

AdS₂ holography

Mind the Gap!

Iosif Bena

IPhT, CEA Saclay

with

Nick Warner, Emil Martinec, Jan deBoer, Micha Berkooz, Simon Ross, Gianguido Dall'Agata, Stefano Giusto, Rodolfo Russo, Guillaume Bossard, Masaki Shigemori, Monica Guică, Nikolay Bobev, Bert Vercoe, Andrea Puhm, David Turton, **Stefanos Katmadas**, **Johan Blåbäck**, **Pierre Heidmann**



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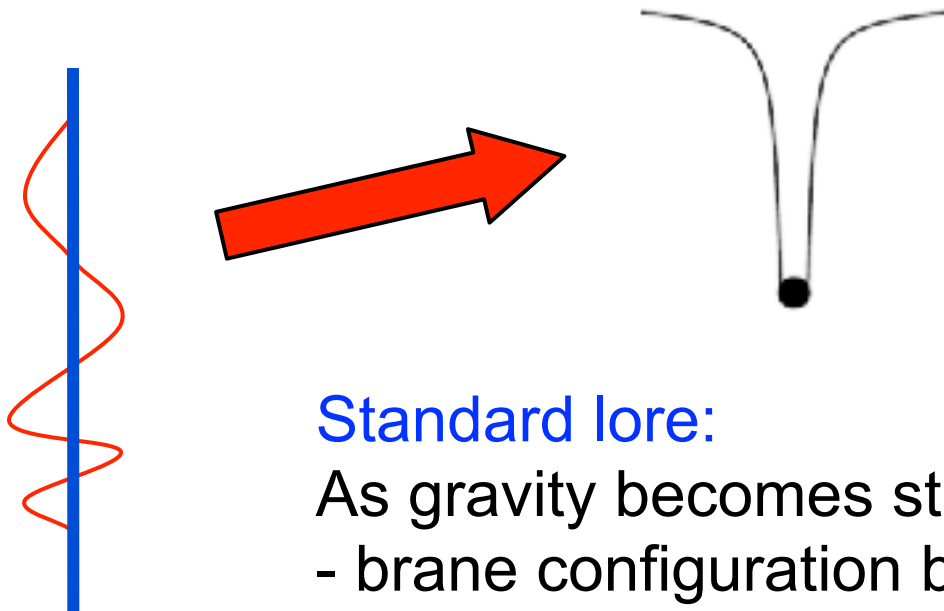
cea
SACLAY

Strominger and Vafa (1996):

*Black Hole Microstates at **Zero Gravity*** (branes + strings)

Correctly match B.H. entropy !!!

One Particular Microstate at **Finite Gravity**:



Standard lore:

As gravity becomes stronger,

- brane configuration becomes smaller
- horizon develops and engulfs it
- recover standard black hole

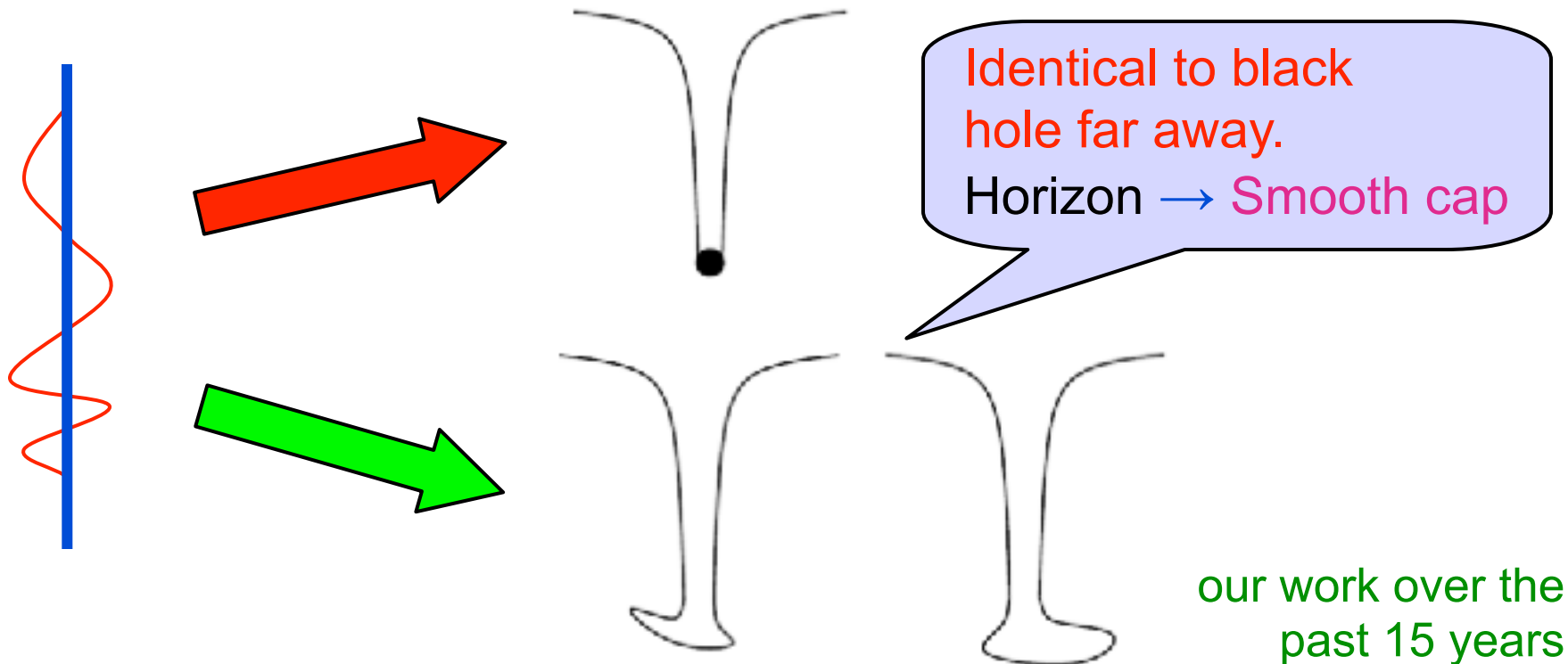
Susskind
Horowitz, Polchinski
Damour, Veneziano

