From

Double to Extended Field Theory,

Stringy Geometries and Gauged Supergravities

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Frontiers in String Phenomenology,
Ringberg, 2014

Duff, Siegel, Tseytlin, (1990-1993)

Hull, Zwiebach (2009)

Hohm, Hull, Zwiebach (2010)

G.A, Andriot, Baron, Bedoya, Berkeley, Berman, Betz, Blair, Blumenhagen, Dall Agata, Dibitetto, Cederwall, Coimbra, Copland, Geissbuller, Fernandez-Melgarejo, Fuchs, Graña, Hassler, Hohm, Hull, Jeon, Kleinschmidt, Kwak, Larfors, Lee, Lust, Malek, Marques, Musaev, Nibbelink, Nuñez, Park, Patalong, Penas, Perry, Renecke, Roest, Rosabal, Rudolph, Samtleben,Thompson, Waldram, West, Zweibach, …

Hitchin, Gualtieri, Petrini, Strickland-Constable, Waldram, ...(GG)

O.Hohm talk

Motivations:

- Low energy effective field theories for strings, 10(11) dimensional “sugras” miss “stringy dualities”. D(E)FT, could capture duality information

- Metric and 2-form field (RR fields) geometrically unified (Berman's talk)

- Like Riemann Geometry describes Gravity, D(E)FT could provide a “Geometry” for strings

- New configurations, non derivable from effective 10 dimensional sugra theories could be reached from D(E)FT. Relevant for Phenomenology, susybreaking, moduli stabilization..

Plan:

- T(U)-duality as a symmetry of a field theory: D(E)FT
- D(E)FT, “dynamical fluxes” formulation
- DFT(EFT) a “Geometry” for strings?
- Generalized Scherk-Schwarz reductions. Fluxes and gaugings
- Link with (bosonic) sector of Supergravities
- Comments, problems and outlook

Double Field Theory

$\mathrm{O(n,n)}$ (T-duality) explicit in a field theory

↓

Extended Field Theory

$E_{n(n)}$ (U-duality) explicit in a field theory

Double(Extended) Field Theory

- Coordinate space
- Fields
- Symmetries
- Action

DFT

coordinates

$p^i \leftrightarrow y_i$ $\quad$ $\tilde{p}^i \leftrightarrow \tilde{y}^i$ $\quad$ dual coordinates

$i = 1, \ldots, n$

$P_M = (p_i, \tilde{p}^i) \leftrightarrow \mathbb{Y} = (y^i, \tilde{y}_i)$ $\quad$ internal, fundamental representation of O(n,n)

x^μ $\quad$ space-time $\quad$ d

$\tilde{x}^\mu$ $\quad$ space time duals, fictitious

$X^M = (x^\mu, \tilde{x}^\mu, \mathbb{Y})$ $\quad$ O(D,D) $\quad$ D=d+n

$$d = 4 \longrightarrow 4 + \cancel{4} + 12 \longrightarrow 4 + 12$$

p_i

DFT

6 KK momentum

T-duality

$$\mathcal{P}^M = \begin{pmatrix} \tilde{p}_i \\ p^i \end{pmatrix}$$

12D fund. of $O(6,6)$

KK +winding=6+6=12

↓

string charge

U-duality

$$\mathcal{P}^M = \begin{pmatrix} p \, \tilde{p} \, \mathbf{Q} \end{pmatrix}$$

56D fund. of $E_{7,7}$

$\mathbf{Q}$ → D1+D3+D5 charge=6+20+6

$\mathbf{Q}$ → NS charge=6

$\mathbf{Q}$ → KK monopole=6

EFT

$\mathbb{Y}^M \leftrightarrow \mathcal{P}^M$ 56D fund. of $E_{7,7}$ $M = 1, \ldots, 56$

$X = (x^\mu, \mathbb{Y}) \longrightarrow 4 + 56$

fields

DFT

$\Phi_{MN..}(X^M)$ restrict to $\mathcal{H}_{MN}(X), d(X)$

Generalized metric

Invariant dilaton

$$\mathcal{H}_{MN} = \begin{pmatrix} g^{ij} & -g^{ik}b_{kj} \\ b_{ik}g^{kj} & g_{ij} - b_{ik}g^{kl}b_{lj} \end{pmatrix} \in O(D,D)$$

$$e^{-2d} = \sqrt{g}e^{-2\phi}$$

Massless bosonic modes of D=10 string theory $(\tilde{x}^i = 0, \quad i = 1, \ldots D)$

EFT

$$\mathcal{H}_{MN} = \mathcal{H}_{MN}(\phi, g, b, C_p)$$

Symmetries

$$\mathcal{L}_\xi V^M = L V^M + Y^{MN}_{PQ}\partial_N \xi_P V^Q$$

Y^{MN}_{PQ} departure form Lie derivative is fixed by requiring $\mathcal{L}_\xi V^M$ inside the group

i.e O(D,D) $\mathcal{L}_\xi \ \eta_{MN} = 0 \longrightarrow Y^M{}_P{}^N{}_Q = \eta^{MN}\eta_{PQ}$ $\eta_{MN} = \begin{pmatrix} 0 & \delta^i{}_j \\ \delta_i{}^j & 0 \end{pmatrix}$

leads to

$$\mathcal{L}_\xi V^M = \xi^P \partial_P V^M - \partial_P \xi^M V^P + \partial^M \xi_P V^P$$

$$\delta_\xi \mathcal{H}^{MN} = \xi^P \partial_P \mathcal{H}^{MN} + \left(\partial^M \xi_P - \partial_P \xi^M\right)\mathcal{H}^{PN} + \left(\partial^N \xi_P - \partial_P \xi^N\right)\mathcal{H}^{MP}$$

$$\delta_\xi e^{-2d} = \partial_M \left(\xi^M e^{-2d}\right)$$

if $\tilde{\partial}^i(\dots) = 0$ $\xi^M = (\tilde{\xi}_i, \xi^i)$

$$\mathcal{L}_\xi g_{ij} = L_\lambda g_{ij}$$

$$\mathcal{L}_\xi b_{ij} = L_\lambda b_{ij} + 2\ \partial_{[i}\tilde{\epsilon}_{j]}$$

Usual diffeos and gauge transformations in sugra

In general

$$Y^M{}_N{}^P{}_Q = \delta^M_Q \delta^P_N - \alpha P_{(adj)}{}^M{}_N{}^P{}_Q + \beta \delta^M_N \delta^P_Q \, ,$$

	$Y^M{}_Q{}^N{}_P$	α	β
$O(n,n)$	$\eta^{MN}\eta_{PQ}$	2	0
$E_{4(4)} = SL(5)$	$\epsilon^{iMN}\epsilon_{iPQ}$	3	$\frac{1}{5}$
$E_{5(5)} = SO(5,5)$	$\frac{1}{2}(\gamma^i)^{MN}(\gamma_i)_{PQ}$	4	$\frac{1}{4}$
$E_{6(6)}$	$10d^{MNR}\bar{d}_{PQR}$	6	$\frac{1}{3}$
$E_{7(7)}$	$12K^{MN}{}_{PQ} + \delta_P^{(M}\delta_Q^{N)} + \frac{1}{2}\epsilon^{MN}\epsilon_{PQ}$	12	$\frac{1}{2}$

Table 1: Invariant Y-tensor and proportionality constants for different dimensions. Here η_{MN} is the $O(n,n)$ invariant metric, ϵ_{iMN} is the $SL(5)$ alternating tensor, $(\gamma^i)^{MN}$ are 16×16 MW representation of the $SO(5,5)$ Clifford algebra, d^{MNR} and K^{MNPQ} are the symmetric invariant tensors of $E_{6(6)}$ and $E_{7(7)}$ respectively, and ϵ^{MN} is the symplectic invariant in $E_{7(7)}$.

Berman, Cederwall, Kleinschmidt,Thompson (2013)

Consistency

Two successive gauge transformations parameterized by ξ_1 and ξ_2, acting on a given field ξ_3, must reproduce a new gauge transformation parameterized by some given $\xi_{12}(\xi_1, \xi_2)$ acting on the same vector

define
$$\Delta_\xi \equiv \delta_\xi - \mathcal{L}_\xi$$

$$\Delta_{123}{}^M = -\Delta_{\xi_1}\left(\mathcal{L}_{\xi_2}\xi_3^M\right) = \left([\mathcal{L}_{\xi_1},\ \mathcal{L}_{\xi_2}] - \mathcal{L}_{\xi_{12}}\right)\xi_3^M = 0$$

with
$$\xi_{12} = \mathcal{L}_{\xi_1}\xi_2$$

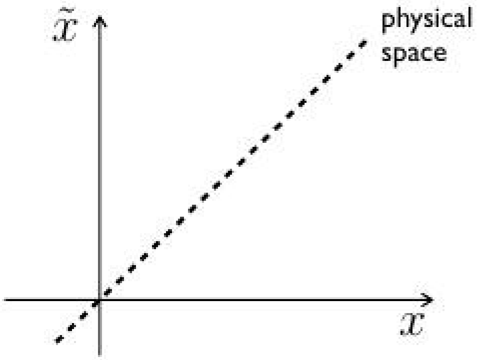
requires
$$\Delta_{123}{}^M = Y^P{}_R{}^Q{}_S\left(2\partial_P\xi^R_{[1}\ \partial_Q\xi^M_{2]}\ \xi^S_3 - \partial_P\xi^R_1\ \xi^S_2\ \partial_Q\xi^M_3\right) = 0$$

DFT is a constrained theory

a solution is
$$Y^M{}_P{}^N{}_Q\ \partial_M\partial_N(\dots) = \eta^{MN}\partial_M\partial_N(\dots) = \tilde{\partial}^i\partial_i(\dots) = 0$$

“section condition” or “strong constraint”

$$Y^M{}_P{}^N{}_Q\;\partial_M\partial_N(\dots) = \eta^{MN}\partial_M\partial_N(\dots) = \tilde{\partial}^i\partial_i(\dots) = 0$$



Fields depend on half of the coordinates

not really doubled !

