Non-abelian T-dualities and generalised fluxes

or: (generalised) geometry and (classical) strings

based on arXiv:1803.03971 with Dieter Lüst

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$\Delta p \cdot \Delta q \geq \frac{1}{2}\hbar$

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MAX-PLANCK-GESELLSCHAFT
TAGUNGSSTÄTTE SCHLOSS RINGBERG

motivation

- non-abelian T-dualities
    - beyond toroidal compactifications/abelian isometries
        - generic group isometries: non-abelian T-duality
        - 'Poisson-Lie' manifolds: Poisson-Lie T-duality

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    - generic group isometries: non-abelian T-duality
    - 'Poisson-Lie' manifolds: Poisson-Lie T-duality
  - for abelian T-duality
    - duality group $O(d, d)$
    - 'duality covariant' frameworks (DFT, gen. geometry)
    - connecting geometric and non-geometric backgrounds

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  $\rightarrow$ similar things for non-abelian T-dualities?
- deformations of integrable string σ-model
  - η- resp. Yang-Baxter deformations
    originally motivated by algebraic structures
    behind Poisson-Lie T-duality

    $\rightarrow$ connections to non-abelian T-duality

overview

1. review: non-abelian and Poisson-Lie T-duality
2. non-abelian T-duality group
3. generalised fluxes of the Poisson-Lie σ-model
4. application to Yang-Baxter deformations

(abelian) T-duality

- string σ-model

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- Hamiltonian density H:

$$\mathcal{L} = \dot{X} \cdot P - H, \quad \text{with} \quad H = (X', P)\mathcal{H}(G,B)\begin{pmatrix} X' \\ P \end{pmatrix}$$

with generalised metric $\mathcal{H}(G,B) = \begin{pmatrix} G - BG^{-1}B & BG^{-1} \\ -G^{-1}B & G^{-1} \end{pmatrix}$

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- natural action of $\varphi \in \mathrm{O}(d,d)$ on $\mathcal{H}(G,B)$:

$$\mathcal{H}(G,B) \quad \xrightarrow{T} \quad \varphi \cdot \mathcal{H}(G,B) \cdot \varphi^T$$

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  - *B*-shifts - shifts of B by a constant skewsymmetric σ
  - β-shifts - shifts of β by a constant skewsymmetric r

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  - B-shifts - shifts of B by a constant skewsymmetric σ
  - β-shifts - shifts of β by a constant skewsymmetric r
- different parameterisation of $\mathcal{H}$: $g + \beta = \frac{1}{G+B}$
  - g^{-1}: open string metric
  - β^{MN} open string non-commutativity parameters, conjugate (dual) B-field

non-abelian T-duality (NATD)

- σ-model with isometry group $\mathcal{G}$

$$S \sim \int \mathrm{d}^2\sigma \; E_{ab}(g^{-1}\partial_+ g)^a (g^{-1}\partial_- g)^b$$

with $g: \Sigma \to \mathcal{G}$, $\mathfrak{g}$ Lie algebra of $\mathcal{G}$, ${f^c}_{ab}$ structure constants
$E = G_0 + B_0$ constant

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- same procedure as in abelian case [De la Ossa et. al 93]
    - substitute $g^{-1}\partial_\pm g \to j_\pm$
    - add Lagrangian multiplier term $x_a(\partial_{(+}j_{-)} + [j_+, j_-])^a$
    - integrate out $j_\pm$

$$S \sim \int \mathrm{d}^2\sigma \; \left(\frac{1}{E_{ab} - x_c {f^c}_{ab}}\right) \partial_+ x_a \partial_- x_b .$$

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- caveats:
  - sugra solution generating rather than 'duality'
  - 'duality' does not always conserve conformal inv.
  - $\mathcal{G}$-isometric model dual to a non-isometric model?

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$$[t_a, \bar{t}^b] = \bar{f}_a{}^{bc} t_c + f^b{}_{ca} \bar{t}^c .$$

  - Jacobi identity: $f^c{}_{mn} \bar{f}_c{}^{ab} = f^{(a}{}_{c(m} \bar{f}_{n)}{}^{b)c}$.

