corrections for Bosonic Strings and Closed SFT

α' corrections for bosonic string (Riemann-sq.) ? (\mathbb{Z}_2 invariant $b \rightarrow -b$)

Closed bosonic SFT \Rightarrow deformed gauge algebra for *cubic* theory

$$[\xi_1, \xi_2]_+^M = [\xi_1, \xi_2]_C^M + \frac{1}{2} \bar{\mathcal{H}}^{KL} K_{[1K}{}^P \partial^M K_{2]LP}$$

with $K_{MN} = 2\partial_{[M}\xi_{N]}$ and background generalized metric $\bar{\mathcal{H}}_{MN}$

\Rightarrow α' -deformed diffeomorphisms as implied by (perturbative) redefinition

$$h'_{ij} = h_{ij} - \frac{1}{4} \alpha' \partial_k h_i{}^p \partial^k h_{jp} + \dots,$$

agrees with earlier results on duality-invariant Riemann-sq.

[Meissner (1996), Hohm & Zwiebach (2011)]

More general \mathbb{Z}_2 even/odd deformations (with parameters γ^\pm)

$$[\xi_1, \xi_2]_{\alpha'}^M = [\xi_1, \xi_2]_C^M + \frac{1}{2} (\gamma^+ \bar{\mathcal{H}}^{KL} - \gamma^- \eta^{KL}) K_{[1K}{}^P \partial^M K_{2]LP}$$

Cubic Action

$$\begin{aligned}
S &= S^{(2,2)} + S^{(3,2)} \\
&+ \frac{1}{4} \mathcal{R}_{\underline{M} \underline{N} \bar{K} \bar{L}} \mathcal{R}_{\underline{M} \underline{N} \bar{K} \bar{L}} + \frac{1}{4} \phi \mathcal{R}_{\underline{M} \underline{N} \bar{K} \bar{L}} \mathcal{R}_{\underline{M} \underline{N} \bar{K} \bar{L}} \\
&- \frac{1}{8} \left(\Gamma_{\underline{P} \bar{M} \bar{N}} \Gamma_{\bar{M} \underline{K} \underline{L}} \partial_{\underline{P}} \Gamma_{\bar{N} \underline{K} \underline{L}} - \Gamma_{\bar{P} \underline{M} \underline{N}} \Gamma_{\underline{M} \bar{K} \bar{L}} \partial_{\bar{P}} \Gamma_{\underline{N} \bar{K} \bar{L}} \right. \\
&\quad \left. - \Gamma_{\bar{M} \underline{K} \underline{L}} \Gamma_{\bar{N} \underline{K} \underline{L}} \partial_{\bar{M}} \Gamma_{\bar{N}} + \Gamma_{\underline{M} \bar{K} \bar{L}} \Gamma_{\underline{N} \bar{K} \bar{L}} \partial_{\underline{M}} \Gamma_{\underline{N}} \right) \\
&- \frac{1}{2} \mathcal{R}_{\underline{M} \underline{N} \bar{K} \bar{L}} \Gamma_{\bar{K} \underline{M} \underline{P}} \Gamma_{\bar{L} \underline{N} \underline{P}} + \frac{1}{2} \mathcal{R}_{\underline{K} \underline{L} \bar{M} \bar{N}} \Gamma_{\underline{K} \bar{M} \bar{P}} \Gamma_{\underline{L} \bar{N} \bar{P}} \\
&- \frac{1}{2} m_{\underline{M} \bar{N}} \mathcal{R}_{\underline{M} \underline{K} \bar{P} \bar{Q}} \partial^{\bar{N}} \Gamma_{\underline{K} \bar{P} \bar{Q}} + \frac{1}{2} m_{\underline{M} \bar{N}} \mathcal{R}_{\underline{P} \underline{Q} \bar{N} \bar{K}} \partial^{\underline{M}} \Gamma_{\bar{K} \underline{P} \underline{Q}} \\
&+ \frac{1}{2} \mathcal{R}_{\underline{M} \underline{N} \bar{K} \bar{L}} \partial^{\underline{P}} m_{\underline{M} \bar{K}} \partial_{\underline{P}} m_{\underline{N} \bar{L}} .
\end{aligned}$$

Part III: Exceptional Field Theory

Cremmer-Julia [1979]: torus reduction of $D = 11$ SUGRA

→ $E_{6(6)}$ [$D = 5$], $E_{7(7)}$ [$D = 4$], $E_{8(8)}$ [$D = 3$]

Larger mathematical framework that explains/makes it manifest?

