AdS_2 Holography for nonBPS Black Holes

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Extreme nonBPS Black Holes

- A controlled ***environment*** to study black holes: ***supergravity***.
- The simplest black hole ***solutions*** are ***extremal*** black holes. They are ***ground states***: lowest energy (with given charges).
- BPS black holes are ***solutions to SUGRA*** that ***preserve (some) SUSY***.
- Many features simplify for BPS black holes: geometry and microscopic description; ***indices*** are useful.
- This talk: extremal ***nonBPS*** black holes in 4D. ***Solutions to SUGRA*** that do ***not*** preserve SUSY.

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Black Hole Spectroscopy

- Detailed study of black holes: ***the spectrum***.

- The SUGRA spectrum: ***quadratic fluctuations*** around classical solution.

- Simplifying feature in extremal case: ***AdS$_2 \times S^2$ near horizon geometry*** dominates.

- So we can study SUGRA on AdS$_2 \times S^2$.

- Technical complication: for nonBPS black holes the ***spectrum has no SUSY***.

AdS_2 Holography

- Another complication: $\text{AdS}_2/\text{CFT}_1$ holography is confusing.
- So ***interpretation*** of black hole spectrum is subtle.
- Recent progress: the ***nAdS$_2$/nCFT$_1$ correspondence***.
- Much research: nCFT_1s such as the SYK model and its relatives.
- Here: focus on AdS_2 gravity.
- We first just ***compute*** the spectrum, leaving interpretation for later in the talk.

The 4D Massless Scalar Field

- Example: ***massless scalar field*** on $\mathrm{AdS}_2 \times S^2$

$$\nabla_4^2 \phi = (\nabla_A^2 + \nabla_S^2)\phi = 0$$

- ***Partial wave expansion***: $\nabla_S^2 \to -l(l+1)$ with $l = 0, 1, \ldots$.

- So the ***effective 2D mass***es are $m_2^2 = l(l+1)$.

- This gives ***conformal weights***:

$$h = \frac{1}{2} + \sqrt{\frac{1}{4} + m_2^2 \ell_2^2} = l + 1$$

- Result for the ***spectrum***: $(h, j) = (k+1, k)$ with $k = 0, 1, \ldots$.

Einstein-Maxwell Theory

- Bosonic fields of $\mathcal{N} = 2$ gravity multiplet:

$$16\pi G_N \mathcal{L} = \mathcal{R} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

- ***Linearize*** around AdS$_2 \times S^2$, ***fix gauge***, then introduce ***partial waves***: $l = 1, 2, \ldots$ for vector, $l = 2, 3, \ldots$ for tensor.

- ***Dualize*** all fields to scalars, ***diagonalize*** 4×4 scalar mass matrix, ***compute*** conformal weights.

- The spectrum has all ***integral weights***:
$(h, j) = 2(k + 2, k + 2)\ , 2(k + 3, k + 1)$ with $k = 0, 1, \ldots$.

The BPS Spectrum

- Decompose a general theory with $\mathcal{N} \geq 2$ SUSY in terms of $\mathcal{N} = 2$ fields: one SUGRA multiplet, $\mathcal{N} - 2$ (massive) gravitini, n_V vector multiplets, n_H hyper multiplets.

- The spectrum for this field content has $(h, j) =$

$$\begin{aligned}
\text{Supergravity}: &\quad 2[(k+2,k+2)\,,2(k+\tfrac{5}{2},k+\tfrac{3}{2})\,,(k+3,k+1)] \\
\text{Gravitino}: &\quad 2[(k+\tfrac{3}{2},k+\tfrac{3}{2})\,,2(k+2,k+1)\,,(k+\tfrac{5}{2},k+\tfrac{1}{2})] \\
\text{Vector}: &\quad 2[(k+1,k+1)\,,2(k+\tfrac{3}{2},k+\tfrac{1}{2})\,,(k+2,k)] \\
\text{Hyper}: &\quad 2[(k+\tfrac{1}{2},k+\tfrac{1}{2})\,,2(k+1,k)\,,(k+\tfrac{3}{2},k-\tfrac{1}{2})]
\end{aligned}$$

  Each tower has $k = 0, 1, \ldots$.