Strominger and Vafa (1996):

*Black Hole Microstates at **Zero Gravity*** (branes + strings)

Correctly match B.H. entropy !!!

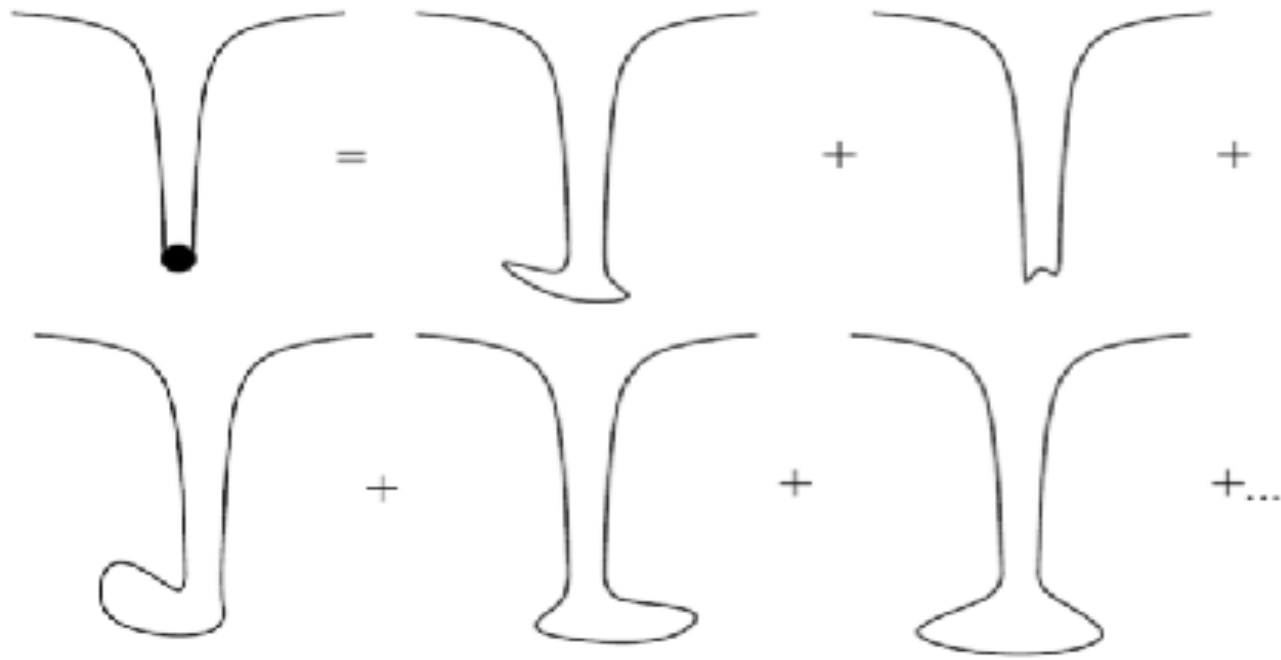
One Particular Microstate at **Finite Gravity**:



BIG QUESTION: Are *all* black hole microstates becoming geometries with no horizon ?

