Relative Entropy and Proximity of QFTs

Jonathan J. Heckman
UNC Chapel Hill

hep-th/14??.???? w/ V. Balasubramanian and A. Maloney

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Motivation

Given a string compactification, we get:

- A collection of fields $\{\phi\}$
- An effective action $S_{eff}[\phi]$

Question: How well can we determine:

A list of “nearby” string compactifications

Related: What about the spacetime?

JJH ’13, Hebecker ’13

Related Motivation

String theory yields a landscape of EFTs

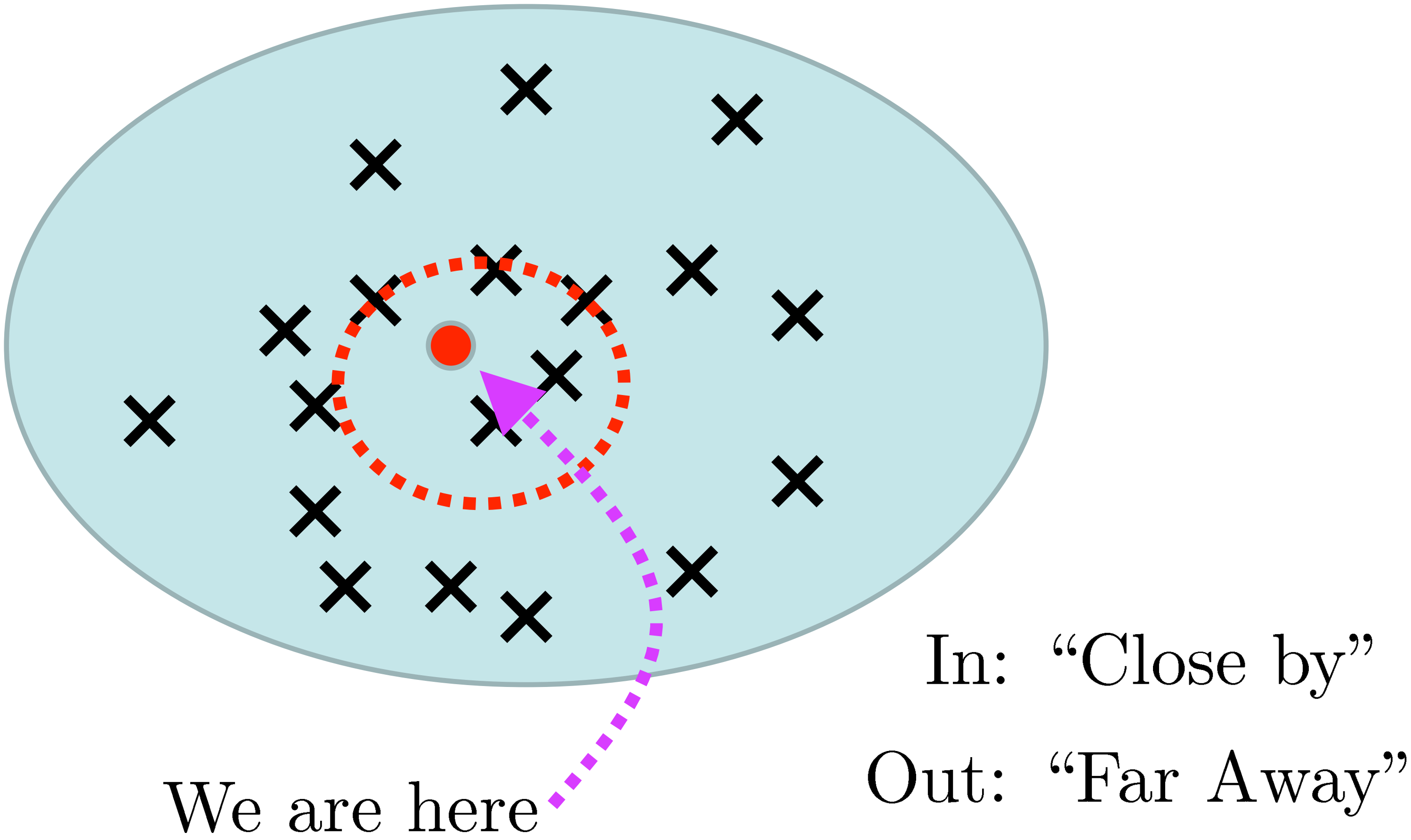
$$\text{EFT}_1,\ldots,\text{EFT}_{10^{500}}$$

