T-duality and α'-corrections

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Based on
- E. Lescano, C.N., A. Rodriguez, 1808.xxxxxx
- D. Marqués, C.N., 1507.00652
- W.Baron, J.Fernández-Melgarejo, D.Marqués, C.N., 1702.05489

Find a duality covariant gauge principle that determines the α'-expansion of the string effective field theories

Double field theory has such a gauge principle that fixes the leading order terms

A deformation of gauge structure of Double Field Theory fixes the first order corrections of

- bosonic string
- bosonic sector of heterotic string and lower dimensional gauged (super)gravities

D. Marqués, C.N., 1507.00652

W.Baron, J.Fernández-Melgarejo, D.Marqués, C.N., 1702.05489

N=1 Supersymmetric deformation of double field theory

$\rightarrow$ fixes all terms of massless spectrum of heterotic string $\mathcal{O}(\alpha')$

E. Lescano, C.N., A. Rodriguez, 1808.xxxxxx

Frame formulation of DFT

- W. Siegel (1993) and O. Hohm & S. Kwak (2010)

- Symmetries:

  - Global $G=O(D,D;\mathbb{R})$ with invariant metric η_{MN}

  - Local $H=O_-(1, D-1;\mathbb{R}) \times O_+(D-1, 1;\mathbb{R})$ generated infinitesimally by $\Lambda_A{}^B$ with invariant metrics η_{AB} and $\mathcal{H}_{AB}$

$$P_{AB} = \frac{1}{2}\left(\eta_{AB} - \mathcal{H}_{AB}\right), \quad \bar{P}_{AB} = \frac{1}{2}\left(\eta_{AB} + \mathcal{H}_{AB}\right)$$

  - Generalized diffeomorfisms generated infinitesimally by ξ^M through $\hat{\mathfrak{L}}_\xi$

Fields organized in a generalized frame $E_M{}^A$

$$\eta_{MN} = E^A{}_M \eta_{AB} E^B{}_M \qquad \mathcal{H}_{MN} = E^A{}_M \mathcal{H}_{AB} E^B{}_M$$

and an O(D,D) invariant generalized dilaton d

DFT is defined on a doubled space $\mathrm{X}^{\mathrm{M}} = (\tilde{\mathrm{x}}_i\,, \mathrm{x}^i\,)$ but there is a (duality invariant) strong constraint

$$\partial_M \partial^M \cdots = 0\,, \quad \partial_M \ldots \partial^M \ldots = 0$$

Gauge invariance

DFT is invariant under duality covariant gauge transformations

$$\delta E_A{}^M = \hat{\mathfrak{L}}_\xi E_A{}^M + E_B{}^M \Lambda^B{}_A$$

$$\delta e^{-2d} = \partial_M (\xi^M e^{-2d})$$

strong constraint

$$\delta G = L_\xi G$$
$$\delta B = L_\xi B + d\tilde{\xi}$$
$$\delta \phi = L_\xi \phi$$

$$S = \int d^D x \sqrt{G}\, e^{-2\phi} \left[R + \alpha\, \partial^i \phi\, \partial_i \phi + \beta\, H_{ijk} H^{ijk} \right]$$

strong constraint

$$S_{NSNS} = \int d^D x \sqrt{G}\, e^{-2\phi} \left[R + 4\, \partial^i \phi\, \partial_i \phi - \frac{1}{12} H_{ijk} H^{ijk} \right]$$

Can the gauge principle be deformed so that it requires and fixes the higher derivative corrections?

The heterotic string has such principle: Green-Schwarz deformation of the gauge transformation of $B_{\mu\nu}$

$$\delta B_{\mu\nu} = \frac{1}{2}\partial_{[\mu}\Lambda_a{}^b \omega^{(-)a}_{\nu]b}, \qquad \omega^{(-)b}_{\mu a} = \omega_{\mu a}{}^b - \frac{1}{2}\hat{H}_{\mu a}{}^b$$

requires higher derivative corrections to

$$\hat{H}_{\mu\nu\rho} = H_{\mu\nu\rho} + \frac{3}{2}\left(\omega^{(-)b}_{[\mu a}\partial_\nu \omega^{(-)a}_{\rho]b} + \frac{2}{3}\omega^{(-)b}_{[\mu a}\omega^{(-)c}_{\nu b}\omega^{(-)a}_{\rho]c}\right)$$

But the action has additional terms that are not required by Green-Schwarz transformation

$$L = e^{-2\phi}\left(R + 4g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - \frac{1}{12}\hat{H}_{\mu\nu\rho}\hat{H}^{\mu\nu\rho} - \frac{1}{8}R^{(-)}_{\mu\nu a}{}^{b}R^{(-)\mu\nu}{}_{b}{}^{a} \right)$$

T-duality mixes G and B, we expect that a duality covariant generalization of the GS transformation determines the (Riemann)^2 terms

$$\delta B_{\mu\nu} = \frac{1}{2}\partial_{[\mu}\Lambda_a{}^b\,\omega^{(-)a}_{\nu]b}$$

⇓

$$\tilde{\delta}_\Lambda E_M{}^C = a\,\partial_{[\underline{M}}\Lambda_{\underline{A}}{}^{\underline{B}}\,\omega_{\overline{N}]\underline{B}}{}^{\underline{A}}E^{NC} - b\,\partial_{[\overline{M}}\Lambda_{\overline{A}}{}^{\overline{B}}\,\omega_{\underline{N}]\overline{B}}{}^{\overline{A}}E^{NC}$$

a and b arbitrary coefficients of $\mathcal{O}(\alpha')$

Deform the duality covariant gauge transformations with

Generalized Green Schwarz transformation

$$\delta E_M{}^C = \hat{\mathcal{L}}_\xi E_M{}^C + E_M{}^B \Lambda_B{}^A + \tilde{\delta}_\Lambda E_M{}^C$$

It preserves the constraints and closes

Is there a gauge invariant action?

