# Axions and U(1)s in F-theory


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based on:

Axions: 1404.4268 & 1008.4133

Massive U(1)s: 1406.5180 with L. Anderson, I. García-Etxebarria, J. Keitel

# Introduction

- In recent years there has been vast progress in the study of F-theory effective actions in four and six dimensions using an approach via M-theory.

- The understanding of geometric properties and the physics in the effective actions are going hand in hand.
  - classical moduli action
  - fluxes and charged spectrum
  - massless and massive U(1)'s, gauge theory branches and resolutions
  - $\alpha'$ corrections ....

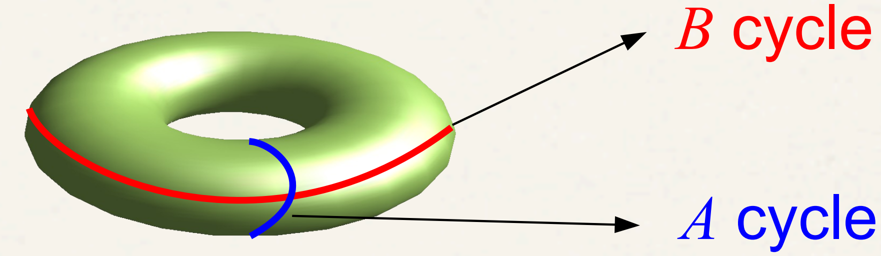
- Main motivations:
  - phenomenology: Grand Unified Theories, ...
  - general effective theories within string theory (`minimal models')
  - formulating string theory away from weak coupling (M-theory)

# Computing the F-theory effective actions

- F-theory encodes physics of seven-branes in higher-dim. geometry: [Vafa]
  - singularities of the elliptic fibration: $y^2 = x^3 + f(u)x + g(u)$
  - seven-brane locations (gauge group, matter, ...): $\Delta = 27g^2 + 4f^3$

- No twelve-dimensional low-enegry effective action for F-theory.

**Analyze and define F-theory via M-theory**




(1) A-cycle: if small than M-theory becomes Type IIA
(2) B-cycle: T-duality $\Rightarrow$ Type IIA becomes Type IIB
(3) grow extra dimension: send $T^2$- volume T-dual $\Rightarrow$ B-cycle becomes large

$\Rightarrow$ M-theory to F-theory limit connects 6d and 5d effective theories
4d and 3d effective theories

# F-theory effective actions via M-theory

- effective actions can be computed via M-theory / 11-dimensional supergravity on the resolved Calabi-Yau manifolds

F-theory on singular $M_{8/6}$

**4d/6d** effective theory with non-Abelian gauge symmetry $G$ and non-Abelian tensors

1-dim. compactification

**3d/5d** effective theory pushed to Coulomb branch

M-theory on resolved $\tilde{M}_{8/6}$

**3d/5d** effective theory with only abelian gauge symmetries

compare

explicitly compute characteristic data determining the action

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a) can be a **circle** (standard approach)
b) can be an **interval** for Spin(7)
[Bonetti,TG,Pugh] [Bonetti,TG,Palti,Pugh]
c) can be an **fluxed circle**

# Comments on M-theory / F-theory limit

- Concrete way to perform computation in a genuine F-theory setup:
  - compared to 2008 enormous progress (mostly on extracting discrete data):

    (1) chiral spectrum in 6d, four-form fluxes and chirality in 4d:

    5d / 3d Chern-Simons terms $\Longleftrightarrow$ 6d / 4d chiral matter

    deep relations to AdS-CFT refs [Landsteiner etal.] [Loganayagam etal.] [Golkar,Son]
    relation to $S^3 \times S^1$, $S^5 \times S^1$ partition functions [Di Pietro,Komargodski]

    (2) U(1) symmetries and their selection rules → review talk by T. Weigand
    → talk by J. Keitel

  - compute continuous quantities: forced to improve understanding of M-theory
    - corrections to Kähler potential and gauge coupling (GUT unification...)
      Yukawas → mostly in local perspective or by dualities
    - possible approach: corrections to 11d supergravity → talk by M. Weissenbacher