Question: Is there a notion of distance:

$$Distance(EFT_i, EFT_j) \geq 0?$$

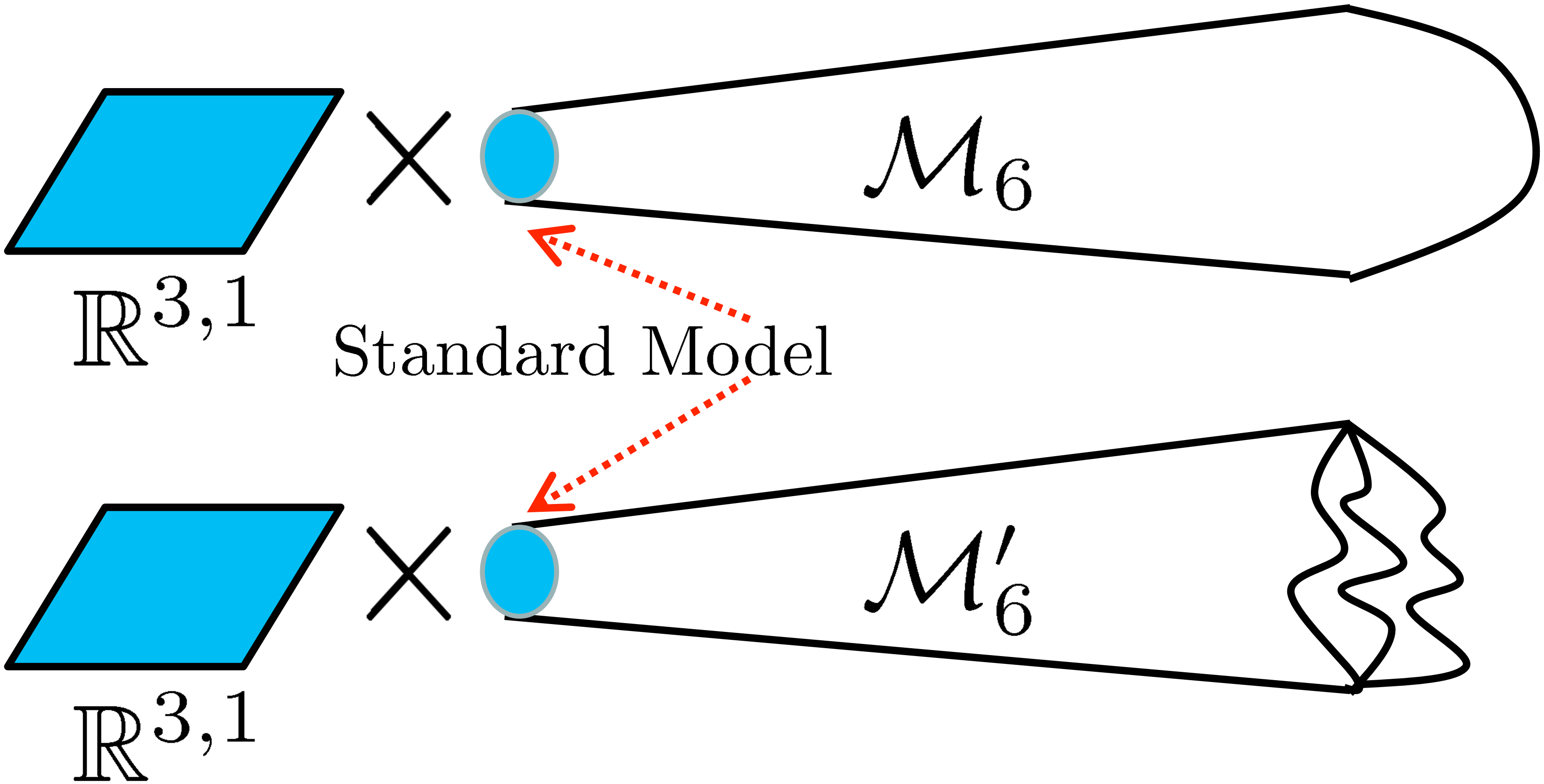
Douglas, "Spaces of Quantum Field Theories" 1005.2779