The zeroth order action in the frame formulation of DFT

$$S_{DFT} = \int dX\, e^{-2d}\, \mathcal{R}(E,d)$$

$$\mathcal{R} = \left(2\partial_A F_B - F_A F_B\right)\left(\mathcal{H}^{AB} - \eta^{AB}\right) - \frac{1}{4}\mathcal{H}^{AD} F_{AB}{}^{C} F_{DC}{}^{B}$$

$$-\frac{1}{12} F_{AB}{}^{C} F_{DE}{}^{F} \mathcal{H}^{AD} \mathcal{H}^{BE} \mathcal{H}_{CF} - \frac{1}{6} F^{ABC} F_{ABC}$$

with

$$F_{ABC} = 3\partial_{[A} E_B{}^N E_{C]N} = \omega_{[ABC]}, \quad F_A = \partial^B E_B{}^N E_{AN} + 2E_A{}^M \partial_M d = \omega_{BA}{}^B$$

$$\omega_{\overline{M}\underline{AB}} = F_{\overline{M}\underline{AB}} = F^{(-)}_{MAB} \qquad \omega_{\underline{M}\overline{AB}} = F_{\underline{M}\overline{AB}} = F^{(+)}_{MAB}$$

The coefficients are fixed by double Lorentz invariance

Gauge invariance of the action under generalized GS transformations requires corrections

$$\delta_\Lambda \mathcal{R} = 0 \quad \text{but} \quad \tilde{\delta}_\Lambda \mathcal{R} \neq 0$$

$$S_{DFT} = \int dX \, e^{-2d} \left(\mathcal{R}(E,d) + a\mathcal{R}^{(-)} + b\mathcal{R}^{(+)} \right)$$

Have to find some $\mathcal{R}^{(\pm)}$ that satisfy:

- $\mathcal{R}^{(\pm)}$ transform as scalars under generalized diffeos
- $$\tilde{\delta}_\Lambda \mathcal{R} + \delta_\Lambda \left(a\mathcal{R}^{(-)} + b\mathcal{R}^{(+)} \right) = 0$$

These two requirements have a unique solution

$$\mathcal{R}^{(-)} = \partial_A \partial_B F_{CDE} F_{FGH} \left(P^{CF} P^{DG} \overline{P}^{AE} \overline{P}^{BH} + P^{CF} P^{DG} \overline{P}^{AH} \overline{P}^{BE} \right)$$

$$+ \partial_A F_{BCD} \partial_E F_{FGH} \left(\frac{1}{2} P^{AE} P^{BF} P^{CG} P^{DH} - P^{BF} P^{CG} P^{AD} P^{EH} - \frac{1}{2} P^{BF} P^{CG} P^{AE} P^{DH} \right)$$

$$+ (2\partial_A F_B - F_A F_B) F_{CDE} F_{FGH} P^{CF} P^{AE} P^{DG} P^{BH}$$

$$+ 2\partial_A F_{BCD} F_{FGH} F_E \left(P^{BF} P^{CG} P^{AD} P^{EH} + P^{BF} P^{CG} P^{AH} P^{DE} \right)$$

$$- \partial_A F_{BCD} F_{EFG} F_{HIJ} \left(P^{BH} P^{CI} P^{AE} P^{DF} P^{GJ} + 4 P^{BE} P^{CH} P^{FI} P^{AG} P^{DJ} - P^{BE} P^{CF} P^{AH} P^{DI} P^{GJ} \right)$$

$$+ F_{ABC} F_{DEF} F_{GHI} F_{JKL} (P^{BG} P^{EJ} P^{AD} P^{HK} \overline{P}^{CL} \overline{P}^{FI} - P^{AD} P^{EJ} P^{HK} P^{BG} \overline{P}^{CF} \overline{P}^{IL}$$

$$+ P^{AD} P^{BE} P^{FK} P^{GJ} P^{CH} P^{IL} + \frac{4}{3} P^{AD} P^{BG} P^{FK} P^{CJ} P^{EH} P^{IL})$$

and $\mathcal{R}^{(+)} = \mathcal{R}^{(-)}[P \leftrightarrow \overline{P}]$

$$S_{DFT} = \int dX \, e^{-2d} \left(\mathcal{R}(E,d) + a\mathcal{R}^{(-)} + b\mathcal{R}^{(+)} \right)$$

Does this theory reproduce the string effective actions?

Duality covariance
vs
gauge covariance

Parametrize generalized fields

$$E_M{}^A = \frac{1}{\sqrt{2}} \begin{pmatrix} \tilde{e}_a{}^\mu & -g^{ab}\tilde{e}_b{}^\mu \\ \tilde{e}_\mu{}^b g_{ba} - \tilde{e}_a{}^\rho \tilde{B}_{\rho\mu} & \tilde{e}_\mu{}^a + g^{ab}\tilde{e}_b{}^\rho \tilde{B}_{\rho\mu} \end{pmatrix},$$

$$e^{-2d} = \sqrt{-\tilde{g}}\, e^{-2\tilde{\phi}} \,, \quad \tilde{g}_{\mu\nu} = \tilde{e}_\mu{}^a g_{ab} \tilde{e}_\nu{}^b$$

Deformation of $\tilde{\delta}\, E_M{}^C \Rightarrow \tilde{\delta}\, \tilde{e}_\mu{}^a, \tilde{\delta}\tilde{B}_{\mu\nu}$

Then $\tilde{g}_{\mu\nu}$ and $\tilde{B}_{\mu\nu}$ are not the standard gauge covariant fields