Black hole $\stackrel{?}{=}$ ensemble of horizonless microstate configurations

Mathur 2003



Only way to solve QM-GR conflict

Mathur 2009, Almheiri, Marolf, Polchinski, Sully 2012

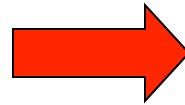
Analogy with ideal gas

Thermodynamics

(Air = ideal fluid)

$$P V = n R T$$

$$dE = T dS + P dV$$



Statistical Physics

(Air -- molecules)

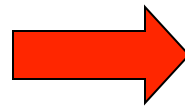
e^S microstates

typical

atypical

Thermodynamics

Black Hole Solution



Statistical Physics

Microstate geometries

Long distance physics

Gravitational lensing

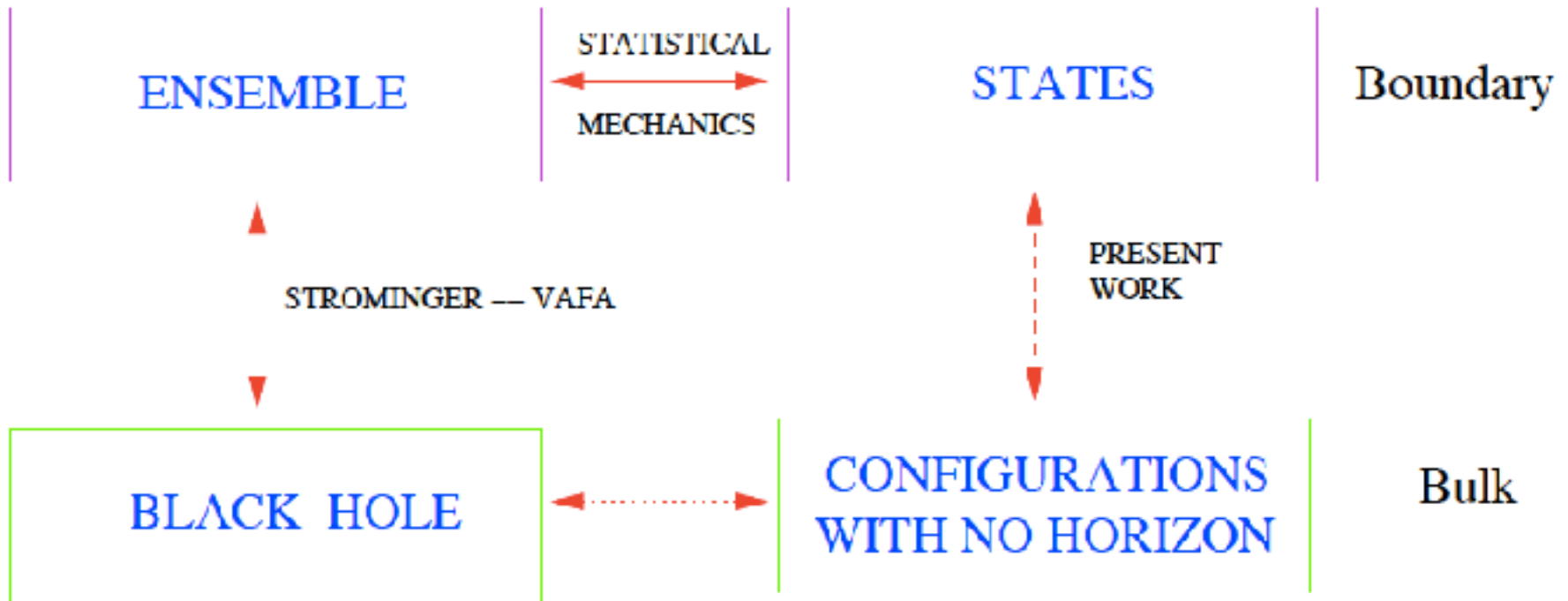
Physics at horizon

Information loss

Gravity waves ?

AdS-CFT formulation:

e.g. Bena, Warner, 2007



Not some **hand-waving** idea - **provable** by rigorous calculations in String Theory

Word of caution

- To replace classical BH by BH-sized object
 - Gravastar, quark-star, boson-star
 - Infinite density firewall hovering just above horizon
 - Gas of wormholes
 - Bose-Einstein condensate of gravitons
 - LQG configuration...