Why a Distance Would Help



Further Motivation

A local model may have different global embeddings



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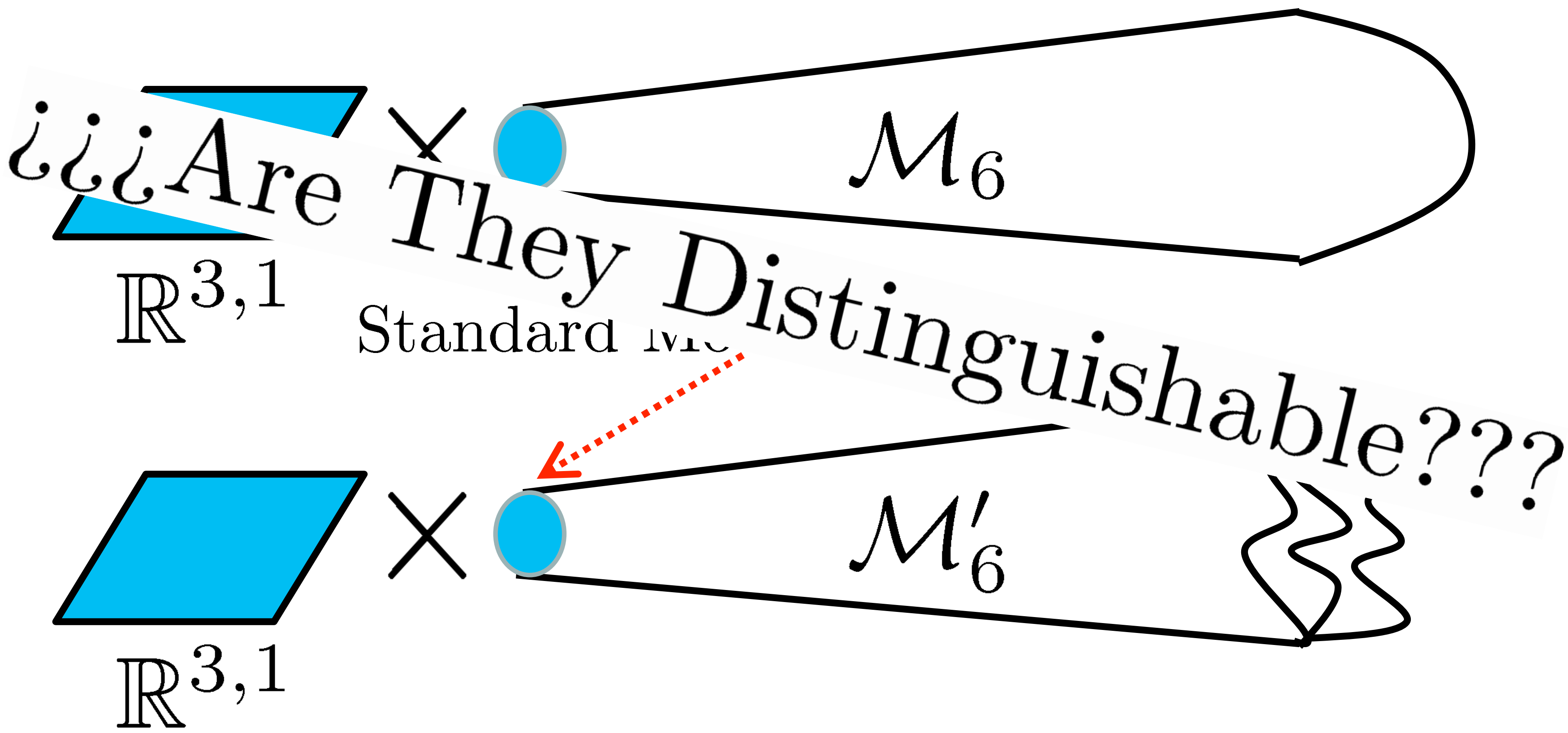


Basic Idea

We ask a basic question:

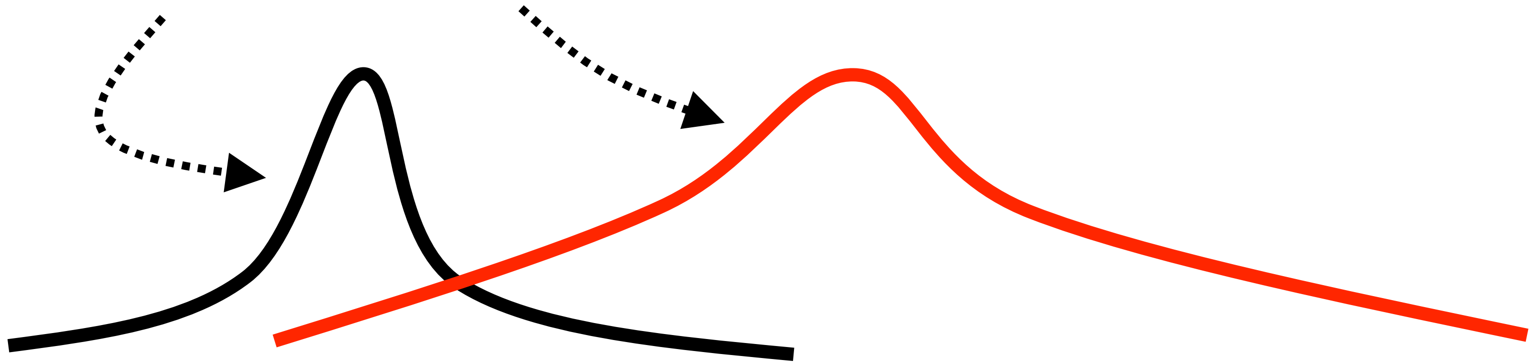
How many measurements would it take to tell the difference between $S^{(1)}[\phi]$ and $S^{(2)}[\phi]$?