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{a}{4}\omega_{\mu a}^{(-)b}\omega_{\nu b}^{(-)a} - \frac{b}{4}\omega_{\mu a}^{(+)b}\omega_{\nu b}^{(+)a} \quad \rightarrow \quad \delta\, g_{\mu\nu} = L_\xi g_{\mu\nu}$$

$$\delta\, \tilde{B}_{\mu\nu} = L_\xi \tilde{B}_{\mu\nu} + 2\partial_{[\mu}\tilde{\xi}_{\nu]} + \frac{a}{2}\omega_{[\mu a}^{(-)b}\partial_{\nu]}\Lambda_b{}^a - \frac{b}{2}\omega_{[\mu a}^{(+)b}\partial_{\nu]}\Lambda_b{}^a$$

- For $a = -\alpha'$, $b = 0$ $\rightarrow$ Green-Schwarz transformation in the heterotic string

Duality covariance vs gauge covariance

- The generalized frame/metric are not gauge covariant in the standard sense but they are duality covariant

- The components $\tilde{g}_{\mu\nu}$ and $\tilde{B}_{\mu\nu}$ transform as usual under T-duality or Buscher rules, but their induced gauge transformations are deformed

- The duality covariant fields $\tilde{g}_{\mu\nu}$, $\tilde{B}_{\mu\nu}$ are related to the gauge covariant fields $g_{\mu\nu}$, $B_{\mu\nu}$ through non-covariant field redefinitions

Bosonic and heterotic strings

Rewriting the α'-deformed DFT action in terms of the standard gauge covariant fields & strong constraint

$$\mathcal{R}(E,d) + a\mathcal{R}^{(-)} + b\mathcal{R}^{(+)} =$$

$$R + 4g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{12}\hat{H}_{\mu\nu\rho}\hat{H}^{\mu\nu\rho} + \frac{a}{8}R^{(-)}_{\mu\nu a}{}^{b}R^{(-)\mu\nu}{}_{b}{}^{a} + \frac{b}{8}R^{(+)}_{\mu\nu a}{}^{b}R^{(+)\mu\nu}{}_{b}{}^{a}$$

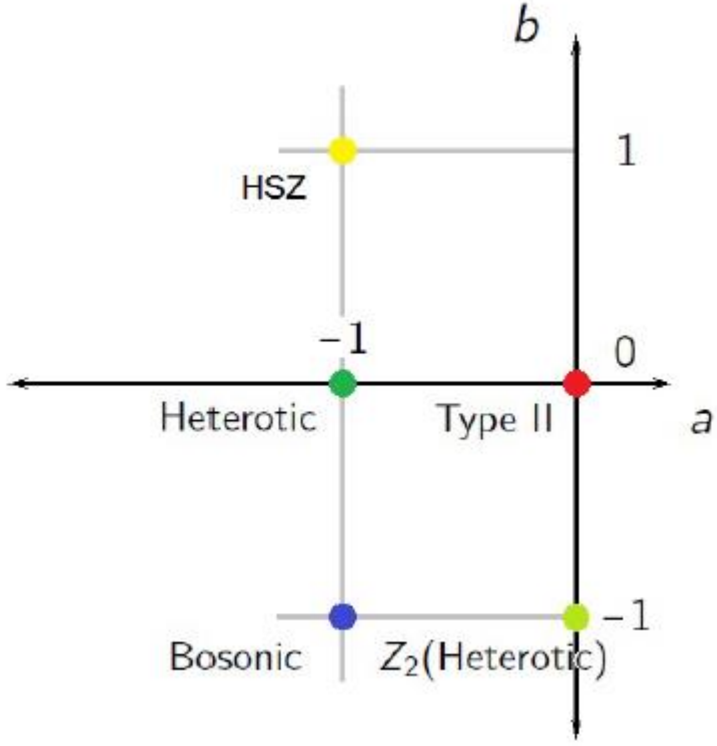
$$\hat{H}_{\mu\nu\rho} = H_{\mu\nu\rho} - \frac{3}{2}a\,\Omega^{(-)}_{\mu\nu\rho} + \frac{3}{2}b\,\Omega^{(+)}_{\mu\nu\rho}$$

$$\omega^{(\pm)a}_{\nu b} = \omega_{\nu b}{}^{a} \pm \frac{1}{2}H_{\mu a}{}^{b}$$

$a = -\alpha'$, $b = 0$ is the heterotic string (Bergshoeff & de Roo, 1989)

$a = b = -\alpha'$ is the bosonic string

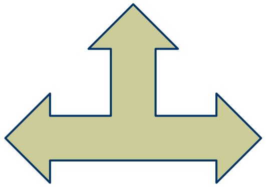
The two-parameter family of deformations produces



$$\delta E_M{}^C = \hat{\mathfrak{L}}_\xi E_M{}^C + E_M{}^B \Lambda_B{}^A + \tilde{\delta}_\Lambda E_M{}^C$$

$$\tilde{\delta}_\Lambda E_M{}^C = a\, \partial_{[\underline{M}} \Lambda_{\underline{A}}{}^{\underline{B}} F_{\overline{N}]\underline{B}}{}^{\underline{A}} E^{NC} - b\, \partial_{[\overline{M}} \Lambda_{\overline{A}}{}^{\overline{B}} F_{\underline{N}]\overline{B}}{}^{\overline{A}} E^{NC}$$

Close to first order in α' and produce duality covariant theories



Higher orders in α' | Include other fields

Extend duality group G = O(D, D; $\mathbb{R}$) $\rightarrow$ G = O(D, D+N; $\mathbb{R}$)

+ generalized Scherk-Schwarz compactification

Split coordinates into external and internal:

D = n + d

Organize the dof as $V^M = (\tilde{V}_\mu, V^\mu, V^m)$

$(\tilde{V}_\mu, V^\mu)$ transform in the fundamental of $G_e = O(n,n) \in G$

V^m transforms in the fundamental of $G_i = O(d,d+N) \in G$

H group now splits in

$H_e = O(n-1,1) \times O(1,n-1)$, $H_i = O(d) \times O(d+N)$

Generalized Scherk-Schwarz compactification ~ Gauged DFT

$$f_{MNP} = f_{[MNP]}, \qquad f_{[MN}{}^{R} f_{P]R}{}^{Q} = 0$$

- Deform generalized Lie derivatives

$$\hat{\mathfrak{L}}_{\xi} V^{M} \to \hat{\mathfrak{L}}_{\xi} V^{M} + f_{PQ}{}^{M} \xi^{P} V^{Q}$$

$$\Rightarrow F_{ABC} \to F_{ABC} + f_{ABC}$$

- New constraint

$$f_{MN}{}^{P} \partial_{P} \ldots = 0$$

The action will be deformed: non-abelian gauge fields, scalar potential, …

Impose the constraints

$$\partial_M = \left(\tilde{\partial}^\mu, \partial_\mu, \partial_m\right) \quad \rightarrow \quad \tilde{\partial}^\mu \ldots = \partial_m \ldots = 0,$$

$$f_{MNP} = \begin{cases} f_{mnp} \text{ if } (M, N, P) = (m, n, p) \\ 0 \qquad \text{otherwise} \end{cases}$$

Reparametrize the generalized fields in terms of lower dimensional fields

$$E_M{}^A = \frac{1}{\sqrt{2}} \begin{pmatrix} \tilde{e}_a{}^\mu & 0 & 0 \\ -\tilde{e}_a{}^\rho (\tilde{B}_{\rho\mu} + \frac{1}{2} \tilde{A}_\rho^p \tilde{A}_{\mu p}) & \tilde{e}_\mu{}^a & \tilde{A}_\mu^p \tilde{\Phi}_p{}^\alpha \\ -\tilde{e}_a{}^\rho \tilde{A}_{\rho m} & 0 & \tilde{\Phi}_m{}^\alpha \end{pmatrix}$$

Trivialize the transformations through non-covariant field redefinitions

The action

$$S=\int d^{n}X\sqrt{-g}\,e^{-2\phi}\Big(\;R+4g^{\mu\nu}\partial_{\mu}\phi\,\partial_{\nu}\phi-\frac{1}{12}\hat{H}_{\mu\nu\rho}\hat{H}^{\mu\nu\rho}$$

$$-\frac{1}{4}F^{m}_{\mu\nu}F^{n\mu\nu}M_{mn}+\frac{1}{8}\nabla_{\mu}M_{mn}\nabla^{\mu}M^{mn}-V_{0}+a\,L^{(-)}+b\,L^{(+)}\Big)$$

The zeroth order potential has the standard form (Schon & Weidner)

$$V_{0}=\frac{1}{12}f_{mp}{}^{r}f_{nq}{}^{s}M^{mn}M^{pq}M_{rs}+\frac{1}{4}f_{mp}{}^{q}f_{nq}{}^{p}M^{mn}+\frac{1}{6}f_{mnp}f^{mnp}$$

$$L^{(a,b)}\left(R_{\mu\nu\rho\sigma},\hat{H}_{\mu\nu\rho},F_{\mu\nu}{}^{m},M_{mn},f_{mnp},\nabla_{\mu}\right)$$

There is a correction to the scalar potential

$$V = V_0 + aV^{(-)} + bV^{(+)}$$

$$V^{(\pm)} = f^{mpq} f^{np'q'} f^{m'rs} f^{n'r's} P^{(\pm)}_{qq'} P^{(\pm)}_{rr'} P^{(\mp)}_{ss'} (P^{(\pm)}_{mm'} P^{(\pm)}_{nn'} P^{(\mp)}_{pp'} - P^{(\mp)}_{mm'} P^{(\mp)}_{nn'} P^{(\pm)}_{pp'})$$

$$+ f^{mns} f^{m'pr} f^{n'p'q} f^{q'r's'} P^{(\pm)}_{qq'} P^{(\pm)}_{rr'} P^{(\mp)}_{ss'} (P^{(\pm)}_{mm'} P^{(\pm)}_{nn'} P^{(\mp)}_{pp'} + \frac{4}{3} P^{(\mp)}_{mm'} P^{(\mp)}_{nn'} P^{(\pm)}_{pp'})$$

Cannot be eliminated through field redefinitions $\rightarrow$ genuine deformation of scalar potential (affects structure of the vacuum)

Reproduce gauge sector of heterotic string $b=0$

Take n=10, d=0, N=496 and the gaugings f_{mnp} are the structure constants of SO(32) or $E_8\times E_8$

$$S_{hete} = \int d^{10}x\sqrt{-g}\,e^{-2\phi}\left[R + 4g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{12}\hat{H}_{\mu\nu\rho}\hat{H}^{\mu\nu\rho} - \frac{1}{4}F^m_{\mu\nu}F^{m\mu\nu}\right.$$

$$\left. + \frac{\alpha'}{8}\left(R^{(-)}_{\mu\nu a}{}^{b}R^{(-)\mu\nu}{}_{b}{}^{a} - \frac{1}{2}T_{\mu\nu}\,T^{\mu\nu} - \frac{3}{2}T_{\mu\nu\rho\sigma}\,T^{\mu\nu\rho\sigma}\right)\right]$$

$$T_{\mu\nu} = F_{\mu}{}^{\rho m}F_{\rho\nu m}\,, \quad T_{\mu\nu\rho\sigma} = F_{[\mu\nu}{}^{m}F_{\rho\sigma]m}$$

$$A^{\alpha}_{\mu} \leftrightarrow \omega^{(-)ab}_{\mu} \qquad \left(F_{\mu\nu}\right)^4 \leftrightarrow \left(Riem\right)^4$$

A.Coimbra, R.Minasian, H.Triendl, D. Waldram
O. Bedoya, D. Marqués, CN

N=1 Supersymmetric α'-deformed DFT

- DFT reformulation of $\mathcal{N}=1$ supergravity coupled to n abelian vector multiplets in D=10 O. Hohm & S. Kwak (2012)