3 very stringent tests:

1. Same growth with G_N !!!

Horowitz

BH size **grows** with G_N ; “normal objects” **shrink**

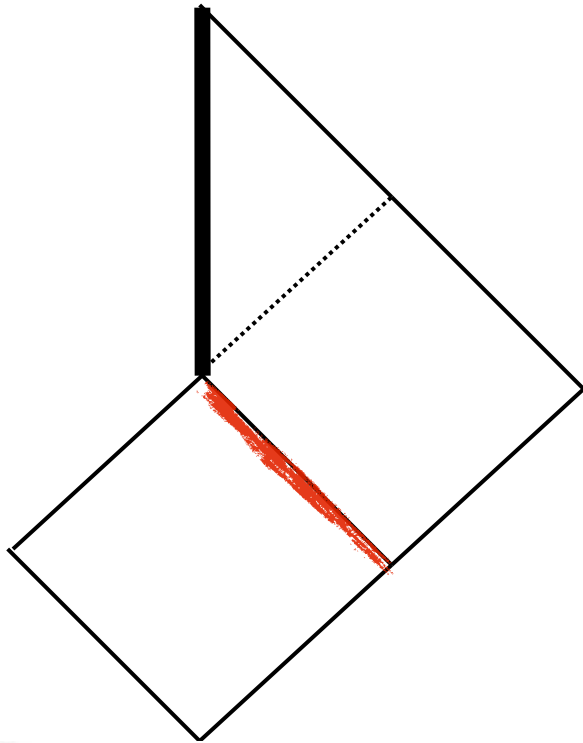
- BH **microstate** geometries **pass** this test
- **Highly nontrivial** mechanism: $G_N = g_s^2$
- D-branes = solitons, **tension** $\sim 1/g_s \rightarrow$ lighter as G_N increases



To build structure@horizon, non-perturbative degrees of freedom you must use !

2. Mechanism not to fall into BH

Very difficult !!!



GR Dogma:

**Thou shalt not put anything
at the horizon !!!**

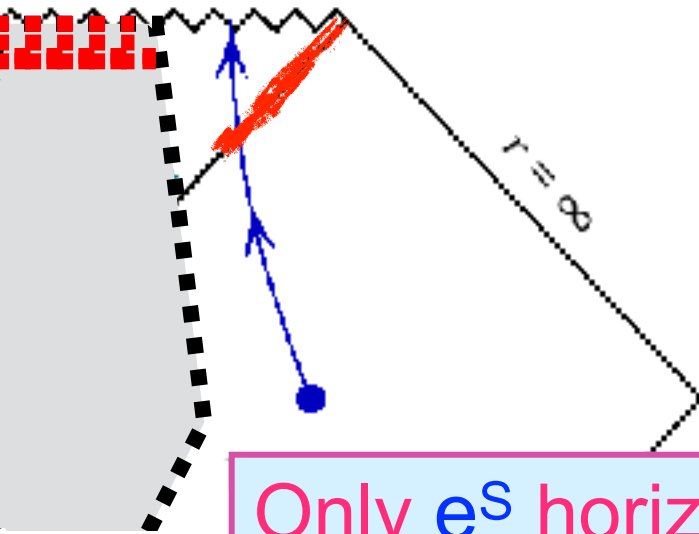
- Null \rightarrow speed of light.
- If massive: ∞ boost \rightarrow ∞ energy
- If massless: dilutes with time
- Nothing can live there !
(or carry degrees of freedom)
- No membrane, no spins, no “quantum stuff”
- No (fire)wall

*If support mechanism have you not,
go home and find one*



3. Avoid forming a horizon

- Collapsing shell forms horizon Oppenheimer and Snyder (1939)
- If curvature is low, no reason not to trust classical GR
- By the time shell becomes **curved-enough for quantum effects to become important**, horizon in causal past



Backwards in time - **illegal** !

BH has e^S microstates with no horizon

Small tunneling probability = e^{-S}

Will tunnel with probability **ONE** !!!

