Geometry and Strings 2018 - Ringberg

The Refined Swampland Distance Conjecture in Calabi-Yau Moduli Spaces

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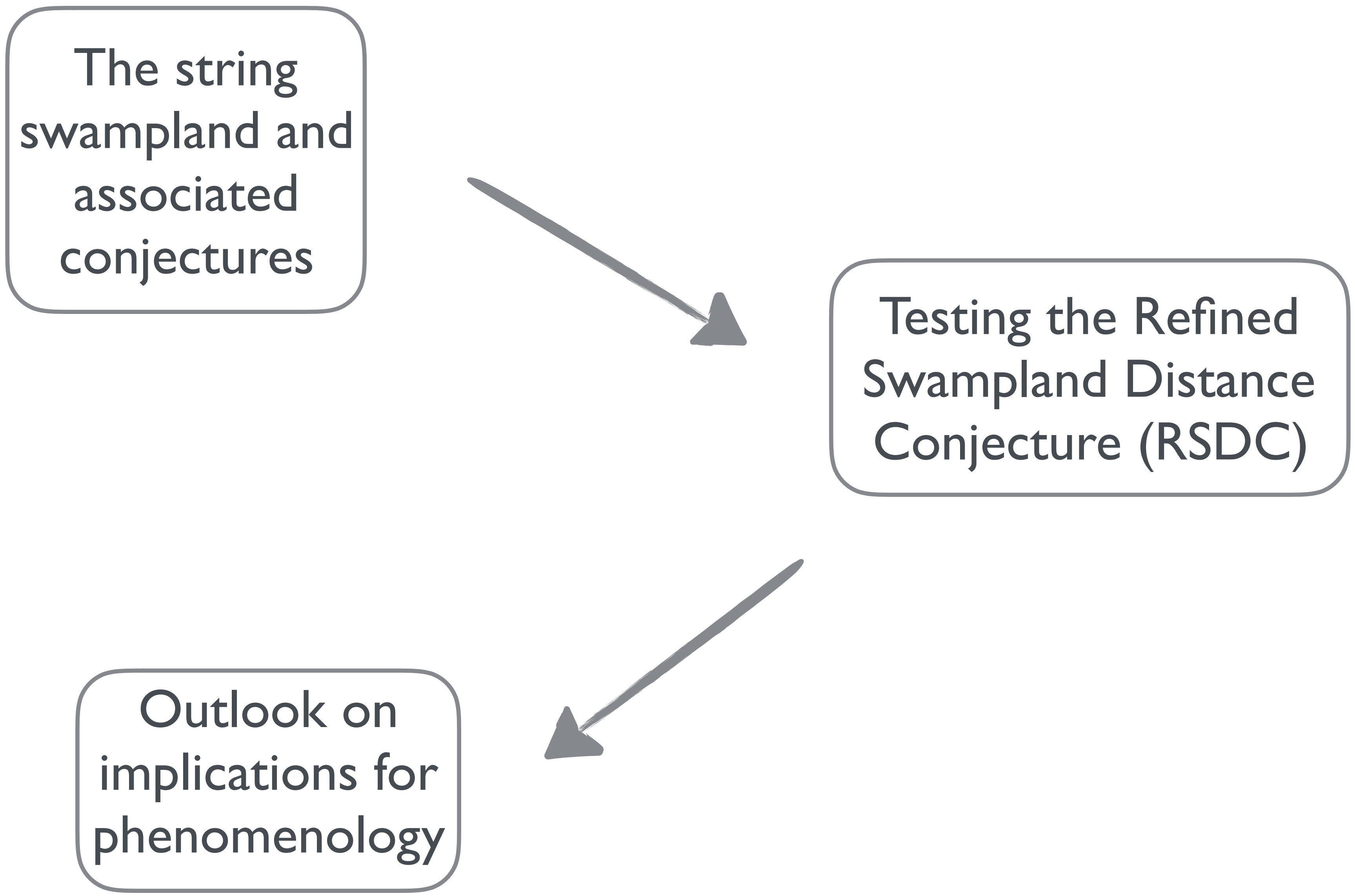
Ralph Blumenhagen, DK, Lorenz Schlechter, Florian Wolf — JHEP 06 (2018) 052

Outline


The string swampland and associated conjectures
Testing the Refined Swampland Distance Conjecture (RSDC)
Outlook on implications for phenomenology

The Swampland and Related Ideas

String theory has just celebrated its **50th birthday** 🎉 in Okinawa! String Phenomenology is a well-developed subject, addressing many problems in particle physics and cosmology from a top-down perspective.

Many detailed constructions have been developed to obtain: dS vacua, inflation, GUTs, etc…

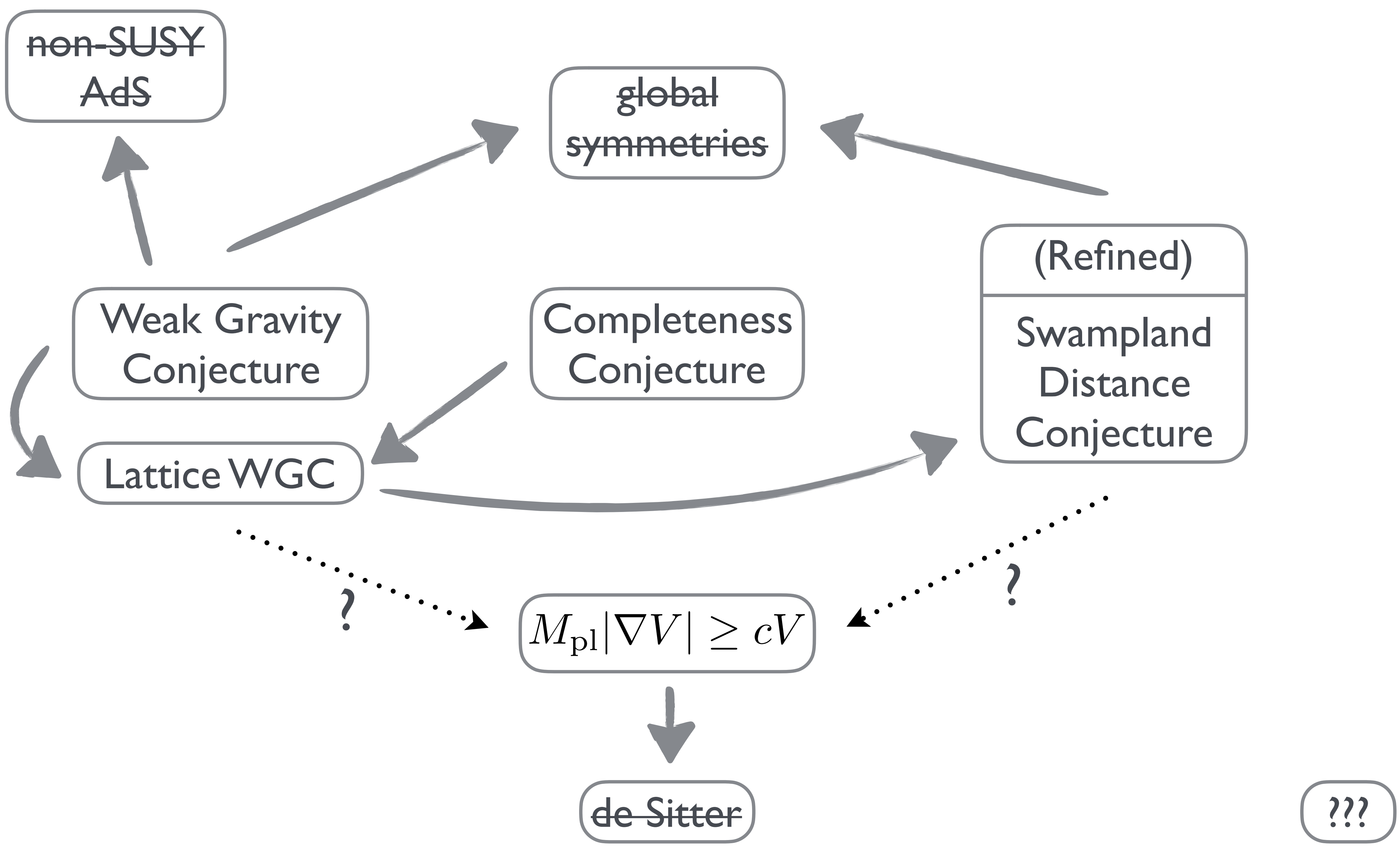
Yet **general ideas about quantum gravity** and its realization in string theory appear to **challenge many of these models.**

The **(string) swampland** is the set of (seemingly consistent) effective field theories, which cannot be obtained from a consistent string construction.