    ⇒ significant progress needed to study moduli stabilization, susy breaking, inflation in F-theory

# Goals of this talk

- I like to discuss axions and U(1) gauge symmetries in F-theory compactifications on Calabi-Yau fourfolds and threefolds:

- Stepwise introduce:

  (1) Systematics of axions in F-theory

  (2) Axion decay constants and their moduli dependence

  (3) U(1) gauge symmetries that are massive by 'eating' axions

  (4) Massive U(1) gauge symmetries to describe physics of elliptic fibrations without section

# (1) Axions and their decay constants in F-theory

# Generalities: Axions in Type IIB

- Axions are generically present in Type IIB string compactifications:
  - zero-modes of R-R, NS-NS form fields: form-field axions

$$C_0 \qquad C_2 = c^a\,\omega_a \qquad C_4 = \rho_\alpha\,\tilde{\omega}^\alpha \qquad B_2 = b^a\,\omega_a$$

$\longrightarrow$ $C_0\,,\ c^a,\ b^a$ non-trivially transform under SL(2,Z)

  - can arise from D7-branes: Wilson line axions

$A_{D7} = a^p\,\xi_p \longleftarrow$ one-forms on brane world-volume

  - symmetry points in geometrical moduli spaces: geometrical axions

    example: 'large complex structure point' in complex structure moduli space of Calabi-Yau manifold

# Axions in F-theory via M-theory

- F-theory on elliptically fibered Calabi-Yau fourfold $Y_4$ with base $B_3$ obtained obtained via M-theory

- Axions are arising from M-theory three-form:

$$C_3^M = A^\alpha \wedge \omega_\alpha + iG^a \bar{\Psi}_a - i\bar{G}^a \Psi_a$$

(1,1) - forms on CY fourfold two legs in $B_3$

(2,1) - forms on CY fourfold one leg in fiber, two legs in $B_3$

$A^\alpha \longrightarrow$ yields $C_4$ axions $\rho_\alpha$

$G^a \longrightarrow$ unifies form-field and Wilson line axions: $c^a, \; b^a, \; a^p$

- geometrical axions arise from complex structure moduli space of $Y_4$
$\Rightarrow$ $C_0$ is geometrical axion in F-theory
$\Rightarrow$ no shift symmetry in general F-theory setting

# Axion decay constants I

- focus on the (2,1)-form fields $G^a$ in the following (assume $h^{2,1}(B_3) = 0$):
  - complex fields $G^a$ live on complex $N$-dimensional torus:

$$T^{2N} = \frac{H^{2,1}(Y_4, \mathbb{C})}{H^3(Y_4, \mathbb{Z})} \qquad N = h^{2,1}(Y_4) - h^{2,1}(B_3)$$

$\longrightarrow$ complex structure on $T^{2N}$ is induce by complex structure of $Y_4$

  - choice of basis: $\Psi_a = \frac{1}{2}\mathrm{Im}(h^{ab})^{-1}(\beta^b - h^{bc}\alpha_c)$ $\quad (\alpha_b, \beta^b)$ integral three-forms

$\longrightarrow$ $h^{ab}$ is a holomorphic function of the complex structure moduli of $Y_4$

  - couplings of classical effective theory does only depend on $G^a - \bar{G}^a$

$\longrightarrow$ shift symmetry for $\mathrm{Im}\, G^a$ (e.g. R-R two-forms, D7-Wilson lines)

# Axion decay constants II

- metric for $G^a$ , axion decay constants:

$$f_{ab}^2 = \frac{i}{\mathcal{V}} \int_{Y_4} J \wedge \bar{\Psi}_a \wedge \Psi_b$$