Kraus, Mathur; Bena, Mayerson, Puhm, Vercoocke

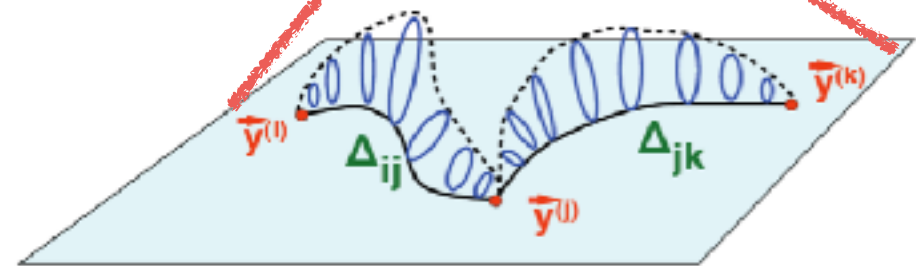
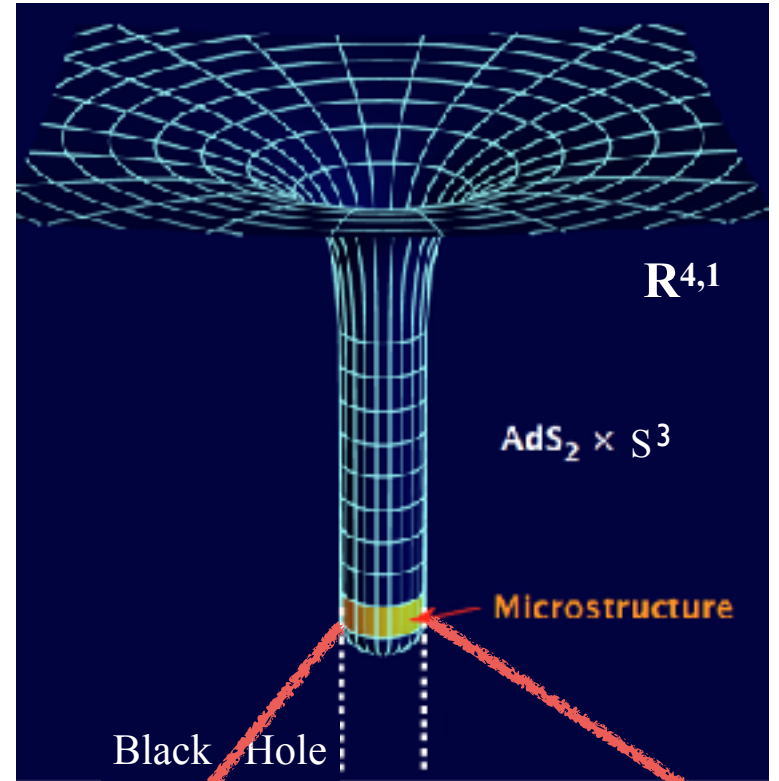
Only e^S horizon-sized microstates can do it !

*If quantum tunneling you are brushing aside,
incorrect physics you are doing*



Microstates geometries: M2-M2-M2 frame

11d/CY - black hole in 5d



2-cycles + magnetic flux

- Where is the BH charge ?

$$L = q A_0$$

magnetic

$$L = \dots + A_0 F_{12} F_{34} + \dots$$

- Where is the BH mass ?

$$E = \dots + F_{12} F^{12} + \dots$$

- BH angular momentum

$$J = E \times B = \dots + F_{01} F_{12} + \dots$$

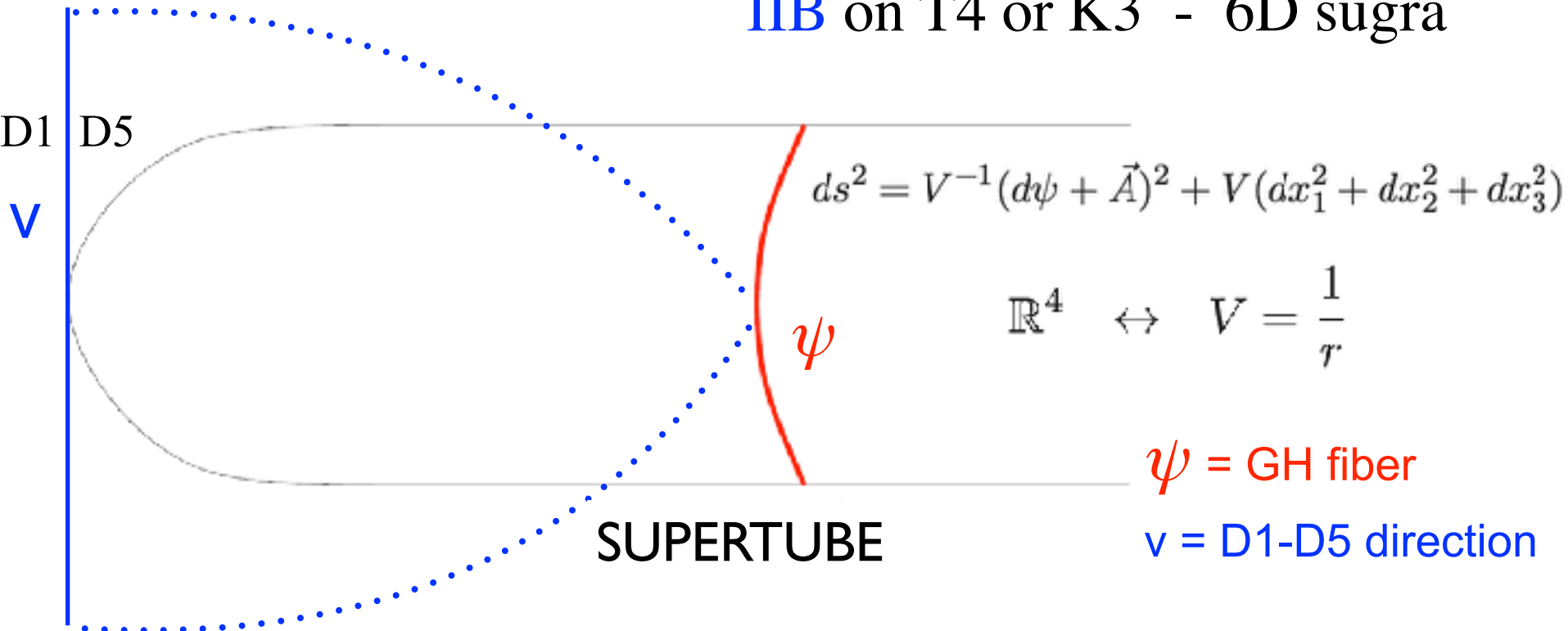
Charge dissolved in fluxes.