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- $(\mathfrak{d}, \mathfrak{g}, \mathfrak{g}^\star)$ - Manin triple decomposition
  - $2d$-dimensional Lie algebra $\mathfrak{d}$ with $O(d,d)$-metric
  - two complementary Lagrangian subalgebras $\mathfrak{g}$, $\mathfrak{g}^\star$

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- examples:
  - semi-abelian bialgebra: $\mathfrak{g} \oplus \mathfrak{u}(1)^d$
  - any solution of the classical Yang-Baxter equation $r^{ab} t_a \wedge t_b$ ($\leftrightarrow$ Poisson bivector on $\mathcal{G}$) via $\bar{f}_c{}^{ab} = f^{(a}{}_{cd} r^{b)d}$

Poisson-Lie T-duality 1

[Klimcik, Severa '95; Sfetsos, von Unge, Hlavaty, Hull/Reid-Edwards, ...]

- σ-model for $l: \ \Sigma \to \mathcal{D}$ (group of a Lie bialgebra $\mathfrak{d}$)

$$S \sim S_{WZW}(l) - \langle l^{-1}\partial_\sigma l, \hat{\mathcal{H}}(l^{-1}\partial_\sigma l)\rangle$$

with $\hat{\mathcal{H}}(G, B)$: generalised metric

  - non-abelian generalisation of $\mathcal{L} = \dot{X} \cdot P - H$
  - independent of choice of Manin triple decomp. of $\mathfrak{d}$

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  - non-abelian generalisation of $\mathcal{L} = \dot{X} \cdot P - H$
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- dynamics determined by decomposition into *orthogonal* subbundles, defined by generalised metric $\mathcal{H}(G, B)$

$$0 = \langle l^{-1}\partial_\pm l, \mathfrak{d}^\pm\rangle \,, \qquad \mathfrak{d} = \mathfrak{d}^+ \perp \mathfrak{d}^-$$

$$\mathfrak{d}^\pm = \text{span}\left\{t_a \pm (G \pm B)_{ab}\bar{t}^b\right\}$$

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- Reduction to a non-doubled action
  - choose a Manin triple decomposition (groups $\mathcal{G}$, $\bar{\mathcal{G}} \subset \mathcal{D}$)
  - Ansatz $l = \bar{g}g^{-1}$, integrate out $\bar{g}$

Poisson-Lie T-duality 2

- Manin triple $(\mathfrak{d}, \mathfrak{g}, \mathfrak{g}^\star) \to$ Poisson bivector on $\mathcal{G}$

$$\Pi^{ab}(g) = \bar{f}_c{}^{ab} x^c - \frac{1}{2} \bar{f}_c{}^{k(a} f^{b)}{}_{dk} x^c x^d + \dots , \quad \text{for } g = \exp(x^a t_a) \in \mathcal{G},$$

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- Poisson-Lie σ-model (determined by $G_0 + B_0$ and $(\mathfrak{d}, \mathfrak{g}, \mathfrak{g}^\star)$)

$$S \sim \int \mathrm{d}^2\sigma \; (g^{-1}\partial_+ g)^a \left(\frac{1}{\frac{1}{G_0+B_0} + \Pi(g)} \right)_{ab} (g^{-1}\partial_- g)^b$$

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- Poisson-Lie T-duality - choose other Manin triple decomposition $(\mathfrak{d}, \mathfrak{g}^\star, \mathfrak{g})$ [Klimcik, Severa 95]

$$S_{PL} \sim \int \mathrm{d}^2\sigma \, (\bar{g}^{-1}\partial_+ \bar{g})_a \left(\frac{1}{G_0 + B_0 + \bar{\Pi}(\bar{g})} \right)^{ab} (\bar{g}^{-1}\partial_- \bar{g})^b$$

Poisson-Lie T-duality - summary

- 'doubled' string σ-model

  input: Lie bialgebra $\mathfrak{d}$, $\mathcal{H}(G, B) \rightarrow$ dynamics via $\mathfrak{d} = \mathfrak{d}^+ \perp \mathfrak{d}^-$