The **swampland as a blessing**: Knowing which field theories cannot be realized could actually lead to falsifiable predictions!!!

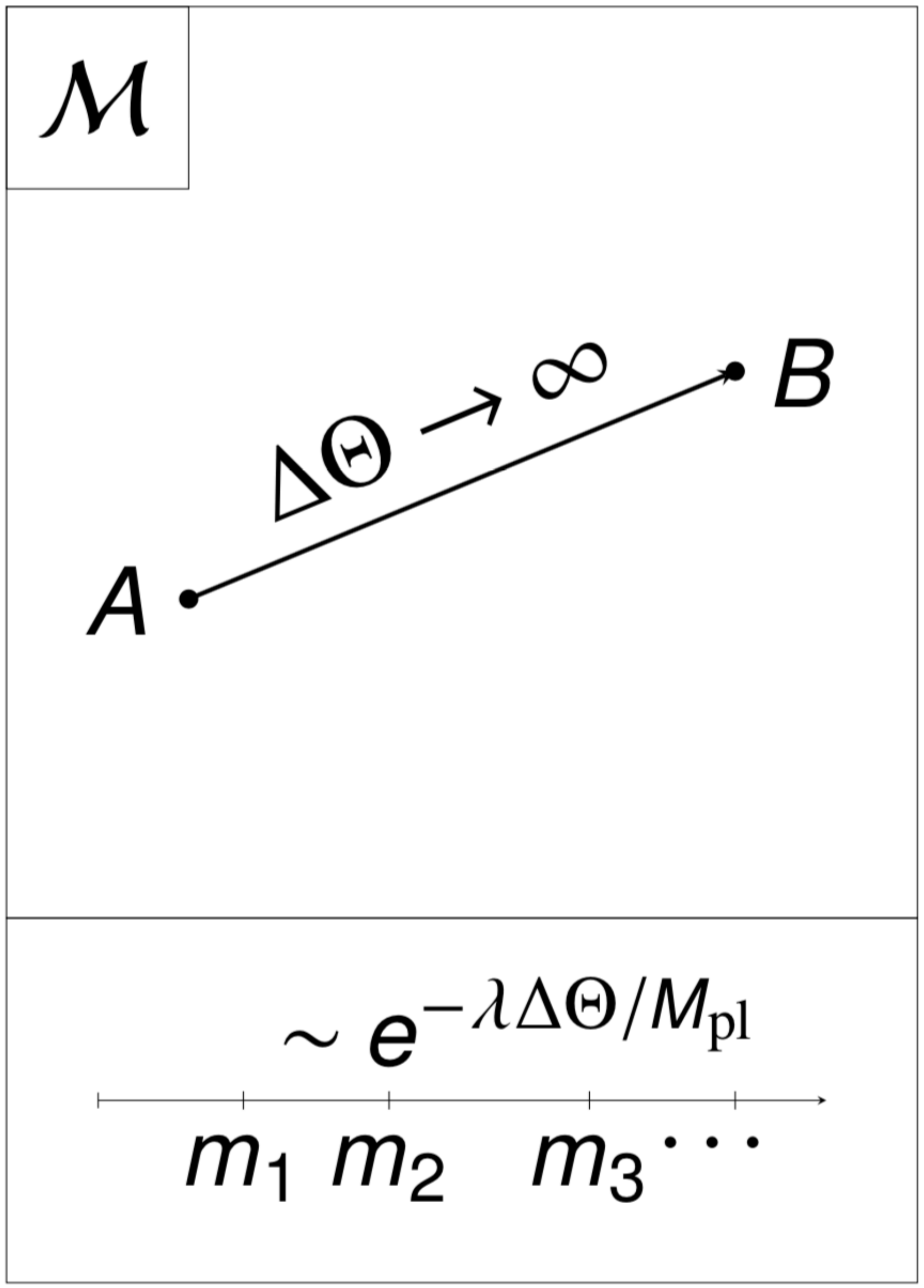
We want to **map out the boundary** of the swampland and explore the geometry of the landscape!

A Web of Conjectures…



The Swampland Distance Conjecture

[Ooguri, Vafa '06]



- Asymptotic displacements A → B in continuous moduli space of quantum gravity
- Conjectured **universal behavior of mass scale of an infinite tower of states**

$$\Theta = \int_{\tau_A}^{\tau_B} d\tau \sqrt{G_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau}}$$

- Casts **doubt on validity of EFT** for large field displacements

$$\Theta > \Theta_\lambda = \frac{1}{\lambda} = \mathcal{O}(1) M_{\rm pl}$$

Evidence

- Well known for **string theory on tori** (IIB on S^1) [Ooguri, Vafa '06]

$$M_{\rm KK} \sim \frac{1}{R^{\frac{8}{7}}} \qquad M_{\rm W} \sim R^{\frac{6}{7}} \qquad G_{\rm RR} \sim \frac{1}{R^2} \qquad \Theta \sim \log\left(\frac{R_B}{R_A}\right)$$

- **Holds for N > 8 supercharges** (moduli space is coset) [Cecotti '15]
- **Evidence also for N = 8 supercharges** [Grimm, Palti, Valenzuela '18] [Blumenhagen, DK, Schlechter, Wolf '18]
- Evidence from **semi-classical arguments**, relating it to WGC: [DK, Palti '16]
- (Sub-)Lattice WGC predicts infinite tower of states with $m \sim qgM_{\rm pl}$
- In gravitational theory, scalar fields can grow at most logarithmically [Nicolis '08]

$$\Delta\phi < \frac{1}{\alpha}\log(r/r_*)$$

- Together with magnetic WGC bound on the energy density $g(r) > \rho(r)^{\frac{1}{2}}$

Find that gauge coupling = mass drops at least exponentially in $\Delta\phi$

The Refined Swampland Distance Conjecture

[Baume, Palti '16; DK, Palti '16]

- SDC holds globally in simple moduli spaces (toroidal compactification)
- Generically expect the SDC to be **badly violated** at finite distances

Refined SDC quantifies this:

The universal exponential behavior sets in for

finite displacements Θ_0

of order the Planck scale or earlier

Evidence

- Even **less evidence than for the SDC**
- The semi-classical argument gives a **hint**: Free Scalars can only support sub-Planckian variations. Inside sources $\Delta\Theta > M_{\rm pl}$ is indeed possible, but only logarithmic growth!
- Solid **evidence from string theory** has been **lacking**