- Symmetries:

  - Global $G=O(10, 10+N; \mathbb{R})$

  - Local $H = O(9, 1; \mathbb{R})_L \times O(1, 9+N; \mathbb{R})_R$

  - Generalized diffeomorfisms

  - Supersymmetry generated infinitesimally by a Majorana fermion ε transforming as a spinor of $O(9, 1)_L$

Generalized fields

- $E_{\mathcal{A}}{}^{\mathcal{M}}$ generalized vielbein parameterizing the coset

$$\frac{\mathrm{O}(10,10+\mathrm{N})}{\mathrm{O}(9,1)_{\mathrm{L}} \times \mathrm{O}(1,9+\mathrm{N})_{\mathrm{R}}}$$

- d an O(10,10+N) scalar dilaton

- $\Psi_{\bar{A}}$ a Majorana spinor *gravitino* transforming as a spinor of $\mathrm{O}(9, 1)_{\mathrm{L}}$, as a vector of $\mathrm{O}(1, 9+\mathrm{N})_{\mathrm{R}}$, and as a scalar of O(10,10+N)

– ρ a Majorana spinor *dilatino* transforming as a spinor of $\mathrm{O}(9,1)_{\mathrm{L}}$ and as a scalar of O(10,10+N)

$$\mathcal{A} = (a, \bar{a}, \bar{\alpha}) = 0,\ldots,19+N$$

$$a = 0,\ldots,9; \quad \bar{a} = 0,\ldots,9; \quad \bar{\alpha} = 1,\ldots,(N = 496)$$

Double Lorentz and SUSY transformations

$$\delta_\Lambda E_{\mathcal{A}}{}^{\mathcal{M}} = E_{\mathcal{B}}{}^{\mathcal{M}} \Lambda^{\mathcal{B}}{}_{\mathcal{A}},$$

$$\delta_\Lambda \Psi_{\bar{A}} = \Psi_{\bar{B}} \Lambda^{\bar{B}}{}_{\bar{A}} - \frac{1}{4} \Lambda_{bc} \gamma^{bc} \Psi_{\bar{A}},$$

$$\delta_\Lambda \rho = -\frac{1}{4} \Lambda_{bc} \gamma^{bc} \rho$$

$$\delta_\varepsilon E_a{}^{\mathcal{M}} = -\frac{1}{2} \bar{\varepsilon} \gamma_a \Psi_{\bar{B}} E^{\bar{B}\mathcal{M}},$$

$$\delta_\varepsilon E_{\bar{A}}{}^{\mathcal{M}} = \frac{1}{2} \bar{\varepsilon} \gamma_b \Psi_{\bar{A}} E^{b\mathcal{M}},$$

$$\delta_\varepsilon d = -\frac{1}{4} \bar{\varepsilon} \rho,$$

$$\delta_\varepsilon \Psi_{\bar{A}} = \nabla_{\bar{A}} \varepsilon,$$

$$\delta_\varepsilon \rho = -\gamma^a \nabla_a \varepsilon$$

$O(9,1)_L$ γ–matrices $\{\gamma_a, \gamma_b\} = 2P_{ab}$

The algebra of transformations closes up to terms with two fermions and it leaves the following action invariant

The action

$$S = \int dX e^{-2d}\left(\mathcal{R}(E,d) - \Psi^{\bar{A}}\gamma^{b}\nabla_{b}\Psi_{\bar{A}} - \rho\gamma^{a}\nabla_{a}\rho - 2\overline{\Psi}^{\bar{A}}\nabla_{\bar{A}}\rho\right)$$

reduces to D=10, N=1 supergravity coupled to n gauge vectors with the parametrization

$$E_{\mathcal{A}}{}^{\mathcal{M}} = \frac{1}{\sqrt{2}}\begin{pmatrix} -e_{a\mu} - e_{a}{}^{\rho}C_{\rho\mu} & e_{a}{}^{\mu} & -e_{a}{}^{\rho}A_{\rho}{}^{\alpha} \\ e_{\bar{a}\mu} - e_{\bar{a}}{}^{\rho}C_{\rho\mu} & e_{\bar{a}}{}^{\mu} & -e_{\bar{a}}{}^{\rho}A_{\rho}{}^{\alpha} \\ \sqrt{2}e_{\bar{\alpha}}{}^{\alpha}A_{\alpha\mu} & 0 & \sqrt{2}e_{\bar{\alpha}}{}^{\alpha} \end{pmatrix},$$

$$C_{\mu\nu} = B_{\mu\nu} + \frac{1}{2}A_{\mu}^{\alpha}A_{\nu\alpha}$$

$$d = \phi - \frac{1}{2}\log\sqrt{-g},$$

$$\Psi_{\bar{A}} = \left(\Psi_{\bar{a}}, \chi_{\bar{\alpha}}\right)$$

$$\tilde{\delta}_\Lambda E_M{}^C \Rightarrow$$ The algebra of transformations does not close

Parametrize O(10, 10+m) generalized fields in terms of O(10,10) multiplets and introduce O(10,10) vector $\mathfrak{W}_M{}^{\bar{\alpha}}$

W. Baron, E. Lescano, D. Marqués, arXiv:1808.xxxxx

$$A_\mu^\alpha \leftrightarrow \omega_\mu^{(-)ab} \qquad \left(F_{\mu\nu}\right)^4 \leftrightarrow \left(Riem\right)^4$$