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$$\to S \sim \int \mathrm{d}^2\sigma \; (g^{-1}\partial_+ g)^a \underbrace{\left(\frac{1}{\frac{1}{G_0+B_0} + \Pi(g)}\right)_{ab}}_{=E_{ab}(g)} (g^{-1}\partial_- g)^b$$

dynamics: with $\quad j = g^{-1}\mathrm{d}g, \quad \bar{j}_{\pm,a} = \pm E^{(T)}_{ab}(g) j^b_\pm$

$$\underbrace{\mathrm{d}j^c + \frac{1}{2} f^c{}_{ab} j^a \wedge j^b = 0,}_{\text{Bianchi id.}} \qquad \underbrace{\mathrm{d}\bar{j}^c + \frac{1}{2} \bar{f}_c{}^{ab} \bar{j}_a \wedge \bar{j}_b = 0}_{\text{e.o.m.}}$$

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- Buscher-like duality $\mathfrak{g} \oplus \mathfrak{g}^\star \leftrightarrow \mathfrak{g}^\star \oplus \mathfrak{g}$:

  Bianchi id. $\leftrightarrow$ e.o.m., $\qquad E(e) \leftrightarrow E^{-1}(e)$

a non-abelian T-duality group

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  2. preserve the algebraic closure of $\mathfrak{g}$ and $\mathfrak{g}^\star$, i.e.

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- action on Poisson-Lie σ-model.
  - $(\mathfrak{d}, \mathfrak{g}', \mathfrak{g}'^\star) = (\mathfrak{d}, \varphi(\mathfrak{g}), \varphi(\mathfrak{g}^\star))$,
    different group $\mathcal{G}'$, different Poisson structure Π' on $\mathcal{G}'$

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    different group $\mathcal{G}'$, different Poisson structure Π' on $\mathcal{G}'$
  - standard O(d, d)-action on the generalised metric

$$\mathcal{H}(G_0', B_0') = \varphi^{-1}.\mathcal{H}(G_0, B_0).$$

  - insights into group via standard O(d, d) subgroups

non-abelian T-duality B-shifts 1

- B-shift as $\mathrm{O}(d,d)$-transformation: $\varphi_B = \begin{pmatrix} \mathbb{1} & \sigma \\ 0 & \mathbb{1} \end{pmatrix}$

$$t_a \ , \ \bar{t}^a \quad \xrightarrow{\varphi \in O(d,d)} \quad t'_a = t_a + \sigma_{ab}\bar{t}^b, \ \ \bar{t}'^a = \bar{t}^a$$

imposing closure

$$[t'_a, t'_b] = F^c{}_{ab} t'_c + H_{abc}\bar{t}'^c$$

$$\text{with} \quad F^c{}_{ab} = f^c{}_{ab} + \sigma_{k(a}\overline{f}_{b)}{}^{kc}$$

$$\text{and} \quad H_{abc} = \underbrace{\sigma_{(a|d}\sigma_{|b|e}\overline{f}_{|c)}{}^{de}}_{cYBe} - \underbrace{\sigma_{k(a}f^k{}_{bc)}}_{2-cocycle} \stackrel{!}{=} 0,$$

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  - $\mathfrak{g}^\star$ is abelian: $\rightarrow$ $\sigma_{ab}(g^{-1}\mathrm{d}g)^a \wedge (g^{-1}\mathrm{d}g)^b$ closed

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    gauge transformation of the **H**-flux:

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  - generic case: B-shifts connect groups $\mathcal{G}$, s.t. original Poisson bivector is still Poisson

    $$S_B \propto \int \mathrm{d}^2\sigma (g'^{-1}\partial_+ g')^a \left[\frac{1}{\frac{1}{G_0+B_0+\sigma} + \Pi'(g')}\right]_{ab} (g'^{-1}\partial_- g')^b,$$

non-abelian T-duality β-shifts 1

- β-shift as $\mathrm{O}(d,d)$-transformation: $\varphi_\beta = \begin{pmatrix} \mathbb{1} & 0 \\ r & \mathbb{1} \end{pmatrix}$
  - imposing closure condition

$$[\bar{t}'^a, \bar{t}'^b] = \overline{F}_c{}^{ab} \bar{t}'^c + R^{abc} t'_c$$

$$\text{with} \quad \overline{F}_c{}^{ab} = \overline{f}_c{}^{ab} + r^{k(a} f^{b)}{}_{kc}$$

$$\text{and} \quad R^{abc} = \underbrace{r^{(a|d} r^{|b|e} f^{|c)}{}_{de}}_{cYBe} - \underbrace{r^{k(a} \overline{f}_k{}^{bc)}}_{2-cocycle} \stackrel{!}{=} 0,$$

