AdS_3 at the String Scale

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Based on work with Kevin Ferreira, Rajesh Gopakumar, Chris Hull, and Juan Jottar.

Motivation

At the tensionless point in moduli space, string theory on AdS is dual to a (nearly) free conformal field theory.

The conserved currents of the free CFT correspond to massless higher spin fields in AdS, and the tensionless string theory contains a Vasiliev higher spin theory as a (closed) subsector.

[Fradkin & Vasiliev, '87]
[Vasiliev, '99…]

[Sundborg, '01], [Witten, '01], [Mikhailov, '02],
[Klebanov & Polyakov, '02], [Sezgin & Sundell, '03..]

HS theory — CFT duality

[MRG, Gopakumar '13 & '14]

Concrete realisation of this idea in context of AdS_3 :

large $\mathcal{N}=4$

hs theory based on

$\mathrm{shs}_2[\lambda]$

$\longleftrightarrow$

$$\frac{\mathfrak{su}(N+2)_k \oplus \mathfrak{so}(4N+4)_1}{\mathfrak{su}(N)_{k+2} \oplus \mathfrak{u}(1)_\kappa} \oplus \mathfrak{u}(1)_\kappa \ .$$

Wolf space cosets

[Sevrin, Troost, Van Proeyen, Schoutens, Spindel, .. '88/'89]

in 't Hooft limit with $\lambda = \dfrac{N+1}{N+k+2}$.

hs theory in string theory

[MRG, Gopakumar '14]

large $\mathcal{N}=4$		small $\mathcal{N}=4$
$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$	$\xrightarrow{\lambda \to 0}$	$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$
hs theory based on $\mathrm{shs}_2[\lambda]$		string theory
$\updownarrow$		$\updownarrow$
Wolf space cosets	$\subset$	symmetric orbifold
$\frac{\mathfrak{su}(N+2)_k \oplus \mathfrak{so}(4N+4)_1}{\mathfrak{su}(N)_{k+2} \oplus \mathfrak{u}(1)_\kappa} \oplus \mathfrak{u}(1)_\kappa$.		$\mathrm{Sym}_{N+1}(\mathbb{T}^4) \equiv (\mathbb{T}^4)^{\otimes(N+1)}/S_{N+1}$

large stringy symmetry

Direct understanding

The identification between higher spin theories and string theory is, so far, however rather indirect, i.e. can only be seen via the dual CFT.

Try to find more direct description of it. This requires a world-sheet approach since the higher spin symmetry is only expected to emerge in the tensionless (stringy) limit — far away from usual supergravity regime.

Dual CFT

To start with, let us consider bosonic case, i.e. WZW model based on sl(2,R).

[Maldacena, Ooguri '00]

The dual (`spacetime') CFT lives on the boundary of AdS3, and we have the identifications

$$L_0^{\rm CFT} = J_0^3 \ , \qquad L_1^{\rm CFT} = J_0^- \ , \qquad L_{-1}^{\rm CFT} = J_0^+ \ ,$$

with a similar relation for the right-movers.

The spacetime energy and spin are then given as

$$E = h + \bar{h} \ , \qquad s = h - \bar{h} \ .$$

spacetime conformal dimension of left- and right-movers

Massless higher spins

On the other hand, the AdS mass is

$$m^2_{\mathrm{AdS}_3} = (E - |s|)(E + |s| - 2)$$

Given that spacetime conformal dimensions are non-negative, massless higher spin fields only arise for

$$E = \pm s \qquad h = 0 \text{ or } \bar{h} = 0.$$

chiral fields of spacetime CFT

Physical states

This description is covariant, i.e. we need to impose physical state condition

$$L_n^{\text{tot}}\Phi = 0 \quad n > 0$$

$$(L_0^{\text{tot}} - 1)\Phi = 0 \ .$$

In particular, the second condition (mass-shell) condition implies that

$$\frac{C}{k-2} + h_0 + N = 1 \ .$$

Casimir of sl(2,R)

World-sheet conformal dim. of internal CFT

Representations I

The sl(2,R) ground state representations that appear in the world-sheet spectrum are the

Discrete lowest weight reps:

$$\mathcal{D}^+_j : \qquad C = -j(j-1) \ , \quad J_0^- |j,j\rangle = 0$$

↑

quasi-primary from spacetime CFT perspective!