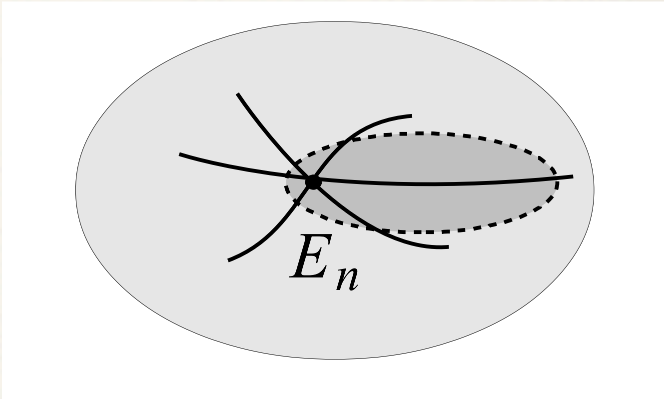
$J,\ \mathcal{V}$ is Kähler form and volume of

$\longrightarrow$ $f_{ab}$ varies non-trivially over Kähler and complex structure moduli space

- Question: Can the axion decay constants be large at special points?
  - simplest example: $N=1$

write locally: $\Psi = \frac{1}{2}\mathrm{Im}(\tau)^{-1}\tilde{\omega} \wedge (dx - \tau dy)$

$$f^2 = \frac{1}{\mathcal{V}_\mathrm{b}} \int_{\mathcal{B}} (\mathrm{Im}\,\tau)^{-1} J_\mathrm{b} \wedge \tilde{\omega}^2$$




$\longrightarrow$ $f_{ab}$ can receive large contributions from strong coupling regions on $B_3$, also needed for GUTs in F-theory

# Axion potential

- Large axion decay constants might allow for interesting models of natural axion inflation

  [Freese,Frieman,Olinto]
  → talks G. Shiu, L. McAllister

- Axion potential in F-theory:
  - arises from M5-brane instantons in dual M-theory picture
  - $G^a$ - dependence through a theta-function $\Theta(G,h)$ on the torus $T^{2N}$

$$W_{\mathrm{M5}}(z,G,T) = f(z)\,\Theta(G,h)\,e^{-T}$$

$$\Theta(G,h) = \sum_{n_a\in\Gamma} \exp\left(\tfrac{1}{2} i h^{ab} n_a n_b + i n_a G^a\right)$$

[Witten] [Ganor] [TG]
[TG,Kerstan,Palti,Weigand]
[Kerstan,Weigand]

→ related to talk by C. Angelantonj

  - axion decay constants large where higher harmonics become relevant, i.e. for $h^{ab} \ll 1$ $\longrightarrow$ no parametrically safe realization, fine-tuning in special setups?

# Summarizing remarks

- it is of key importance to find properties of F-theory vacua that distinguish them from weakly coupled Type IIB
  - key example: 10 10 5 Yukawa → exceptional enhancement → GUTs
    → talk by F. Marchesano
  - axion decay const. of $G^a$- axions → computable strong coupling corrections → holomorphic function $h^{ab}$ is reminiscent of 3d duality of axions ↔ vectors
  - size of axion decay constants have to be determined at various point in complex structure moduli space
- dynamics of axions is crucially depending on scalar potential
  - region of large axion decay constants (compared to Planck mass), requires to include higher harmonics (similar in spirit to [Banks,Dine,Fox,Gorbatov])

# (2) Massive U(1) gauge symmetries

# Massive U(1)s from geometric Stückelberg

- weakly coupled Type IIB theory: D7-brane U(1)s can admit Stückelberg coupling

$$S_{\rm St} = \int_{\rm D7^-} C_6 \wedge F_{\rm U(1)}$$

→ dualize $C_6$ : shift gauging of $C_2$ axions $c^a$

$$\mathcal{D}c^a = dc^a + m^a A_{\rm U(1)}$$

purely geometric

[Jockers,Louis]

- geometrically massive U(1)s in F-theory: [TG,Weigand] [TG,Kerstan,Palti,Weigand]
  - gauging of (2,1)-form axions:

$$dC_3^M = idG^a \wedge \bar\Psi_a + A_{\rm U(1)} \wedge d\omega_{U(1)} + \text{c.c.}$$
$$= \mathcal{D}G^a \wedge \bar\Psi_a + \text{c.c.}$$

