From non-geometric heterotic backgrounds to little string theories via F-theory

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in collaboration with: C. Mayrhofer. JHEP 11 (2017) 064

I. García-Etxebarria, D. Lüst, S. Massai, C. Mayrhofer. JHEP 08 (2016) 175

Overview

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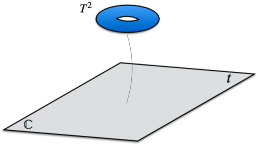

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cplx str. and Kähler moduli $\tau(t), \rho(t)$

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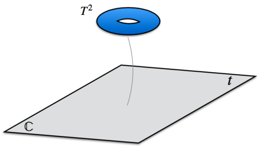

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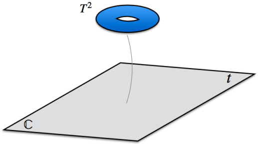

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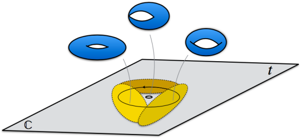
- Non-trivial fibrations must degenerate at points on the base, signaling defects, called *T-fects*. [Lüst, Massai,Vall-Camel]



T-fects induce monodromies in duality group, e.g. $\rho \to \frac{a\rho+b}{c\rho+d}$

- Extend to heterotic strings. [McOrist, Morrison, Sethi; Malmendier, Morrison; Gu, Jockers]

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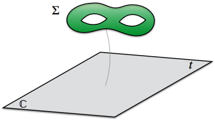

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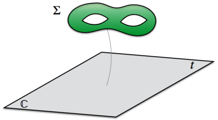

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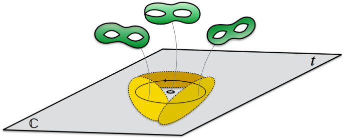

points where Σ degenerates are location of *T-fects*, e.g. NS5-branes

- Possible degenerations of genus 2 curves are classified. [Namikawa-Ueno]
  Namikawa-Ueno list provides a large number of *T-fects*.
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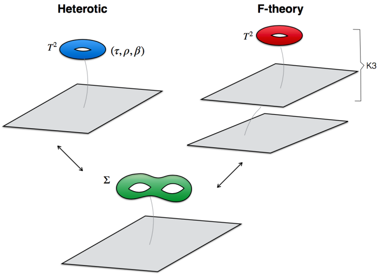

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- $E_8 \times E_8$ heterotic (HE) analyzed by AF, García-Extebarria, Lüst, Massai, Mayrhofer.
  Now focus on $\mathrm{Spin}(32)/\mathbb{Z}_2$ heterotic (HO). AF, Mayrhofer.

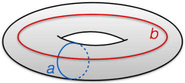
Outline

- Overview ✓
- Heterotic string in $8d$ and $6d$
  - Moduli space of heterotic on T^2
  - From 8d to 6d: Fibration of genus 2 curve
- F-theory and vacua with varying moduli
  - Heterotic/F-theory duality in 8d
  - From genus 2 fibrations to dual K3 fibrations
  - Resolution of singularities
- Results
  - Geometric models: small instantons on ADE singularities
  - Non-geometric models and dualities
  - General properties
- Final comments

Heterotic in 8d and 6d

Moduli space of heterotic on T^2

Complex structure $\tau = \dfrac{\int_b \omega}{\int_a \omega}$



Kähler $\rho = \int_{T^2} B + i\omega \wedge \bar{\omega}\,,$

Wilson lines (WL) $\beta^I = \int_a A^I + i \int_b A^I\,,\ I = 1, \ldots, 16$

consider only one WL in $SU(2)$

HE: $E_8 \times E_8 \xrightarrow{\beta} E_7 \times E_8$

HO: $\mathrm{Spin}(32)/\mathbb{Z}_2 \xrightarrow{\beta} \mathrm{Spin}(28) \times SU(2)/\mathbb{Z}_2$

Moduli of heterotic on T^2 with one WL in $SU(2)$: (τ, ρ, β)