No singular sources.

Klebanov-Strassler

Microstates geometries: D1-D5-P frame

IIB on T4 or K3 - 6D sugra



- Starting solution: $AdS_3 \times S^3$ **Add wiggles**
- Arbitrary $F(\psi)$ - 8 supercharges - **supertube**
 Lunin, Mathur; Lunin, Maldacena, Maoz; Taylor, Skenderis
- Arbitrary $F(\psi, v)$ - 4 supercharges - **superstratum**
 Bena, Giusto, Russo, Shigemori, Warner

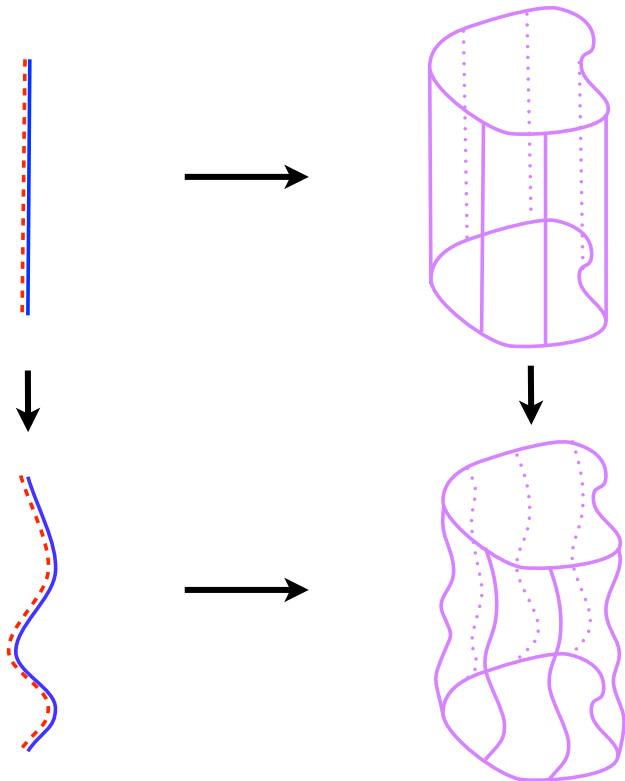
Entropy of wiggles

Bena, Shigemori, Warner

Supertubes - eight $F(\psi)$, ($c = 8$) $S \sim (c L_0)^{1/2} \sim (Q_1 Q_5)^{1/2}$
(with massaging $S \sim Q^{5/4}$)

Superstrata - four $F(\psi, \mathbf{v})$, ($c = \infty$), quantize: $S = 2\pi(Q_1 Q_5 Q_p)^{1/2}$

Double supertube transition. In general non-geometric.



Largest family of solutions known to mankind

Arbitrary functions of **two** variables: $\infty \times \infty$ parameters
 Bena, Giusto, Russo, Shigemori, Warner