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$$\Downarrow$$

$$\mathfrak{W}_M{}^{AB} \in O(10,10)$$

Generalized fields

- $\hat{E}_{\mathcal{A}}{}^{\mathcal{M}} = \hat{E}_{\mathcal{A}}{}^{\mathcal{M}}(E_A{}^M, \mathfrak{W}_{M\bar{\alpha}}) \in O(10,10+m),$

$$E_A{}^M, \mathfrak{W}_{M\bar{\alpha}} \in O(10,10)$$

- $\hat{d}$ O(10,10+m) scalar dilaton

- $\hat{\Psi}_{\bar{A}} = \hat{\Psi}_{\bar{A}}(\Psi_{\bar{a}}, \Psi_{\bar{\alpha}}) \in O(10,10+m)$

$$\Psi_{\bar{a}}, \Psi_{\bar{\alpha}} \in O(10,10)$$

– $\hat{\rho}$ O(10,10+m) scalar dilatino

$$\mathcal{A} = (a, \bar{a}, \bar{\alpha}) = 0,\ldots,19+m$$

$$a = 0,\ldots,9; \quad \bar{a} = 0,\ldots,9; \quad \bar{\alpha} = 1,\ldots,45$$

Double Lorentz and SUSY transformations

$$\delta_{\hat\Lambda}\hat{E}_{\mathcal{A}}{}^{\mathcal{M}} = \hat{E}_{\mathcal{A}}{}^{\mathcal{M}}\hat{\Lambda}^{\mathcal{B}}{}_{\mathcal{A}},$$

$$\delta_{\hat\Lambda}\hat{\Psi}_{\bar{A}} = \hat{\Psi}_{\bar{B}}\hat{\Lambda}^{\bar{B}}{}_{\bar{A}} - \frac{1}{4}\hat{\Lambda}_{bc}\gamma^{bc}\hat{\Psi}_{\bar{A}},$$

$$\delta_{\hat\Lambda}\hat{\rho} = -\frac{1}{4}\hat{\Lambda}_{bc}\gamma^{bc}\hat{\rho}$$

$$\delta_{\varepsilon}\hat{E}_{a}{}^{\mathcal{M}} = -\frac{1}{2}\bar{\varepsilon}\gamma_{a}\hat{\Psi}_{\bar{B}}\hat{E}^{\bar{B}\mathcal{M}},$$

$$\delta_{\varepsilon}\hat{E}_{\bar{A}}{}^{\mathcal{M}} = \frac{1}{2}\bar{\varepsilon}\gamma_{b}\hat{\Psi}_{\bar{A}}\hat{E}^{b\mathcal{M}}, \quad \delta_{\varepsilon}\hat{d} = -\frac{1}{4}\bar{\varepsilon}\hat{\rho},$$

$$\delta_{\varepsilon}\hat{\Psi}_{\bar{B}} = \hat{\nabla}_{\bar{B}}\varepsilon, \quad \delta_{\varepsilon}\hat{\rho} = -\gamma^{a}\hat{\nabla}_{a}\varepsilon$$

Standard undeformed O(10,10+m) covariant transformations induce deformed O(10,10) covariant transformations

Choosing gauge group = Lorentz group

$$\delta\hat{E}_{M}{}^{C} = \hat{\mathfrak{L}}_{\hat\xi}\hat{E}_{M}{}^{C} + \hat{E}_{M}{}^{B}\hat{\Lambda}_{B}{}^{A} \rightarrow \mathfrak{L}_{\xi}E_{M}{}^{C} + E_{M}{}^{B}\Lambda_{B}{}^{A} + \tilde{\delta}_{\Lambda}E_{M}{}^{C}$$

$$[t_{\alpha}, t_{\beta}] = f_{\alpha\beta}{}^{\gamma}t_{\gamma} \quad \rightarrow \quad \hat{\xi}^{\alpha} = (t^{\alpha})^{\bar{A}\bar{B}}\Lambda_{\bar{A}\bar{B}}$$

$$\Rightarrow \quad \mathfrak{W}_{A\bar{B}}{}^{\bar{C}} = F_{A\bar{B}}{}^{\bar{C}} + \frac{1}{2}\Psi_{\bar{B}}\gamma_A\Psi_{\bar{C}}, \qquad \hat{\Psi}_{\bar{\alpha}} = \hat{\Psi}_{\bar{A}\bar{B}} = \hat{\nabla}_{[\bar{A}}\Psi_{\bar{B}]}$$

$$\hat{F}_{ABC} = 3\partial_{[A}\hat{E}_B{}^N\hat{E}_{C]N} = F_{ABC} + F^{(2)}_{ABC} + F^{(3)}_{ABC} + \ldots,$$

$$\hat{F}_A = \partial^B\hat{E}_B{}^N\hat{E}_{AN} + 2\hat{E}_A{}^M\partial_M\hat{d} = F_A + F^{(2)}_A + F^{(3)}_A + \ldots$$

$$S = \int dX e^{-2d}\left(\mathcal{R}(\hat{E},d) - \overline{\hat{\Psi}}{}^{\bar{A}}\gamma^b\hat{\nabla}_b\hat{\Psi}_{\bar{A}} - \bar{\hat{\rho}}\gamma^a\hat{\nabla}_a\hat{\rho} - 2\overline{\hat{\Psi}}{}^{\bar{A}}\hat{\nabla}_{\bar{A}}\hat{\rho}\right)$$

$$\Downarrow$$

$$S = \int dX e^{-2d}\left(\mathcal{R}(F) + L_F^{(0)} + \mathcal{R}^{(+)}(\mathfrak{W}) + L_F^{(1)}\right)$$

$$S = \int dX e^{-2d} \Big(\mathcal{R}(F) + L_F^{(0)}(F, \Psi, \rho) + \mathcal{R}^{(+)}(F^*) + L_F^{(1)}(F, \Psi, \rho) \Big)$$