Narain moduli space $\mathcal{M}_{\text{het}} = O(3) \times O(2) \backslash O(3,2) / O(3,2,\mathbb{Z})$

duality group $O(3,2,\mathbb{Z})$ e.g. $\tau \to \frac{\rho}{\beta^2 - \tau\rho}, \quad \rho \to \frac{\tau}{\beta^2 - \tau\rho}, \quad \beta \to \frac{-\beta}{\beta^2 - \tau\rho}$

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restricting to $SO^+(3,2,\mathbb{Z})$, map $\mathcal{M}_{\text{het}}$ to $\mathbb{H}_2/Sp(4,\mathbb{Z})$: moduli space of genus 2

$$\mathbb{H}_2 = \left\{ \Omega = \begin{pmatrix} \tau & \beta \\ \beta & \rho \end{pmatrix} \,\middle|\, \det \operatorname{Im}(\Omega) > 0,\ \operatorname{Im}(\rho) > 0 \right\}$$

Siegel upper half-plane of genus 2

Ω: period matrix

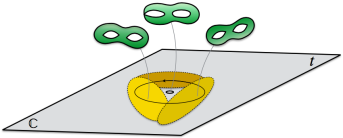
$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \in Sp(4,\mathbb{Z}), \qquad \Omega \to (A\Omega + B)(C\Omega + D)^{-1}, \qquad \text{e.g. } \Omega \to -\Omega^{-1}$$

From 8d to 6d: Fibration of genus 2 curve Σ

Construct 6d vacua by letting moduli (τ, ρ, β) vary along $\mathbb{C} \ni t$
Use geometrical object encoding moduli to handle identifications under duality group around closed paths, i.e. use Σ

Eq. of motion $\Rightarrow$ $\Sigma(t)$ holomorphic in t

$\Sigma(t)$ must degenerate at points in $\mathbb{C}$



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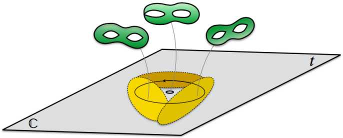

Degenerations of genus 2 fibrations with monodromy in $Sp(4, \mathbb{Z})$ classified by Namikawa-Ueno (NU)

NU give local equation (sextic), with singularity at $t = 0$ and provide the monodromy

Ex. $\mathrm{III} - \mathrm{III} - 0$ $\quad Y^2 = X(X-1)(X^2+t)\left((X-1)^2+t\right)$

$$\text{monodromy} \begin{pmatrix} 0 & \text{-1} \\ 1 & 0 \end{pmatrix}, \quad \Omega \to -\Omega^{-1}$$

F-theory and vacua with varying moduli

F-theory/Heterotic duality in 8d

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HE: $y^2 = x^3 + (a\, u^4 v^4 + c\, u^3 v^7)\, x\, z^4 + (b\, u^6 v^6 + d\, u^5 v^7 + u^7 v^5)\, z^6$

x, y, z, and u, v: homogeneous coordinates of the fiber ambient variety $\mathbb{P}_{2,3,1}$, and the base $\mathbb{P}^1$

singularities: II^* (E_8) at $v = 0$, III^* (E_7) at $u = 0$, $\to \mathrm{II}^*$ for $c = 0$ $\Rightarrow$ no WL

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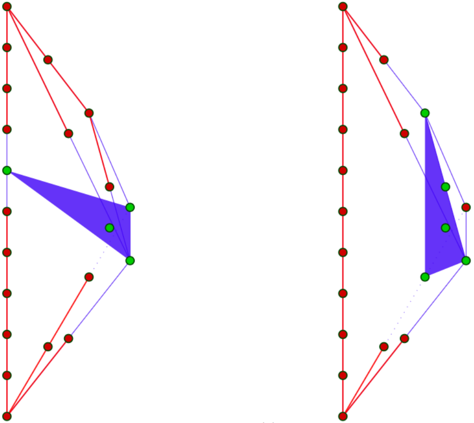
HO: $y^2 = x^3 + v(u^3 + a\,u\,v^2 + b\,v^3)\,x^2 z^2 + v^7(c\,u + d\,v)\,x\,z^4$

singularities: I_2 ($SU(2)$) at $cu + dv = 0$, I_{10}^* ($SO(28)$) at $v = 0$, $\to \mathrm{I}_{12}^*$ for $c = 0$

HE and HO K3's are birationally equivalent

HE/HO T-duality



(a) Reflexive section dividing polytope into E_7- and E_8-top.