- ***Short representations of*** $SU(2|1,1)$: a ***chiral primary*** with $h = j$ assures the integral (or 1/2-integral) weights.

- Each chiral primary has 3 descendants due to two supercharges.

BPS vs nonBPS

- Extremal black holes in $\mathcal{N}=4$ SUGRA satisfy BPS condition iff

$$I_4 = (\vec{P} L \vec{P})\,(\vec{Q} L \vec{Q}) - (\vec{P} L \vec{Q})^2 > 0$$

- L is a T***-duality pairing***: e.g. string winding w and momentum n on the same S^1.

- The "dangerous" term is an S***-duality pairing***: a charge and its magnetic dual, like a $D0$ and a $D6$.

- Canonical nonBPS BH: KK-theory with both electric and magnetic charge (equivalent to pure D0/D6)

- The two branes preserve opposite SUSYs, the pair preserves none.

Kaluza-Klein Black Holes

- KK gravity multiplet (pure gravity in 5D, reduced to 4D)

$$16\pi G_N \mathcal{L} = \mathcal{R} + \frac{1}{2}(\partial\Phi)^2 - \frac{1}{4}e^{-\sqrt{3}\Phi}F_{\mu\nu}F^{\mu\nu}$$

- Constant Φ requires equally large electric and magnetic fields.
- ***Linearize*** around background,...,***diagonalize*** scalar mass matrix.
- The spectrum: $(h,j) =$
  $(k{+}3,k)\,,(k{+}3,k{+}1)\,,(k{+}2,k{+}1)\,,(k{+}2,k{+}2)\,,(k{+}1,k{+}2)$
  with $k = 0, 1, \ldots$.
- m_2^2 so that ***weight*** h ***integral*** also for nonBPS black holes!
- The $(3,0)$ operator was also identified as the squashing parameter of Kerr-black holes.

Fermions and KK Black Holes

- Fermions in $\mathcal{N} = 2$ hypermultiplets have ***Pauli couplings***:

$$\mathcal{L}_{\rm hyper} = -2\overline{\zeta}_A \gamma^\mu D_\mu \zeta^A - \frac{1}{2}\left(\overline{\zeta}^A \hat{F} \zeta^B \epsilon_{AB} + {\rm h.c.} \right) \ .$$

- The field strength $\hat{F}$ is different for BPS and nonBPS.

- ***Linearize*** around background,...,***diagonalize*** fermion mass matrix.

- The nonBPS spectrum for hyperfermions:
$(h, j) = 2[(k+2, k+\frac{1}{2}) \ , (k+1, k+\frac{1}{2})]$ with $k = 0, 1, \ldots$.

- Conformal weights h for fermions are ***integral*** on the nonBPS branch.

- They are ***not*** $1/2$***-integral***!

AdS$_2$ from AdS$_3$

- Illuminating to embed AdS$_2 \times S^2$ in AdS$_3 \times S^2$.
- Example: the MSW $(0, 4)$ theory = M-theory on CY with M5 branes wrapped on 4-cycles.
- The unwrapped $M5$ direction gives an “effective string” CFT$_2$.
- The MSW string has AdS$_3$ near horizon geometry.
- The SUGRA spectrum on AdS$_3$: towers of $(h, \bar{h}, \bar{j})$.
- A CFT$_2$ with short multiplets in R sector.

CFT_1s from CFT_2s

- Null reduction to AdS$_2$ (***BPS sector***): ***ignore*** h, keep $(\bar{h}, \bar{j})$.
- Looks like a CFT$_1$ with operators in short multiplets.
- Null reduction to AdS$_2$ reduction (***nonBPS sector***): ***ignore*** $\bar{h}$, keep $(h, \bar{j})$.
- Looks like a CFT$_1$ with operators unrelated by SUSY.
- Details check out: null reduction of AdS$_3$ SUGRA spectrum (known since 90's) yields the BPS/nonBPS SUGRA spectra on AdS$_2 \times S^2$.
- nonBPS sector inherits integral conformal weight from BPS protection of CFT$_2$.

