Vacua of β-supergravity

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arXiv:1306.4381, 1402.5972 and work in progress
by D. A. and André Betz

Frontiers in String Phenomenology,
01/08/2014, Ringberg Castle, Tegernsee, Germany

Introduction

NSNS sector in 10D and 4D supergravities

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10D standard supergravity: $\frac{\mathcal{L}_{\rm NSNS}}{e^{-2\phi}\sqrt{|g|}} = \mathcal{R}(g) + 4(\partial\phi)^2 - \frac{H^2}{2}$,

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4D gauged supergravity: also non-geometric fluxes $Q_a{}^{bc}, R^{abc}$

all fluxes are components of the embedding tensor

$\Leftrightarrow$ some "structure constants" in gauging algebra

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Terms/fluxes related by 4D T-duality chain

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10D origin of Q and R? Naively not present in $\mathcal{L}_{\rm NSNS}$

+ argued to descend from 10D non-geometric backgrounds...

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Fields glue on overlaps:
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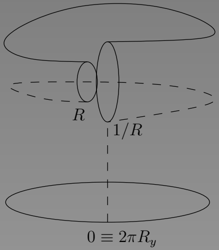

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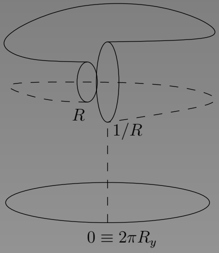

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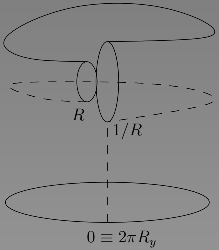

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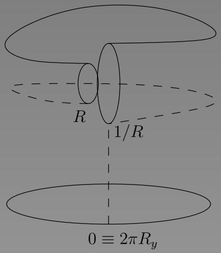

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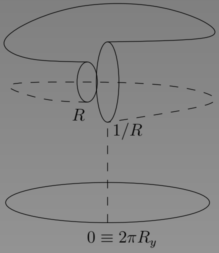

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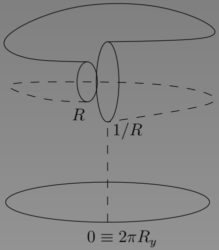

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Relation to conditions for supersymmetric vacua with SU(3)×SU(3) structure.

β-supergravity

Lagrangian $\tilde{\mathcal{L}}_\beta(f, Q, R)$

Reformulation of $\mathcal{L}_{\mathrm{NSNS}}$ via a field redefinition

Build on earlier results: arXiv:1303.0251 by D. A.

arXiv:1106.4015 by D. A., M. Larfors, D. Lüst, P. Patalong

arXiv:1202.3060, arXiv:1204.1979 by D. A., O. Hohm, M. Larfors, D. Lüst, P. Patalong

See also related work in: arXiv:1210.1591, arXiv:1211.0030, arXiv:1304.2784

by R. Blumenhagen, A. Deser, E. Plauschinn, F. Rennecke, C. Schmid

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Lagrangian $\tilde{\mathcal{L}}_\beta(f, Q, R)$

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β w.r.t. Q, R: motivations from Gen. Complex Geom./sugra

hep-th/0609084, arXiv:0708.2392 by P. Grange, S. Schäfer-Nameki

arXiv:0807.4527 by M. Graña, R. Minasian, M. Petrini, D. Waldram

Apply the field redefinition to $\mathcal{L}_{\text{NSNS}}(g, b, \phi)$

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arXiv:0807.4527 by M. Graña, R. Minasian, M. Petrini, D. Waldram
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10D theory with non-geometric fluxes (β-supergravity),
uplift of 4D gauged supergravity ✓

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Structures: natural with Generalized Geometry formalism

David
ANDRIOT

10D examples of non-geometries

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Famous toroidal example: NSNS sector:

$$T^3 + H_{123} \xleftrightarrow{T_1} \text{twisted torus } ({f^1}_{23}) \xleftrightarrow{T_2} \text{non} - \text{geometric config.}$$

hep-th/0211182 by S. Kachru, M. B. Schulz, P. K. Tripathy, S. P. Trivedi

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$$g = h_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{h_0} \end{pmatrix} ,\ b = h_0 \begin{pmatrix} 0 & -Kz^3 & 0 \\ Kz^3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ,\ h_0(z^3) = \frac{1}{1 + (Kz^3)^2}$$

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arXiv:1402.5972 by D. A. and André Betz

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David ANDRIOT

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β-supergravity has 10D non-geometric fluxes,
it restores geometry $\Rightarrow$ alternative description, compactif. ✓

Bianchi identities and *NS*-branes

Bianchi identities for the fluxes

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4D, constant fluxes: gaugings of gauged supergravity

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Automatically satisfied with 10D expressions of f, Q, R

More involved expressions for Bianchi identities

arXiv:1205.1522 by R. Blumenhagen, A. Deser, E. Plauschinn and F. Rennecke

NS-branes as sources

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$$NS5\text{-brane} \xleftrightarrow[\text{+T-d.}]{\text{smearing}} KK\text{-monopole} \xleftrightarrow[\text{+T-d.}]{\text{smearing}} 5^2_2 \text{ or } Q\text{-brane}$$

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$NS5$-brane: $\mathrm{d}H \sim \mathrm{vol}_4\ \delta^{(4)}(r_4)$