If you can't tell the difference, compactifications are indistinguishable
$\Rightarrow$ Coarse graining on the landscape

Info and Distinguishability

This issue has been studied in the context of statistical inference and information theory

Suppose $p(z)$ and $q(z)$ two prob. distributions:



Odds of thinking we sampled from p rather than q?

Relative Entropy

Suppose $p(z)$ and $q(z)$ two prob. distributions:

Shannon Entropy: $-\int dz\, p(z) \log p(z)$

Kullback-Leibler Divergence / Relative Entropy:

$$D_{KL}(p||q) \equiv \int dz\, p(z) \log \frac{p(z)}{q(z)}$$

Properties of D_{KL}

- $D_{KL}(p||q) \geq 0$

- $D_{KL}(p||q) = 0$ iff $p = q$ almost surely

- measured in "nats" rather than "bits"

Interpretations of D_{KL}

Learning:

If $p(z)$ is the "true" dist, but we think it's $q(z)$

$D_{KL}(p||q) =$ info we'd gain from learning $p(z)$

Chernoff Bound:

Sample N times from $q(z)$

$\Pr[\text{gen}^{ed} \text{ from } p(z)] \leq \exp(-N D_{KL}(p||q))$

Proximity

$D_{KL}(p||q)$ says how "close" q and p are

But it's not a metric... (not symmetric)

Infinitesimal Version: $p(z) = q(z|\lambda) + \frac{\partial q}{\partial \lambda^i} \delta\lambda^i$

$D_{KL}(p||q) = G_{ij}\, \delta\lambda^i \delta\lambda^j$

Info Metric: $G_{ij} = \int\, dz\, q \frac{\partial \log q}{\partial \lambda^i} \frac{\partial \log q}{\partial \lambda^j}$

The Proposal (I / II)

V. Balasubramanian, JJH, A. Maloney, to appear

Consider Euclidean Signature theory

Suppose we know $S[\phi]$

This defines a probability distribution:

$p[\phi] = \frac{\exp(-S[\phi])}{Z}$, i.e. a Boltzmann factor

Partition Function: $Z = \int \mathcal{D}\phi \exp(-S[\phi])$

The Proposal (II / II)

Suppose we have two theories, i.e. two distns:

$$p[\phi] = \frac{\exp(-S_p[\phi])}{Z_p} \text{ and } q[\phi] = \frac{\exp(-S_q[\phi])}{Z_q}$$

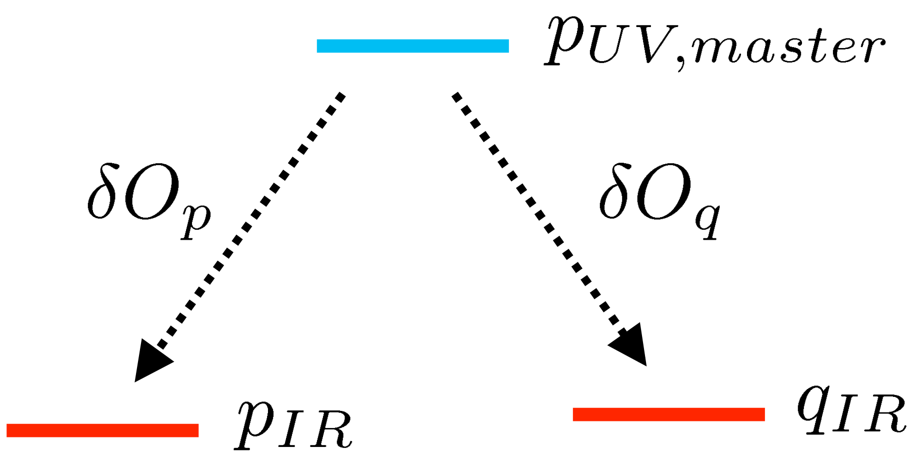
$$\text{Proximity} = D_{KL}(p||q) = \int \mathcal{D}\phi \, p[\phi] \log\left(\frac{p[\phi]}{q[\phi]}\right)$$

"Sample" = Field configuration in spacetime

Note...

p and q could even have different field content!

