High-energy string collisions

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High energies in string theory

- Explore string theory as a quantum theory of gravity

  Large energies cause a large backreaction of the spacetime.

- Explore the dynamics of the full spectrum of string states

  An infinite number of states of the string spectrum contribute to and are excited in the scattering process.

Well-defined framework: S-matrix, unitarity, UV complete.

String-string collisions

$$\alpha' s \gg 1 \ , \qquad R_g^{d-3} \sim G_d \sqrt{s} \ .$$

- $b \gg b_T \gg R_g$ *elastic scattering*;

- $b_T \geq b \gg R_g$ *string excitations*;

- $b < R_g$, $\begin{cases} R_g \ll l_s & \textit{creation of closed strings}; \\ R_g \gg l_s & \textit{formation of a black-hole}. \end{cases}$

String-brane collisions

$$\alpha' s \gg 1 \ , \qquad \left(\frac{R}{l_s}\right)^{7-p} \sim \ g\,N \ .$$

- $b \gg b_T \gg R$ *elastic scattering*;

- $b_T \geq b \gg R$ *string excitations*;

- $b < R$, $\begin{cases} R \ll l_s & \textit{creation of open strings;} \\ R \gg l_s & \textit{infall of the string into the singularity.} \end{cases}$

The eikonal operator

Start with the Regge limit of the tree-level amplitude (the disk)

$$\mathcal{A}_1(s,t) \sim \Gamma\left(-\frac{\alpha'}{4}t\right) e^{-i\pi\frac{\alpha' t}{4}} \; (\alpha' s)^{1+\frac{\alpha' t}{4}}$$

$$s = E^2 \; , \qquad t = -(p_1 + p_2)^2 \; .$$

Leading behaviour $\Leftrightarrow$ single-Reggeon exchange.

It diverges at high energy $\Rightarrow$ resum the leading terms of the higher-order amplitudes.

Leading term of the annulus amplitude in the Regge limit

$$\frac{\mathcal{A}_2(s,t)}{2E} = \frac{i}{2}\int \frac{d^{8-p}\mathbf{k}_1}{(2\pi)^{8-p}} \, \frac{\mathcal{A}_1(s,t_1)}{2E} \, \frac{\mathcal{A}_1(s,t_2)}{2E} \, V_2(t_1,t_2,t)$$

$$\mathbf{q}^2 = -t \; , \qquad \mathbf{k}_1^2 = -t_1 \, , \qquad \mathbf{k}_2^2 \equiv (\mathbf{q}-\mathbf{k})^2 = -t_2 \; .$$

The momenta in the integral are transverse to both the brane and the collision axis.

Essential step: simple operator representation for the two-Reggeon vertex

$$V_2(t_1, t_2, t) = \int_0^{2\pi} \int_0^{2\pi} \frac{d\sigma_1}{2\pi} \frac{d\sigma_2}{2\pi} \langle 0| : e^{i\mathbf{k}_1 X(\sigma_1)} :: e^{i\mathbf{k}_2 X(\sigma_2)} : |0\rangle \ .$$

The operators $X(\sigma)$ are closed string position operators at $\tau = 0$ and without zero modes in the light-cone gauge.

Leading term of a surface with h boundaries $\Rightarrow$ h-reggeon exchange

$$\frac{\mathcal{A}_h(s,t)}{2E} \sim \frac{1}{h!} \frac{i^{h-1}}{(2E)^h} \prod_{i=1}^{h-1} \int \frac{d^{8-p}\mathbf{k}_i}{(2\pi)^{8-p}} \mathcal{A}_1(s,t_1) ... \mathcal{A}_1(s,t_h) V_h(\mathbf{k}_1, \mathbf{k}_2, ..., \mathbf{k}_h)$$

Vertex for the emission of h Reggeon

$$V_h(\mathbf{k}_1, \ldots, \mathbf{k}_h) = \langle 0| \prod_{i=1}^{h} \int_0^{2\pi} \frac{d\sigma_i}{2\pi} : e^{i\mathbf{k}_i X(\sigma_i)} : |0\rangle$$

In impact parameter space

$$\mathcal{A}(E,b) = \int \frac{d^{8-p}\mathbf{q}}{(2\pi)^{8-p}}\, \mathrm{e}^{i\mathbf{b}\cdot\mathbf{q}}\, \mathcal{A}(E,t)$$

we can sum explicitly the leading terms of all the surfaces with h boundaries

$$i \sum_{h=1}^{\infty} \frac{\mathcal{A}_h(s,\mathbf{b})}{2E} \sim \langle 0| \left[e^{2i\hat{\delta}(s,b)} - 1 \right] |0\rangle$$

The result is the eikonal operator

$$S(s,b) = e^{2i\hat{\delta}(s,b)} \ ,$$

$$2\hat{\delta}(s,b) = \int_0^{2\pi} \frac{d\sigma}{2\pi} \frac{:\mathcal{A}_1\left(s, \mathbf{b} + \mathbf{X}(\sigma)\right):}{2E}$$

Amati, Ciafaloni e Veneziano (1987);
GD, Di Vecchia, Russo e Veneziano (2010).