(b) Reflexive section dividing polytope into SO(28)- and trivial-top.

From genus 2 fibrations to dual K3 fibrations

Map relating heterotic moduli (τ, ρ, β) to K3 coefficients a, b, c, d

no WL, $c = 0$, thru $SL(2, \mathbb{Z})$ modular invariant j [Cardoso, Curio, Lüst, Mohaupt]

$$j(\tau)j(\rho) = -1728^2 \frac{a^3}{27d}, \qquad (j(\tau) - 1728)(j(\rho) - 1728) = 1728^2 \frac{b^2}{4d}$$

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one WL, $c \neq 0$, thru $Sp(4, \mathbb{Z})$ Siegel modular forms [Clingher, Doran; Malmendier, Morrison]

$$a = -\tfrac{1}{48}\psi_4(\Omega)\,,\ b = -\tfrac{1}{864}\psi_6(\Omega)\,,\ c = -4\chi_{10}(\Omega)\,,\ d = \chi_{12}(\Omega)\,,\ \Omega = \begin{pmatrix} \tau & \beta \\ \beta & \rho \end{pmatrix}$$

K3 fibrations from genus 2 degenerations

Namikawa-Ueno give genus 2 degenerations as sextics singular at $t = 0$

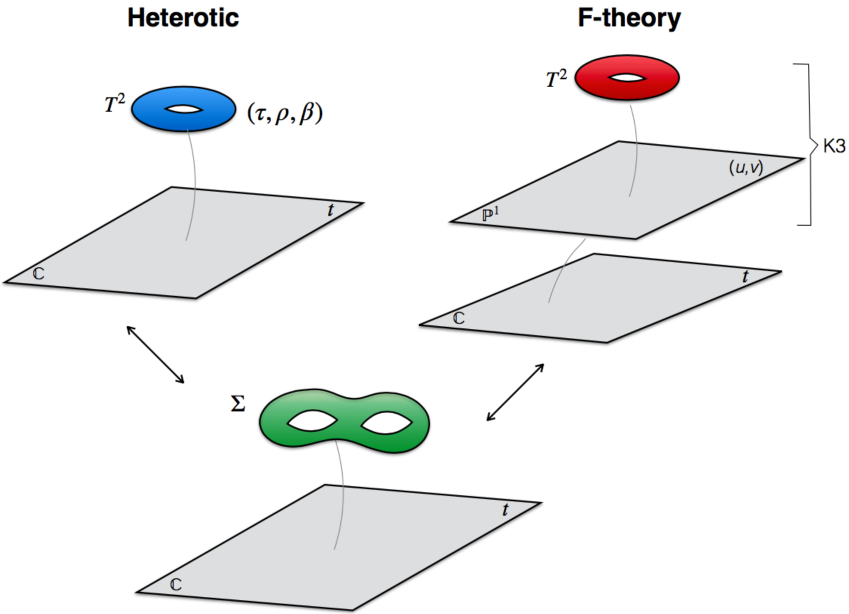
degenerate genus 2 curve $\quad \Sigma(t) : Y^2 = \sum_{i=0}^{6} c_i(t) X^i$

polynomials of $c_i(t) \to$ Igusa-Clebsch invariants $I_{\text{weight}} \to$ modular forms of $\Sigma(t)$

complex structure of K3 written in terms of Igusa-Clebsch invariants

$$a = -3I_4, \quad b = 2(I_2 I_4 - 3I_6), \quad c = -2^3 3^5 I_{10}, \quad d = -2\, 3^5 I_2 I_{10}$$

functions of t, vanishing degree at $t = 0$: $\mu(a),\ \mu(b),\ \mu(c),\ \mu(d)$



Heterotic
F-theory
T^2
(τ, ρ, β)
t
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T^2
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(u,v)
$\mathbb{P}^1$
t
$\mathbb{C}$
Σ
t
$\mathbb{C}$