$\mathrm{O(D,D)}$ rotation to only x dependence

Are there other solutions to consistency constraints?

DFT action

$$S_{DFT} = \int dX e^{-2d}\Big(4\,\mathcal{H}^{MN}\partial_M\partial_N d - \partial_M\partial_N\mathcal{H}^{MN} - 4\,\mathcal{H}^{MN}\partial_M d\,\partial_N d + 4\partial_M\mathcal{H}^{MN}\,\partial_N d$$
$$+\frac{1}{8}\,\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\,\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\,\partial_K\mathcal{H}_{NL} + \Delta_{(\mathrm{SC})}\mathcal{R}\Big)$$

- up to two derivatives
- up to cubic terms in the metric
- O(D,D) invariant
- Invariant under generalized diffeos if strong contrained
- If $\partial_M = (0, \partial_i)$

$$S_{DFT} \;\rightarrow\; S_{sugra} = \int dx\sqrt{g}e^{-2\phi}\left(\mathbf{R} + 4(\partial\phi)^2 - \tfrac{1}{12}H^2\right)$$

DFT action with dynamical fluxes

Geissbuhler, (2011)
Marques, Nuñez, Penas,
G.A, Marques, Nuñez (2014)

A frame formulation:

$M \in O(D,D)$ Hohm, Kwak (2010)

$E_{\bar{A}}{}^{M}$ generalized frame $\in O(D,D)/H$

$\bar{A} \in H = O(1,D-1) \times O(D-1,1)$

$$\mathcal{H}_{MN} = E^{\bar{A}}{}_{M}\ S_{\bar{A}\bar{B}}\ E^{\bar{B}}{}_{N}$$

$$\eta_{MN} = E^{\bar{A}}{}_{M}\ \eta_{\bar{A}\bar{B}}\ E^{\bar{B}}{}_{N}$$

can be parametrized as

$$E^{\bar{A}}{}_{M} = \begin{pmatrix} e_{\bar{a}}{}^{i} & e_{\bar{a}}{}^{j} b_{ji} \\ 0 & e^{\bar{a}}{}_{i} \end{pmatrix}\ , \qquad S_{\bar{A}\bar{B}} = \begin{pmatrix} s^{\bar{a}\bar{b}} & 0 \\ 0 & s_{\bar{a}\bar{b}} \end{pmatrix}$$

with $g_{ij} = e^{\bar{a}}{}_{i} s_{\bar{a}\bar{b}} e^{\bar{b}}{}_{j}$ and $s_{\bar{a}\bar{b}} = \mathrm{diag}(- + \cdots +)$

Generalized (dynamical) fluxes $\mathcal{F}_{\bar{A}\bar{B}\bar{C}}(X)$

$$\mathcal{L}_\xi E_{\bar{A}}{}^M = \xi^P \partial_P E_{\bar{A}}{}^M + (\partial^M \xi_P - \partial_P \xi^M)\ E_{\bar{A}}{}^P$$

transforms as a vector

in particular

$$\begin{aligned} \mathcal{L}_{E_{\bar{A}}} E_{\bar{B}}{}^M &= \mathcal{F}_{\bar{A}\bar{B}}{}^{\bar{C}} E_{\bar{C}}{}^M \\ \mathcal{L}_{E_{\bar{A}}} e^{-2d} &= -\mathcal{F}_{\bar{A}}\, e^{2d} \end{aligned}$$

fluxes

then

$$\begin{aligned} \mathcal{F}_{\bar{A}\bar{B}\bar{C}} &= E_{\bar{C}M} \mathcal{L}_{E_{\bar{A}}} E_{\bar{B}}{}^M = 3\Omega_{[\bar{A}\bar{B}\bar{C}]} \\ \mathcal{F}_{\bar{A}} &= -e^{2d} \mathcal{L}_{E_{\bar{A}}} e^{-2d} = \Omega^{\bar{B}}{}_{\bar{B}\bar{A}} + 2E_{\bar{A}}{}^M \partial_M d \end{aligned}$$

with $\Omega_{\bar{A}\bar{B}\bar{C}} = E_{\bar{A}}{}^M \partial_M E_{\bar{B}}{}^N E_{\bar{C}N} = -\Omega_{\bar{A}\bar{C}\bar{B}}$ generalized Weitzsenbock connection

DFT action

$$S_{DFT} \;=\; \int dX e^{-2d}\, \mathcal{R}$$

$$\begin{aligned}\mathcal{R} \;&=\; \mathcal{F}_{\bar{A}\bar{B}\bar{C}}\,\mathcal{F}_{\bar{D}\bar{E}\bar{F}}\,\left[\frac{1}{4}S^{\bar{A}\bar{D}}\eta^{\bar{B}\bar{E}}\eta^{\bar{C}\bar{F}} - \frac{1}{12}S^{\bar{A}\bar{D}}S^{\bar{B}\bar{E}}S^{\bar{C}\bar{F}} - \frac{1}{6}\eta^{\bar{A}\bar{D}}\eta^{\bar{B}\bar{E}}\eta^{\bar{C}\bar{F}}\right]\\ &+ \mathcal{F}_{\bar{A}}\mathcal{F}_{\bar{B}}\,\left[\eta^{\bar{A}\bar{B}} - S^{\bar{A}\bar{B}}\right]\end{aligned}$$

under generalized diffeos

$$\begin{aligned}\delta_\xi \mathcal{F}_{\bar{A}\bar{B}\bar{C}} &= \xi^{\bar{D}}\partial_{\bar{D}}\mathcal{F}_{\bar{A}\bar{B}\bar{C}} + \Delta_{\xi\bar{A}\bar{B}\bar{C}}\,, \\ \delta_\xi \mathcal{F}_{\bar{A}} &= \xi^{\bar{D}}\partial_{\bar{D}}\mathcal{F}_{\bar{A}} + \Delta_{\xi\bar{A}}\end{aligned}$$

if closure is satisfied $\Delta_{123}{}^M = 0$

$$\begin{aligned}\Delta_\xi \mathcal{F}_{\bar{A}\bar{B}\bar{C}} &= \Delta_{\xi\bar{A}\bar{B}\bar{C}} = E_{\bar{C}M}\Delta_\xi(\mathcal{L}_{E_{\bar{A}}}E_{\bar{B}}{}^M) = 0 \\ \Delta_\xi \mathcal{F}_{\bar{A}} &= \Delta_{\xi\bar{A}} = -e^{2d}\Delta_\xi(\mathcal{L}_{E_{\bar{A}}}e^{-2d}) = 0\,.\end{aligned}$$

then $\delta_\xi \mathcal{R} = \mathcal{L}_\xi \mathcal{R} = \xi^P\partial_P\mathcal{R}$ scalar ! ⟶ the action is invariant

Is $\mathcal{R}$ a generalized Ricci scalar?

A Double Geometry?

Can we “generalize” Riemann geometry to a “Double Geometry”?

diffeo	$L \to \mathcal{L}$	generalized diffeo?
covariant derivative	$\nabla_i \to \nabla_M$	generalized covariant derivative?
connections	${\Gamma_{ij}}^k \to {\Gamma_{MN}}^P$	
	$\ldots \to \ldots$	generalized ….?
Ricci tensor	$R_{ijlk} \to \mathcal{R}_{MNPQ}$	
scalar curvature	$R \to \mathcal{R}$	generalized scalar curvature?

Double Geometry

$$L \to \mathcal{L}$$

$$\nabla_M V^N = \partial_M V^N + \Gamma_{MP}{}^N V^P \qquad \nabla_M E_{\bar{A}}{}^N = W_{M\bar{A}}{}^{\bar{B}} E_{\bar{B}}{}^N$$ generalized covariant derivative

metric compatibility, torsionless...

$$\nabla_A H^{BC} = 0$$

$$\mathcal{T}_{\bar{A}\bar{B}}{}^M = (\mathcal{L}^{\nabla}_{E_{\bar{A}}} - \mathcal{L}_{E_{\bar{A}}}) E_{\bar{B}}{}^M = 0 \quad \Rightarrow \quad 2\Gamma_{[AB]}{}^C = Y^C{}_B{}^P{}_Q \Gamma_{PA}{}^Q$$

only part of $\Gamma_{MP}{}^N$ (and $W_{M\bar{A}}{}^{\bar{B}}$)is determined

$$\check{P}_N{}^R\ \hat{P}_S{}^Q\ \Gamma_{MR}{}^S = \hat{P}_R{}^Q \partial_M \check{P}_N{}^R \qquad \hat{P}_{MN} = \frac{1}{2}(\eta_{MN} - \mathcal{H}_{MN})\,, \qquad \check{P}_{MN} = \frac{1}{2}(\eta_{MN} + \mathcal{H}_{MN})\,,$$