Continuous reps:

$$\mathcal{C}(p,\alpha) : \qquad C = \tfrac{1}{4} + p^2 \ , \quad |j,m\rangle \text{ with } m \in \alpha + \mathbb{Z}$$

No-ghost theorem

Because of the Maldacena-Ooguri (unitarity) bound,

$$\text{MO-bound:} \qquad \frac{1}{2} < j < \frac{k-1}{2}$$

[Hwang '91]
[Evans, MRG, Perry '98]
[Maldacena, Ooguri '00]

the spectrum is bounded from above. Additional states are spectrally flowed images of these two classes of representations

[Maldacena, Ooguri '00]
see also [Henningson et.al. '91]

They are not Virasoro highest weight, and are therefore best described in terms of the spectral flow automorphism.

Spectral flow automorphism

Basic idea: work with original representation space, but define on it a new action (by automorphism):

$$\hat{J}^{\pm}_n \equiv \alpha_w(J^{\pm}_n) = J^{\pm}_{n\mp w}$$

$$\hat{J}^3_n \equiv \alpha_w(J^3_n) = J^3_n + \tfrac{k}{2} w \delta_{n,0} \qquad (w \in \mathbb{N})$$

$$\hat{L}_n \equiv \alpha_w(L_n) = L_n - w J^3_n - \tfrac{k}{4} w^2 \delta_{n,0} \; .$$

Since the automorphism is outer, get a new representation in this manner: spectrally flowed rep.

Long Strings

Here the interpretation is that w is the winding number of the string around the boundary of AdS.

In particular, the w=1 continuous representation describes the long string running near the boundary of AdS. It is stable since

tension is compensated by the
NS flux of the AdS space.

Physical spectrum

With these preparations at hand, we can now study the physical spectrum of the theory.

In particular, we can look systematically for massless (higher spin) fields, i.e., physical states with h=0, say.

Let us begin by analysing the unflowed discrete representations.

Unflowed discrete reps

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In this case, the mass-shell condition becomes

$$-\frac{j(j-1)}{k-2}+N=1$$

where we have set $h_0 = 0$. This can be rewritten as

$$j^2 - j - (k-2)(N-1) = 0$$

Unflowed discrete reps

At level N the sl(2,R) spin is at least

$$h = j - N \quad \stackrel{h=0}{\Longrightarrow} \quad j = N \ .$$

Plugging into the above equation then leads to

$$N^2 - N - (k-2)(N-1) = 0$$

There is one obvious solutions:

$$N = j = 1 : \quad \text{graviton}$$

Unflowed discrete reps

The other solution of the quadratic equation arises for

$$N = k - 2 \ .$$

However, since N=j, this implies

$$j = k - 2 \geq \frac{k-1}{2} \qquad \begin{array}{l}(\text{for } j = N = 2, 3, \ldots, \\ \ \text{i.e., } k = 4, 5, \ldots)\end{array}$$

Not allowed by the MO-bound!

Thus there are no massless higher spin fields from discrete unflowed representations. The same conclusion also holds for the spectrally flowed discrete reps.

Flowed representations

For the spectrally flowed continuous representations, the mass-shell condition becomes $[\alpha_w(L_n) = L_n - wJ^3_n - \frac{k}{4}w^2\delta_{n,0}]$

$$\frac{C}{k-2} - wm - \frac{k}{4}w^2 + N = 1 \quad \text{where} \quad C = \frac{1}{4} + p^2$$

is the Casimir of the ground state representation and m the magn. quantum number. Demanding h=0 with

$$h = m + \frac{k}{2}w = 0 \quad \Longrightarrow \quad m = -\frac{wk}{2}\ , \qquad [\alpha_w(J^3_n) = J^3_n + \tfrac{k}{2}w\delta_{n,0}]$$