Resolution of singularities I

F-theory K3 fibration over $\mathbb{C}$ equivalent to elliptic fibration over 2d complex base B, represented by Weierstraß model

$$y^2 = x^3 + f\,x\,z^4 + g\,z^6\,,$$

f, g sections of some line bundles over B
Calabi-Yau condition: f, g are K_B^{-4}, K_B^{-6}
K_B: canonical bundle of B

to begin f, g polynomials of $(u, v, t) \in \mathbb{P}^1 \times \mathbb{C}$

e.g. in HE: $f = a(t)u^4v^4 + c(t)u^3v^7, \quad g = b(t)u^6v^6 + d(t)u^5v^7 + u^7v^5$

Elliptic fiber becomes singular when discriminant $\Delta = 4f^3 + 27g^2 = 0$

Blow-up base if singularity is non-minimal, i.e. order$(f) \geq 4$ and order$(g) \geq 6$

Resolution of singularities II

In HE and HO $\nexists$ non-minimal points at $v = 0$, work at patch (u, t) to begin

HE: non-minimal point at $u = t = 0$ $\rightarrow$ introduce blow-ups

HO: singularity at $t = 0$ of type I_{2k}, $k = \mu(c)$, supports algebra $\mathfrak{sp}(k)$
non-minimal point at $u = t = 0$ (or $u = u_0, t = 0$) $\rightarrow$ introduce blow-ups

Resolution can be accomplished, i.e. finite number n_T of blow-ups, iff

$\mu(a) < 4$ or $\mu(b) < 6$ or $\mu(c) < 10$ or $\mu(d) < 12$

Resolution example: NU degeneration $[\mathrm{I}_9 - \mathrm{I}_6^* - 0] \equiv [\mathrm{I}_9 - \mathrm{I}_6^*]$

$Y^2 = \left((X-1)^2 + t^9\right)(X^2 + t^8)(X^2 + t) \ \rightarrow \ a(t),\ b(t), c(t), d(t)$ of dual K3

$\beta \rightarrow -\beta\,, \rho \rightarrow \rho + 9\,, \tau \rightarrow \tau + 6\,, \qquad M_\tau = \begin{pmatrix} -1 & -6 \\ 0 & -1 \end{pmatrix}$ D_{10} singularity

From monodromy and BI $dH \sim (\mathrm{tr} R^2 - \mathrm{tr}\, F^2)$, expect resolution to give theory of 21 small instantons on D_{10} singularity [Aspinwall, Morrison; Blum, Intriligator]

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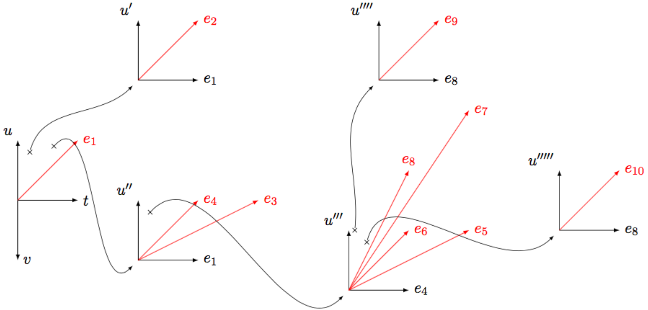
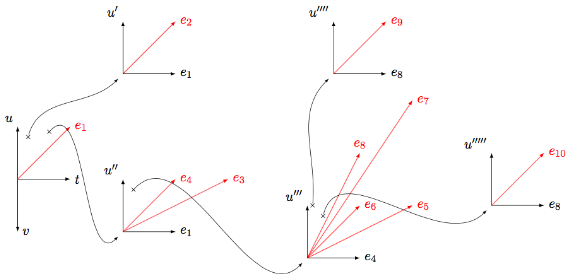
Schematic resolution in HO