Two main effects

- Deflection of the trajectory
- Excitation of the internal degrees of freedom of the string: tidal forces

When $b \gg R \gg l_s\sqrt{\ln(\alpha' s)}$

$$2\ \hat{\delta}(s, \mathbf{b} + \hat{\mathbf{X}}) \sim \frac{1}{2E}\left[\mathcal{A}_1(s,b) + \frac{1}{2}\frac{\partial^2 \mathcal{A}_1(s,b)}{\partial b^i \partial b^j}\ \overline{\hat{X}^i \hat{X}^j} + \ldots\right]$$

where $\bar{Q} \equiv \frac{1}{2\pi}\int_0^{2\pi} d\sigma : Q(\sigma) :$. The string position operators are

$$X^i = i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\left(\frac{A_n^i}{n}\mathrm{e}^{in\sigma} + \frac{\bar{A}_n^i}{n}e^{-in\sigma}\right) \quad , \qquad [A_n^i, A_m^j] = n\delta^{ij}\delta_{n+m,0}\ .$$

Elastic loop amplitudes $\Rightarrow$ eikonal operator $\Rightarrow$ inelastic transitions.

The eikonal operator gives a precise microscopic description of the excited string state resulting from the high energy collision.

The eikonal operator and the Green-Schwarz vertex

We need to clarify on which Hilbert space $\mathcal{H}$ the eikonal operator acts.

$\mathcal{H}$: *space of the physical states in a lightcone gauge with the spatial direction of the lightcone aligned to the direction of large momentum*

Light-cone vectors

$$\sqrt{2}e^{-} = \lim_{E_1\to\infty} \frac{p_1}{E_1} = - \lim_{E_2\to\infty} \frac{p_2}{E_2} \,, \qquad e^{+}e^{-} = 1.$$

The basis also contains eight spacelike vectors ϵ^{i} orthogonal to $e^{\pm}$.

Eikonal phase in momentum space

$$W(s,\bar{q}) = \mathcal{A}(s,\bar{q}) \int \frac{d\sigma}{2\pi} : e^{i\bar{q}X(\sigma)} : \ .$$

- Regge limit of the 3-string GS vertex with two states on-shell and one off-shell;
- contraction of the off-shell string with a closed string propagator and the boundary state.

The eikonal operator and the inelastic amplitudes

Matrix elements of the eikonal operator $\Rightarrow$ high-energy behaviour of the inelastic amplitudes.

The amplitudes involve $\left\{ \begin{array}{l} \text{four (string} - \text{string)} \\ \\ \text{two (string} - \text{brane)} \end{array} \right.$ arbitrary string states.

Two interesting features of the eikonal operator

- the string modes appear as a simple shift of the impact parameter b by the string position operator X;
- it does not contain the light-cone modes of the fermionic fields.

The light-cone basis

$$W(s,\bar{q}) \;=\; \mathcal{A}(s,t)\int_0^{2\pi}\frac{d\sigma}{2\pi}\,:\mathrm{e}^{i\bar{q}\hat{X}}:\equiv \mathcal{A}(s,t)\sum_{n,m=0}^{\infty}\Delta_{n,m}(\bar{q})\,\bar{\Delta}_{n,m}(\bar{q})\ .$$

The operators $\Delta_{n,m}$ generate all the transitions between an initial level m and a final level n.

For example (for simplicity we display only the holomorphic part)

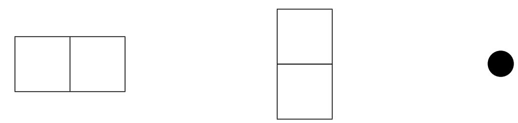
$$\begin{aligned}\Delta_{1,0} &= -\sqrt{\frac{\alpha'}{2}}\,\bar{q}^i\,A^i_{-1}\ ,\\ \Delta_{2,0} &= \frac{\alpha'}{4}\bar{q}^i\bar{q}^j\,A^i_{-1}A^j_{-1}-\sqrt{\frac{\alpha'}{8}}\bar{q}^i\,A^i_{-2}\ .\end{aligned}$$

In the light-cone gauge the massless NS state is a vector of $SO(8)$

$$|\epsilon\rangle = \epsilon_i B^i_{-1/2}|0\rangle\ .$$

In general $SO(8)$ polarization tensors ω corresponding to Young diagrams and normalized, $\omega\cdot\omega = 1$.

Transitions to the first level

64 states 

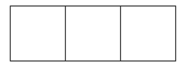
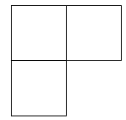
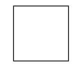
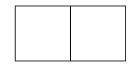
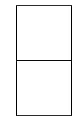
$SO(8)$ representation	Matrix element $\langle\omega\|\Delta_{10}\|\epsilon\rangle$
$\|\omega^{(2)}\rangle = \omega^{(2)}_{ij} A^i_{-1} B^j_{-\frac{1}{2}}\|0\rangle$	$-\sqrt{\frac{\alpha'}{2}}\,\epsilon^i\,\omega^{(2)}_{ij}\,\bar{q}^j$
$\|\omega^{(1,1)}\rangle = \omega^{(1,1)}_{ij} A^i_{-1} B^j_{-\frac{1}{2}}\|0\rangle$	$\sqrt{\frac{\alpha'}{2}}\,\epsilon^i\,\omega^{(1,1)}_{ij}\,\bar{q}^j$
$\|\omega^{(0)}\rangle = \frac{1}{\sqrt{8}} A^i_{-1} B^i_{-\frac{1}{2}}\|0\rangle$	$-\frac{\sqrt{\alpha'}}{4}\,\epsilon\bar{q}$