$$\begin{aligned}
L_F^{(1)} = {} & \overline{\Psi}^{\bar{B}\bar{C}} \gamma^b \partial_b \Psi_{\bar{B}\bar{C}} - \frac{1}{2} \overline{\Psi}^{\bar{A}} \gamma^b F_M{}^{\bar{B}\bar{C}} F_{b\bar{B}\bar{C}} \partial^M \Psi_{\bar{A}} - \frac{1}{12} \overline{\Psi}^{\bar{B}\bar{C}} F_{bcd} \gamma^{bcd} \Psi_{\bar{B}\bar{C}} \\
& + \frac{1}{2} \overline{\Psi}^{\bar{B}\bar{C}} F_b \gamma^b \Psi_{\bar{B}\bar{C}} + \frac{1}{12} \overline{\Psi}^{\bar{A}} F^{(3)}_{bcd} \gamma^{bcd} \Psi_{\bar{A}} \\
& - \frac{1}{4} \overline{\Psi}^{\bar{A}} \Big(F_d F^{d\bar{B}\bar{C}} F_{b\bar{B}\bar{C}} + \partial_a \Big(F^{a\bar{B}\bar{C}} F_{b\bar{B}\bar{C}} \Big) \Big) \gamma^b \Psi_{\bar{A}} + \frac{1}{12} \bar{\rho} F^{(3)}_{bcd} \gamma^{bcd} \rho \\
& - \frac{1}{2} \bar{\rho} \gamma^b F_M{}^{\bar{B}\bar{C}} F_{b\bar{B}\bar{C}} \partial^M \rho - \frac{1}{2} \bar{\rho} \Big(F_d F^{d\bar{B}\bar{C}} F_{b\bar{B}\bar{C}} + \partial_a \Big(F^{a\bar{B}\bar{C}} F_{b\bar{B}\bar{C}} \Big) \Big) \gamma^b \rho \\
& - 2 \overline{\Psi}^{\bar{B}\bar{C}} F_{M\bar{B}\bar{C}} \partial^M \rho + \frac{1}{2} \overline{\Psi}^{\bar{A}} F^{(3)}_{\bar{A}bc} \gamma^{bc} \rho - \overline{\Psi}^{\bar{B}\bar{C}} F^{(2)}_{\bar{B}\bar{C}bc} \gamma^{bc} \rho
\end{aligned}$$

$$S_{DFT} \to S_{BdR}$$

Summary and conclusions

- The traditional formulation of DFT has a duality covariant gauge symmetry principle based on a generalized Lie derivative that determines the two-derivative effective action uniquely.
- The duality covariant transformations can be extended to include deformations that account for the first order α'- corrections to the heterotic string effective action containing all the massless fields
- A trivial generalized Scherk-Schwarz compactification gives a manifestly duality covariant reformulation of half-maximal gauged sugras in arbitrary dimensions including $\mathcal{O}(\alpha')$- corrections

Future

- To examine the exact deformation by BLM further. It should have information on non-perturbative aspects of string theory

- To find the duality covariant extension of the $(\text{Riemann})^4$ and higher of type II theories (probably need S-duality)

Thank you

Lichnerowicz principle

A.Coimbra, R.Minasian, H.Triendl, D. Waldram

$$\left(\gamma^a \nabla_a \gamma^b \nabla_b + \nabla_{\bar{A}} \nabla^{\bar{A}}\right)\varepsilon = -\frac{1}{4}\mathcal{R}\varepsilon$$

$$\left[\nabla_{\bar{A}}, \gamma^b \nabla_b\right]\varepsilon = \frac{1}{4}\gamma^b \mathcal{R}_{\bar{A}b}\varepsilon$$

⇓

$$\left(\gamma^a \hat{\nabla}_a \gamma^b \hat{\nabla}_b + \hat{\nabla}_{\bar{A}} \hat{\nabla}^{\bar{A}}\right)\varepsilon = -\frac{1}{4}\left(\mathcal{R} + \mathcal{R}^{(+)}\right)\varepsilon$$

$$\left[\hat{\nabla}_{\bar{A}}, \gamma^b \hat{\nabla}_b\right]\varepsilon = \frac{1}{4}\gamma^b \left(\mathcal{R}_{\bar{A}b} + \mathcal{R}^{(+)}_{\bar{A}b}\right)\varepsilon$$

Closure of the algebra

- These gauge transformations preserve the constraints and close

$$[\delta_{(\xi_1,\Lambda_1)},\delta_{(\xi_2,\Lambda_2)}] = \delta_{(\xi_{21},\Lambda_{21})}$$

w.r.t. the modified brackets

$$\xi_{12}^M = [\xi_1,\xi_2]_{(C)}^M - \frac{a}{2}\Lambda_{[1\underline{A}}{}^{\underline{B}}\Lambda_{2]\underline{B}}{}^{\underline{A}} + \frac{b}{2}\Lambda_{[1\overline{A}}{}^{\overline{B}}\Lambda_{2]\overline{B}}{}^{\overline{A}}$$

$$\Lambda_{12A}{}^{B} = 2\xi_{[1}^P\partial_P\Lambda_{2]A}{}^{B} - 2\Lambda_{[1A}{}^{C}\Lambda_{2]C}{}^{B}$$
$$+ a\partial_{[\overline{A}}\Lambda_1{}^{\underline{CD}}\partial_{\overline{B}]}\Lambda_{2\underline{DC}} + a\partial_{[\underline{A}}\Lambda_1{}^{\underline{CD}}\partial_{B]}\Lambda_{2\underline{DC}}$$
$$- ba\partial_{[\underline{A}}\Lambda_1{}^{\overline{CD}}\partial_{\underline{B}]}\Lambda_{2\overline{DC}} - b\partial_{[\overline{A}}\Lambda_1{}^{\overline{CD}}\partial_{\overline{B}]}\Lambda_{2\overline{DC}}$$

$$[\xi_1,\xi_2]_{(c)}^M = \xi_1^P\partial_P\xi_2^M - \xi_2^P\partial_P\xi_1^M - \frac{1}{2}\xi_1^P\partial^M\xi_{2P} + \frac{1}{2}\xi_2^P\partial^M\xi_{1P}$$