→ **fill this gap!**

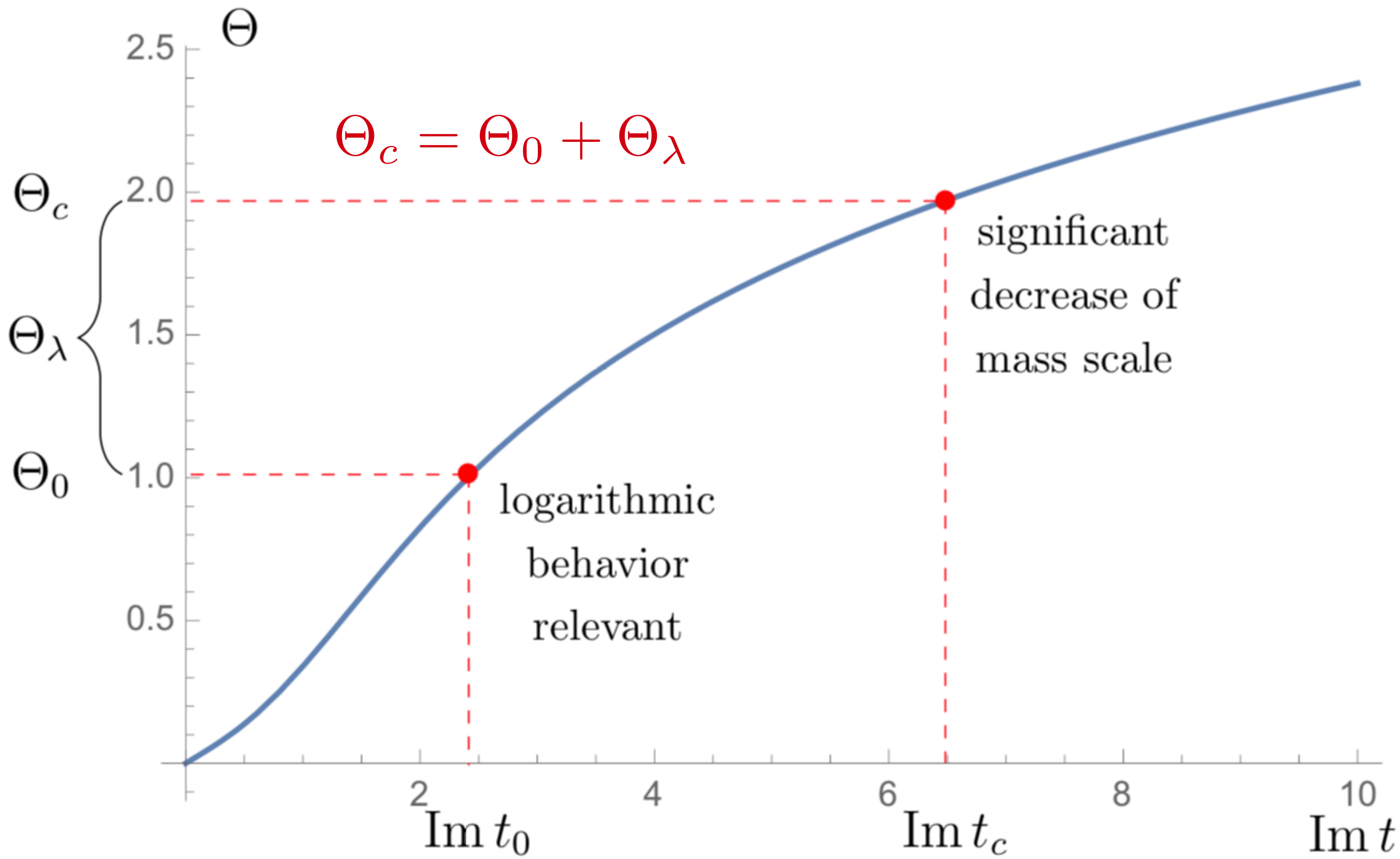
Additional Evidence?

- The RSDC applies to moduli, i.e. flat directions. For Pheno, we really want it to apply it to fields with a potential (Inflaton,…).
- In fact, there is evidence that a similar mechanism is at work.
- (F-term) axion monodromy inflation: [Silverstein, Westphal '08; Marchesano, Shiu, Uranga '14] [Palti, Baume '16; Blumenhagen, Valenzuela, Wolf '17]
- Break axion shift symmetry by fluxes, but corrections to the effective potential controlled even in the trans-Planckian regime $\Delta\Theta > M_{\rm pl}$
- Axions do not control mass scales, should be safe from SDC
- For trans-Planckian axion, the axion valley moves into saxion direction (backreaction). $s(\theta) = \lambda\theta$
- This implies the behavior predicted by the refined SDC

$$\Theta = \int K_{\theta\theta}^{1/2}(s) d\theta \sim \int \frac{d\theta}{s(\theta)} \sim \frac{1}{\lambda}\log(\theta)$$

Objectives

- Test the Refined Swampland Distance Conjecture in CY moduli spaces



Prediction: $\Theta_0 < \mathcal{O}(1) M_{\rm pl}$ | $\Theta_\lambda < \mathcal{O}(1) M_{\rm pl}$

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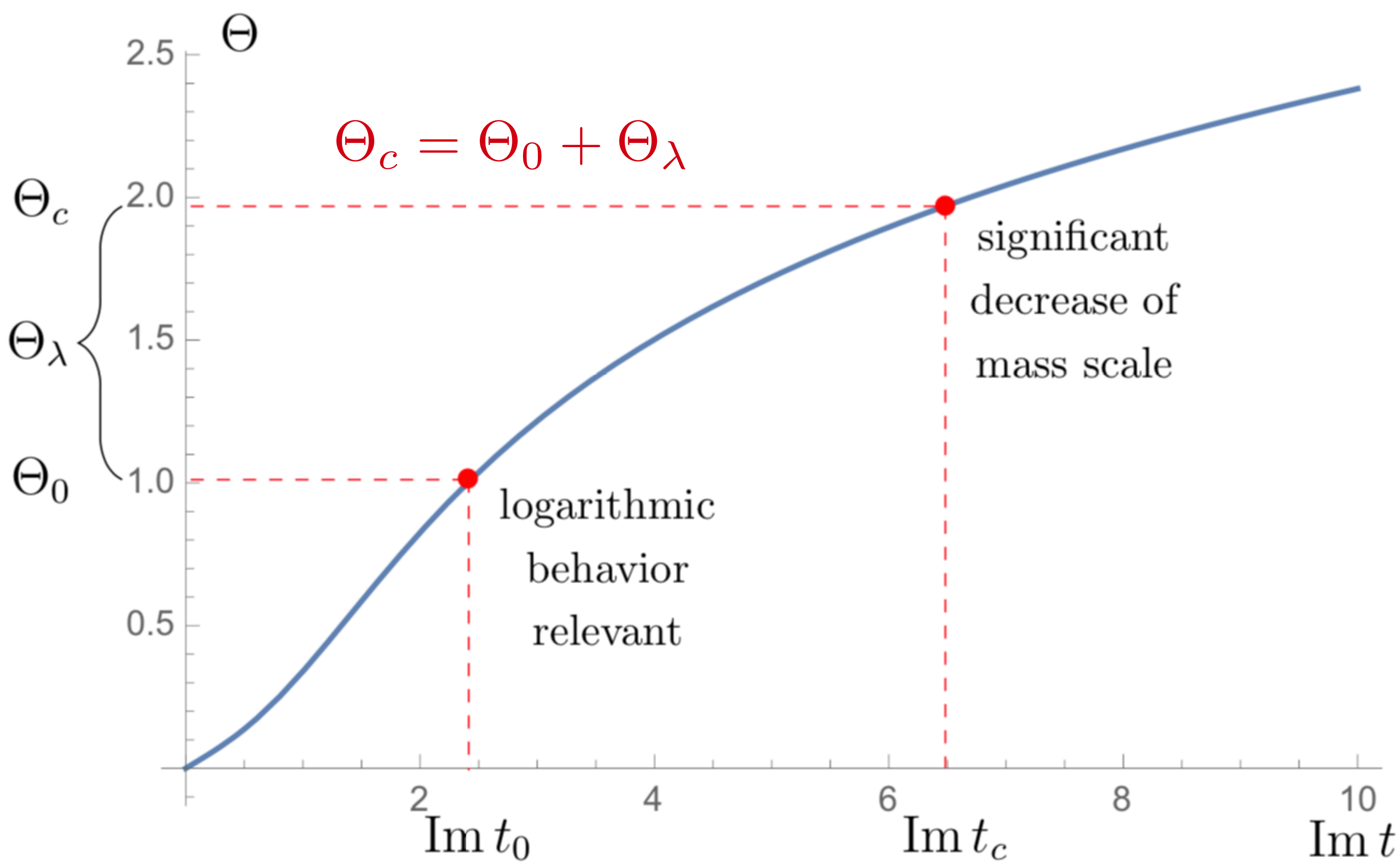


Prediction: $\Theta_0 < \mathcal{O}(1) M_{\rm pl}$ $\Theta_\lambda < \mathcal{O}(1) M_{\rm pl}$

Calabi Yau Moduli Spaces

- IIA/IIB string theory on a Calabi-Yau M manifold with

$$h^{11} = \dim\left(H^{1,1}\right)$$
$$h^{21} = \dim\left(H^{2,1}\right)$$

- Low energy EFT: N=2 supergravity
- Moduli space of deformations of M splits into

Kähler x **complex structure**

IIA: h^{11} vector multiplets h^{21} hypermultiplets

IIB: h^{11} hypermultiplets h^{21} vector multiplets

- **Mirror symmetry:** duality between IIA on M and IIB on W (mirror CY)
  - Exchanges Kähler and CS moduli spaces

Calabi Yau Moduli Spaces

- Metric on moduli space is determined by **Kähler potential** $g_{\alpha\bar{\beta}} = \partial_\alpha \partial_{\bar{\beta}} \mathcal{K}$

$$\mathcal{K}_K = -\log\left(-\frac{i}{6}\kappa^{abc}(t_a - \bar{t}_a)(t_b - \bar{t}_b)(t_c - \bar{t}_c) + \xi + \mathcal{O}\left(e^{-2\pi i t_a}\right)\right)$$