The transitions to the remaining 64 NS states of the first level

$$|\omega^{(1,1,1)}\rangle = \frac{1}{\sqrt{6}}\omega^{(1,1,1)}_{ijk} B^i_{-\frac{1}{2}} B^j_{-\frac{1}{2}} B^k_{-\frac{1}{2}}|0\rangle \ , \qquad |\omega^{(1)}\rangle = \omega_i B^i_{-\frac{3}{2}}|0\rangle \ ,$$

are subleading in energy.

Transitions to the second level

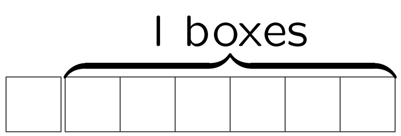
352 states   $2 \times$    $\bullet$

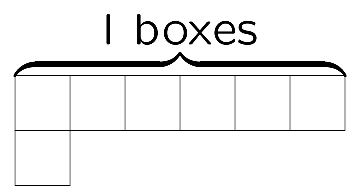
$SO(8)$ representation	Matrix element
$\lvert\omega^{(3)}\rangle = \frac{1}{\sqrt{2}}\omega^{(3)}_{ijk} A^i_{-1} A^j_{-1} B^k_{-\frac{1}{2}} \lvert 0\rangle$	$\frac{\alpha'}{\sqrt{8}}\epsilon^i\, \omega^{(3)}_{ijk}\bar{q}^j \bar{q}^k$
$\lvert\omega^{(2,1)}\rangle = \sqrt{\frac{2}{3}}\omega^{(2,1)}_{ij;k} A^i_{-1} A^j_{-1} B^k_{-\frac{1}{2}} \lvert 0\rangle$	$-\frac{\alpha'}{\sqrt{6}}\epsilon^i\, \omega^{(2,1)}_{ij;k}\bar{q}^j \bar{q}^k$
$\lvert\omega^{(2)}\rangle = \frac{1}{\sqrt{2}}\omega^{(2)}_{ij} A^i_{-2} B^j_{-\frac{1}{2}} \lvert 0\rangle$	$\frac{\sqrt{\alpha'}}{2}\epsilon^i\, \omega^{(2)}_{ij}\bar{q}^j$
$\lvert\omega^{(1,1)}\rangle = \frac{1}{\sqrt{2}}\omega^{(1,1)}_{ij} A^i_{-2} B^j_{-\frac{1}{2}} \lvert 0\rangle$	$-\frac{\sqrt{\alpha'}}{2}\epsilon^i\, \omega^{(1,1)}_{ij}\bar{q}^j$
$\lvert\omega^{(1)}\rangle = -\frac{\omega_i}{4\sqrt{35}}\left[8 A^i_{-1} A^j_{-1} B^j_{-\frac{1}{2}} - A^j_{-1} A^j_{-1} B^i_{-\frac{1}{2}}\right] \lvert 0\rangle$	$-\frac{\alpha'}{\sqrt{35}}\left(\epsilon\bar{q}\omega\bar{q} + \frac{\alpha' t}{8}\epsilon\omega\right)$
$\lvert\lambda^{(1)}\rangle = \frac{\lambda_i}{4} A^j_{-1} A^j_{-1} B^i_{-\frac{1}{2}} \lvert 0\rangle$	$-\frac{\alpha' t}{8}\epsilon\lambda$
$\lvert\omega^{(0)}\rangle = \frac{1}{4} A^i_{-2} B^i_{-\frac{1}{2}} \lvert 0\rangle$	$-\frac{\sqrt{\alpha'}}{4\sqrt{2}}\epsilon\bar{q}$

Pattern of the representations and couplings

Transitions from the massless sector.

At level l one obtains all the $SO(8)$ representations and couplings present at level $l-1$ together with two new rank-$(l+1)$ $GL(8)$ tensors of symmetry type $(l+1)$ and $(l,1)$





The amplitudes contain

$$\epsilon^k \omega_{k i_1 \ldots 1_{n-1}} \bar{q}^{i_1} \ldots \bar{q}^{i_{n-1}} t^a \ , a \in \mathbb{N} \ ,$$

$$\epsilon^k \bar{q}_k \omega_{i_1 \ldots 1_n} \bar{q}^{i_1} \ldots \bar{q}^{i_n} t^b \ , \qquad b \in \mathbb{N} \ .$$

What is the covariant dynamics responsible for the simple properties of the eikonal operator?

Massive polarizations

For every massive state with a non-zero time-like momentum p_r^μ (spatial part $\vec{p}_r$) we define the longitudinal polarization vector v_r

$$v_r^\mu = -\frac{m_r}{|\vec{p}_r|}\hat{t}^\mu + \frac{E_r}{|\vec{p}_r|}\frac{p_r^\mu}{m_r} = \frac{|\vec{p}_r|}{m_r}\hat{t}^\mu + \frac{E_r}{m_r}\hat{p}_r^\mu \ , \qquad v_r p_r = 0 \ , \qquad v_r^2 = 1 \ ,$$

where $\hat{t}$ is the unit vector in the time direction.