$$
\begin{aligned}
&\delta g_{\mu\nu} = L_\xi g_{\mu\nu} \\
&\delta A_\mu{}^m = L_\xi A_\mu{}^m + \partial_\mu \lambda^m + f_{pq}{}^m \lambda^p A_\mu{}^q \\
&\delta B_{\mu\nu} = L_\xi B_{\mu\nu} + 2\partial_{[\mu}\xi_{\nu]} + A_{[\mu}{}^m \partial_{\nu]}\lambda_m \\
&\qquad -\frac{1}{2}\left(a\omega_{[\mu}^{(-)ab} - b\omega_{[\mu}^{(+)ab}\right)\partial_{\nu]}\Lambda_{ab} \\
&\qquad -\frac{1}{2}\left(a\omega_{[\mu}^{(-)\alpha\beta} - b\omega_{[\mu}^{(+)\alpha\beta}\right)\partial_{\nu]}\Lambda_{\alpha\beta} \\
&\delta M_{mn} = L_\xi M_{mn} - 2 f_{p(m}{}^q M_{n)q} \lambda^p \\
&\delta\phi = L_\xi \phi
\end{aligned}
$$

The generalized Green-Schwarz transformation implies

$$\hat{H}_{\mu\nu\rho} = H_{\mu\nu\rho} - 3\Omega^{(g)}_{\mu\nu\rho} - \frac{3}{2}a\Omega^{(e,-)}_{\mu\nu\rho} + \frac{3}{2}b\Omega^{(e,+)}_{\mu\nu\rho} - \frac{3}{2}a\Omega^{(i,-)}_{\mu\nu\rho} + \frac{3}{2}b\Omega^{(i,+)}_{\mu\nu\rho}$$

$$\Omega^{(g)}_{\mu\nu\rho} = A^m_{[\mu}\partial_\nu A_{\rho]m} - \frac{1}{3} f_{mnp} A^m_{[\mu} A^n_\nu A^p_{\rho]} \,,$$

$$\Omega^{(e,\pm)}_{\mu\nu\rho} = \omega^{(\pm)b}_{[\mu a}\partial_\nu \omega^{(\pm)a}_{\rho]b} + \frac{2}{3}\omega^{(\pm)b}_{[\mu a}\omega^{(\pm)c}_{\nu b}\omega^{(\pm)a}_{\rho]c}$$

$$\Omega^{(i,\pm)}_{\mu\nu\rho} = \omega^{(\pm)\beta}_{[\mu\alpha}\partial_\nu \omega^{(\pm)\alpha}_{\rho]\beta} + \frac{2}{3}\omega^{(\pm)\beta}_{[\mu\alpha}\omega^{(\pm)\gamma}_{\nu\beta}\omega^{(\pm)\alpha}_{\rho]\gamma}$$

$$\delta\,\tilde{g}_{\mu\nu} = L_\xi \tilde{g}_{\mu\nu} - \frac{a}{2}\omega^{(-)b}_{(\mu a}\partial_{\nu)}\Lambda_b{}^a - \frac{b}{2}\omega^{(+)b}_{(\mu a}\partial_{\nu)}\Lambda_b{}^a$$

$$\delta\,\tilde{B}_{\mu\nu} = L_\xi \tilde{B}_{\mu\nu} + 2\partial_{[\mu}\tilde{\xi}_{\nu]} + \frac{a}{2}\omega^{(-)b}_{[\mu a}\partial_{\nu]}\Lambda_b{}^a - \frac{b}{2}\omega^{(+)b}_{[\mu a}\partial_{\nu]}\Lambda_b{}^a$$

The action

- The new piece of information is the second line where $L^{(\pm)}$ contain a huge number of terms

- I will discuss some simple special cases

- Taking n=26, d=0, N=0 and $(a,b)=(-\alpha', -\alpha')$ one recovers the bosonic string effective action

$$S_{bos} = \int d^{26}x\sqrt{-g}e^{-2\phi}\Big(R + 4g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{12}\hat{H}_{\mu\nu\rho}\hat{H}^{\mu\nu\rho}$$

$$- \frac{\alpha'}{8}R^{(-)}_{\mu\nu a}{}^{b}R^{(-)\mu\nu}{}_{b}{}^{a} - \frac{\alpha'}{8}R^{(+)}_{\mu\nu a}{}^{b}R^{(+)\mu\nu}{}_{b}{}^{a}\Big)$$

- The GGS transformations induce

$$\delta \tilde{g}_{\mu\nu} = L_\xi \tilde{g}_{\mu\nu} + \frac{1}{2}(a\omega^{(-)ab}_{(\mu} + b\omega^{(+)ab}_{(\mu})\partial_{\nu)}\Lambda_{ab}$$

$$+ \frac{1}{2}(a\omega^{(-)\alpha\beta}_{(\mu} + b\omega^{(+)\alpha\beta}_{(\mu})\partial_{\nu)}\Lambda_{\alpha\beta}$$

where

$$\omega^{(-)\beta}_{\mu\alpha} = \Phi_{\underline{\alpha}}{}^m \nabla_\mu \Phi_m{}^{\underline{\beta}}, \quad \omega^{(+)\beta}_{\mu\alpha} = \Phi_{\bar{\alpha}}{}^m \nabla_\mu \Phi_m{}^{\bar{\beta}}$$

which requires non-covariant field redefinitions such that

$$\delta g_{\mu\nu} = L_\xi g_{\mu\nu}$$

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + \frac{a}{4}\omega^{(-)ab}_{\mu}\omega^{(-)}_{\nu ab} + \frac{b}{4}\omega^{(+)}_{\mu ab}\omega^{(+)ab}_{\nu}$$

$$+ \frac{a}{4}\omega^{(-)}_{\mu\alpha\beta}\omega^{(-)\alpha\beta}_{\nu} + \frac{b}{4}\omega^{(+)}_{\mu\alpha\beta}\omega^{(+)\alpha\beta}_{\nu}$$