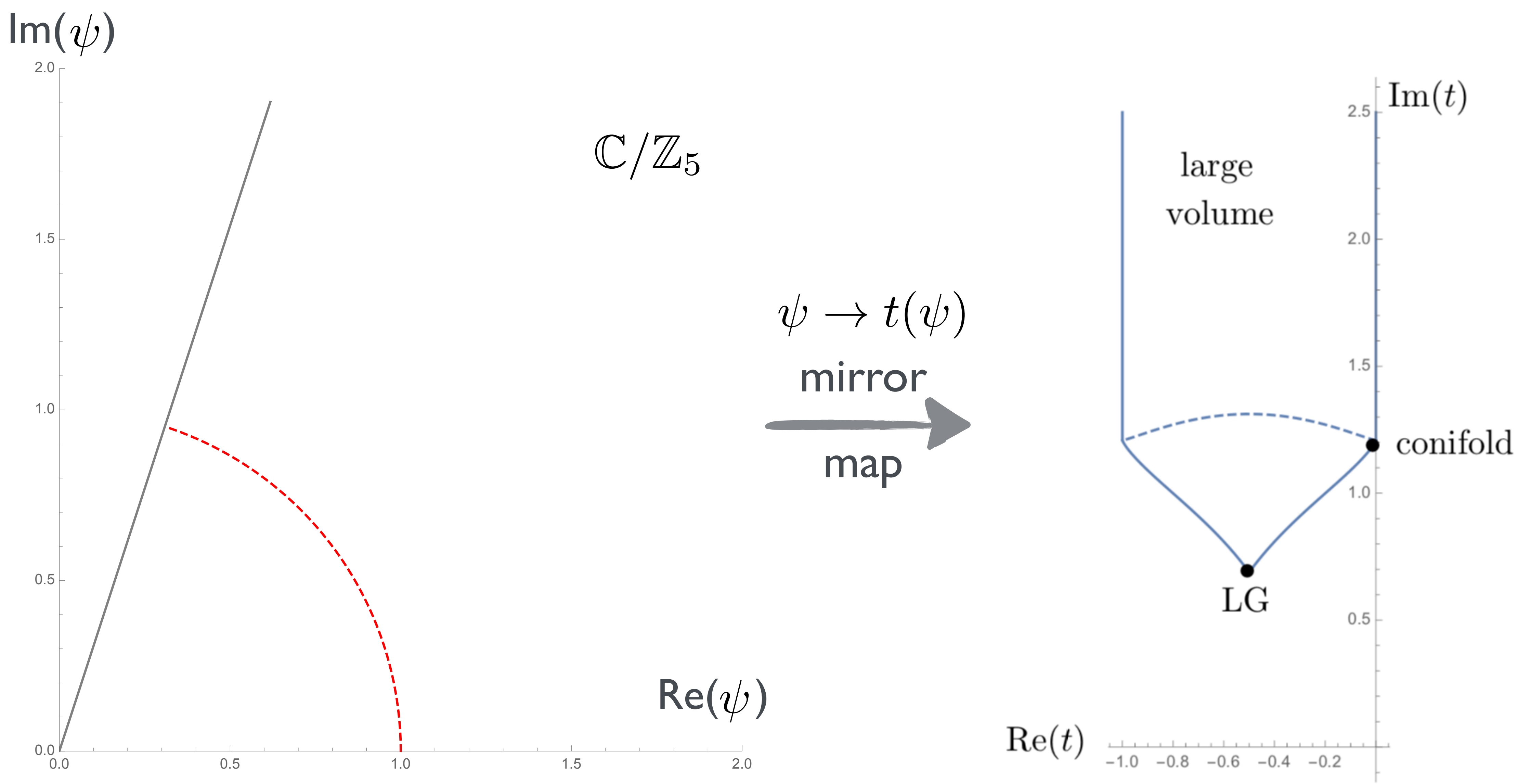
$$t_a = \int_{\Sigma_a} B + i \int_{\Sigma_a} J \qquad a = 1, \ldots, h^{11} \qquad \text{compl. Kähler moduli}$$

$$\mathcal{K}_{CS} = -\log(-i\overline{\Pi}\Sigma\Pi) \qquad \Pi_i(\Phi_\alpha) = \int_{A_i} \Omega(\Phi_\alpha) \qquad i = 1, \ldots, 2h^{2,1} + 2 \ \ \text{periods}$$

- The **Kähler side receives** perturbative and non-perturbative **corrections**
- The classical result for the complex structure side is **exact**
- We focus on the Kähler side because of the obvious associated **tower of Kaluza-Klein states** (similar results apply for the CS sector)
- Use mirror symmetry as tool to compute the fully corrected Kähler potential and explore non-geometric regions of moduli space $\mathrm{Im}(t_j) = \mathcal{O}(1)$

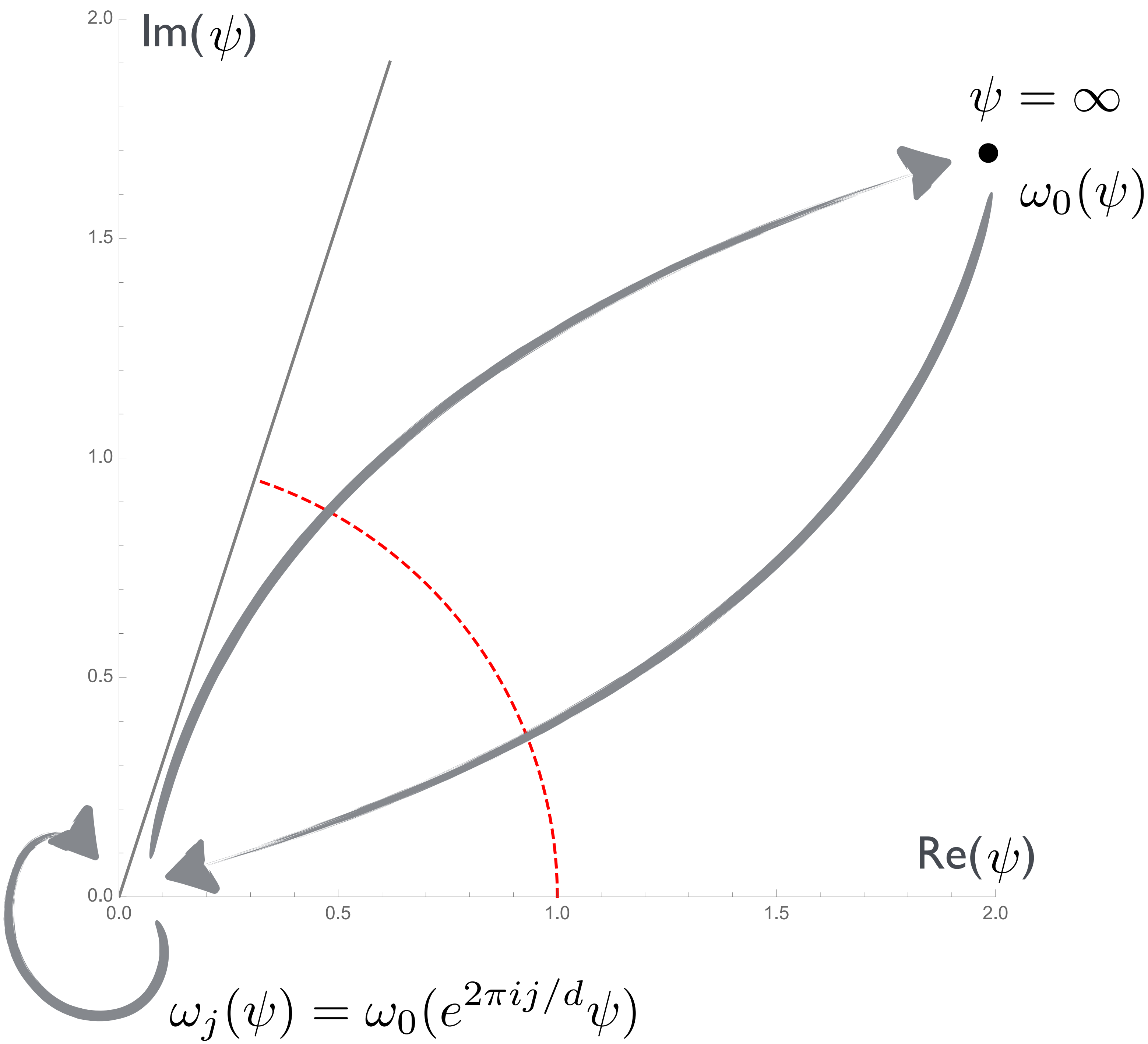
CY moduli spaces and the RSDC

- **Example: (mirror) quintic** $x_1^5 + x_2^5 + x_3^5 + x_4^5 + x_5^5 + 5\psi x_1 x_2 x_3 x_4 x_5 = 0$