Using p_r, v_r and eight spacelike unit vectors $\epsilon_{r,k}$ transverse both to p_r and v_r we can form a convenient basis for the polarization tensors of the massive states.

We will use this basis together with the physical state conditions and the high energy limit to express the covariant amplitudes in terms of spacelike tensors orthogonal to the collision axis.

The covariant dynamics: Reggeon exchange

Two-point amplitudes in the background of a stack of branes

$$\mathcal{A}(s,t) \sim \Pi_R^{D_p}(s,t)\, C_{12R}\, \bar{C}_{12R}\ .$$

Four-point amplitudes

$$|t'| = |p_1 + p_3|^2 \ll |s'| = |p_1 + p_2|^2\ ,$$

$$\mathcal{A}(s',t') \sim \Pi_R(s',t')\, C_{13R}\, C_{24R}\ .$$

Three steps

- Regge limit $\sim$ limit of short worldsheet distances
- Identification of the intermediate states giving the leading behaviour
- Sum over intermediate states $\sim$ single local operator

Notation for the vertex operators

$$\mathcal{V}_{(\mathbf{S},\bar{\mathbf{S}})} = \frac{\kappa_{10}}{2\pi} \mathbf{V_S}\ \bar{\mathbf{V}}_{\bar{\mathbf{S}}} = \frac{\kappa_{10}}{2\pi} \epsilon_{\mu_1\ldots\mu_k} V_S^{\mu_1\ldots\mu_k} \quad \bar{\epsilon}_{\nu_1\ldots\nu_l} V_{\bar{S}}^{\nu_1\ldots\nu_l} \ ,$$

$$\epsilon_{\mu_1\ldots\mu_k}\epsilon^{\mu_1\ldots\mu_k} = \bar{\epsilon}_{\mu_1\ldots\mu_l}\bar{\epsilon}^{\mu_1\ldots\mu_l} = 1 \ .$$

V_S is a polynomial in $\partial^r X^\mu$, $\partial^s \psi^\nu$ times an exponential e^{ipX}

First step

$$\left\langle \prod_{i=1}^{4} \mathrm{e}^{ip_i X_i} \right\rangle \sim \mathrm{e}^{-\frac{\alpha' t'}{4} \ln|z|^2 - \frac{\alpha' s'}{4} \ln|1-z|^2} \ , \qquad z = \frac{z_{13} z_{24}}{z_{14} z_{23}} \ .$$

In the limit $\alpha' s' \gg 1$, $\frac{t'}{s'} \ll 1$ it is dominated by $\alpha' s' |z|^2 \leq 1$.

Disk amplitude with two closed strings

$$A_{12} = \frac{\alpha'}{8\pi} \int d^2 z \, \langle 0 | \mathcal{V}^{(-1,-1)}_{(S_1,\bar{S}_1)} \, \mathcal{V}^{(0,0)}_{(S_2,\bar{S}_2)} z^{L_0 - 1} \bar{z}^{L_0 - 1} | D_p \rangle \ .$$

Second step

$$A_{12} = \frac{\alpha'}{8\pi} \sum_{(l,n_l,\bar{n}_l)} \int d^2 z \, (\mathbf{z\bar{z}})^{\mathbf{l-1-\frac{\alpha' t}{4}}} \left\langle \mathcal{V}^{(-1,-1)}_{(S_1,\bar{S}_1)} \, \mathcal{V}^{(0,0)}_{(S_2,\bar{S}_2)} \, \mathcal{V}^{(-1,-1)}_{l,n_l,\bar{n}_l} \right\rangle_{\mathcal{S}} \left\langle \mathcal{V}^{(-1,-1)}_{l,n_l,\bar{n}_l} \right\rangle_{D_p} \, .$$

Two sources for the factors of the energy

a) contractions of $\partial^r X^+$ with the exponentials

$$\sqrt{\frac{2}{\alpha'}} \partial^r X^+ \, \mathrm{e}^{ip_1 X} \sim \sqrt{\alpha'} E \; \partial_z^{r-1} \left(\frac{1}{z}\right) \, \mathrm{e}^{ip_1 X} \; ,$$

b) contractions of $\partial^r X^+$, $\partial^s \psi^+$ with the polynomial part of the vertices

$$i \sqrt{\frac{2}{\alpha'}} \partial^r X^+ \, i \partial^s X^\rho \sim \alpha' s \, \frac{v^\rho}{m} \, \partial_z^{r-1} \partial_w^{s-1} \left(\frac{1}{z-w}\right)^2 \quad ,$$

$$\partial^r \psi^+ \partial^s \psi^\rho \sim \sqrt{\alpha'} E \sqrt{\frac{2}{\alpha'}} \frac{v^\rho}{m} \, \partial_z^r \partial_w^s \left(\frac{1}{z-w}\right) \; .$$