RG as motion on space of couplings



$\Rightarrow$ Just need a "master theory" (e.g. strings)

Perturbation Theory

Perturbation Theory

Suppose $S_p - S_q = \int d^D x \, \delta\lambda^i O_i(x) \equiv \Theta$

To leading order in perturbation theory, we get:

$$D_{KL}(p||q) = \langle(\Theta - \langle\Theta\rangle_p)(\Theta - \langle\Theta\rangle_p)\rangle_p$$

Regulators...

$$D_{KL}(p||q) = \int d^D x \int d^D y \langle O_i(x) O_j(y) \rangle_p \delta\lambda^i \delta\lambda^j$$

$$= \mathrm{Vol}(\mathcal{M}_D) \int d^D x \langle O_i(x) O_j(0) \rangle_p \delta\lambda^i \delta\lambda^j$$

Two Divergences:

Info Density

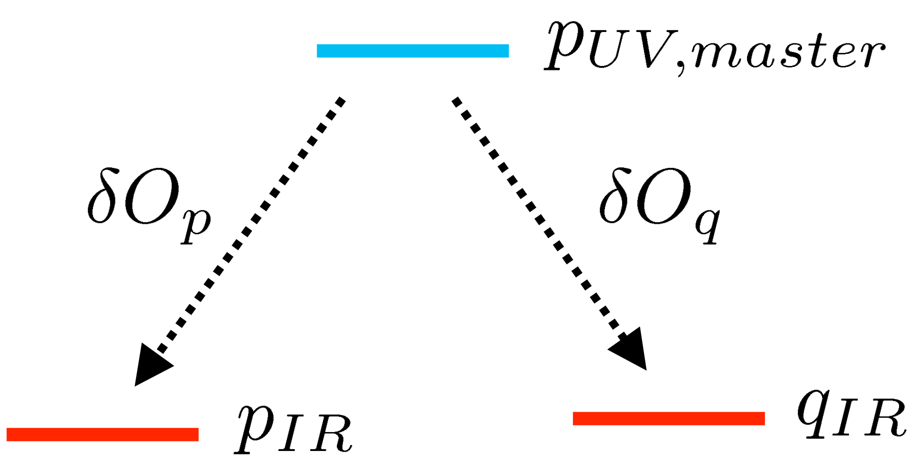
i) IR (just use $\mathcal{D}_{KL} \equiv D_{KL}/\mathrm{Vol}(\mathcal{M}_D)$)

ii) UV from contact terms (i.e. $x \sim 0$ region)

Scheme Dependence

So, $D_{KL}(p||q)$ depends on a scheme

specify it once for master theory...

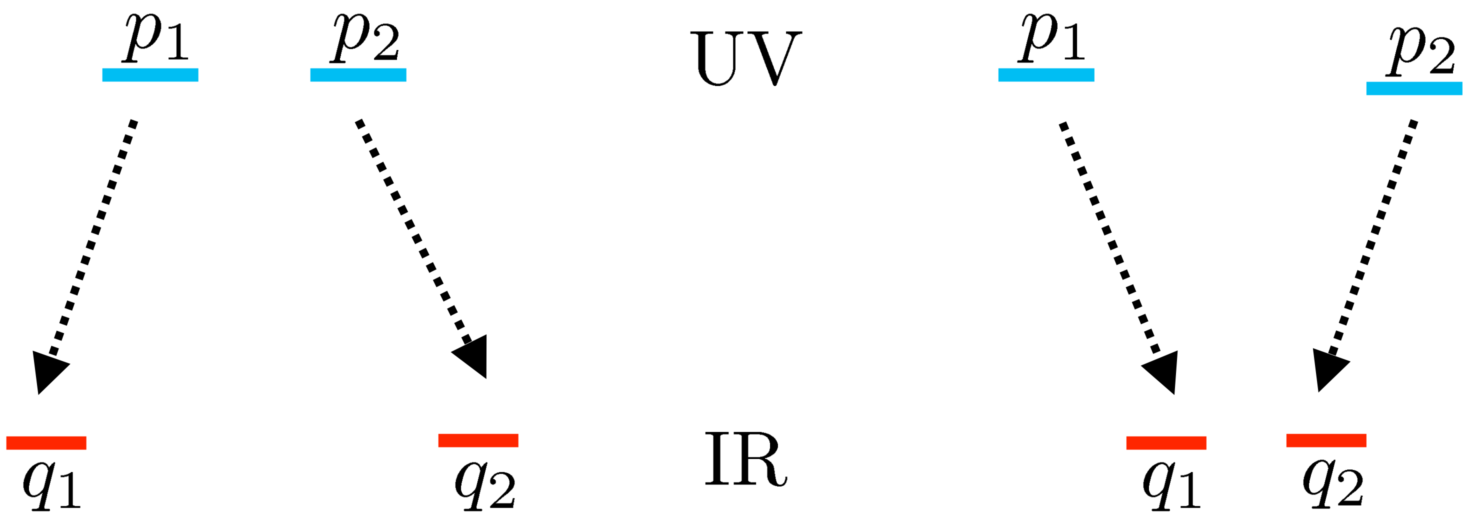


RG: $\frac{\partial D_{KL}(p||q)}{\partial(\log\mu)^m}$ tells us info as a f^n of scale