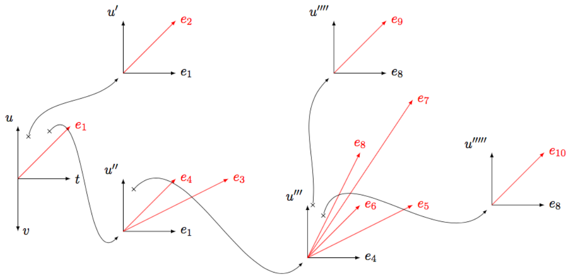
ten blow-ups, number of tensor multiplets $n_T = 10$



Resolution procedure allows to obtain self-intersection numbers of blow-up divisors, and read off algebras plus matter content.

	e_2						e_9	
	$\mathfrak{sp}(13)$						$\mathfrak{sp}(1)$	
	1						1	
	|						|	
$\mathfrak{sp}(21)$	$\mathfrak{so}(68)$	$\mathfrak{sp}(26)$	$\mathfrak{so}(52)$	$\mathfrak{sp}(18)$	$\mathfrak{so}(36)$	$\mathfrak{sp}(10)$	$\mathfrak{so}(20)$	$\mathfrak{sp}(1)$
1*	4	1	4	1	4	1	4	1
$t = 0$	e_1	e_3	e_4	e_5	e_6	e_7	e_8	e_{10}

Whenever a resolution is attained the end result is a represented by a tree-like diagram.



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	$\mathfrak{sp}(13)$ 1						$\mathfrak{sp}(1)$ 1	
	|						|	
$\mathfrak{sp}(21)$ 1*	$\mathfrak{so}(68)$ 4	$\mathfrak{sp}(26)$ 1	$\mathfrak{so}(52)$ 4	$\mathfrak{sp}(18)$ 1	$\mathfrak{so}(36)$ 4	$\mathfrak{sp}(10)$ 1	$\mathfrak{so}(20)$ 4	$\mathfrak{sp}(1)$ 1

Notation: each blow-up divisor is identified by algebra it supports, written above integer equal to minus self-intersection number. 1* means $t = 0$ isn't blow-up. Diagram reflects pattern of intersections. Hypers $\frac{1}{2}$(**fund**, **fund**) for adjacent $\mathfrak{sp}$-$\mathfrak{so}$.

Results

Geometric models

Moduli monodromy and Bianchi identity indicate genus 2 degenerations expected to describe small instantons on ADE singularities:

sing.	NU type	local model	$\mu(a)$	$\mu(b)$	$\mu(c)$	$\mu(d)$
A_{p-1}	$[\mathrm{I}_{n-p-0}]$	$Y^2=\left(t^n+X^2\right)\left(t^p+(X-\alpha)^2\right)(X-1)$	0	0	$p+n$	$p+n$
D_{p+4}	$[\mathrm{I}_n-\mathrm{I}_p^*]$	$Y^2=\left(t^n+(X-1)^2\right)\left(t^{p+2}+X^2\right)(X+t)$	2	3	$6+p+n$	$6+p+n$
E_6	$[\mathrm{IV}^*-\mathrm{I}_n]$	$Y^2=\left(t^4+X^3\right)\left(t^n+(X-1)^2\right)$	$4+n$	4	$8+n$	$8+n$
E_7	$[\mathrm{III}^*-\mathrm{I}_n]$	$Y^2=X\left(t^3+X^2\right)\left(t^n+(X-1)^2\right)$	3	$6+n$	$9+n$	$9+n$
E_8	$[\mathrm{II}^*-\mathrm{I}_n]$	$Y^2=\left(t^5+X^3\right)\left(t^n+(X-1)^2\right)$	$5+n$	5	$10+n$	$10+n$

↑ # of instantons

In all cases resolution agrees with known results. [Aspinwall, Morrison; Blum, Intriligator]