The leading behaviour of the amplitude is due to the intermediate states

$$\mathcal{Q}_l = Q_l \bar{Q}_l \ , \qquad Q_l = \frac{1}{\sqrt{l!}} \psi^+ \left(i \sqrt{\frac{2}{\alpha'}} \partial X^+ \right)^l \mathrm{e}^{ikX} \ , \qquad l \geq 0 \ .$$

$$\mathcal{A}_{12} = \frac{\alpha'}{8\pi\kappa_{10}} \frac{\pi^{\frac{9-p}{2}}}{\Gamma(\frac{7-p}{2})} R_p^{7-p} \sum_{l=0}^{\infty} \int d^2 z \ (z\bar{z})^{l-2-\frac{\alpha' t}{4}} \ \left\langle \mathcal{V}^{(-1,-1)}_{(S_1,\bar{S}_1)} \mathcal{V}^{(0,0)}_{(S_2,\bar{S}_2)} \mathcal{Q}_l \right\rangle_{\mathcal{S}} \ .$$

Third step: perform the sum and integrate

$$\mathcal{A}_{12} = \Pi_R^{D_p}(s,t)\, C_{S_1,S_2,R}\, \bar{C}_{\bar{S}_1,\bar{S}_2,\bar{R}} \ ,$$

$$C_{S_1,S_2,R} = \left\langle V_{S_1}^{(-1)} V_{S_2}^{(0)} V_R^{(-1)} \right\rangle \ ,$$

$$\Pi_R^{D_p} = \mathcal{A}_1(s,t) = \frac{\pi^{\frac{9-p}{2}}}{\Gamma(\frac{7-p}{2})} R_p^{7-p}\, \Gamma\left(-\frac{\alpha' t}{4}\right) \mathrm{e}^{-i\pi\frac{\alpha' t}{4}} (\alpha' s)^{1+\frac{\alpha' t}{4}} \ .$$

The Reggeon vertex

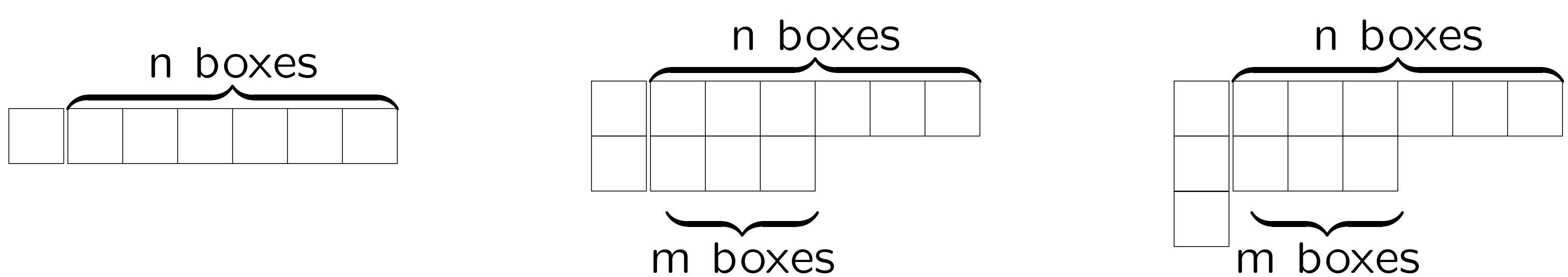
$$V_R^{(-1)} = \frac{\psi^+}{\sqrt{\alpha'}E}\left(\sqrt{\frac{2}{\alpha'}}\frac{i\partial X^+}{\sqrt{\alpha'}E}\right)^{\frac{\alpha' t}{4}} \mathrm{e}^{-iqX} \ .$$

Although it carries an off-shell momentum, the Reggeon vertex is a superconformal primary of dimension one half in the high-energy limit
Ademollo, Bellini, Ciafaloni (1989);
Brower, Polchinski, Strassler, Tan (2007)

Picture zero

$$V_R^{(0)} = \left[-\frac{2}{\alpha'}\frac{\partial X^+ \partial X^+}{\alpha' E^2} - iq\psi\frac{\psi^+\partial X^+}{\alpha' E^2} - \frac{\alpha' t}{4}\frac{\psi^+\partial\psi^+}{\alpha' E^2}\right]\left(\sqrt{\frac{2}{\alpha'}}\frac{i\partial X^+}{\sqrt{\alpha'}E}\right)^{\frac{\alpha' t}{4}-1} \mathrm{e}^{-iqX_L} \, .$$

Transitions from the massless sector



From the Reggeon to the eikonal operator via DDF

We can write the phase of the eikonal operator as follows

$$W_R(s,q) = \Pi_R^{D_p} \sum_{i,\bar{i},j,\bar{j}} C_{(S_i,S_{\bar{i}}),(S_j,S_{\bar{j}}),R} \, |S_i, S_{\bar{i}}\rangle \langle S_j, S_{\bar{j}}| \ ,$$

Two special bases

- Covariant (manifest $SO(9-p)$)
- DDF (manifest $SO(8-p)$)

Using the DDF basis one can show that

$$W_R(s,q) \ = \ \mathcal{A}(s,t) \int_0^{2\pi} \frac{d\sigma}{2\pi} \ : \mathrm{e}^{i\bar{q}X} : \ .$$