$$\begin{aligned}
 ds_{10}^2 &= \frac{1}{\sqrt{\alpha}} ds_6^2 + \sqrt{\frac{Z_1}{Z_2}} ds_4^2, \\
 ds_6^2 &= -\frac{2}{\sqrt{\mathcal{P}}} (dv + \beta) \left[du + \omega + \frac{\mathcal{F}}{2}(dv + \beta) \right] + \sqrt{\mathcal{P}} ds_4^2, \\
 e^{2\sigma} &= \frac{Z_1^2}{\mathcal{P}}, \\
 B &= \frac{Z_4}{\mathcal{P}} (du + \omega) \wedge (dv + \beta) + a_4 \wedge (dv + \beta) + \delta_2, \\
 C_0 &= \frac{Z_4}{Z_1}, \\
 C_2 &= \frac{Z_2}{\mathcal{P}} (du + \omega) \wedge (dv + \beta) + a_1 \wedge (dv + \beta) + \gamma_2, \\
 C_4 &= \frac{Z_4}{Z_2} \widehat{\text{vol}}_4 + \frac{Z_4}{\mathcal{P}} \gamma_2 \wedge (du + \omega) \wedge (dv + \beta) + x_3 \wedge (dv + \beta) + \mathcal{C}, \\
 C_6 &= \widehat{\text{vol}}_4 \wedge \left[-\frac{Z_1}{\mathcal{P}} (du + \omega) \wedge (dv + \beta) + a_2 \wedge (dv + \beta) + \gamma_1 \right] \\
 &\quad + \frac{Z_4}{\mathcal{P}} \mathcal{C} \wedge (du + \omega) \wedge (dv + \beta), \\
 \alpha &\equiv \frac{Z_1 Z_2}{Z_1 Z_2 - Z_4^2}, \quad \mathcal{P} \equiv Z_1 Z_2 - Z_4^2.
 \end{aligned}$$

$$\begin{aligned}
 & - \frac{Rr}{\sqrt{2} k_2 (m_1^2 - 1)} \frac{m_1 (k_2 + m_1 + 1) \Delta_{k_2 + m_1 - 1, m_1 - 1} + (k_2 + m_1 - 1) \Delta_{k_2 + m_1}}{(r^2 + a^2)^2} \\
 & - \frac{R}{\sqrt{2} k_2 (m_1^2 - 1) a^2 \sin \theta \cos \theta} \left[2(m_1 - 1) \Delta_{k_2 + m_1 - 3, m_1 - 1} \right. \\
 & \quad \left. + (m_1 - 1)(m_1 - 2) \Delta_{k_2 + m_1 - 1, m_1 - 1} + m_1 (k_2 - 2) \Delta_{k_2 + m_1 - 1, m_1 + 1} \right. \\
 & \quad \left. - m_1 (m_1 - 1) \Delta_{k_2 + m_1 + 1, m_1 - 1} + (m_1^2 (k_2 - 1) + 1) \Delta_{k_2 + m_1 + 1, m_1 + 1} \right], \\
 & - \frac{R}{\sqrt{2}} \frac{\Delta_{k_2 + m_1 + 1, m_1 + 1}}{\Sigma} \sin^2 \theta - \frac{R}{\sqrt{2} k_2 (m_1^2 - 1) a^2} \left[2(m_1 - 1) \Delta_{k_2 + m_1 - 3, m_1 - 1} \right. \\
 & \quad \left. + (m_1^2 - 2m_1 + k_2 - 1) \Delta_{k_2 + m_1 - 1, m_1 - 1} + m_1 (k_2 - 2) \Delta_{k_2 + m_1 - 1, m_1 + 1} \right. \\
 & \quad \left. + m_1 (k_2 - m_1 - 1) \Delta_{k_2 + m_1 + 1, m_1 - 1} + (k_2 (m_1^2 + m_1 - 1) - m_1 (m_1 + 1)) \Delta_{k_2 + m_1 + 1, m_1 + 1} \right] \\
 & - \frac{R}{\sqrt{2}} \frac{\Delta_{k_2 + m_1 + 1, m_1 + 1}}{\Sigma} \cos^2 \theta - \frac{R}{\sqrt{2} k_2 (m_1^2 - 1) a^2} \left[(k_2 - 1)(m_1 - 1) \Delta_{k_2 + m_1 - 1, m_1 - 1} \right. \\
 & \quad \left. - 2(m_1 - 1) \Delta_{k_2 + m_1 - 3, m_1 - 1} - (m_1 - 1)(m_1 - 2) \Delta_{k_2 + m_1 - 1, m_1 - 1} \right. \\
 & \quad \left. + (m_1 - 1)(k_2 - 3) \Delta_{k_2 + m_1 - 1, m_1 + 1} + m_1 (m_1 - 1) \Delta_{k_2 + m_1 + 1, m_1 - 1} \right. \\
 & \quad \left. + (m_1 - 1)(m_1 (k_2 - 1) + 1) \Delta_{k_2 + m_1 + 1, m_1 + 1} \right].
 \end{aligned}$$



String theory
 input crucial
 Giusto, Russo, Turton

Habemus
 Superstratum !!!