Remainder of Talk: Applications

Quantifying Fine-Tuning

$$F_{ij} \equiv \frac{D_{KL}(p_i^{UV}||p_j^{UV})}{D_{KL}(q_i^{IR}||q_j^{IR})}$$



Fine-Tuned

$F_{12} < 1$

Natural

$F_{12} > 1$

Proximity for CFTs

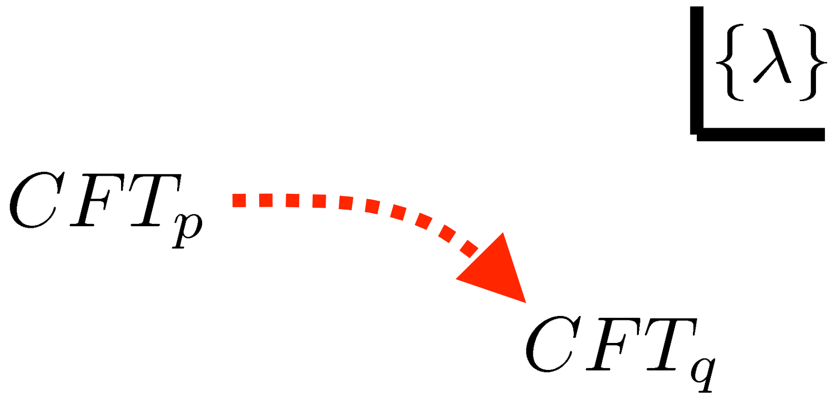
Special Case: p a CFT

$$D_{KL}(p||q) = \int d^D x \int d^D y \langle O_i(x) O_j(y) \rangle_p \delta\lambda^i \delta\lambda^j$$

$$\langle O_i(x) O_j(y) \rangle_p = \frac{G_{ij}^{Zamolodchikov}}{||x-y||^{2\Delta_O}}$$

$$\Rightarrow G_{ij}^{Information} \propto G_{ij}^{Zamolodchikov}$$

¡¡Zamolodchikov Metric!!



Two CFTs connected by marg. pert^{n}s

Formal notion of proximity between CFTs...

¿¿Zamolodchikov Metric??

So why didn't Douglas like this?

Not all CFTs connected by marginal pert^{n}s...

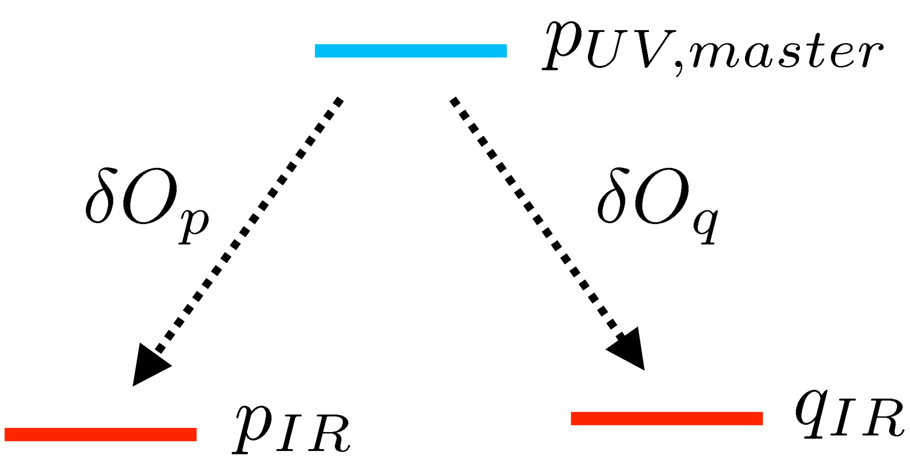
Example: Two isolated $c = 4/5$ Models:
Douglas, "Spaces of Quantum Field Theories" 1005.2779

i) Tetracritical Ising

ii) 3-State Potts

But Recall...

p and q could even have different field content!