Fermions in nonBPS CFT$_1$

- In the MSW construction the CFT$_1$ has a 5D origin.

- Fermions in 5D have $1/2$-integral angular momentum $\bar{j}$ so R-weight $\bar{h}$ also $1/2$-integral.

- Fermions also have $1/2$-integral 5D helicity $h - \bar{h}$.

- Therefore the L-weight h ***is integral***.

nAdS$_2$/nCFT$_1$ Correspondence

- Excitations (with finite energy) always violate AdS$_2$ boundary conditions.

- So the ***spectrum does not refer to states***.

- The spectrum refers to ***irrelevant operators*** that deform away from the IR CFT$_1$ fixed point.

$$\delta\mathcal{L} = \frac{1}{\Lambda^{h+\bar{h}-2}}\mathcal{O}^{(h,\bar{h})}(z,\bar{z})$$

- The nCFT$_1$ describes ***a kinematic regime where one chirality dominates***.

Application: Quantum BH Entropy

- The leading corrections to the black hole entropy are logarithmic:
$$S = \frac{A}{4G} + \frac{1}{2} D_0 \log A + \ldots \; .$$
- The coefficient D_0 can be computed from the low energy theory: ***only massless fields contribute.***
- All contributions from quadratic fluctuations around the classical geometry take the schematic form
$$e^{-W} = \int \mathcal{D}\phi \, e^{-\phi\Lambda\phi} = \frac{1}{\sqrt{\det\Lambda}} \; .$$
- The quantum corrections are encoded in the heat kernel
$$D(s) = \mathrm{Tr}\, e^{-s\Lambda} = \sum_i e^{-s\lambda_i} = \text{poles} + D_0 + \text{regular} \; .$$

Simple Heat Kernels in 2D

- A ***massless*** scalar field on AdS_2 involves a continuous spectrum:

$$K_A^0(s) = \frac{1}{2\pi}\int_0^\infty e^{-(p^2+\frac{1}{4})s} p \tanh \pi p \, dp = \frac{1}{4\pi s}\left(1 - \frac{1}{3}s + \frac{1}{15}s^2 + ..\right)$$

- A field with conformal weight h (mass $m^2 = h(h-1)$) and $SU(2)$ quantum number j (degeneracy $2j+1$):

$$K_A(h,j;s) = K_A^0 \, e^{-h(h-1)s}(2j+1) \; .$$

- The sum over the tower of AdS_2 fields essentially gives the heat kernel on S^2.

Quantum BH Entropy (BPS branch)

- The AdS$_2 \times S^2$ theory (BPS branch) gives

$$\delta S = \frac{1}{12}\left(23 - 11(\mathcal{N} - 2) - n_V + n_H\right) \log A_H \ .$$

- Special case $\mathcal{N} = 4$ SUSY: ***no logarithmic correction***.

- Special case $\mathcal{N} = 8$ SUSY: $\delta S = -4 \log A$.

- These results agree with ***microscopic*** theory.

Quantum Entropy (nonBPS branch)

- The logarithmic correction to the entropy on the nonBPS branch:

$$\delta S = \frac{1}{48}\left(65 - 87(\mathcal{N}-2) + 17 n_V + n_H\right)\log A_H\ .$$

- Special case $\mathcal{N}=8$ SUSY: $\delta S = -\frac{13}{2}\log A$.

- The detailed results pose a challenge to microscopic theory.

Summary

We computed the spectrum of extended SUGRA on the nonBPS branch of $AdS_2 \times S^2$.

Some highlights:

- Conformal weights are integral (protected?).
- Conformal weights of fermions are also integral (a magnetic effect?) .
- Interpretation: unprotected chiral sector of (0,4) CFT_2.
- Interpretation: $nAdS_2/nCFT_1$ correspondence.
- Application: quantum corrections to black hole entropy.