→ non-closed (1,1)-form

$$\int_{\mathcal{C}} dJ = \int_{\partial\mathcal{C}} J = v^{U(1)}$$

  - non-Kähler resolutions: include massive U(1)s in effective theory
  - beautiful explicit geometric realization [A. Braun,Collinucci,Valandro]

# F-theory on manifolds without section

- Calabi-Yau threefold with genus-one fibration without section:
  - base $B_2$ is not a submanifold locally described by embedding equations
  - dilaton-axion $\tau(u)$ can be still extracted at each point of $B_2$
    $\Rightarrow$ F-theory well-defined on this space, despite having no Weierstrass model?
  - M-theory on Calabi-Yau threefolds without sections is well defined and can be analyzed using 11d sugra $\Rightarrow$ 5d effective description of F-theory setup!

- Our Proposal: [Anderson,García-Etxebarria,TG,Keitel]

  (1) F-theory setup contains geometrically massive U(1)s and charged spectrum under these U(1)s

  (2) Derivation from M-theory requires a fluxed circle reduction from 6d $\rightarrow$ 5d
  $\Rightarrow$ new massive and massless 5d combinations of circle Kaluza-Klein vector and 6d massive U(1)
  $\Rightarrow$ agreement of 5d effective theories after integrating out massive fields

- recent analysis of geometries without section [V.Braun,Morrison] [Morrison,Taylor]

# Local checks via T-duality

- metric for $T^2$-fibration $Y_3$ without section can locally only brought to form:

$$ds^2(X) = g_{i\bar{j}} du^i d\bar{u}^j + \frac{v^0}{\mathrm{Im}\tau}|X - \tau Y|$$

$$X = dx + \tilde{X} \qquad Y = dy + \tilde{Y}$$

non-trivial $T^2$ KK-vectors

- non-trivial field strength in simplest situation

$$\langle dX \rangle = -n\tilde{\omega} \qquad \langle dY \rangle = 0$$

- M-theory on $Y_3$ connects to Type IIB theory on extra $S^1$ : [Witten]
  - non-vanishing flux $G_3 = F_3 - \tau H_3$ :

    $F_3 = -n\,\tilde{\omega} \wedge dy$
    flux along the circle

  - Fluxed circle reduction induces gauging with Kaluza-Klein vector $A^0$

$$\mathcal{D}c = dc + n\,A^0$$

⟶ gauging of the $C_2$ axion arising from the expansion in $\tilde{\omega}$

# Fluxes circle reduction with massive U(1)

- combining 6d Stückelberg gauging with gauging from fluxed circle reduction:

$$\mathcal{D}c = dc + m\, A_{U(1)} + n\, A^0$$

- Massive $\tilde{A}^0$ and massless $A^{\text{mass}}$ linear combinations:

$$\mathcal{L} = f^2 |\mathcal{D}c|^2 \quad \rightarrow \quad \mathcal{L}_{\text{mass}} = f^2 |\underbrace{m A_{\text{U}(1)} + n A^0}_{A^{\text{mass}}}|$$

- 5d effective theory for massless modes can be compared with M-theory
  - need to compute the charges of the 6d states on $S^1$ under massless $\tilde{A}^0$

$$\tilde{q}_i = q_j N_i^j \qquad N_i^j = \begin{pmatrix} m & n \\ -n & m \end{pmatrix} \rightarrow$$

geometry of manifolds with bi-section:

$m = 2\lambda \quad n = -1$

# Integrating out all massive states

- 5d massive states: - charged 6d matter in the 5d Coulomb branch
  - Kaluza-Klein states to all 6d multiplets