$\Rightarrow$ Just need a "master theory" (e.g. strings)

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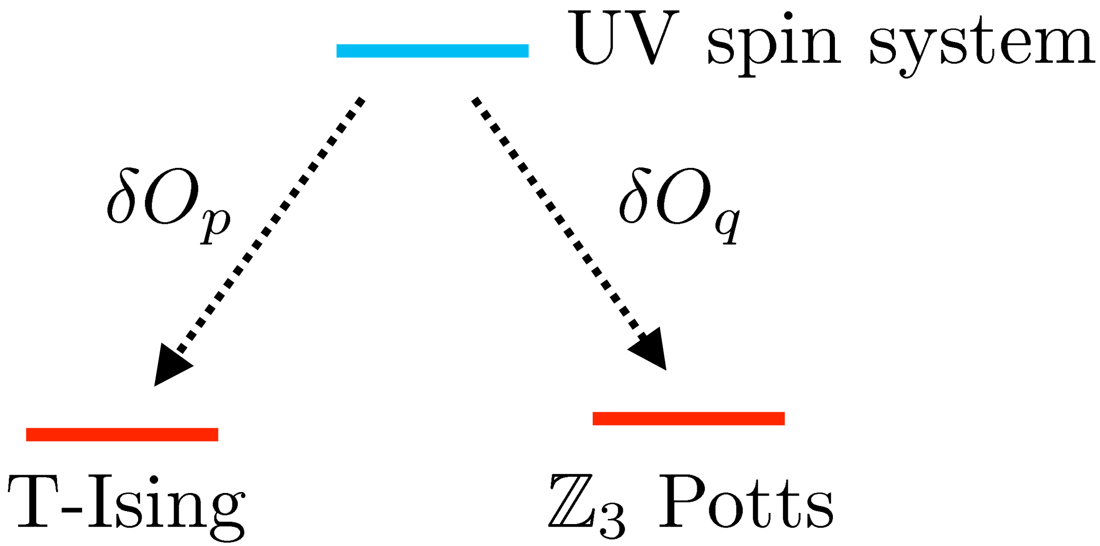

$\Rightarrow$ Just need a "master theory" (spin system)

Special Case: p a CFT

$$D_{KL}(p||q) = \int d^D x \int d^D y \langle O_i(x) O_j(y) \rangle_p \delta\lambda^i \delta\lambda^j$$

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What About Central Charge?

Treat $g_{\mu\nu}$ as a parameter: $p[\phi|g_{\mu\nu}]$

Take $q[\phi|g_{\mu\nu}] = p[\phi|g_{\mu\nu} + \delta g_{\mu\nu}]$

$$D_{KL}(p||q) = \int_x \int_y \langle T^{\mu\nu}(x) T^{\rho\sigma}(y) \rangle_p \delta g_{\mu\nu} \delta g_{\rho\sigma}$$

Note $\langle T^{\mu\nu}(x) T^{\rho\sigma}(y) \rangle = C_T \frac{I^{\mu\nu,\rho\sigma}(x,y)}{||x-y||^{2D}}$

Info $\propto C_T$: measures local degrees of freedom

Proximity for Flux Vacua

A Toy Model

Consider a theory of one real scalar ϕ

$S[\phi] = \int d^4x \left(\frac{1}{2}(\partial\phi)^2 - V(\phi)\right)$

Suppose we have fluxes $\overrightarrow{N}$ and $\overrightarrow{M}$

with eff. potentials $V_{\overrightarrow{N}}(\phi)$ and $V_{\overrightarrow{M}}(\phi)$

Suppose further:

$V(\phi) = \frac{1}{\Lambda^{2l-4}}(\phi - \phi_1)^2 \cdots (\phi - \phi_l)^2$

ϕ_i depend on choice of flux vector

Question

¿What is proximity $D_{KL}(\vec{N}||\vec{M})$?

Approximation: Assume a "discretuum"
such that ϕ_i in $V(\phi)$ are parameters:
$V(\phi|\{\phi_1, ..., \phi_l\}) = \frac{1}{\Lambda^{2l-4}}(\phi - \phi_1)^2 \cdots (\phi - \phi_l)^2$

a la Bousso Polchinski hep-th/0004134

So, $p[\phi|\{\phi_1, ..., \phi_l\}]$

Computation

$$D_{KL}(\overrightarrow{N}||\overrightarrow{M}) = G_{ij}\,\delta\phi^i\delta\phi^j$$

$$G_{ij} = \int\int\; d^4x\,d^4y\,\langle\frac{\partial V(\phi(x))}{\partial\phi^i}\,\frac{\partial V(\phi(y))}{\partial\phi^j}\rangle$$