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- action on Poisson-Lie σ-model
  - $\underline{\mathfrak{g}^\star \text{ is abelian:}}$ $\rightarrow$ $r^{ab} t_a \wedge t_b$ classical r-matrix

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    Π Poisson structure to new dual algebra $r^{k(a} f^{b)}{}_{kc}$

$$S_\beta \sim \int \mathrm{d}^2\sigma (g^{-1}\partial_+ g)^a \left[\frac{1}{\frac{1}{G_0+B_0} + r + \Pi(g)} \right]_{ab} (g^{-1}\partial_- g)^b,$$

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  - generic case:

    β-shifts connect different Poisson structures to the group $\mathcal{G}$

    $$S_\beta \sim \int \mathrm{d}^2\sigma (g^{-1}\partial_+ g)^a \left[\frac{1}{\frac{1}{G_0+B_0} + r + \Pi'(g)} \right]_{ab} (g^{-1}\partial_- g)^b,$$

generalised fluxes of the Poisson-Lie σ-model

definitions and Bianchi identities

[Shelton et. al 05; Grana et al. 09; Blumenhagen et al. 12]

- definitions in a non-holonomic basis $t_a = e_a{}^i \partial_i \equiv "\partial_a"$

$$
\begin{aligned}
\mathbf{H}_{abc} &= \partial_{(a} B_{bc)} + f^d{}_{(ab} B_{c)d} \\
\mathbf{f}^c{}_{ab} &= e^c{}_j \left(e_a{}^i \partial_i e_b{}^j - e_b{}^i \partial_i e_a{}^j \right) = f^c{}_{ab} \\
\mathbf{Q}_c{}^{ab} &= \partial_c \beta^{ab} + f^{(a}{}_{mc} \beta^{b)m} \\
\mathbf{R}^{abc} &= -\beta^{m(a} \partial_m \beta^{bc)} + f^{(a}{}_{mn} \beta^{b|m} \beta^{|c)n} .
\end{aligned}
$$

definitions and Bianchi identities

[Shelton et. al 05; Grana et al. 09; Blumenhagen et al. 12]

- definitions in a non-holonomic basis $t_a = e_a{}^i \partial_i \equiv "\partial_a"$

$$\mathbf{H}_{abc} = \partial_{(a} B_{bc)} + f^d{}_{(ab} B_{c)d}$$
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- fluxes not independent - Bianchi identities

$$0 = \mathbf{H}_{k(ab} \mathbf{f}^k{}_{cd)}, \qquad 0 = \mathbf{f}^a{}_{k(b} \mathbf{f}^k{}_{cd)} + \mathbf{H}_{k(bc} \mathbf{Q}_{d)}{}^{ak}$$
$$0 = \mathbf{R}^{kab} \mathbf{H}_{kcd} + \mathbf{Q}_k{}^{ab} \mathbf{f}^c{}_{cd} - \mathbf{f}^{(a}{}_{k(c} \mathbf{Q}_{d)}{}^{b)k}$$
$$0 = \mathbf{Q}_k{}^{(ab} \mathbf{R}^{cd)k}, \qquad 0 = \mathbf{Q}_k{}^{(ab} \mathbf{Q}_d{}^{c)k} + \mathbf{f}^{(a}{}_{kd} \mathbf{R}^{bc)k}.$$

generalised fluxes of Poisson-Lie σ-model

- open string variables

$$g + \beta = \frac{1}{G_0 + B_0} + \Pi(g) = g + \underbrace{\beta_0 + \Pi(g)}_{\beta}$$

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- claim: generalised fluxes from β

$$\begin{aligned} \mathbf{H}_{abc} &\quad \text{complicated} \\ \mathbf{f}^c{}_{ab} &= f^c{}_{ab} \\ \mathbf{Q}_c{}^{ab} &= \overline{f}_c{}^{ab} + \beta_0^{d(a} f^{b)}{}_{dc} \\ \mathbf{R}^{abc} &= -\beta_0^{m(a} \overline{f}_m{}^{bc)} + \beta_0^{(a|m} \beta_0^{|b|n} f^{|c)}{}_{mn}, \end{aligned}$$