Generalized Ricci tensor,

$$[\nabla_M, \nabla_P] V^P + \frac{1}{2} \nabla_A (Y^A{}_M{}^B{}_P \nabla_B V^P) = \mathcal{R}_{MP} V^P$$

covariant if consistency constraints are satisfied, partially determined

	Riemannian geometry	Double geometry
Frame compatibility	$W = \Omega + \Gamma$	$W = \Omega + \Gamma$
$O(D, D)$ compatibility	− − − − − − − −	$\Gamma_{MNP} = -\Gamma_{MPN}$ $W_{M\bar{A}\bar{B}} = -W_{M\bar{B}\bar{A}}$
Metric compatibility	$\partial_i g_{jk} = 2\Gamma_{i(j}{}^{l} g_{k)l}$	$\partial_M \mathcal{H}_{PQ} = 2\Gamma_{M(P}{}^{N}\mathcal{H}_{Q)N}$
Vanishing torsion	$\Gamma_{[ij]}{}^{k} = 0$ $W_{[\bar{a}\bar{b}]}{}^{\bar{c}} = 2f_{\bar{a}\bar{b}}{}^{\bar{c}}$	$\Gamma_{[MNP]} = 0$ $W_{[\bar{A}\bar{B}\bar{C}]} = 3\mathcal{F}_{\bar{A}\bar{B}\bar{C}}$
Measure compatibility	$\Gamma_{ki}{}^{k} = \frac{1}{\sqrt{g}}\partial_i\sqrt{g}$ $W_{\bar{b}\bar{a}}{}^{\bar{b}} = f_{\bar{b}\bar{a}}{}^{\bar{b}}$	$\Gamma_{PM}{}^{P} = e^{2d}\partial_M e^{-2d}$ $W_{\bar{B}\bar{A}}{}^{\bar{B}} = -\mathcal{F}_{\bar{A}}$
Determined part	Totally fixed $\Gamma_{ij}{}^{k} = \frac{1}{2}g^{kl}(\partial_i g_{jl} + \partial_j g_{il} - \partial_l g_{ij})$	Only some projections
Covariance failure	$\Delta_\xi \Gamma_{ij}{}^{k} = \partial_i \partial_j \xi^k$	$\Delta_\xi \Gamma_{MNP} = 2\partial_M \partial_{[N}\xi_{P]}$ $+\ \Omega_{RNP}\Omega^{R}{}_{MS}\xi^S$

Table 1: A list of conditions is given for objects in Riemannian and double geometry, with their corresponding implications on the connections. Every line assumes that the previous ones hold.

	Riemannian geometry	Double geometry
Torsion	$T_{ij}{}^{k} = 2\Gamma_{[ij]}{}^{k}$	$\mathcal{T}_{MN}{}^{P} = 2\Gamma_{[MN]}{}^{P} + \Gamma^{P}{}_{MN}$
Riemann tensor	Determined $R_{ijl}{}^{k} = 2\partial_{[i}\Gamma_{j]l}{}^{k}$ $+2\Gamma_{[i\vert m}{}^{k}\Gamma_{\vert j]l}{}^{m}$	Undetermined $\mathcal{R}_{MNPQ} = R_{MNPQ} + R_{PQMN}$ $+\Gamma_{RMN}\Gamma^{R}{}_{PQ} - \Omega_{RMN}\Omega^{R}{}_{PQ}$
Ricci tensor	Determined $R_{ij} = R_{ikj}{}^{k}$	Undetermined $\mathcal{R}_{MN} = \hat{P}_{P}{}^{Q}\mathcal{R}_{MQN}{}^{P}$
EOM	$R_{ij} = 0$	$\hat{P}_{(M}{}^{R}\check{P}_{N)}{}^{S}\ \mathcal{R}_{RS} = 0$
Ricci Scalar	$R = g^{ij}R_{ij}$	$\mathcal{R} = \frac{1}{4}\hat{P}^{MN}\mathcal{R}_{MN}$

Table 1: A list of definitions of curvatures is given for Riemannian and double geometry.

Action from generalized Ricci scalar:

$$S_{DFT} = \int dX e^{-2d}\,\mathcal{R}$$

$$\begin{aligned}\mathcal{R} &= \mathcal{F}_{\bar{A}\bar{B}\bar{C}}\,\mathcal{F}_{\bar{D}\bar{E}\bar{F}}\left[\frac{1}{4}S^{\bar{A}\bar{D}}\eta^{\bar{B}\bar{E}}\eta^{\bar{C}\bar{F}} - \frac{1}{12}S^{\bar{A}\bar{D}}S^{\bar{B}\bar{E}}S^{\bar{C}\bar{F}} - \frac{1}{6}\eta^{\bar{A}\bar{D}}\eta^{\bar{B}\bar{E}}\eta^{\bar{C}\bar{F}}\right]\\ &\quad + \mathcal{F}_{\bar{A}}\mathcal{F}_{\bar{B}}\left[\eta^{\bar{A}\bar{B}} - S^{\bar{A}\bar{B}}\right]\end{aligned}$$

G. A, Baron, Marques, Nuñez, (2011)
Geissbuller

Scherk-Schwarz dimensional reductions

$$D = d + n$$

split coordinates

$$X = (\mathbb{X}, \mathbb{Y})$$

$$\mathbb{Y}^A = (\tilde{y}_m, y^m) \qquad m = 1, \ldots, n$$ internal

$$\mathbb{X} = (\tilde{x}_\mu, x^\mu) \qquad \mu = 1, \ldots, d$$ s-time

ansatz

$$\xi^M(X) = \widehat{\xi}^I(\mathbb{X}) \, U_I{}^M(\mathbb{Y}) \qquad E^{\bar{A}}{}_M(X) = \widehat{E}^{\bar{A}}{}_I(\mathbb{X}) \, U^I{}_M(\mathbb{Y}) \; , \qquad d(X) = \widehat{d}(\mathbb{X}) + \lambda(\mathbb{Y})$$

twist

$$U_I{}^M(\mathbb{Y}) \in O(n,n)$$
$$\lambda(\mathbb{Y})$$

encode dependence on internal coordinates

Dynamical fluxes split

$$X = (\mathbb{X}, \mathbb{Y})$$

$$\widehat{F} = \widehat{F}(\mathbb{X})$$

$$\begin{aligned} \mathcal{F}_{\bar{A}\bar{B}\bar{C}}(X) &= \widehat{F}_{\bar{A}\bar{B}\bar{C}} + f_{IJK}\ \widehat{E}_{\bar{A}}{}^{I}\widehat{E}_{\bar{B}}{}^{J}\widehat{E}_{\bar{C}}{}^{K} \, , \\ \mathcal{F}_{\bar{A}}(X) &= \widehat{F}_{\bar{A}} + f_I\ \widehat{E}_{\bar{A}}{}^{I} \end{aligned}$$

with

$$\begin{aligned} f_{IJK} &= 3\tilde{\Omega}_{[IJK]} \ , \\ f_I &= \tilde{\Omega}^{J}{}_{JI} + 2U_I{}^{M}\partial_M\lambda \end{aligned}$$

$$\tilde{\Omega}_{IJK} = U_I{}^{M}\partial_M U_J{}^{N} U_{KN} \, ,$$

$$\Delta = 0 \qquad \longrightarrow$$

$$\partial_I \widehat{V}\ \partial^I \widehat{W} = 0$$

strong constraint

$$\left([f_I, f_J] + f_{IJ}{}^{K} f_K\right) + (\partial f) = 0$$

choose

$$f_{IJK} = \text{constant} \ , \qquad f_I = \text{constant} \, .$$

($\mathbf{220},$ $\mathbf{12}$ for $O(6,6)$)

$$\begin{aligned} \mathcal{F}_{\bar{A}\bar{B}\bar{C}}(X) &= \mathcal{F}_{\bar{A}\bar{B}\bar{C}}(\mathbb{X}) \\ \mathcal{F}_{\bar{A}}(X) &= \mathcal{F}_{\bar{A}}(\mathbb{X}) \end{aligned}$$

Gauged DFT

M. Grana, D. Marques, (2012)

$$S_{GDFT} = v \int d\mathbb{X} e^{-2\widehat{d}} \Big[-\frac{1}{4}\Big(\widehat{F}_{IK}{}^{L} + f_{IK}{}^{L}\Big)\Big(\widehat{F}_{JL}{}^{K} + f_{JL}{}^{K}\Big)\widehat{\mathcal{H}}^{IJ}$$

$$-\frac{1}{12}\Big(\widehat{F}_{IJ}{}^{K} + f_{IJ}{}^{K}\Big)\Big(\widehat{F}_{LH}{}^{G} + f_{LH}{}^{G}\Big)\widehat{\mathcal{H}}^{IL}\widehat{\mathcal{H}}^{JH}\widehat{\mathcal{H}}_{KG}$$

$$-\frac{1}{6}\Big(\widehat{F}_{IJK} + f_{IJK}\Big)\Big(\widehat{F}^{IJK} + f^{IJK}\Big) + \Big(\widehat{\mathcal{H}}^{IJ} - \eta^{IJ}\Big)\widehat{F}_I\widehat{F}_J\Big] .$$

$$v = \int d\mathbb{Y} e^{-2\lambda}$$

$$\mathcal{L}_\xi V^M = U_I{}^M \ \widehat{\mathcal{L}}_{\widehat{\xi}}\widehat{V}^I \, ,$$