Saddle Point: $p[\phi] \sim \frac{1}{l}\left(\frac{e^{-S_1}}{Z_1} + \cdots + \frac{e^{-S_l}}{Z_l}\right)$

$$S_k[\phi] = \int d^4x\,\left(\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m_k^2\phi^2\right)$$

Answer

$$D_{KL}(\overrightarrow{N}||\overrightarrow{M}) = G_{ij}\,\delta\phi^i\delta\phi^j$$

$$G_{ij} = \delta_{ij} \times \frac{\mathrm{Vol}(\mathcal{M}_4)}{l} \times m_i^2$$

$$m_i^2 = V''(\phi)|_i = \frac{2}{\Lambda^{2l-4}} \prod_{k\neq i}(\phi_k - \phi_i)^2$$

Proximity for Inflatons

Suppose...

Suppose we have single scalar slow-roll with:
Large field range, and effective action:

$$S_{eff}[\phi] = \int d^4x \left(\frac{1}{2}(\partial\phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4 \right)$$

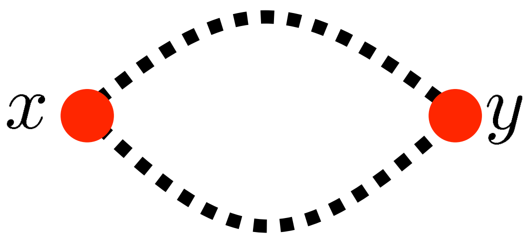
Proximity of $\lambda = 0$ and $\lambda \neq 0$?

¿How much info do higher order ops give?

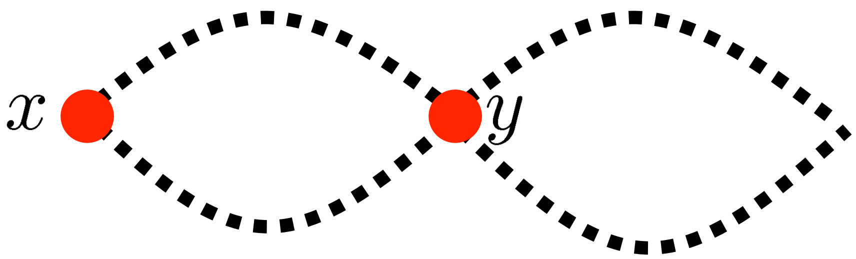
Information Metric

We have two parameters: m^2 and λ...

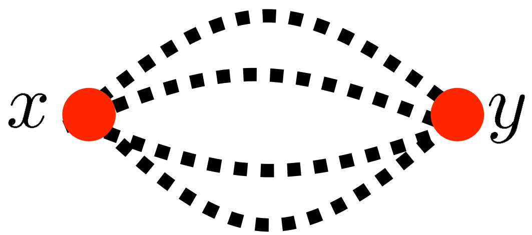
$G_{m^2 m^2}$:



$G_{m^2 \lambda}$:



$G_{\lambda\lambda}$:



Information Metric

We have two parameters: m^2 and λ...

$$G_{m^2 m^2} \sim \mathrm{Vol}(\mathcal{M}_4) \times \log m^2$$

$$G_{m^2 \lambda} \sim \mathrm{Vol}(\mathcal{M}_4) \times (\Lambda_{UV}^2 + m^2 \log m^2)$$

$$G_{\lambda\lambda} \sim \mathrm{Vol}(\mathcal{M}_4) \times (\Lambda_{UV}^4 + \log^3 m)$$

$\lambda = 0$ vs $\lambda \neq 0$

$$D_{KL}(\lambda = 0 || \lambda \neq 0) \sim \lambda^2 \times \left(\frac{\Lambda_{UV}}{\Lambda_{IR}}\right)^4 \sim \lambda^2 \times \left(\frac{M_{pl}}{m_{inf}}\right)^4$$

But to not spoil $m^2\phi^2$ slow roll, we already need:

$\lambda < \left(\frac{m_{inf}}{\Delta\phi}\right)^2$ so if $\Delta\phi \sim 10M_{pl}$...

$$\Rightarrow D_{KL} < \left(\frac{M_{pl}}{\Delta\phi}\right)^4 \sim 10^{-4}$$

Conclusions / Future

Conclusions

- To study landscape, would like $D(EFT_i, EFT_j)$
- Proposal from statistical inference: $D_{KL}(p||q)$
- Recovers G^{Zam}, and other intuitive measures
- Applications: Flux Vacua, Fine-Tuning, Inflatons,...

Future

- Formal: Use this to prove a / c / F-theorems?
- Pheno: SM versus MSSM w/ heavy superpartners?
- Cosmo: Apply to inflationary measure problem?