Periods

- Well-known method to obtain **Kähler potential on CS side and mirror map:**



Tedious, but can be done in a case by case analysis for h[11] small

[Berglund, Candelas, de la Ossa, Font, Hübsch, Jancic, Quevedo '93]

[Hosono, Klemm, Theisen, Yau '93]

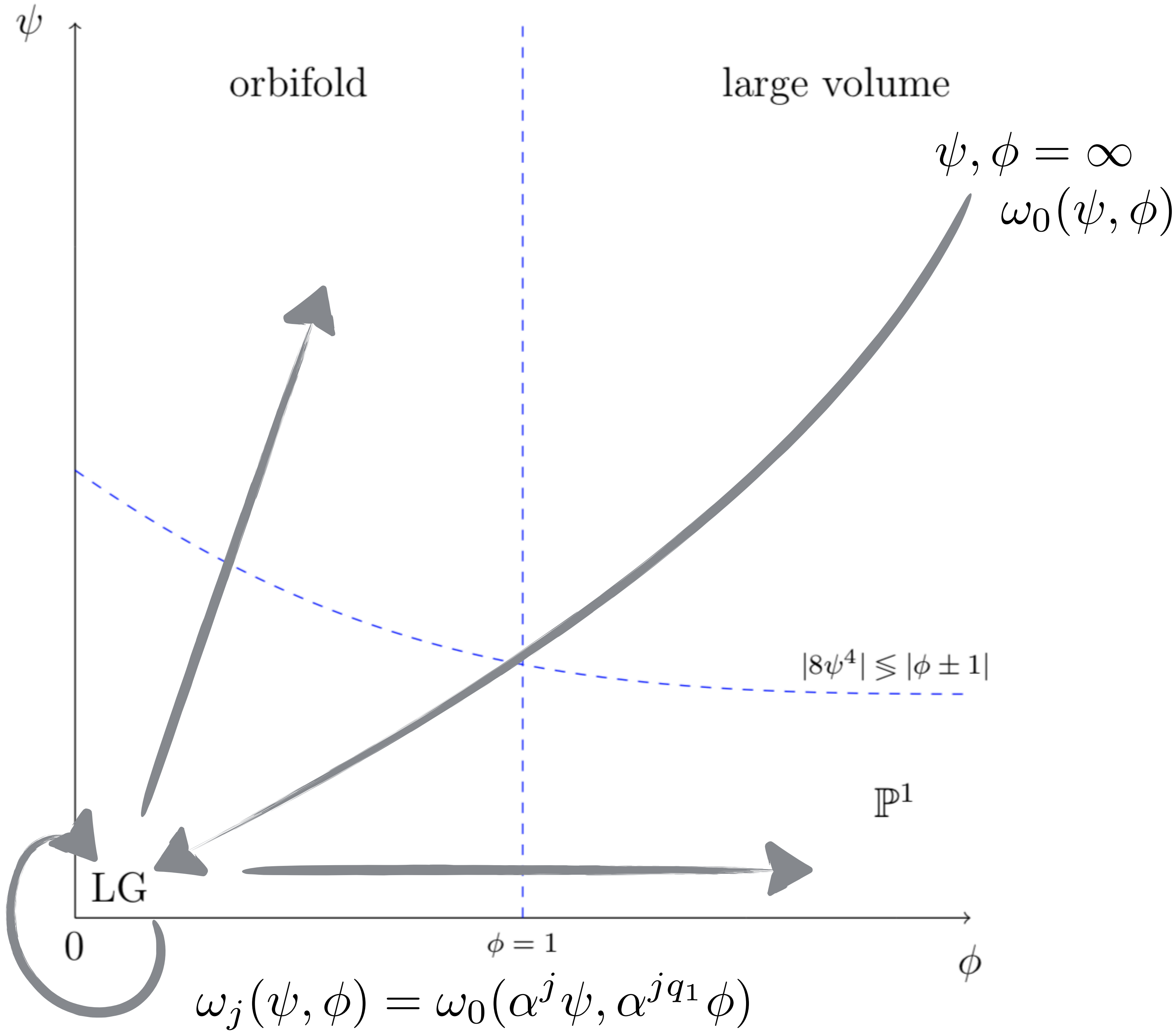
[Candelas, de la Ossa, Font, Katz, Morrison '94]

$$t = t(\psi)$$

$$\Pi = m\,\omega$$

Periods for 2-dimensional moduli spaces

$$P = x_1^8 + x_2^8 + x_3^4 + x_4^4 + x_5^4 + \psi x_1 x_2 x_3 x_4 x_5 + \phi x_1^4 x_2^4$$



Analytic continuation is subtle, but periods can be written in terms of hypergeometric functions in different ways and standard techniques apply

$\mathbb{P}^4_{11222}[8]$ and $\mathbb{P}^4_{11226}[12]$

[Berglund et al '94]

$8|\psi|^4 > |\phi \pm 1|$

$$\omega_0(\psi,\phi) = \sum_{l=0}^{\infty} \frac{(q_1 l)!(d\psi)^{-q_1 l}(-1)^l}{l! \prod_{i=2}^{5} \left(\frac{k_i}{d}(q_1 - q_i)l\right)!} U_l(\phi)$$

$$U_\nu(\phi) = \frac{e^{\frac{i\pi\nu}{2}}\Gamma\left(1+\frac{\nu}{2}(k_2-1)\right)}{2\Gamma(-\nu)} \left[2i\phi \frac{\Gamma(1-\nu/2)}{\Gamma\left(\frac{1+\nu k_2}{2}\right)} \, {}_2F_1\left(\frac{1-\nu}{2}, \frac{1-k_2\nu}{2}; \frac{3}{2}; \phi^2\right) + \right.$$
$$\left. + \frac{\Gamma(-\frac{\nu}{2})}{\Gamma\left(\frac{2+\nu k_2}{2}\right)} \, {}_2F_1\left(-\frac{\nu}{2}, -\frac{k_2\nu}{2}; \frac{1}{2}; \phi^2\right)\right]$$

$8|\psi|^4 < |\phi \pm 1|$

$$\omega_0(\psi,\phi) = -\frac{2}{d}\sum_{n=1}^{\infty} \frac{\Gamma(\frac{2n}{d})\,(-d\psi)^n\, U_{-\frac{2n}{d}}(\phi)}{\Gamma(n)\,\Gamma\left(1-\frac{n}{d}(k_2-1)\right)\,\prod_{i=3}^{5}\Gamma\left(1-\frac{k_i n}{d}\right)}$$

obtain all periods by $\omega_j(\psi,\phi) = \omega_0(\alpha^j\psi, \alpha^{jq_1}\phi)$,

$\mathbb{P}^4_{11222}[8]$ and $\mathbb{P}^4_{11226}[12]$

Alternative representation:

$$\omega_j(\psi,\phi) = -\frac{2}{d}\sum_{r=1}^{d}(-1)^r\, e^{2\pi i j r/d}\ \eta_{j,r}(\psi,\phi)$$

$$\eta_{j,r}(\psi,\phi) = \frac{1}{2}\sum_{n=0}^{\infty} e^{i\pi n(j+1/2)}\ \frac{(2\phi)^n}{n!}\ V_{n,r}(\psi)$$

$$V_{n,r}(\psi) = N_{n,r}\,(d\,\psi)^r\, H_{n,r}(\psi)$$

$$H_{n,r}(\psi) = {}_{(d+1)}F_d\left(1, \frac{n}{2}+\frac{r}{d}, \underbrace{1+\frac{r}{d}-\frac{l_2+1-\frac{n}{2}}{k_2}, 1+\frac{r}{d}-\frac{l_i+1}{k_i}}_{i=3,\ldots,5 \qquad l_i=0,\ldots,k_i-1}; \underbrace{\frac{r+l}{d}}_{l=0,\ldots,d-1}; \prod_{j=1}^{5} k_j^{k_j}\,\psi^d\right)$$

Convenient for continuation to $8|\psi|^4 > |\phi \pm 1|$!