DDF operators

$$|p_T;0\rangle\ ,\qquad p_T^2=1\ ,\qquad k^2=0\ ,\qquad kp_T=1\ .$$

Bosonic and fermionic modes for the NS sector

$$A_{-n,j}=-i\oint_0 dz\ (\epsilon_j)_\mu\left(\partial X^\mu+in(k\psi)\psi^\mu\right)\mathrm{e}^{-inkX(z)}\ ,$$

$$B_{-r,j}=i\oint_0 dz\ (\epsilon_j)_\mu\left(\partial X^\mu(k\psi)-\psi^\mu(k\partial X)+\frac{1}{2}\psi^\mu(k\psi)\frac{(k\partial\psi)}{(k\partial X)}\right)\frac{\mathrm{e}^{-irkX(z)}}{(ik\partial X)^{\frac{1}{2}}}\ ,$$

In the Regge limit

$$\begin{aligned}
V_R^{(0)}(z) &\sim \left(\sqrt{\frac{2}{\alpha'}}\frac{i\partial X^+(z)}{\sqrt{\alpha'}E}\right)^{\frac{\alpha' t}{4}+1}\mathrm{e}^{-iqX(z)}\ ,\\
A_{-n,j}(z) &\sim -i\sqrt{\frac{2}{\alpha'}}\oint_z dw\ (\epsilon_j)_\mu\partial X^\mu\mathrm{e}^{-inkX}\ ,\\
B_{-r,j}(z) &\sim -i\oint_z dw\ (\epsilon_j)_\mu\psi^\mu(ik\partial X)^{\frac{1}{2}}\mathrm{e}^{-irkX}\ .
\end{aligned}$$

The Regge limit of the inelastic amplitudes

Three-point couplings

$$C_{S_1,S_2,R} = \epsilon_{\mu_1\ldots\mu_r}\ \zeta_{\nu_1\ldots\nu_s}\ T^{\mu_1\ldots\mu_r;\nu_1\ldots\nu_s}_{S_1,S_2,R} \quad .$$

Transitions from the ground state

$$\mathcal{A}_{g,(S,\bar{S})} = \Pi^{D_p}_{R} C_{g,S,R} \bar{C}_{g,\bar{S},R} \ .$$

Let us start from the elastic amplitude. The massless vertex in the -1 picture is

$$V^{\mu}_{g}(p) = \psi^{\mu} \mathrm{e}^{ipX}\, \mathrm{e}^{-\varphi} \ ,$$

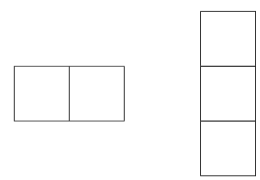
and we find

$$T^{\mu;\rho}_{g,g,R} = \eta^{\mu\rho} \ .$$

$$\mathcal{A}_{g,g} = \Pi^{D_p}_{R} \epsilon_{\mu\nu} \zeta^{\mu\nu} \ .$$

First massive level

The NS sector of the first massive level contains 128 bosonic physical states: a rank-2 traceless totally symmetric tensor S_2 (44 components) and a rank-3 totally antisymmetric tensor A_3 (84 components)



The corresponding vertex operators in the -1 picture are

$$
\begin{aligned}
V_{S_2}^{\rho\alpha} &= i\sqrt{\frac{2}{\alpha'}}\,\psi^\rho \partial X^\alpha \mathrm{e}^{ipX}\,\mathrm{e}^{-\varphi}\ , \\
V_{A_3}^{\rho\alpha\gamma} &= \frac{1}{\sqrt{3!}}\,\psi^\rho \psi^\alpha \psi^\gamma \mathrm{e}^{ipX}\,\mathrm{e}^{-\varphi}\ .
\end{aligned}
$$

Transition g $\to S_2$

$$T^{\mu;\rho\alpha}_{g,S_2,R} = -\sqrt{\frac{\alpha'}{2}}\left[\eta^{\mu\rho}\left(q^\alpha - \frac{2}{\alpha'}\left(1+\frac{\alpha' t}{4}\right)\frac{v^\alpha}{m}\right) + \frac{q^\mu}{m}v^\rho\left(q^\alpha - \frac{t}{2m}v^\alpha\right)\right] \ .$$

$$Q^\alpha = q^\alpha - \frac{t}{2m}v^\alpha \ , \qquad \bar{q}^\alpha = q^\alpha - \frac{m}{2}\left(1+\frac{t}{m^2}\right)v^\alpha \ , \qquad \delta^{\mu\rho}_\perp = \eta^{\mu\rho} + \frac{q^\mu}{m}v^\rho \ .$$

$$T^{\mu;\rho\alpha}_{g,S_2,R} = -\sqrt{\frac{\alpha'}{2}}\left[\delta^{\mu(\rho}_\perp \bar{q}^{\alpha)} + \frac{q^\mu}{2}v^\rho v^\alpha\right] \ .$$

Transition g $\to A_3$

$$T^{\mu;\rho\alpha\gamma}_{g,A_3,R} = \frac{\sqrt{6}}{m}\eta^{\mu[\rho}q^\alpha v^{\gamma]} \ ,$$

$$T^{\mu;\rho\alpha\gamma}_{g,A_3,R} = \frac{\sqrt{6}}{m}\delta^{\mu[\rho}_\perp \bar{q}^\alpha v^{\gamma]} \ .$$

$SO(9) \Rightarrow SO(8)$

Decompose the $SO(9)$ tensors with respect to the transverse $SO(8)$

$$\zeta^{S} \mapsto \sum \zeta^{S,(n_1,n_2,\ldots,n_r)} \ .$$

d_r multiplicity of the representation r of $SO(8)$
c_r number of independent couplings
$(d_r - c_r)$ linear combinations with couplings subleading in energy.