Ex. NU degeneration $[\mathrm{IV}^* - \mathrm{I}_n]$: $k = (8+n)$ instantons on E_6 singularity

$$\beta \to \frac{\beta}{\tau}\,, \quad \rho \to \rho + n - \frac{\beta^2}{\tau}\,, \quad \tau \to -\frac{1+\tau}{\tau}$$

Resolution in HO

$\mathfrak{sp}(k)$	$\mathfrak{so}(4k\text{-}16)$	$\mathfrak{sp}(3k\text{-}24)$	$\mathfrak{su}(4k\text{-}32)$	$\mathfrak{su}(2k\text{-}16)$
$1*$	4	1	2	2

Resolution in HE

		$\mathfrak{sp}(1)$	$\mathfrak{g}_2$		$\mathfrak{f}_4$		$\mathfrak{su}(3)$	
1	2	2	3	1	5	1	3	1

$\mathfrak{e}_6$		$\mathfrak{su}(3)$	
6	1	3	1

$\otimes(n-1)$

$\times$

$\mathfrak{f}_4$		$\mathfrak{g}_2$	$\mathfrak{sp}(1)$		
5	1	3	2	2	1^*

$n \geq 1$

Non-geometric models

Degenerations with non-geometric monodromies in all T-duality frames.
In several cases dual F-theory CY admits a resolution.
In many, emerging theory equal to small instantons on ADE singularities.

Ex. NU $[\mathrm{III}-\mathrm{III}]$ $\quad Y^2 = X(X-1)(X^2+t)\left((X-1)^2+t\right)$

$$\tau \to \frac{\rho}{\beta^2-\tau\rho}, \quad \rho \to \frac{\tau}{\beta^2-\tau\rho}, \quad \beta \to \frac{-\beta}{\beta^2-\tau\rho} \qquad \left(\tau \to -\frac{1}{\tau}, \ \rho \to -\frac{1}{\rho}, \ \text{when } \beta = 0\right)$$

Resolution gives same theory obtained in $[\mathrm{I}_0 - \mathrm{I}_0^*]$, i.e. theory of 6 small instantons on D_4 singularity.

In HO

$\mathfrak{sp}(6)$	$\mathfrak{so}(7)$
1*	1

In HE

		$\mathfrak{sp}(1)$	$\mathfrak{g}_2$	$\mathfrak{sp}(1)$		
1	2	2	2	2	2	1^*

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Duality between $[\mathrm{III}-\mathrm{III}]$ and $[\mathrm{I}_0 - \mathrm{I}_0^*]$ explained relating their monodromies expressed in terms of Dehn twists. AF, García-Extebarria, Lüst, Massai, Mayrhofer

General properties I

In resolvable model, 6d $\mathcal{N}=(1,0)$ theory in IR, valid on tensor branch, captured by a diagram with $n_T{+}1$ nodes, encoding full gauge algebra $\mathcal{G}$ and matter content.

Numbers of hyper and vector multiplets, n_H, n_V, read off from diagram

Each theory characterized by two intrinsic quantities equal in HE and HO:

$h_{\rm R} = \operatorname{rank}\mathcal{G} + n_T$ $\qquad$ $\#$ vectors in 5d

$r_{\rm R} = n_H - n_V + 29n_T - 30k\,, \quad k=\mu(c)$ $\qquad$ gravitational anomaly

in geometric models $r_{\rm R} = \operatorname{rank} G_{\rm ADE}$ [Intriligator]

Ex. NU $[\mathrm{IX}-1]$ $Y^2 = X^5 + t^2$, dual K3 with $a = b = d = 0$, $\mu(c) = 8$

$$\tau \to 1 + \rho - \frac{(1+\beta)^2}{\tau}, \quad \rho \to -\frac{1}{\tau}, \quad \beta \to -\frac{\beta+1}{\tau}$$ order 5

In HE, $n_T = 16$, rank $\mathcal{G} = 22$, $h_\mathrm{R} = 38$, $r_\mathrm{R} = 10$