The Gauged Linear Sigma Model

- Can also compute directly on the Kähler side, using Witten's **gauged linear sigma model** (GLSM) description [Witten '93] [Jockers, Kumar, Lapan, Morrison, Romo '13]
- GLSM is N=(2,2) SUSY gauge theory in 2d. Varying the FI parameters leads to phase transitions, corresponding to phases of Kähler moduli space
- **Kähler potential is given by sphere partition function** $e^{-\mathcal{K}} = Z_{S^2}$

$$Z_{S^2}(\xi, \bar{\xi}, Q, R) = \sum_{m_1 \in \mathbb{Z}} \cdots \sum_{m_s \in \mathbb{Z}} \int_{-i\infty}^{i\infty} da_1 \, \ldots \int_{-i\infty}^{i\infty} da_s \, Z_{\text{class}} \, Z_{\text{gauge}} \, Z_{\text{chiral}}$$

$$Z_{\text{chiral}} = \prod_{i=1}^{M} \frac{\Gamma\left(R_i/2 + \sum_{j=1}^{s} Q_{i,j} \cdot (a_j - m_j/2)\right)}{\Gamma\left(1 - R_i/2 - \sum_{j=1}^{s} Q_{i,j} \cdot (a_j + m_j/2)\right)} \qquad Z_{\text{class}} = \prod_{j=1}^{s} e^{-4\pi i r_j a_j + i\theta_j m_j} \qquad Z_{\text{gauge}} = 1$$

[Doroud, Gomis, Le Floch, Lee '13] [Benini, Cremonesi '15]

- Allows for **direct and algorithmic computation of the Kähler potential** without knowing the periods. Subtleties of analytic continuation are traded for subtleties in the evaluation of the integrals.

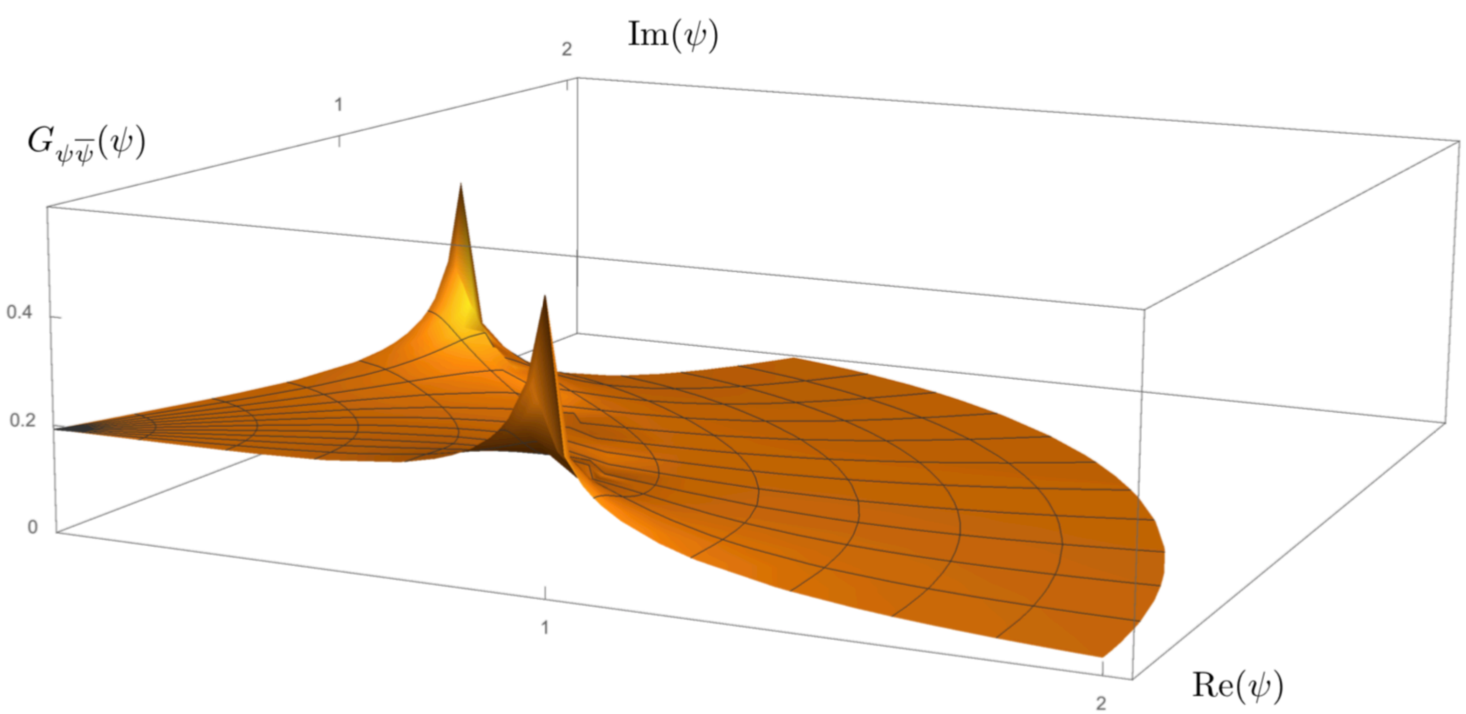
The Quintic

$$x_1^5 + x_2^5 + x_3^5 + x_4^5 + x_5^5 + 5\psi x_1 x_2 x_3 x_4 x_5 = 0$$

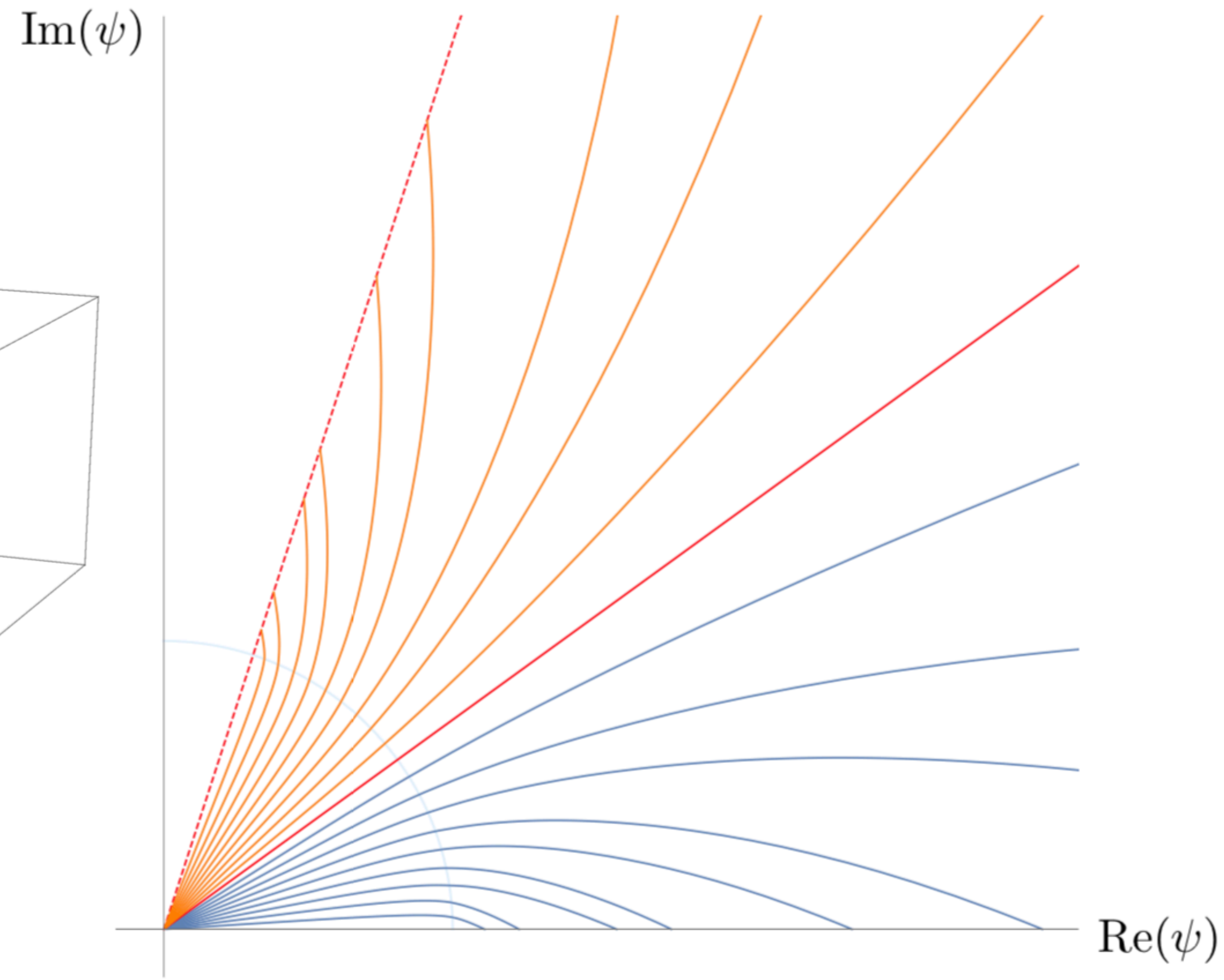
- **Necessary steps:**
- Compute metric, mirror map as described
- Determine the interesting regions in the moduli space (here: Landau-Ginzburg)
- Solve the geodesic equation numerically $\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0$
- Check consistency with the RSDC