[de Wit, Nicolai (1986)]

Truncation of $D = 11$ SUGRA in $4 + 7$ split,

keeping only 'internal' field components and coordinates,

$$G_{MN} = \begin{pmatrix} e^{2\Delta} \eta_{\mu\nu} & 0 \\ 0 & g_{mn}(y) \end{pmatrix}, \quad \text{etc.}$$

extending coordinates to fundamental 56 $\Rightarrow E_{7(7)}$ covariant action

more recently: other groups, geometry, covariant section constraints, etc.

[Hillmann (2009), Berman & Perry (2010), Coimbra, Strickland-Constable & Waldram (2011), etc.]

Complete $D = 11$ SUGRA?? duality transformations in $D = 11$??

E₇₍₇₎ Exceptional Field Theory

Coordinates (x^μ, Y^M) , $\mu = 0, \dots, 3$, M : fundamental 56 of E₇₍₇₎

$$(t_\alpha)^{MN} \partial_M \otimes \partial_N = 0, \quad \Omega^{MN} \partial_M \otimes \partial_N = 0$$

with Ω^{MN} symplectic form of E₇₍₇₎ \subset Sp(56)

Generalized Lie derivative

[Coimbra, Strickland-Constable & Waldram (2011), Berman, Cederwall, Kleinschmidt & Thompson (2012)]

$$\mathbb{L}_\Lambda V^M \equiv \Lambda^K \partial_K V^M - 12 \mathbb{P}^M_N{}^K{}_L \partial_K \Lambda^L V^N + \lambda \partial_P \Lambda^P V^M$$

with projector $\mathbb{P}^M_N{}^K{}_L$ onto adjoint; $\lambda(V)$ density weight.

→ E₇₍₇₎ bracket

$$[\Lambda_1, \Lambda_2]_E^M = 2\Lambda_{[1}^K \partial_K \Lambda_2^M] + 12 (t_\alpha)^{MN} (t^\alpha)_{KL} \Lambda_{[1}^K \partial_N \Lambda_2^L] - \frac{1}{4} \Omega^{MN} \Omega_{KL} \partial_N (\Lambda_1^K \Lambda_2^L)$$

Jacobiator

$$J^M(\Lambda_1, \Lambda_2, \Lambda_3) = (t^\alpha)^{MN} \partial_N \chi_\alpha(\Lambda) + \Omega^{MN} \chi_N(\Lambda),$$

where

$$\chi_\alpha = -\frac{1}{2} (t_\alpha)_{PQ} \Lambda_1^P [\Lambda_2, \Lambda_3]_E^Q + \dots$$

$$\chi_N = \frac{1}{12} \Omega_{PQ} (\Lambda_1^P \partial_N [\Lambda_2, \Lambda_3]_E^Q + [\Lambda_2, \Lambda_3]_E^P \partial_N \Lambda_1^Q + \dots)$$

χ_M ‘covariantly constrained’, satisfying the same constraints as ∂_M

A_μ^M gauge field for *local* Λ^M : $\delta A_\mu^M \equiv \mathcal{D}_\mu \Lambda^M$

Covariant curvature (Jacobiator for bracket \rightarrow tensor hierarchy \rightarrow 2-forms)

[de Wit, Nicolai, Samtleben & Trigiante (2001–2003)]

$$\mathcal{F}_{\mu\nu}^M \equiv F_{\mu\nu}^M - 12 (t^\alpha)^{MN} \partial_N B_{\mu\nu\alpha} - \frac{1}{2} \Omega^{MN} B_{\mu\nu N}$$

3-form field strength $\mathcal{H}_{(3)M}$ of 2-forms by generalized Bianchi identity

$$3 \mathcal{D}_{[\mu} \mathcal{F}_{\nu\rho]}^M = -12 (t^\alpha)^{MN} \partial_N \mathcal{H}_{\mu\nu\rho\alpha} - \frac{1}{2} \Omega^{MN} \mathcal{H}_{\mu\nu\rho N}$$