we get

$$\boxed{\frac{\frac{1}{4}+p^2}{k-2} + \frac{k}{4}w^2 + N = 1}$$

Flowed representations

$$\boxed{\frac{\frac{1}{4}+p^2}{k-2}+\frac{k}{4}w^2+N=1}$$

For spectral flow w=1, the mass-shell condition becomes (for N=0)

$$\frac{1}{4}+p^2=\left(1-\tfrac{k}{4}\right)(k-2)$$

which has the solution

[MRG, Gopakumar, Hull ’17]

$$k=3 \qquad \text{and} \qquad p=0\ .$$

Higher Spin Symmetry

[MRG, Gopakumar, Hull '17]

In fact, an infinite set of higher spin fields becomes massless at this point: for the right-movers we have to solve the right-moving analogue of

$$\frac{C}{k-2} - wm - \frac{k}{4}w^2 + N = 1 \qquad (k=3, p=0, w=1)$$

i.e.

$$\frac{1}{4} - \bar{m} - \frac{3}{4} + \bar{N} = 1$$

which is solved by

$$\bar{m} = -\frac{3}{2} + \bar{N} \ .$$

Thus get a massless higher spin field for every right-moving excitation (and similarly for left-movers)!

Supersymmetric version

The analysis of the supersymmetric version of this theory is similar. There are two interesting cases:

$$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$$

$$\updownarrow$$

$$\mathfrak{sl}(2)_k \oplus \mathfrak{su}(2)_{k'} \oplus \mathfrak{u}(1)^4$$

[N=1 susy WZW models]

Criticality: $k = k'$

$$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$$

$$\updownarrow$$

$$\mathfrak{sl}(2)_k \oplus \mathfrak{su}(2)_{k_+} \oplus \mathfrak{su}(2)_{k_-} \oplus \mathfrak{u}(1)$$

$$\frac{1}{k} = \frac{1}{k_+} + \frac{1}{k_-}$$

Massless higher spins

[Ferreira, MRG, Jottar '17]

The analogue of k=3 in the bosonic case is now

$$k = 1 \qquad \text{[corresponds to } k=3 \text{ for bosonic } \mathfrak{sl}(2)]$$

For this value of the level, an infinite tower of massless higher spin fields appears in the w=1 spectrally flowed continuous representation with p=0.

In fact, a stronger statement is true: at k=1 and p=0, the susy mass-shell condition (in NS sector)

$$\frac{C}{k} - wm - \frac{k}{4}w^2 + N = \frac{1}{2} \qquad \text{where} \qquad C = \frac{1}{4} + p^2$$

Full spectrum

becomes for generic w

$$\frac{1}{4} - w\left(m + \tfrac{w}{4}\right) + N = \frac{1}{2}\ .$$

Solving for m and observing that the actual J_0^3 eigenvalue is

$$h = m + \frac{w}{2} = \frac{N}{w} + \frac{w^2 - 1}{4w}\ .$$

w-twisted modes

ground state energy in w-twisted sector

Symmetric orbifold formula for cycle length w!

Full symmetric orbifold

Thus we recover the full single-particle spectrum of the symmetric orbifold.

[MRG, Gopakumar ’18]
see also [Giribet, et.al. ’18]

However, there are three subtleties:

(1) Fermions and GSO
(2) Which orbifold do we actually get?
(3) Compatibility with OPE structure

Fermions

On the world-sheet, the fermions are GSO-projected and appear in both NS and R sector.

However, the dual CFT should **not** have a GSO projection, and only the perturbative (NS sector) should appear.

The relation is quite subtle since GSO projection depends on cardinality of the flow, and structure of twisted sector on cardinality of the twist. However, everything comes out correctly in the end, using the abstruse identity.

[MRG, Gopakumar '18]

Which orbifold

For $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$ the situation is cleanest: at k=1, criticality leads to

$$k_+ = k_- = 2$$

and thus the bosonic su(2) factors do not contribute at all. Then there are 4 bosons from

$$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$$

3 - - 1 = 4

which are reduced to 2 by physical state condition.