The Quintic

Metric



Geodesics



Results

- **Find distances 0.42-0.45** inside the LG phase
- Θ_λ varies because geodesics curve in axion direction

$$\Theta_0 \leq 0.45$$

$$\Theta_\lambda < 1$$

$\theta_{\text{init}} \cdot 60/\pi$	α_0	α_1	λ^{-1}	Θ_0	Θ_c
3	0.1315	0.2043	0.9605	0.4262	1.3866
4	0.1127	0.2099	0.9865	0.4261	1.4125
5	0.0998	0.2213	0.9780	0.4260	1.4040
6	0.0955	0.2294	0.9567	0.4259	1.3827
7	0.0818	0.2475	0.9611	0.4259	1.3869
8	0.0877	0.2592	0.9275	0.4258	1.3533
9	0.0808	0.2825	0.9253	0.4257	1.3510
10	0.0929	0.3093	0.8969	0.4257	1.3226
11	0.0998	0.3497	0.8845	0.4257	1.3102
12	0.1234	0.1662	0.8657	0.4256	1.2914

- Analyse all CYs with $h^{11} = 1$ given by hypersurfaces in $W\mathbb{C}P$, namely

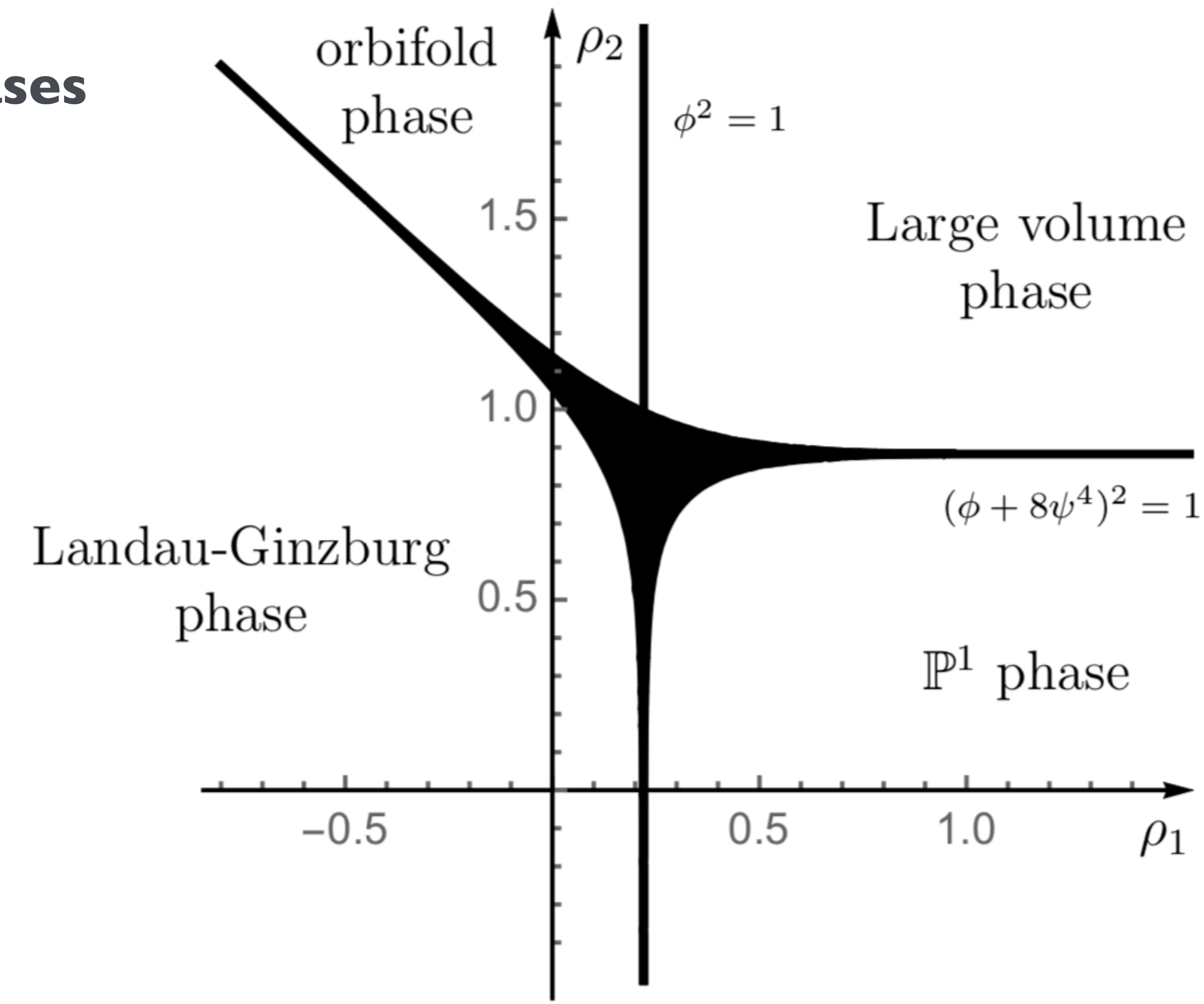
$$\mathbb{P}^4_{11112}[6] \qquad \mathbb{P}^4_{11114}[8] \qquad \mathbb{P}^4_{11125}[10]$$