Complete action:

$$S_{\text{EFT}} = \int d^4x d^{56}Y e \left(\hat{R} + \frac{1}{48} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{8} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}_{\mu\nu}{}^N + e^{-1} \mathcal{L}_{\text{top}} - V(\mathcal{M}_{MN}, g_{\mu\nu}) \right)$$

– Twisted (electric-magnetic) self-duality relations

$$\mathcal{F}_{\mu\nu}{}^M = -\frac{1}{2} e \varepsilon_{\mu\nu\rho\sigma} \Omega^{MN} \mathcal{M}_{NK} \mathcal{F}^{\rho\sigma}{}^K$$

$A_\mu{}^M$ include 7 Kaluza-Klein vectors from $D = 11$ metric

→ 7 dual vectors → dual graviton in *non-linear* duality relation

[no-go theorems: Bekaert, Boulanger, Henneaux (2002)]

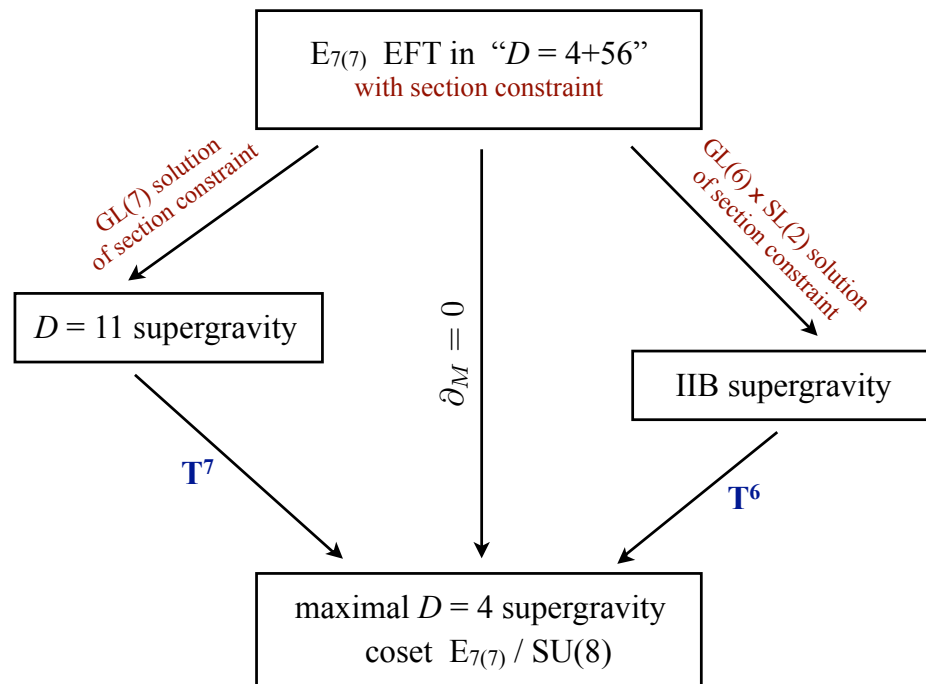
solving constraint → only 7 $B_{\mu\nu N}$ left → ‘dual gravitons’ pure gauge

– Topological terms guarantees consistency with self-duality

– $\mathcal{N} = 8$ supersymmetric extension [Godazgar², Hohm, Nicolai, Samtleben]

consistent supersymmetry variations for $B_{\mu\nu M}$

– $\text{GL}(7) \rightarrow$ M-theory, $\text{GL}(6) \times \text{SL}(2, \mathbb{R}) \rightarrow$ Type IIB → F-theory?



Summary & Outlook

- DFT/EFT provides strikingly economic reformulation of supergravity
- good evidence that it gives more than supergravity:
truly non-geometric backgrounds well-defined thanks to
DFT diffeomorphisms \rightarrow compactification, generalized Scherk-Schwarz
[Aldazabal, Baron, Marques & Nunez, Geissbühler (2011)]
- complete formulation of exceptional field theory for $E_{6(6)}$, $E_{7(7)}$, $E_{8(8)}$
- intriguing new ‘quantum geometry’ beyond lowest order in α'
 - \Rightarrow forget about g and b !
 - \Rightarrow underlying gauge principle of string theory?
- for higher-derivatives corrections so far only partial results:
background-independent extension for bosonic strings?
Field-dependent gauge algebra? Higher order in α' ?
Type II Strings and M-theory extensions?