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see also arXiv:0706.3049 by G. Villadoro and F. Zwirner

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On brane solutions $\Rightarrow$ Poisson equations

KK-monopole: $\Delta_3 f_K = c_K\ \delta^{(3)}(r_3)$

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Generalized derivative $\mathcal{D}$

10D standard supergravity: $\mathcal{D}^2 = 0 \Leftrightarrow$ Bianchi identities

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hep-th/0607015 by J. Shelton, W. Taylor, B. Wecht
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$$\mathcal{D} = 2(\partial_a \cdot \tilde{e}^a \wedge + \beta^{ab}\partial_b \cdot \iota_a - \tfrac{1}{2} f^c{}_{ab}\, \tilde{e}^a \wedge \tilde{e}^b \wedge \iota_c - \tfrac{1}{2} Q_a{}^{bc}\, \tilde{e}^a \wedge \iota_b\, \iota_c$$
$$+ Q_d{}^{dc}\, \iota_c + \tfrac{1}{6} R^{abc}\, \iota_a\, \iota_b\, \iota_c - \partial_a\tilde{\phi}\, \tilde{e}^a \wedge - (\beta^{ab}\partial_b\tilde{\phi} - \mathcal{T}^a)\, \iota_a)$$

$$\mathcal{D}A = 2e^{\tilde{\phi}}(\nabla_a \cdot \tilde{e}^a \wedge - \check{\tilde{\nabla}}^a \cdot \iota_a + \mathcal{T} \vee + R\vee)(e^{-\tilde{\phi}}A)$$

$\mathcal{D}$ Dirac operator for $Spin(d,d) \times \mathbb{R}^+$ cov. derivative:

$$\mathcal{D}\Psi = \Gamma^A D_A \Psi = \Gamma^A \Big(\partial_A + \tfrac{1}{4} \Omega_A{}^D{}_C \Gamma^{BC} \eta_{DB} - \tfrac{1}{2} \Lambda_A \Big) \Psi$$

Generalized derivative $\mathcal{D}$

10D standard supergravity: $\mathcal{D}^2 = 0 \Leftrightarrow$ Bianchi identities

$$\mathcal{D}A = 2e^{\phi}(\mathrm{d} - H\wedge)(e^{-\phi}A)$$
$$= 2\left(\partial_a \cdot e^a \wedge - \tfrac{1}{2} f^c{}_{ab} e^a \wedge e^b \wedge \iota_c - \tfrac{1}{6} H_{abc} e^a \wedge e^b \wedge e^c \wedge - \partial_a\phi\ e^a \wedge\right) A$$

where $V \vee A = V^a \iota_a A = \frac{1}{(p-1)!} V^{m_1} A_{m_1\dots m_p} \mathrm{d}x^{m_2} \wedge \dots \wedge \mathrm{d}x^{m_p}$

In 4D:

hep-th/0607015 by J. Shelton, W. Taylor, B. Wecht
arXiv:0705.3410 by M. Ihl, D. Robbins, and T. Wrase

$$\mathcal{D}_\sharp A = \Big(- \tfrac{1}{6} H_{abc} \tilde{e}^a \wedge \tilde{e}^b \wedge \tilde{e}^c \wedge - \tfrac{1}{2} f^a{}_{bc}\ \tilde{e}^b \wedge \tilde{e}^c \wedge \iota_a$$
$$- \tfrac{1}{2} Q_c{}^{ab}\ \tilde{e}^c \wedge \iota_a\ \iota_b + \tfrac{1}{6} R^{abc}\ \iota_a\ \iota_b\ \iota_c$$
$$- \tfrac{1}{2} f^a{}_{ab}\ \tilde{e}^b \wedge + \tfrac{1}{2} Q_a{}^{ab}\ \iota_b \Big) A$$

$\mathcal{D}_\sharp^2 = 0 \Leftrightarrow$ Bianchi identities and $\frac{1}{3} H_{abc} R^{abc} + \frac{1}{2} f^a{}_{ab} Q_a{}^{ab} = 0$

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Γ^A: Clifford algebra: $\{\Gamma^A, \Gamma^B\} = 2\eta^{AB}$, use the representation

Γ^A : $\Gamma^a = 2\tilde{e}^a \wedge$, $\Gamma_a = 2\iota_a$, spinor Ψ: polyform

SUSY vacua with SU(3)×SU(3) structure

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where $e^A =$ constant, $|\mu|^2 \sim -\Lambda$, and

$$\begin{aligned} \mathrm{IIA} : \Phi_1 = \Phi_+\ ,\ \Phi_2 = \Phi_-\ ,\ \varepsilon = +1\ , \\ \mathrm{IIB} : \Phi_1 = \Phi_-\ ,\ \Phi_2 = \Phi_+\ ,\ \varepsilon = -1\ . \end{aligned}$$

hep-th/0505212, hep-th/0609124 by M. Graña, R. Minasian, M. Petrini and A. Tomasiello

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Study superpotential, and further properties of $\mathcal{D}$...

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4D non-geometric fluxes, 10D non-geometry

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Extension beyond the NSNS sector: RR non-geometric fluxes? exotic *D*-branes? Exceptional geometry/field theory could help

Additional Bianchi identities $\Rightarrow$ *R*-brane?

$\hookrightarrow$ Get new pheno. interesting backgrounds, de Sitter vacua.