$$\widehat{\mathcal{L}}_{\widehat{\xi}}\widehat{V}^I = \mathcal{L}_{\widehat{\xi}}\widehat{V}^I - f^I{}_{JK}\widehat{\xi}^J\widehat{V}^K$$

$$f_{H[IJ}f_{KL]}{}^{H} = 0$$

quadratic constraints

$$\Delta_{123} = 0 \quad \longrightarrow$$

$$\partial_I\widehat{V} \ \partial^I\widehat{W} = 0$$

strong constraint

From gauged DFT to gauged supergravity

$$O(D,D) \rightarrow O(d,d) \times O(n,n)$$

$$\partial_I \widehat{V} \partial^I \widehat{W} = 0 \ \rightarrow \ \tilde{\partial}^\mu \widehat{V} = 0$$

$$f_{IJK} = \begin{cases} f_{ABC} & (I,J,K) = (A,B,C) \\ 0 & \text{otherwise} \end{cases}$$

parametrization

$$\widehat{E}^{\bar{A}}{}_I = \begin{pmatrix} \widehat{e}_{\bar{a}}{}^\mu & -\widehat{e}_{\bar{a}}{}^\rho \widehat{c}_{\rho\mu} & -\widehat{e}_{\bar{a}}{}^\rho \widehat{A}_{A\rho} \\ 0 & \widehat{e}^{\bar{a}}{}_\mu & 0 \\ 0 & \widehat{\Phi}^{\bar{A}}{}_B \widehat{A}^B{}_\mu & \widehat{\Phi}^{\bar{A}}{}_A \end{pmatrix},$$

consider

$$D = 4 + \cancel{4} + 12$$

$$f_{ABC} \equiv \mathbf{220}$$

$$f_A \cancel{\equiv} \mathbf{12}$$

$$O(6,6)$$

space -time fields

$$\mathcal{F}_{\bar{A}\bar{B}\bar{C}} \longrightarrow$$

$$\begin{aligned}
\mathcal{F}^{A}{}_{\mu\nu} &= \partial_\mu \widehat{A}^{A}{}_{\nu} - \partial_\nu \widehat{A}^{A}{}_{\mu} - f_{BC}{}^{A}\widehat{A}^{B}{}_{\mu}\ \widehat{A}^{C}{}_{\nu}\,, \\
\mathcal{G}_{\mu\rho\lambda} &= 3\partial_{[\mu}\widehat{b}_{\rho\lambda]} - f_{ABC}\widehat{A}^{A}{}_{\mu}\widehat{A}^{B}{}_{\rho}\widehat{A}^{C}{}_{\lambda} + 3\partial_{[\mu}\widehat{A}^{A}{}_{\rho}\widehat{A}_{\lambda]A}
\end{aligned}$$

$$D_\mu \widehat{\Phi}_{\bar{B}}{}^{C} = \partial_\mu \widehat{\Phi}_{\bar{B}}{}^{C} - f_{AB}{}^{C}\widehat{A}_{\mu}{}^{A}\widehat{\Phi}_{\bar{B}}{}^{B}$$

gauge transformations

$$\widehat{\mathcal{L}}_{\widehat{\xi}}\widehat{V}^{I} = \mathcal{L}_{\widehat{\xi}}\widehat{V}^{I} - f^{I}{}_{JK}\widehat{\xi}^{J}\widehat{V}^{K} \longrightarrow$$

$$\begin{aligned}
\delta_{\widehat{\xi}}\ \widehat{g}_{\mu\nu} &= L_{\widehat{\epsilon}}\ \widehat{g}_{\mu\nu}\,, \\
\delta_{\widehat{\xi}}\ \widehat{b}_{\mu\nu} &= L_{\widehat{\epsilon}}\ \widehat{b}_{\mu\nu} + (\partial_\mu \widehat{\epsilon}_\nu - \partial_\nu \widehat{\epsilon}_\mu)\ , \\
\delta_{\widehat{\xi}}\ \widehat{A}^{A}{}_{\mu} &= L_{\widehat{\epsilon}}\ \widehat{A}^{A}{}_{\mu} - \partial_\mu \widehat{\Lambda}^{A} + f_{BC}{}^{A}\ \widehat{\Lambda}^{B}\widehat{A}^{C}{}_{\mu}\,, \\
\delta_{\widehat{\xi}}\ \widehat{\mathcal{M}}_{AB} &= L_{\widehat{\epsilon}}\ \widehat{\mathcal{M}}_{AB} + f_{AC}{}^{D}\ \widehat{\Lambda}^{C}\widehat{\mathcal{M}}_{DB} + f_{BC}{}^{D}\ \widehat{\Lambda}^{C}\widehat{\mathcal{M}}_{AD}\,.
\end{aligned}$$

and $S_{DFT} \longrightarrow$

$$S = \int dx \sqrt{\widehat{g}} e^{-2\widehat{\phi}} \Big(\mathbf{R} + 4\, \partial^\mu \widehat{\phi} \partial_\mu \widehat{\phi} - \frac{1}{4} \widehat{\mathcal{M}}_{AB} \mathcal{F}^{A\mu\nu} \mathcal{F}^{B}{}_{\mu\nu} - \frac{1}{12} \mathcal{G}_{\mu\nu\rho} \mathcal{G}^{\mu\nu\rho} + \frac{1}{8} D_\mu \widehat{\mathcal{M}}_{AB} D^\mu \widehat{\mathcal{M}}^{AB} - V \Big) .$$

with

$$V = \tfrac{1}{4} f_{DA}{}^{C}\, f_{CB}{}^{D} \widehat{\mathcal{M}}^{AB} + \tfrac{1}{12} f_{AC}{}^{E}\, f_{BD}{}^{F} \widehat{\mathcal{M}}^{AB}\, \widehat{\mathcal{M}}^{CD}\, \widehat{\mathcal{M}}_{EF} + \tfrac{1}{6} f_{ABC} f^{ABC}$$

Electric bosonic sector of $\mathcal{N} = 4$ gauged supergravity

$f_{\alpha ABC} \to f_{+ABC} \equiv f_{ABC}$

$\mathcal{N} = 4$ sugra

Global symmetry

$$\mathrm{SL}(2,\mathrm{Z})_S \times O(6,6)$$

gaugings

$$\left\{\begin{array}{ll} f_{\alpha ABC} & (\mathbf{2}, \mathbf{220}) \\ f_{+ABC} & \text{electric} \\ f_{-ABC} & \text{magnetic} \\ \xi_{\alpha}^{M} & (\mathbf{2{,}12}) \end{array}\right.$$

$$
\begin{aligned}
V_{\mathcal{N}=4} \quad = \quad & \frac{1}{4}\Big[f_{\alpha MNP} f_{\beta QRS} M^{\alpha\beta}\left(\frac{1}{3}M^{MQ}M^{NR}M^{PS} + \left(\frac{2}{3}\eta^{MQ} - M^{MQ}\right)\eta^{NR}\eta^{PS}\right) \\
& -\frac{4}{9} f_{\alpha MNP} f_{\beta QRS}\epsilon^{\alpha\beta} M^{MNPQRS} + 3\xi_{\alpha}^{M}\xi_{\beta}^{N} M^{\alpha\beta} M_{MN}\Big]
\end{aligned}
$$

$$\Big\downarrow \qquad f_{\alpha ABC} \to f_{+ABC} \equiv f_{ABC}$$

$$V = \tfrac{1}{4} f_{DA}{}^{C}\ f_{CB}{}^{D}\widehat{\mathcal{M}}^{AB} + \tfrac{1}{12} f_{AC}{}^{E}\ f_{BD}{}^{F}\widehat{\mathcal{M}}^{AB}\ \widehat{\mathcal{M}}^{CD}\ \widehat{\mathcal{M}}_{EF}\ +\ \tfrac{1}{6} f_{ABC} f^{ABC}$$

quadratic constraints

$$
\begin{aligned}
f^{P}_{\alpha[AB} f^{L}_{\beta B]P} &= 0 \\
f^{P}_{+AB} f^{L}_{-BP} - f^{P}_{-AB} f^{L}_{+BP} &= 0
\end{aligned}
\qquad \longrightarrow \qquad f_{[AB}{}^{E} f_{CD]E} = 0
$$

$$\partial_{[A}f_{BCD]} - \frac{3}{4}f_{[AB}{}^{E}f_{CD]E} \;=\; -\frac{3}{4}\tilde{\Omega}_{E[AB}\tilde{\Omega}^{E}{}_{CD]} \;= 0$$ closure

constant fluxes

$$f_{[AB}{}^{E}f_{CD]E} \;=\; \tilde{\Omega}_{E[AB}\tilde{\Omega}^{E}{}_{CD]} = 0$$ quadratic constraints

$$\Omega_{EAB}\Omega^{E}{}_{CD} = 0$$ strong constraint

- There is a subset of new solutions not annihilated by the strong constraint.
- Although many non-geometric backgrounds are related to geometric ones through T-duality, there are genuinely non-geometric orbits of fluxes

fluxes=sugra gaugings

$$f_{IJK} = 3\tilde{\Omega}_{[IJK]} \ , \qquad \tilde{\Omega}_{IJK} = U_I{}^M \partial_M U_J{}^N U_{KN} \ ,$$

$$f_I = \tilde{\Omega}^J{}_{JI} + 2 U_I{}^M \partial_M \lambda$$

$$U^A{}_M = \begin{pmatrix} u_a{}^m & u_a{}^n v_{nm} \\ u^a{}_n \beta^{nm} & u^a{}_m + u^a{}_n \beta^{np} v_{pm} \end{pmatrix}$$