- **All results in agreement with the RSDC, quintic is extremal**

$$\Theta_c \equiv \Theta_0 + \Theta_\lambda \leq 1.4$$

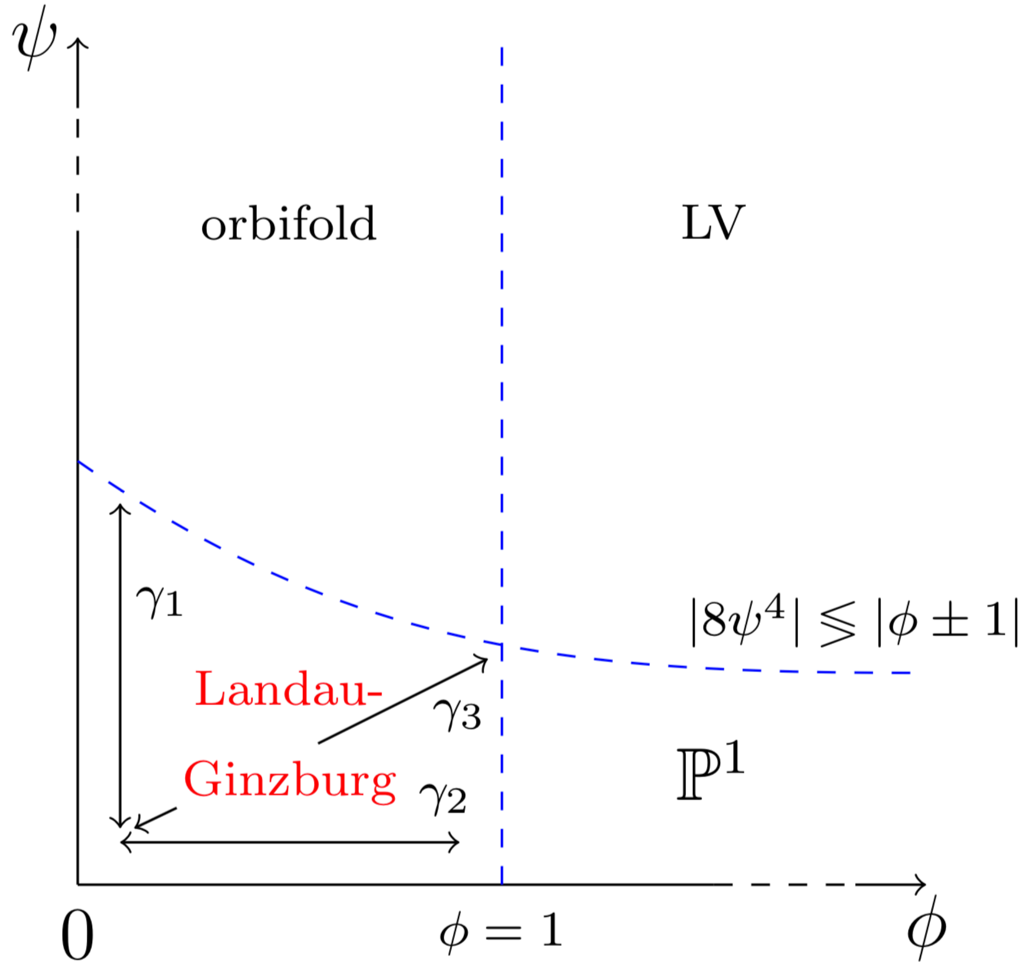
CYs With $h^{11} = 2$

new feature:
hybrid phases



[Aspinwall '94]

$\mathbb{P}^4_{11222}[8]$: LG phase



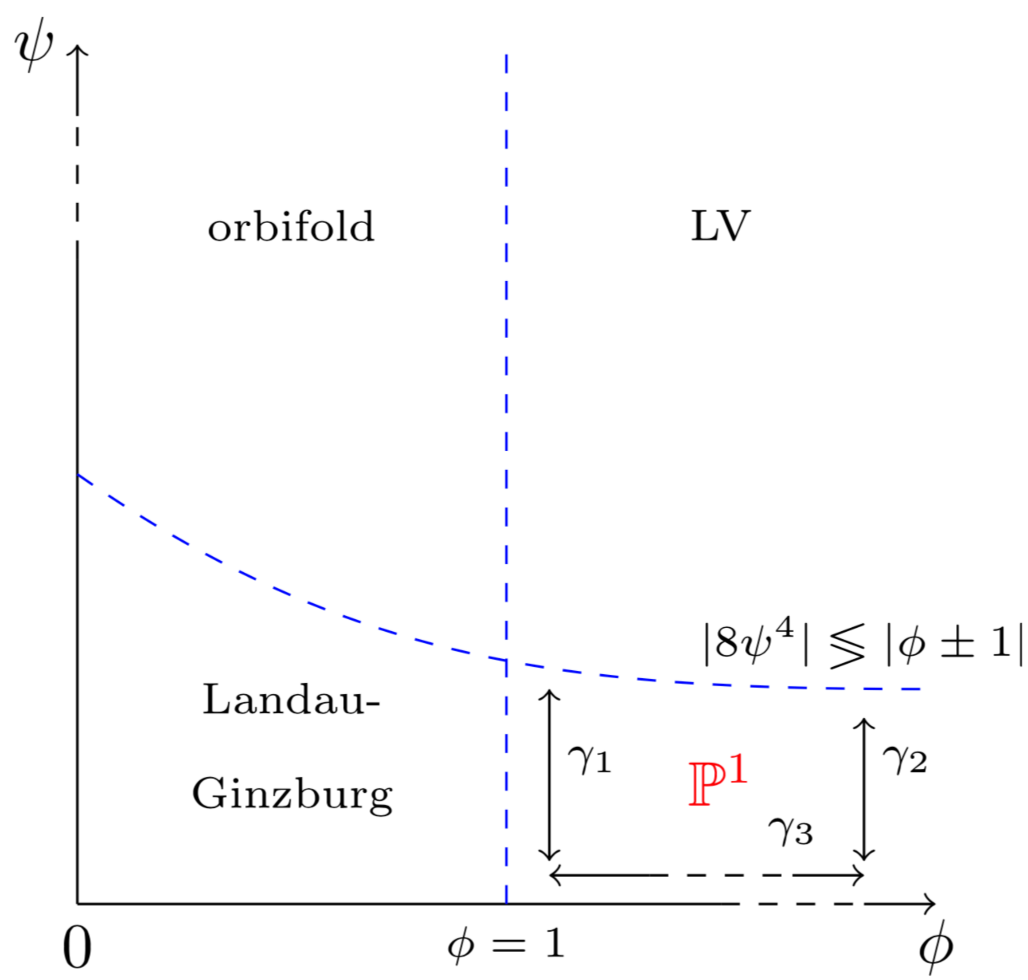
$$\Delta\Theta_1 = \int_{\gamma_1} d\psi \sqrt{G_{\psi\overline{\psi}}(\psi)} = 0.40 \,,$$

$$\Delta\Theta_2 = \int_{\gamma_2} d\phi \sqrt{G_{\phi\overline{\phi}}(\phi)} = 0.24 \,,$$

$$\Delta\Theta_3 = \int_{\gamma_3} d\tau \sqrt{G_{\mu\overline{\nu}}\left(\psi(\tau), \phi(\tau)\right) \frac{dx^\mu}{d\tau}\frac{d\overline{x}^{\overline{\nu}}}{d\tau}} = 0.36$$

Everything consistent with the RSDC!

$\mathbb{P}^4_{11222}[8]$: Hybrid Phase I



$$\Delta\Theta_1 = \int_{\gamma_1} d\psi \sqrt{G_{\psi\overline{\psi}}(\psi)} = 0.24$$

$$G^{\text{asymp}}_{\mathbb{P}^1} \simeq \begin{pmatrix} \frac{0.25}{|\phi|^2 (\log|\phi|)^2} & 0 \\ 0 & \frac{0.5905}{\sqrt{|\phi|}} \end{pmatrix} .$$

$$\Delta\Theta_2 = \int_{\gamma_2} d\psi \sqrt{G^{\text{asymp}}_{\mathbb{P}^1,\psi\overline{\psi}}(\phi)}$$

$$\simeq \sqrt{\frac{0.5905}{\sqrt{|\phi|}}} \cdot \sqrt[4]{\frac{|\phi|}{8}} = 0.46$$

Everything consistent with the RSDC!

The Mirror Quintic: $h^{11} = 101$

$$P = x_1^5 + x_2^5 + x_3^5 + x_4^5 + x_5^5 + \psi x_1 x_2 x_3 x_4 x_5 + 100 \text{ other terms}$$

- Recent advances allow us to compute the **Kähler metric for the Landau-Ginzburg phase of the mirror quintic** [Aleshkin, Belavin '17]
- Computing geodesics in a 101 dimensional space numerically is hopeless
- Group deformations into equivalence classes under coordinate permutations

  $\longrightarrow$ left with 5 sets of deformations of cardinality (1, 20, 30, 30, 20)
- Compute proper lengths of collective displacements
- Compelling: $\Theta_0 \sim \dfrac{1}{\#\text{fields}}$
- No parametric enhancement of Θ_0 in this way.

direction	$\Delta\Theta$
Φ_0	0.4656
Φ_1	0.0082
Φ_2	0.0670
Φ_3	0.0585
Φ_4	0.0089

$$\frac{\Theta_0}{\text{phase}} \#(\text{phases}) \leq M_{\text{pl}} \;?$$

Implications for Cosmology

Large Field Inflation

- Under pressure from several swampland conjectures
- WGC constrains natural inflation
- All models of large field inflation in tension with RSDC
- OOSV $|V'|/V > c = \mathcal{O}(1)$ puts pressure on slow roll [Obied, Ooguri, Spodyneiko, Vafa '18]

Dark Energy

- If dS is in the swampland, what about quintessence?
- Borderline consistent with the OOSV conjecture, RSDC [Agrawal, Obied, Steinhardt, Vafa '18]

Are we missing something fundamental?

Conclusion

- Refined Swampland Distance Conjecture passes many non-trivial tests in Calabi-Yau moduli spaces
- Diameter of non-geometric phases seems to approach zero as $h^{11} \to \infty$
- Our analysis is case by case - it would be good to have a general argument!
- Many of the swampland conjectures turn out to be tightly related. Are there further relations?

Thank You