For S_2

$$\zeta^{S_2,(2)}_{\rho\alpha} = \omega^{(2)}_{\rho\alpha} \ , \quad \zeta^{S_2,(1)}_{\rho\alpha} = \sqrt{2}\,\omega_{(\rho}v_{\alpha)} \ , \quad \zeta^{S_2,(0)}_{\rho\alpha} = \frac{1}{3\sqrt{8}}\left(-\delta^{\rho\alpha}_{\perp} + 8v^{\rho}v^{\alpha}\right) \ ,$$

$$\begin{aligned}
T^{\mu,\rho\alpha}_{g,S_2,R}\ \zeta^{S_2,(2)}_{\rho\alpha} &= -\sqrt{\frac{\alpha'}{2}}\delta^{\mu\rho}_{\perp}\omega^{(2)}_{\rho\alpha}\bar{q}^{\alpha} \ , \\
T^{\mu,\rho\alpha}_{g,S_2,R}\ \zeta^{S_2,(1)}_{\rho\alpha} &= 0 \ , \\
T^{\mu,\rho\alpha}_{g,S_2,R}\ \zeta^{S_2,(0)}_{\rho\alpha} &= -\frac{\sqrt{\alpha'}}{4}\bar{q}^{\mu} \ .
\end{aligned}$$

For A_3

$$\zeta^{A_3,(1,1,1)}_{\rho\alpha\gamma} = \omega^{(1,1,1)}_{\rho\alpha\gamma} \ , \qquad \zeta^{A_3,(1,1)}_{\rho\alpha\gamma} = \sqrt{3}\,\omega^{(1,1)}_{[\rho\alpha}v_{\gamma]} \ .$$

$$\begin{aligned} T^{\mu,\rho\alpha\gamma}_{g,A_3,R}\ \zeta^{A_3,(1,1,1)}_{\rho\alpha\gamma} &= 0 \ , \\ T^{\mu,\rho\alpha\gamma}_{g,A_3,R}\ \zeta^{A_3,(1,1)}_{\rho\alpha\gamma} &= \sqrt{\frac{\alpha'}{2}}\delta^{\mu\rho}_{\perp}\omega^{(1,1)}_{\rho\alpha}\bar{q}^{\alpha} \ . \end{aligned}$$

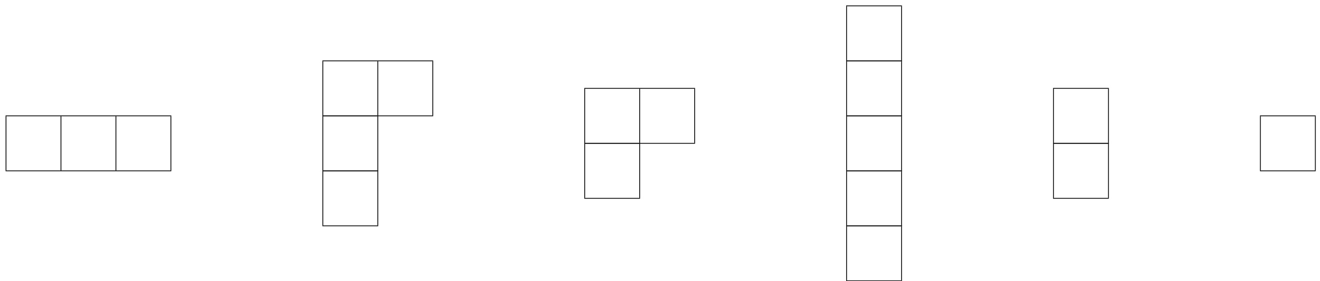
The states in the first massive level that can be excited in the high energy scattering of a massless NS-NS state on a stack of Dp-branes are therefore

$$S = \zeta^{\mathcal{S}_2,(2)}S_2 \ , \quad A = \zeta^{\mathcal{A}_3,(1,1)}A_3 \ , \quad I = \zeta^{\mathcal{S}_2,(0)}S_2 \ ,$$

for a total of 64 degrees of freedom. Perfect agreement with the results obtained in the light-cone gauge.