Only u, v appear in

SS compactifications of Sugras

here we have also $\beta^{[mn]}$

(Andriot' s talk)

$u, v \longrightarrow$ geometric $\qquad f_{abc} = H_{abc} \ , \quad f^a{}_{bc} = \omega_{bc}{}^a$

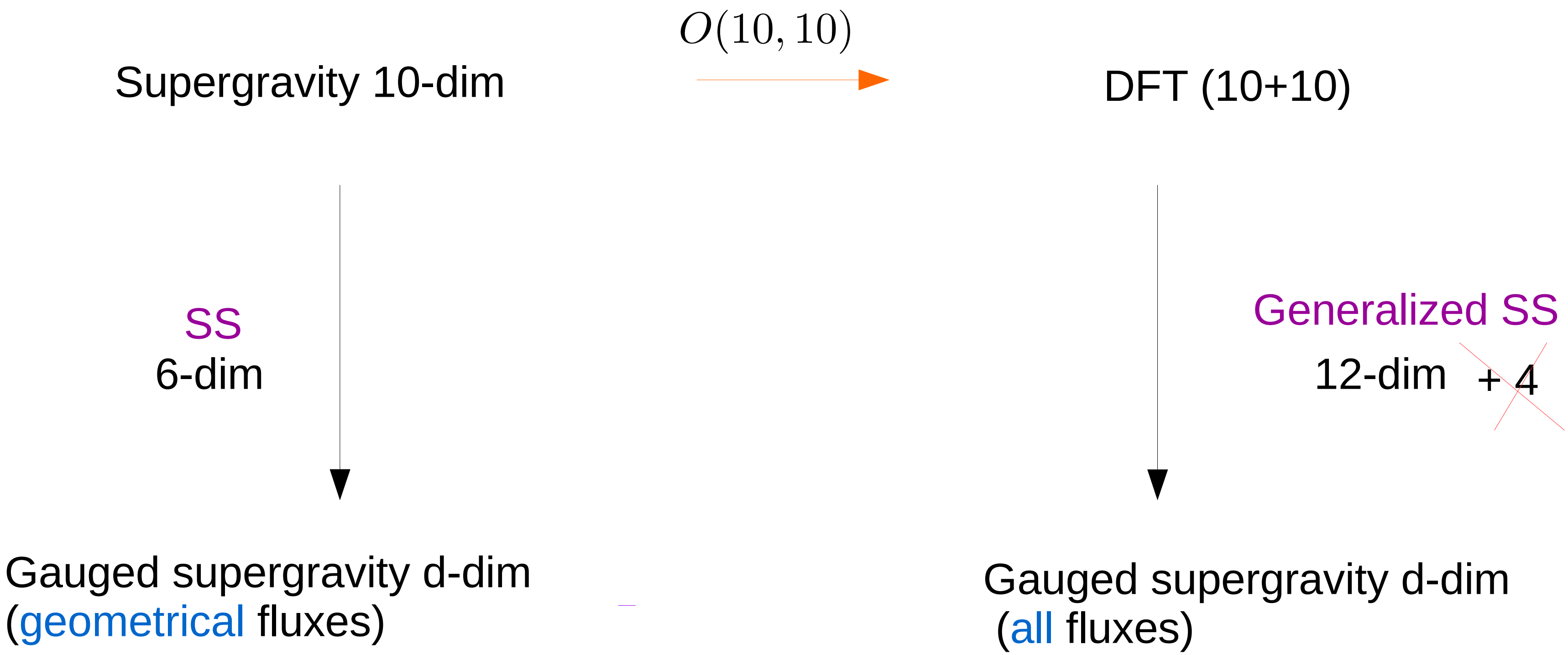
$\beta^{[mn]} \longrightarrow$ non-geometric fluxes $\qquad f^{ab}{}_c = Q_c{}^{ab} \ , \quad f^{abc} = R^{abc}$

$$f_{abc} = H_{abc} \ , \quad f^{a}{}_{bc} = \omega_{bc}{}^{a} \ , \quad f^{ab}{}_{c} = Q_{c}{}^{ab} \ , \quad f^{abc} = R^{abc}$$

more explicitely

$$\begin{aligned}
H_{abc} &= 3\left[\nabla_{[a} v_{bc]} - v_{d[a} \tilde{\nabla}^{d} v_{bc]}\right] , \\
\omega_{ab}{}^{c} &= 2\Gamma_{[ab]}{}^{c} + \tilde{\nabla}^{c} v_{ab} + 2\Gamma^{mc}{}_{[a} v_{b]m} + \beta^{cm} H_{mab} \, , \\
Q_{c}{}^{ab} &= 2\Gamma^{[ab]}{}_{c} + \partial_{c}\beta^{ab} + v_{cm}\tilde{\partial}^{m}\beta^{ab} + 2\omega_{mc}{}^{[a}\beta^{b]m} - H_{mnc}\beta^{ma}\beta^{nb} \, , \\
R^{abc} &= 3\left[\beta^{[\underline{a}m}\nabla_{m}\beta^{\underline{bc}]} + \tilde{\nabla}^{[a}\beta^{bc]} + v_{mn}\tilde{\nabla}^{n}\beta^{[ab}\beta^{c]m} + \beta^{[\underline{a}m}\beta^{\underline{b}n}\tilde{\nabla}^{\underline{c}]} v_{mn}\right] + \beta^{am}\beta^{bn}\beta^{cl} H_{mnl} \, ,
\end{aligned}$$

$$\nabla_{a}, \qquad \tilde{\nabla}^{n} \qquad \longrightarrow \qquad \Gamma_{ab}{}^{c} = u_{a}{}^{m}\partial_{m}u_{b}{}^{n}u^{c}{}_{n} \ , \qquad \Gamma^{ab}{}_{c} = u^{a}{}_{m}\tilde{\partial}^{m}u^{b}{}_{n}u_{c}{}^{n}$$



Supergravity 10-dim

$O(10,10)$

DFT (10+10)

SS

6-dim

Generalized SS

12-dim ~~+ 4~~

Gauged supergravity d-dim
(geometrical fluxes)

Gauged supergravity d-dim
(all fluxes)

EFT

Berman, Dall Agata, Cederwall, Coimbra, Hohm, Kleinschmidt, Musaev,
Samtleben,Thompson, Waldram, West,Strickland-Constable, Waldram, ...

GA, Graña,Marques, Rosabal ´13, ´14

- Coordinate space

$$(x^\mu, \mathbb{Y}^M)$$

x^μ d=4 space-time

$\mathbb{Y}^M$ $M = 1, \ldots, 56$ internal

- to start with: restrict to internal sector

x^μ expectator

- Fields

$$\begin{tikzcd}
\mathcal{H}_{MN} = \mathcal{H}_{MN}(\phi, g, b, C_p) \arrow[d] & & \\
\mathcal{H}^{MN} = E_{\bar{A}}{}^{M} \mathcal{H}^{\bar{A}\bar{b}} E_{\bar{B}}{}^{N} \arrow[rr] & & E_{\bar{A}}{}^{M} \in \mathbb{R}^{+} \times \frac{E_{7(7)}}{SU(8)} \arrow[d, shift right] \arrow[d, shift left] \\
 & & E_{\bar{A}}{}^{M} = e^{-\Delta} \tilde{E}_{\bar{A}}{}^{M} \arrow[d] \\
 & & \text{Integration measure}
\end{tikzcd}$$

$$V^{M} = v^{\bar{A}}(x) E_{\bar{A}}{}^{M}(Y)$$

- Symmetries

$$\mathcal{L}_\xi V^M = (L_\xi V)^M + Y^M{}_N{}^P{}_Q \partial_P \xi^Q V^N$$

$\mathcal{L}_\xi$ must preserve $E_{7(7)}$ invariants

$\omega_{NQ} = -\omega_{QN}$ symplectic metric

$K_{MNPQ} \equiv P^{MP}{}_{NQ}$ projector to adjoint

$$\longrightarrow \qquad Y^M{}_N{}^P{}_Q = -12 P^{MP}{}_{NQ} + \frac{1}{2}\omega^{MP}\omega_{NQ}$$

Extended (dynamical) fluxes

$$(\mathcal{L}_{E_{\bar{A}}} E_{\bar{B}})^M = F_{\bar{A}\bar{B}}{}^{\bar{C}} E_{\bar{C}}{}^M$$

$$\mathbb{F}_{\bar{A}\bar{B}}{}^{\bar{C}} = 2\Omega_{[\bar{A}\bar{B}]}{}^{\bar{C}} + Y^{\bar{C}}{}_{\bar{B}}{}^{\bar{D}}{}_{\bar{E}}\ \Omega_{\bar{D}\bar{A}}{}^{\bar{E}}$$

$$\Omega_{\bar{A}\bar{B}}{}^{\bar{C}} = \hat{E}_{\bar{A}}{}^M \partial_M \hat{E}_{\bar{B}}{}^N (\hat{E}^{-1})_N{}^{\bar{C}}$$

$$F_{AB}{}^C = D_{AB}{}^C + X_{AB}{}^C$$

$$F_{\bar{A}\bar{B}}{}^{\bar{C}} \in \mathbf{56} + \mathbf{912}$$

Namely

$$P_{(adj)}{}^C{}_B{}^D{}_E\ X_{AD}{}^E = X_{AB}{}^C$$

$$\mathbf{912}$$

$$X_{A[BC]} = X_{AB}{}^B = X_{(ABC)} = X_{BA}{}^B = 0$$

$$D_{AB}{}^C = -\vartheta_A \delta_B^C + 8P_{(adj)}{}^C{}_B{}^D{}_A \vartheta_D$$

$$\in \mathbf{56}$$

Consistency constraints

$$\Delta_{123}{}^{M} = \left([\mathcal{L}_{\xi_1},\ \mathcal{L}_{\xi_2}] - \mathcal{L}_{\xi_{12}} \right) \xi_3^M = 0$$

↓

$$[\mathcal{L}_{\xi_1},\ \mathcal{L}_{\xi_2}] = \mathcal{L}_{[[\xi_1,\ \xi_2]]} \,.$$

closure of the algerba

$$\mathcal{L}_{((\xi_1,\ \xi_2))} = 0 \ , \qquad ((\xi_1,\ \xi_2)) = \xi_{(12)}$$

Leibnitz

A solution: “section condition” or “strong constraint”

$$P_{(adj)\,MN}{}^{QR} \partial_Q \otimes \partial_R = 0$$

fields depend on a 6d slice of the 56d space

Are there other solutions to consistency constraints?