- in general: B-shifts gauge transformations of **H**-flux, β-shifts for **R**-flux

generalised fluxes and cohomology

- relation of $(\mathbf{f}, \mathbf{Q}, \mathbf{R})$ to underlying bialgebra?
- generalisation of results in abelian case? [Bakas, Lüst 13]

$$'\beta \sim H^2(\mathfrak{a}, \mathfrak{a})', \qquad '\mathbf{R} \sim H^3(\mathfrak{a}, \mathfrak{a})'$$

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- $(\mathbf{f}, \mathbf{Q}, \mathbf{R})$ describe quasi-Lie bialgebra structure

$$\big(\mathfrak{g}, \mathbf{Q} \in H^1(\mathfrak{g}, \mathfrak{g} \wedge \mathfrak{g}), (\delta\mathbf{R}) \in [0] \in H^1(\mathfrak{g}, \mathfrak{g} \wedge \mathfrak{g} \wedge \mathfrak{g})\big)$$

Q-flux dual structure constants up to β-shifts,
δ**R** failure of their Jacobi identity

application to
Yang-Baxter deformations

Yang-Baxter deformed principal chiral models

- Yang-Baxter deformation [Klimcik 09]

$$S \sim \int \mathrm{d}^2\sigma \; (g^{-1}\partial_+ g)^a \kappa_{ac} \left(\frac{1}{1-\eta R_g} \right)^c{}_b (g^{-1}\partial_- g)^b ,$$

$R : \; \mathfrak{g} \to \mathfrak{g}$, $R_g = \mathrm{Ad}_g^{-1} \circ R \circ \mathrm{Ad}_g$,
solution of (modified) classical Yang-Baxter equation
$[R(m), R(n)] - R([m, R(n)] - [n, R(m)]) = c^2 [m, n]$

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  - in some cases supergravity solution
  - special cases:
    - *abelian* R-operator - standard β-shift
    - *homogeneous* R-operator - some NATD transformation?
    - *Drinfel'd-Jimbo* solution (η-def.) - q-deformation

Yang-Baxter deformations as β-shifts

- in Poisson-Lie σ-model form:

$$S \sim \int \mathrm{d}^2\sigma \; (g^{-1}\partial_+ g)^a \left(\frac{1}{\kappa^{-1} + r + \Pi(g)} \right)_{ab} (g^{-1}\partial_- g)^b$$

with $r^{ab} = \eta \kappa^{ac} R_c{}^b$, $\qquad \Pi^{ab} = \eta r^{d(a} f^{b)}{}_{cd} x^c + \ldots$

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$$\mathbf{Q}_c{}^{ab} = 0, \qquad \mathbf{R}^{abc} = -\eta^2 c^2 \kappa^{ak} f_k{}^{bc},$$

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- generic R: not a duality, also Π not Poisson

conclusion and outlook

- summary:
  - insights into non-abelian T-group
  - a possible generalised flux interpretation of the Poisson-Lie type σ-models
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Thank you for your attention!

bi-Yang-Baxter deformations

- more general: integrable 2-parameter deformation [Klimcik 14]

$$S = \frac{1}{2}\int \mathrm{d}^2\sigma (g^{-1}\partial_+ g)^a \kappa_{ac} \left(\frac{1}{1-\xi R - \eta R_g}\right)^c{}_b (g^{-1}\partial_- g)^b.$$

- non-geometric fluxes:

$$\begin{aligned} \beta^{ab} &= (\xi+\eta) r^{ab} - \eta \Pi^{ab}(g), \\ \mathbf{Q}_c{}^{ab} &= -\xi f^{(a}{}_{cd} r^{b)d} \\ \mathbf{R}^{abc} &= -(\xi+\eta)^2 r^{(a|m} r^{|b|n} f^{|c)}{}_{mn}. \end{aligned}$$

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- for $\xi = -\eta$: **R**-flux free model, not related to PCM by β-shift