$$m_s = m_{\mathrm{CB}} + m^n_{\mathrm{KK}}$$

→ Coulomb branch mass $\zeta^I q_I$

→ Kaluza-Klein mass (level $n$) $n\, r^{-1}$

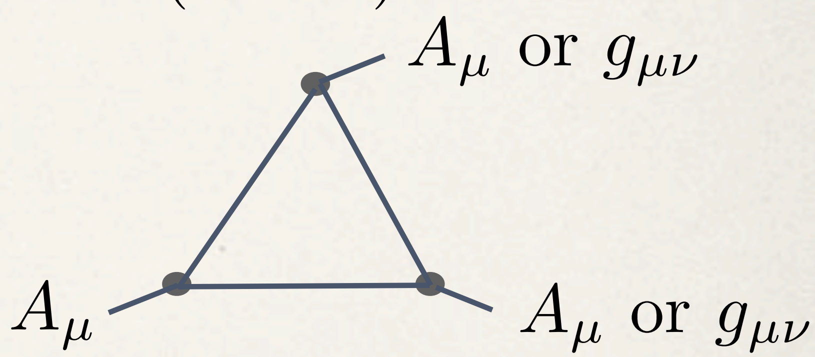
- one-loop Chern-Simons terms `see' massive spectrum:

$$-\frac{1}{48\pi^2} k_{\alpha\beta\gamma} \int_{\mathcal{M}_5} A^\alpha \wedge F^\beta \wedge F^\gamma - \frac{1}{384\pi^2} \kappa_\alpha \int_{\mathcal{M}_5} A^\alpha \wedge \mathrm{Tr}(R \wedge R)$$

$$k_{\alpha\beta\gamma} = \sum_{\text{mass. states}} k_{\mathbf{r}} \cdot q_\alpha\, q_\beta\, q_\gamma\, \mathrm{sign}(m)$$

$$\kappa_\alpha = \sum_{\text{mass. states}} \kappa_{\mathbf{r}} \cdot q_\alpha\, \mathrm{sign}(m)$$

$q_\alpha$ are the charges of the states under the massless 5d U(1)s




| | spin-1/2 | spin-3/2 | tensor |
|---|---|---|---|
| $k_{\mathbf{r}}$ | 1 | 5 | -4 |
| $\kappa_{\mathbf{r}}$ | 1 | -19 | 8 |

[Bonetti,TG,Hohenegger]

# Global checks by computing 1-loop CS terms

- compare 1-loop Chern-Simons terms with intersection numbers and second Chern class for Calabi-Yau examples $Y_3$ without section
  - concrete examples with one massive 6d U(1) have a 'bi-section'
  - to check proposal in global models compute hypermultiplet spectrum charged under massive 6d U(1) by using conifold transition to models with two sections $\Rightarrow$ massless U(1) in 6d $\Rightarrow$ find perfect agreement

    many works on F-theory with multiple U(1)s:

    [TG,Weigand], [Morrison,Park] [Borchmann,Mayrhofer,Palti,Weigand], [V.Braun,TG,Keitel], [Cvetic,Klevers,Piragua,Song] [A.Braun,Collinucci,Valandro] [Kuntzler,Schäfer-Nameki] ...

  - to apply proposal: compute hypermultiplet spectrum by using Chern-Simons terms, i.e. intersection numbers and Chern-classes of $Y_3$

    → more on geometry and the proposal: talk by Iñaki García-Etxebarria

- extension to Calabi-Yau fourfolds with four-form flux is an important open problem $\Rightarrow$ Yukawa couplings and U(1) selection rules?

# Conclusions

- Axions in F-theory
  - F-theory allows to unify axions from bulk supergravity (form-field axions) and seven-brane sector (Wilson line axions) $\Rightarrow$ (2,1)-form axions
  - axion decay constants of (2,1)-form axions depend on complex structure and Kähler moduli $\Rightarrow$ controlled large at special points in complex structure moduli space (special seven-brane configurations, strong coupling effects)
  - (2,1)-form axions: key to understand physics of geometrically massive U(1)s

- Massive U(1)s and F-theory on geometries without section
  - proposal: F-theory effective action for such geometries using M-theory dual
  - crucial to include 6d/4d massive U(1)s perform fluxed circle reduction $\Rightarrow$ Kaluza-Klein vector mixes with U(1)s into massless/massive combination
  - spectrum including modes charged under massive U(1) can still be determined

Workshop:

# Physics and Geometry of F-theory

23 - 26 February 2015

Max-Planck-Institute, Munich