Consistency reads

$$
\begin{aligned}
([\mathcal{L}_{E_{\bar{A}}}, \mathcal{L}_{E_{\bar{B}}}]E_{\bar{C}} - \mathcal{L}_{\mathcal{L}_{E_{\bar{A}}}E_{\bar{B}}}E_{\bar{C}})^M &= \Big([F_{\bar{A}}, F_{\bar{B}}] + F_{\bar{A}\bar{B}}{}^{\bar{E}}F_{\bar{E}}\Big)_{\bar{C}}{}^{\bar{D}} \\
&+ (\partial F)
\end{aligned}
$$

- SS

$$V^M = v^{\bar{A}}(x)E_{\bar{A}}{}^M(Y)$$

- constant fluxes

$$F_{AB}{}^C(\mathbb{Y})$$

- quadratic constraints

$[F_A, F_B] = -F_{AB}{}^C F_C$ $\longrightarrow$ J.I. + $F_{(AB)}{}^E F_{ED}{}^F = 0$

$\mathrm{F}_{(AB)}{}^E$ intertwining tensor.

An Extended Geometry?

Can we "generalize" Riemann geometry to a "Extended Geometry"?

diffeo	$L \to \mathcal{L}$	extended diffeo?
covariant derivative	$\nabla_i \to \nabla_M$	extended covariant derivative?
connections	$\Gamma_{ij}{}^{k} \to \Gamma_{MN}{}^{P}$	
	$\ldots \to \ldots$	extended ….?
Ricci tensor	$R_{ijlk} \to \mathcal{R}_{MNPQ}$	
scalar curvature	$R \to \mathcal{R}$	extended scalar curvature?

generalized covariant derivative

$$\nabla_M V^N = \partial_M V^N + \Gamma_{MP}{}^N V^P \qquad \nabla_M E_{\bar{A}}{}^N = W_{M\bar{A}}{}^{\bar{B}} E_{\bar{B}}{}^N$$

torsion free $\mathcal{T}_{\bar{A}\bar{B}}{}^M = (\mathcal{L}^{\nabla}_{E_{\bar{A}}} - \mathcal{L}_{E_{\bar{A}}}) E_{\bar{B}}{}^M = 0 \quad \Rightarrow \quad 2\Gamma_{[AB]}{}^C = Y^C{}_B{}^P{}_Q \Gamma_{PA}{}^Q$

metric compatibility $\nabla_A H^{BC} = 0$

only part of $\Gamma_{MP}{}^N$ (and $W_{MN}{}^P$) is determined

$$W_{MN}{}^P = -\frac{16}{19} P_{(adj)}{}^P{}_N{}^I{}_M \vartheta_I + \frac{1}{7} X_{MN}{}^P + \Sigma_{MN}{}^P \qquad W \in \mathbf{56} + \mathbf{912} + \mathbf{6480}$$

$$\mathcal{R}_{MN} \equiv \frac{1}{2}\left(R_{MN} + R_{NM} + \Gamma_{RM}{}^P \; Y^R{}_P{}^S{}_Q \; \Gamma_{SN}{}^Q\right) = \mathcal{R}_{NM}$$

$$[\nabla_M, \nabla_P] V^P + \frac{1}{2} \nabla_A (Y^A{}_M{}^B{}_P \nabla_B V^P) = \mathcal{R}_{MP} V^P$$

Coimbra, Strickland-Constable,Waldram(2012)

Ricci scalar

$$\mathcal{R} = H^{AB}\mathcal{R}_{AB}$$

$$\begin{aligned}\frac{1}{4}\mathcal{R} \quad &= \quad \frac{1}{672}\left(H^{AD}H^{BE}H_{CF}X_{AB}{}^{C}X_{DE}{}^{F} + 7H^{AB}X_{AC}{}^{D}X_{BD}{}^{C}\right) \\ &\quad -\frac{1}{672}\left(448H^{AB}P_{(adj)AB}{}^{CD} + \frac{4}{3}H^{CD}\right)\vartheta_C\vartheta_D\end{aligned}$$

$$[F_A, F_B] = -F_{AB}{}^{C}F_C \qquad \longrightarrow \qquad \delta_\xi\mathcal{R} = \mathcal{L}_\xi\mathcal{R} = \xi^P\partial_P\mathcal{R}$$

quadratic constraints

$$F_{AB}{}^{C} = D_{AB}{}^{C} + X_{AB}{}^{C} \qquad\qquad F_{\bar{A}\bar{B}}{}^{\bar{C}} \in \mathbf{56} + \mathbf{912}$$

Internal action

$$S = \frac{1}{4}\int dy\, e^{-2\Delta}\frac{1}{672}\left(H^{AD}H^{BE}H_{CF}X_{AB}{}^{C}X_{DE}{}^{F} + 7H^{AB}X_{AC}{}^{D}X_{BD}{}^{C}\right)$$

$$(\sqrt{H})^{-1/28} = e^{-2\Delta} \qquad \mathcal{L}_{\xi}e^{-2\Delta} = \partial_P(e^{-2\Delta}\xi^P)$$

chose $H^{AB} = \mathcal{M}^{AB}(x)$ and $e^{-2\Delta(y)} = \sqrt{g(y)}$

$$\downarrow$$

$$S = 4V = \frac{1}{672}\left(\mathcal{M}^{AD}(x)\mathcal{M}^{BE}(x)\mathcal{M}_{CF}(x)X_{AB}{}^{C}X_{DE}{}^{F} + 7\mathcal{M}^{AB}(x)X_{AC}{}^{D}X_{BD}{}^{C}\right)$$

Scalar potential for $\mathcal{N}=8$ gauged supergravity

de Wit, H.Samtleben, M.Trigiante (2007)

Extended Field Theory?

Is it possible to build up a $D = 4 + 56$ EFT ?

$(x^\mu, \mathbb{Y}^M)$ x^μ $\mu = 1, 2, 3, 4$ $\mathbb{Y}^M$ $M = 1, \ldots, 56$

Mixed terms terms require extra structure!

$\downarrow$

Tensor hierarchy

- start in $D = 4$ with gaugings → $D = 4$ Gauged Sugra
- then try to uplift to $D = 4 + 56$

$D = 4$ algebra

$$V^{\mathbb{M}} = (\xi^{\mu}, \xi^{M}, \dots)$$

$$\begin{aligned}
(\hat{\mathcal{L}}_{\xi_1}\xi_2)^{\mu} &= (L_{\xi_1}\xi_2)^{\mu} \\
(\hat{\mathcal{L}}_{\xi_1}\xi_2)^{A} &= L_{\xi_1}\xi_2^{A} - \xi_2^{\rho}\partial_{\rho}\xi_1^{A} + F_{BC}{}^{A}\xi_1^{B}\xi_2^{C}
\end{aligned}$$

as in DFT

$$\left(\hat{\Delta}_{\xi_1}\hat{\mathcal{L}}_{\xi_2}V\right)^{\mathbb{A}} = \left[\left(\left[\hat{\mathcal{L}}_{\xi_1}, \hat{\mathcal{L}}_{\xi_2}\right] - \hat{\mathcal{L}}_{\hat{\mathcal{L}}_{\xi_1}\xi_2}\right)V\right]^{\mathbb{A}}$$

$$\begin{aligned}
(\hat{\Delta}_{\xi_1}\hat{\mathcal{L}}_{\xi_2}V)^{\mu} &= 0 \\
(\hat{\Delta}_{\xi_1}\hat{\mathcal{L}}_{\xi_2}V)^{A} &= \left([F_B, F_C] + F_{BC}{}^{E}F_E\right)_{D}{}^{A}\xi_1^{B}\xi_2^{C}V^{D} \\
&\quad -2V^{\rho}F_{(BC)}{}^{A}\partial_{\rho}\xi_1^{B}\xi_2^{C}
\end{aligned}$$