2 bos + 8 fer : $(\mathcal{S}_0)^2$

[MRG, Gopakumar ’18]

Which orbifold

2 bos + 8 fer : $(\mathcal{S}_0)^2$

This theory actually has the expected large $\mathcal{N}=4$ superconformal symmetry.

It is closely related to the CFT dual of string theory on $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$ which for these value of the fluxes was recently proposed to be dual to

$$\mathrm{Sym}(\mathcal{S}_0)$$

[Eberhardt, MRG, Li ’17]
see also earlier work in
[Eberhardt, MRG, Gopakumar, Li ’17]

Which orbifold

For $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$ at k=1, criticality leads to $k' = 1$.
Then the bosonic su(2) factor appears at level -1, and we can use

[Goddard, Olive, Waterson '87]

$$\mathfrak{su}(2)_{-1} \oplus \mathfrak{u}(1) = 4 \text{ symplectic bosons}$$

This leads to the boson counting

$$\mathrm{AdS}_3 \times \underbrace{\mathrm{S}^3 \times \mathrm{S}^1}_{} \times \mathbb{T}^3$$

3   4 sympl.   3   = 6 + 4 sympl.

i.e. to 4 real bosons (and 4 symplectic bosons) after the physical state condition is imposed.

Which orbifold

The 4 symplectic bosons behave as ghosts (at least for the partition function) and remove precisely 4 of the 8 fermions.

(They also lead automatically to the correct ground state energy in the twisted sector.)

Thus we end up with 4+4 free bosons and fermions, i.e. with the

symmetric orbifold of $\mathbb{T}^4$.

[MRG, Gopakumar '18]

OPE structure

The fusion rules of the world-sheet continuous representations are

[Maldacena, Ooguri '01]

$$[w_1] \otimes [w_2] = [w_1 + w_2 - 1] \oplus [w_1 + w_2] \oplus [w_1 + w_2 + 1]$$

whereas the single-particle states of the symmetric orbifold have OPEs

[Jevicki, Mihailescu, Ramgoolam '98]
[Pakman, Rastelli, Razamat '09]

$$[w_1] \otimes [w_2] = [w_1 + w_2 - 1] \oplus [w_1 + w_2 - 3] \oplus [w_1 + w_2 - 5] \oplus \cdots$$

see also [Giribet, et.al. '18]

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see also [Giribet, et.al. '18]

Chiral algebra

However, at least for the chiral fields in w=1 (untwisted sector), the symmetric orbifold OPE is trivial, and in the world-sheet theory the actual OPEs also trivialise since these fields have

$$\bar{h} = 0 \quad \Rightarrow \quad w = 1 \, , \quad p = 0 \, .$$

↑ states in trivial sl(2,R) rep

[Fusion respects tensor product rules of sl(2,R) reps.]

[MRG, Gopakumar, in progress]

Chiral algebra

Thus we can probably only conclude that the chiral symmetry algebra agrees….

Chiral algebra

Thus we can probably only conclude that the chiral symmetry algebra agrees….

… but this symmetry algebra is very large: Higher Spin Square (HSS).

[MRG, Gopakumar ’15]

In particular, this suggests that the presence of this extended higher spin symmetry fixes essentially the structure of the theory.

Conclusions

- Analysed whether string theory on AdS3 has massless higher spin fields, using the WZW world-sheet approach

  massless higher spin fields appear for k=1 from long strings

- In fact, the k=1 theory contains a sector that matches exactly the spectrum of the symmetric orbifold.

Open problems

- However, it seems that the actual spacetime CFT is not exactly the symmetric orbifold (OPEs)

  Understand reason for matching of spectrum:
  representation theory of HSS ?

- Study the behaviour of worldsheet description as one moves away from pure NS-NS background (using hybrid formalism).

[Eberhardt, Ferreira '18]
and work in progress

Thank you…..

Ralph, Ilka, Michael & Dieter