	$\mathfrak{su}(2)$	$\mathfrak{so}(7)$	$\mathfrak{su}(2)$		$\mathfrak{e}_7$			$\mathfrak{sp}(1)$	$\mathfrak{g}_2$		$\mathfrak{f}_4$		$\mathfrak{g}_2$	$\mathfrak{sp}(1)$		
1	2	3	2	1	8	1	2	2	3	1	5	1	3	2	2	1^*

In HO, $n_T = 6$, rank $\mathcal{G} = 32$, $h_\mathrm{R} = 38$, $r_\mathrm{R} = 10$

$\mathfrak{sp}(8)$	$\mathfrak{so}(20)$	$\mathfrak{sp}(4)$	$\mathfrak{so}(12)$		$\mathfrak{su}(2)$	$\mathfrak{so}(7)$
1^*	4	1	4	1	2	3

$h_\mathrm{R} = \mathrm{rank}\,\mathcal{G} + n_T$, $r_\mathrm{R} = n_H - n_V + 29 n_T - 30k$, $k = \mu(c)$

General properties II

Anomaly cancellation gives significant info on resulting 6d $\mathcal{N} = (1, 0)$ theories.

In all models, matter content is such that irreducible tr F^4 terms cancel.

Pure gauge contribution to anomaly polynomial:

$I_{\text{gauge}} = -\frac{1}{8}\eta^{\alpha\beta}\,\text{tr}F_\alpha^2\,\text{tr}F_\beta^2$

F_α: field strength of gauge factor at α node, $\alpha = 0, 1, \ldots, n_T$

$\alpha = 0$ corresponds to $t = 0$ divisor

$\eta^{\alpha\beta}$: intersection matrix, read off from diagram.

Diagonal elements equal to minus self-intersection number.

Non-diagonal elements equal to -1 if nodes linked, to 0 if not.

General properties III

$I_{\text{gauge}} = -\frac{1}{8}\eta^{\alpha\beta}\,\text{tr}F_\alpha^2\,\text{tr}F_\beta^2 \quad , \qquad \alpha = 0, 1, \ldots, n_T$

In all models, $\eta^{\alpha\beta}$ positive semi-definite, with only one zero eigenvalue.

I_{gauge} cancelled by Green-Schwarz-Sagnotti mechanism with n_T tensor multiplets.

Null eigenvalue $\Rightarrow$ linear combination of gauge couplings independent of scalars in tensor multiplets so it defines a mass parameter.

Theories have T-duality.

Mass scale and T-duality suggest that UV completions are LSTs.

Our theories fall into recent classification of LSTs.

[Bhardwaj; Bhardwaj, Del Zotto, Heckman, Morrison, Rudelius, Vafa]

Dropping node corresponding to $t = 0$ gives tensor branch of 6d SCFTs embedded in LSTs. In HO $\mathfrak{sp}(k)$ remains as flavor symmetry.

Final comments

- studied 6d $\mathcal{N} = (1, 0)$ non-geometric heterotic vacua described locally as genus 2 fibrations over $\mathbb{C}$.
- heterotic moduli transform under T-duality around points in the base where fiber degenerates, signaling *T-fects*.
- analyzed T-fects using heterotic/F-theory duality. genus 2 degeneration in Namikawa-Ueno list $\rightarrow$ K3 fibration degeneration.
- applied a toric procedure to resolve singularities of F-theory 3-fold. only 49 out of 120 NU types lead to F-theory duals admitting a resolution by a finite number of base blow-ups.
- observed a kind of duality in which theories living on distinct defects are equal.
- emerging theories living on defects turn out to be little string theories at a generic point on tensor branch.
- open problems: understand nature of degenerations without resolution, extend to 4d ...

▷ spin-off: more on F-theory and heterotic in 8d in progress with C. Mayrhofer, H. Parra

study K3's with 2,3 moduli (Picard number 18, 17)

map K3 moduli to heterotic moduli

das war's