→ quadratic constraints

$$\Big\downarrow$$

$$\xi_{\rho}{}^{A}$$

Tensor hierarchy ($\mathbf{133}$)

$$
\begin{aligned}
(\hat{\mathcal{L}}_{\xi_1}\xi_2)^\mu &= (L_{\xi_1}\xi_2)^\mu \\
(\hat{\mathcal{L}}_{\xi_1}\xi_2)^A &= L_{\xi_1}\xi_2^A - \xi_2^\rho\partial_\rho\xi_1^A + F_{BC}{}^A\xi_1^B\xi_2^C + \xi_2^\rho\xi_{1\rho}{}^A \\
(\hat{\mathcal{L}}_{\xi_1}\xi_2)_\mu{}^A &= (L_{\xi_1}\xi_2)_\mu{}^A + 2\xi_2^\sigma\partial_{[\sigma}\xi_{1\rho]}{}^A + 2F_{(BC)}{}^A(2\xi_{[1}^B\xi_{2]\mu}{}^C + \xi_2^B\partial_\mu\xi_1^C)
\end{aligned}
$$

closes if $\xi_{1\rho}{}^B F_{BC}{}^A = 0$

solved, due to quadratic constraints ($F_{(AB)}{}^E F_{ED}{}^F = 0$), by

$$\xi_\mu{}^A = F_{(BC)}{}^A \xi_\mu{}^{BC} = F_\alpha{}^A \xi_\mu{}^\alpha \qquad \xi_\mu{}^\alpha \in \mathbf{133}$$

Intertwining tensor

$$V^{\mathbb{M}} = (\xi^\mu, \xi^M, \xi_\mu{}^\alpha, \dots)$$

$$\mathbf{0} + \mathbf{56} + \mathbf{133} + \dots$$

Next step in the hierarchy ($\mathbf{912}$)

$$F_\alpha{}^A \left[(\hat{\mathcal{L}}_{\xi_1}\xi_2)_\mu{}^\alpha \;=\; (L_{\xi_1}\xi_2)_\mu{}^\alpha + 2\xi_2^\sigma\partial_{[\sigma}\xi_{1\mu]}{}^\alpha - 2(t^\alpha)_{BC}\left(2\xi^B_{[1}\xi_{2]\mu}{}^\beta F_\beta{}^C + \xi_2^B\partial_\mu\xi_1^C\right) + \Gamma_\mu{}^\alpha \right]$$

$$\Gamma_\mu{}^\alpha F_\alpha{}^A = 0$$

$$\left(\hat{\Delta}_{\xi_1}\hat{\mathcal{L}}_{\xi_2}\xi_3\right)_\mu{}^\alpha = 0$$

$$\xi_{1\rho\mu}{}^\alpha \in \mathbf{912}$$

$$\Gamma_\mu{}^\alpha = \xi_2^\rho\xi_{1\rho\mu}{}^\alpha - F_{A\beta}{}^\alpha\xi_{2\mu}{}^\beta\xi_1^A$$

intertwining tensor

$$V^{\mathbb{M}} = (\xi^\mu, \xi^M, \xi_\mu{}^\alpha, \xi_{\mu\nu}{}^{\mathcal{M}}, \ldots)$$

$$\mathbf{0} + \mathbf{56} + \mathbf{133} + +\mathbf{912}\ldots$$

$$
\begin{aligned}
\left(\hat{\mathcal{L}}_{\xi_1}\xi_2\right)^{\mu} &= (L_{\xi_1}\xi_2)^{\mu} \\
\left(\hat{\mathcal{L}}_{\xi_1}\xi_2\right)^{A} &= L_{\xi_1}\xi_2^A - \xi_2^\rho\partial_\rho\xi_1^A + F_{BC}{}^{A}\xi_1^B\xi_2^C + \xi_2^\rho\xi_{1\rho}{}^{\gamma}F_{\gamma}{}^{A} \\
\left(\hat{\mathcal{L}}_{\xi_1}\xi_2\right)_{\mu}{}^{\alpha} &= (L_{\xi_1}\xi_2)_{\mu}{}^{\alpha} - 2\xi_2^\rho\partial_{[\rho}\xi_{1\mu]}{}^{\alpha} - \ 2(t^\alpha)_{BC}\left(2\xi^B_{[1}\xi_{2]\mu}{}^{\gamma}F_{\gamma}{}^{C} + \xi_2^B\partial_\mu\xi_1^C\right) \\
&\quad + \xi_2^\rho\xi_{1\rho\mu}{}^{\alpha} - F_{A\beta}{}^{\alpha}\xi_{2\mu}{}^{\beta}\xi_1^A \\
\left(\hat{\mathcal{L}}_{\xi_1}\xi_2\right)_{\mu\nu}{}^{\alpha} &= (L_{\xi_1}\xi_2)_{\mu\nu}{}^{\alpha} - 3\xi_2^\rho\partial_{[\rho}\xi_{1\mu\nu]}{}^{\alpha} \\
&\quad +2F_{A\beta}{}^{\alpha}\left(\xi_{2[\mu}{}^{\beta}\partial_{\nu]}\xi_1^A - \xi_2^A\partial_{[\mu}\xi_{1\nu]}{}^{\beta} - \xi^A_{[1}\xi_{2]\mu\nu}{}^{\beta} + \xi_{1[\mu}{}^{\beta}\xi_{2\nu]}{}^{A}\right) \\
&\vdots
\end{aligned}
$$

$$V^{\mathbb{M}} = (\xi^\mu, \xi^M, \xi_\mu{}^\alpha, \xi_{\mu\nu}{}^{\mathcal{M}}, \xi_{\mu\nu\xi}{}^{\mathbf{M}}, \dots)$$

Extended tangent space

$$\mathbf{0} + \mathbf{56} + \mathbf{133} + \mathbf{912} + (\mathbf{133} + \mathbf{8645}) + \dots$$

Cosistent with: F. Riccioni, D. Steele and P. West, The E(11) origin of all maximal supergravities: The Hierarchy of field-strengths, JHEP 0909, 095 (2009).

Generalized diffeomorphisms in gauged maximal supergravity

$$\xi^{\mathbb{A}} \quad = \quad (\xi^{\mu}, \xi^{A}, \xi_{\mu}{}^{<AB>}, \xi_{\mu\nu}{}^{<ABC>}, \xi_{\mu\nu\rho}{}^{<ABCD>}, \dots)$$

vectors

$< \cdots >$ projector onto irreps. $\xrightarrow{E_{7(7)}}$ $(\xi^{\mu}, \xi^{A}, \xi_{\mu}{}^{\alpha}, \xi_{\mu\nu}{}^{\mathcal{A}}, \xi_{\mu\nu\rho}{}^{\mathbf{A}}, \dots)$

$$\left(\hat{\mathcal{L}}_{\xi_1}\xi_2\right)^{\mathbb{A}} = \xi_1^{\mathbb{B}}\partial_{\mathbb{B}}\xi_2^{\mathbb{A}} - \xi_2^{\mathbb{B}}\partial_{\mathbb{B}}\xi_1^{\mathbb{A}} + W^{\mathbb{A}}{}_{\mathbb{B}}{}^{\mathbb{C}}{}_{\mathbb{D}}\partial_{\mathbb{C}}\xi_1^{\mathbb{D}}\xi_2^{\mathbb{B}} + F_{\mathbb{B}\mathbb{C}}{}^{\mathbb{A}}\xi_1^{\mathbb{B}}\xi_2^{\mathbb{C}}$$

Gen. diff

$$\partial_{\mathbb{A}} = (\partial_{\mu}, 0, \dots)$$

Intertwinig tensor.

$$F_{\alpha}{}^{A}\ , \quad F_{\mathcal{A}}{}^{\alpha}\ , \quad F_{\mathbf{A}}{}^{\mathcal{A}}\ , \quad \dots \qquad\qquad F_{\mathcal{A}}{}^{\alpha}F_{\alpha}{}^{A} = F_{\mathbf{A}}{}^{\mathcal{A}}F_{\mathcal{A}}{}^{\alpha} = \cdots = 0$$

Extended tangent space

$$V^{\mathbb{M}} = (\xi^{\mu}, \xi^{M}, \xi_{\mu}{}^{\alpha}, \xi_{\mu\nu}{}^{\mathcal{M}}, \xi_{\mu\nu\xi}{}^{\mathbf{M}}, \dots)$$

vectors

$$\Big\downarrow$$

$$\mathbb{E}_{\bar{\mathbb{A}}}{}^{\mathbb{M}} \supset (e_{\bar{a}}{}^{\mu}, \Phi_{\bar{A}}{}^{M}, A_{\mu}{}^{M}, B_{\mu\nu}{}^{\alpha}, C_{\mu\nu\xi}{}^{\mathcal{M}}, D_{\mu\nu\xi\rho}{}^{\mathbf{M}}, \dots)$$

frame parametrization

$$(\delta_{\xi} V)^{\mathbb{M}} = (\hat{\mathcal{L}}_{\xi} V)^{\mathbb{M}}$$

gauge trasformations

$$\Big\downarrow$$

$$\delta\mathbb{E}_{\bar{\mathbb{A}}}{}^{\mathbb{M}} \supset (\delta e_{\bar{m}}{}^{\mu}, \delta\Phi_{\bar{A}}{}^{M}, \delta A_{\mu}{}^{M}, \delta B_{\mu\nu}{}^{\alpha}, \delta C_{\mu\nu\xi}{}^{\mathcal{M}}, \delta D_{\mu\nu\xi\rho}{}^{\mathbf{M}}, \dots)$$