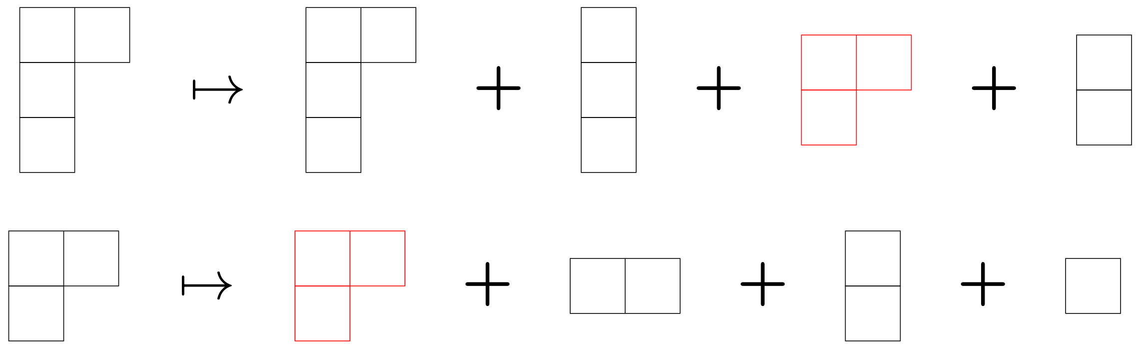
Second massive level

The NS sector of the second massive level contains 1152 bosonic physical states in the following six irreducible representations of $SO(9)$



$$
\begin{aligned}
V_Z^{\rho\alpha\gamma} &= -\frac{1}{\sqrt{2}}\frac{2}{\alpha'}\partial X^\rho \partial X^\alpha \psi^\gamma \, \mathrm{e}^{ipX}\, \mathrm{e}^{-\varphi} \quad , \\
V_Y^{\rho\alpha\gamma\omega} &= i\sqrt{\frac{3}{8}}\sqrt{\frac{2}{\alpha'}}\partial X^{(\rho}\psi^{\alpha)}\psi^\gamma\psi^\omega \, \mathrm{e}^{ipX}\, \mathrm{e}^{-\varphi} \quad , \\
V_U^{\rho\alpha\gamma} &= -\frac{1}{\sqrt{6}}\left[\frac{2}{\alpha'}\partial X^\rho \partial X^\alpha \psi^\gamma + 2\partial\psi^{(\rho}\psi^{\alpha)}\psi^\gamma\right] \mathrm{e}^{ipX}\, \mathrm{e}^{-\varphi} \quad , \\
V_X^{\rho\alpha\gamma\omega\xi} &= \frac{1}{\sqrt{5!}}\psi^\rho\psi^\alpha\psi^\gamma\psi^\omega\psi^\xi \, \mathrm{e}^{ipX}\, \mathrm{e}^{-\varphi} \quad , \\
V_V^{\gamma\omega} &= \frac{i}{5}\sqrt{\frac{2}{7}}\sqrt{\frac{2}{\alpha'}}\left[\left(\widehat{\eta}_{\rho\alpha}\partial X^\rho\psi^\alpha\right)\psi^\gamma\psi^\omega - \frac{7}{2}\left(\partial^2 X^\gamma\psi^\omega - 2\partial X^\gamma \partial\psi^\omega\right)\right] \mathrm{e}^{ipX}\, \mathrm{e}^{-\varphi} \\
V_W^{\gamma} &= \frac{\widehat{\eta}_{\alpha\rho}}{8\sqrt{22}}\left[-\frac{2}{\alpha'}\partial X^\gamma\partial X^\alpha\psi^\rho + 5\frac{2}{\alpha'}\partial X^\rho\partial X^\alpha\psi^\gamma + 11\partial\psi^\rho\psi^\alpha\psi^\gamma\right] \mathrm{e}^{ipX}\, \mathrm{e}^{-\varphi} \quad .
\end{aligned}
$$

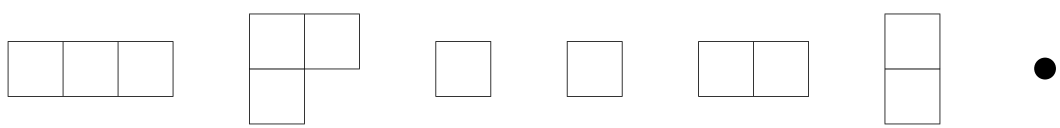
An example of a degenerate representation



$$H_1 = \frac{1}{2}\left(\zeta^{Y,(2,1)}\,Y - \sqrt{3}\zeta^{U,(2,1)}\,U\right) \;, \qquad C_{g,H_1,R} = 0 \;,$$
$$H_2 = \frac{1}{2}\left(\sqrt{3}\zeta^{Y,(2,1)}\,Y + \zeta^{U,(2,1)}\,U\right) \;, \qquad C_{g,H_2,R} = -\frac{\alpha'}{\sqrt{6}}\epsilon^{\alpha}\omega^{(2,1)}_{\alpha\rho;\gamma}\bar{q}^{\rho}\bar{q}^{\gamma} \;.$$

Proceeding in a similar way, we find a total of 352 degrees of freedom

$$F \;, \quad H_2 \;, \quad B_3 \;, \quad B_4 \;, \quad S_2 \;, \quad A_2 \;, \quad I_2 \;,$$



Conclusions

- The leading eikonal operator represents one of the rare examples of resummation of the complete perturbative series of string theory.
- We clarified its structure, analyzing it from various point of view. We provided a simple derivation of the eikonal using the Reggeon vertex and the DDF operators.
- We derived the asymptotic high-energy behaviour of the covariant string amplitudes with massive states, explaining how to take into account the longitudinal polarizations in a simple way.
- Subtle interplay between α' and m in the high-energy couplings. Interesting lessons for higher spin interactions?
- A step towards deriving the complete form of the eikonal operator at the first subleading order in $\frac{R}{b}$.