$$\mathbb{F}_{\bar{\mathbb{A}}\bar{\mathbb{B}}}{}^{\bar{\mathbb{C}}} = (\hat{\mathcal{L}}_{\mathbb{E}_{\bar{\mathbb{A}}}} \mathbb{E}_{\bar{\mathbb{B}}})^{\mathbb{C}} (\mathbb{E}^{-1})_{\mathbb{C}}{}^{\bar{\mathbb{C}}}$$

dynamical fluxes

$$\mathbb{F}_{\bar{\mathbb{A}}\bar{\mathbb{B}}}{}^{\bar{\mathbb{C}}} = (\hat{\mathcal{L}}_{\mathbb{E}_{\bar{\mathbb{A}}}}\mathbb{E}_{\bar{\mathbb{B}}})^{\mathbb{C}}(\mathbb{E}^{-1})_{\mathbb{C}}{}^{\bar{\mathbb{C}}} \longrightarrow$$

$$\begin{aligned}
\mathbb{F}_{\bar{a}\bar{b}}{}^{\bar{c}} &\sim \omega_{[\bar{a}\bar{b}]}{}^{\bar{c}} \\
\mathbb{F}_{\bar{a}\bar{b}}{}^{\bar{C}} &\sim F_{\mu\nu}{}^{C} \\
\mathbb{F}_{\bar{a}\bar{b}\bar{c}}{}^{\bar{\gamma}} &\sim \mathcal{H}_{\mu\nu\xi}{}^{\alpha} \\
\mathbb{F}_{\bar{A}\bar{B}}{}^{\bar{C}} &\sim F_{AB}{}^{C} \\
\mathbb{F}_{\bar{a}\bar{B}}{}^{\bar{C}} &\sim D_{\mu}\Phi_{\bar{B}}{}^{C} \\
&\vdots
\end{aligned}$$

$$\begin{aligned}
D_{\mu}\Phi_{\bar{B}}{}^{C} &= \partial_{\mu}\Phi_{\bar{B}}{}^{C} - F_{AB}{}^{C}A_{\mu}{}^{A}\Phi_{\bar{B}}{}^{B} \\
\mathcal{H}_{\mu\nu}{}^{C} &= 2\partial_{[\mu}A_{\nu]}{}^{C} - F_{[AB]}{}^{C}A_{\mu}{}^{B}A_{\nu}{}^{C} + B_{\mu\nu}{}^{\alpha}F_{\alpha}{}^{C} \\
\mathcal{H}_{\mu\nu\rho}{}^{\alpha} &= 3\left[\partial_{[\mu}B_{\nu\rho]}{}^{\alpha} - C_{\mu\nu\rho}{}^{\mathcal{A}}F_{\mathcal{A}}{}^{\alpha} + 2(t^{\alpha})_{BC}\left(A_{[\mu}{}^{B}\partial_{\nu}A_{\rho]}{}^{C} + A_{[\mu}{}^{B}B_{\nu\rho]}{}^{\beta}F_{\beta}{}^{C} \; + \frac{1}{3}\, F_{DE}{}^{B}A_{[\mu}{}^{D}A_{\nu}{}^{E}A_{\rho]}{}^{C}\right)\right]
\end{aligned}$$

$$\hat{\Delta}_{\bar{\mathbb{A}}\bar{\mathbb{B}}\bar{\mathbb{C}}}{}^{\mathbb{D}} = \left([\hat{\mathcal{L}}_{\mathbb{E}_{\bar{\mathbb{A}}}}, \hat{\mathcal{L}}_{\mathbb{E}_{\bar{\mathbb{B}}}}]\mathbb{E}_{\bar{\mathbb{C}}} - \hat{\mathcal{L}}_{\hat{\mathcal{L}}_{\mathbb{E}_{\bar{\mathbb{A}}}}\mathbb{E}_{\bar{\mathbb{B}}}}\mathbb{E}_{\bar{\mathbb{C}}}\right)^{\mathbb{D}} \xrightarrow{\text{BI}}$$

$$\begin{aligned}
\hat{\Delta}_{\bar{d}\bar{a}\bar{b}}{}^{\bar{c}} &\sim R_{[\mu\nu\rho]}{}^{\sigma} = 0 \\
\hat{\Delta}_{\bar{d}\bar{a}\bar{b}}{}^{\bar{C}} &\sim (3D_{[\mu}F_{\nu\rho]}{}^{M} - \mathcal{H}_{\mu\nu\rho}{}^{M}) = 0 \\
&\vdots
\end{aligned}$$

Democratic formulation of maximal (bosonic) gauged Sugra in $D = 4$

An Extended space-time geometry?

$D = 4 + 56$

$D = d + dim\, E_{n+1(n+1)}$

$$\hat{\xi}^{\mathbb{M}}(x, Y) = (\hat{\xi}^{\mu}, \hat{\xi}^{M}, \hat{\xi}_{\mu}{}^{<MN>}, \hat{\xi}_{\mu\nu}{}^{<MNP>}, \dots)$$

$$\left(\hat{\mathcal{L}}_{\xi_1}\xi_2\right)^{\mathbb{A}} = \xi_1^{\mathbb{B}}\partial_{\mathbb{B}}\xi_2^{\mathbb{A}} - \xi_2^{\mathbb{B}}\partial_{\mathbb{B}}\xi_1^{\mathbb{A}} + W^{\mathbb{A}}{}_{\mathbb{B}}{}^{\mathbb{C}}{}_{\mathbb{D}}\partial_{\mathbb{C}}\xi_1^{\mathbb{D}}\xi_2^{\mathbb{B}} + F_{\mathbb{B}\mathbb{C}}{}^{\mathbb{A}}\xi_1^{\mathbb{B}}\xi_2^{\mathbb{C}}$$

Extended gauged sugra

$\partial_{\mathbb{M}} = (\partial_{\mu}, \dots).\quad D = 4$

$\downarrow$ uplift?

$$\left(\hat{\mathcal{L}}_{\hat{\xi}_1}\hat{\xi}_2\right)^{\mathbb{M}} = \hat{\xi}_1^{\mathbb{P}}\partial_{\mathbb{P}}\hat{\xi}_2^{\mathbb{M}} - \hat{\xi}_2^{\mathbb{P}}\partial_{\mathbb{P}}\hat{\xi}_1^{\mathbb{M}} + Y^{\mathbb{M}}{}_{\mathbb{P}}{}^{\mathrm{N}}{}_{\mathbb{Q}}\partial_{\mathrm{N}}\hat{\xi}_1^{\mathbb{Q}}\hat{\xi}_2^{\mathbb{P}}$$

$\partial_{\mathbb{M}} = (\partial_{\mu}, \partial_{M}, 0, \dots).$

$D = 4 + 56$

Obstructions to uplifting at different levels $M, < MN >, < MNP >, \dots)$ depending on n

See Berman et al., Hohm et al. ,

hierarchy $\longrightarrow$ $M, < MN >, < MNP >, \ldots$

$$F_\alpha{}^A \left(\hat{\mathcal{L}}_{\xi_1}\xi_2\right)_\mu{}^\alpha = \ldots \longrightarrow Y^M{}_P{}^N{}_Q \partial_N \left(\hat{\mathcal{L}}_{\hat{\xi}_1}\hat{\xi}_2\right)_\mu{}^{PQ} = \ldots$$

Intertwinig tensor.

Intertwining operartor

i.e.

$$F_{(AB)}{}^C = Y^M{}_P{}^N{}_Q \partial_N U_{(A}{}^Q U_{B)}{}^P (U^{-1})_M{}^C$$

$$\left(\hat{\mathcal{L}}_{\xi_1}\xi_2\right)^M \longrightarrow \xi_\mu{}^{AB} F_{(AB)}{}^C U_C{}^M = \frac{1}{2} Y^M{}_P{}^N{}_Q \partial_N \hat{\xi}_\mu{}^{PQ} + Y^M{}_{[P}{}^N{}_{Q]} \partial_N U_A{}^Q U_B{}^P \xi_\mu{}^{AB}$$

Obstruction in $E_{7(7)}$

$E_{7(7)}$ $\quad Y^M{}_P{}^N{}_Q = -12 P^{MN}{}_{(PQ)} + \underline{\frac{1}{2}\omega^{MN}\omega_{[PQ]}}$

$E_{n(n)}$ $\quad Y^M{}_P{}^N{}_Q = C_n P^{MN}{}_{(PQ)}$

$< MN >$ obstruction in $E_{6(6)}$

Summary and Outlook

- Low energy effective field theories for strings miss “stringy dualities”,
  D(E)FT could capture duality information

- Like Riemann Geometry describes Gravity, DFT(EFT) could provide a “Geometry” for strings

  Non-Geometry geometrized

- Twisted (SS) compactifications lead to full gauged supergravities

  DFT → (electric bosonic sector of) half-maximal gauge supergravity

  EFT (under construction?, O. Hohm talk)

  → Scalar potential of maximal gauged supergravity
  Democratic maximal gauged supergravity

- All (electric) “gaugings” are obtained from DFT(EFT). New configurations,
  not derivable from effective 10 dimensional sugra theories can be reached from D(E)FT

- Can the strong constraint be generically relaxed? i.e.: SS (see Betz,B.L,R)

- Can we really include windings?

  Strong constraint: no windings

  SS: only zero mode

- Is there a double(extended) geometry?:

  patchings involving all coordinates? (see Hohm,L.Z,.; Berman et al., Hull,..)

  truly double manifold with a globally defined basis?

- A consistent truncation of string (field?) theory?

- Truly stringy states? .

- Massive states? Quantum corrections?