Deep superstrata

DI-D5-P black string in 6D

- J can be **arbitrarily small**
Bena, Giusto, Martinec Russo, Shigemori,
Turton, Warner '16 (PRL editor's selection)

- First BTZ microstates

- **CFT dual state known**

- Certain superstrata (1,0,n)
Wave equation separable !

Bena, Turton, Walker, Warner

- Can compute many things:

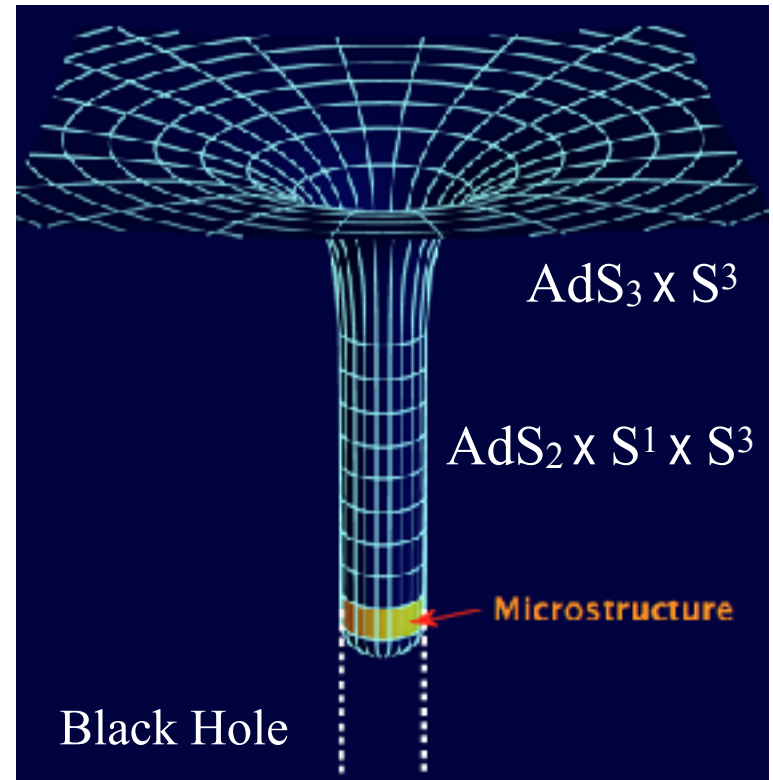
Geodesics Tyukov, Walker, Warner

Mass gaps Bena, Heidmann, Turton

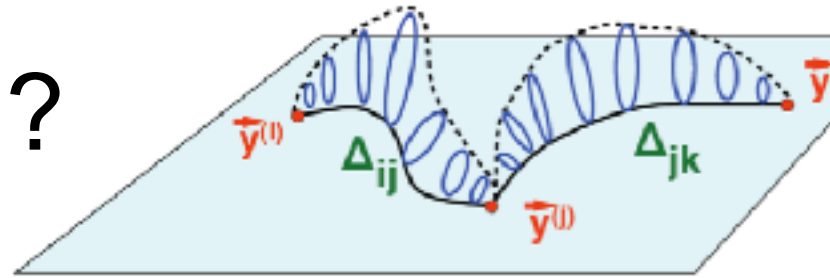
Wightman functions Raju, Shrivastava

Green fns, Thermalization, Chaos, dip-ramp-plateau

Bena, Guica, Heidmann, Monten, Warner (to appear)



Why not collapsing ?



- 5(+6)d : smooth solutions + **quantized** magnetic flux on topologically-nontrivial **2-cycles**
 - cycles smaller \rightarrow increases energy
 - bubbling = **only** mechanism to avoid collapse in semiclassical limit Gibbons, Warner
 - If **any** state in the **e^S -dimensional** BH Hilbert space has a semiclassical limit, it **must** be a microstate geometry !
- 4(+6)d : multicenter solutions Denef
 - smooth GH centers with negative charge \rightarrow centers with **negative D6 charge** and **negative mass**
 - common in String Theory (e.g. orientifolds); **nowhere else**
 - **Highly unusual** matter from a 4d perspective
 - Usual matter does not hang around, just falls in BH

Quantum Gravity in AdS₂

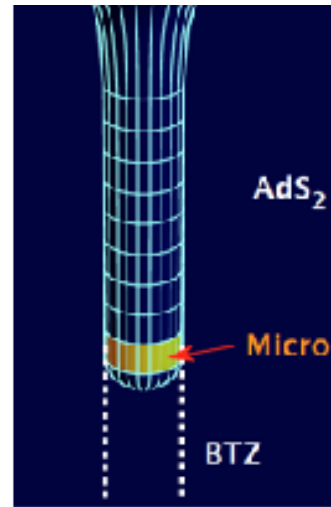
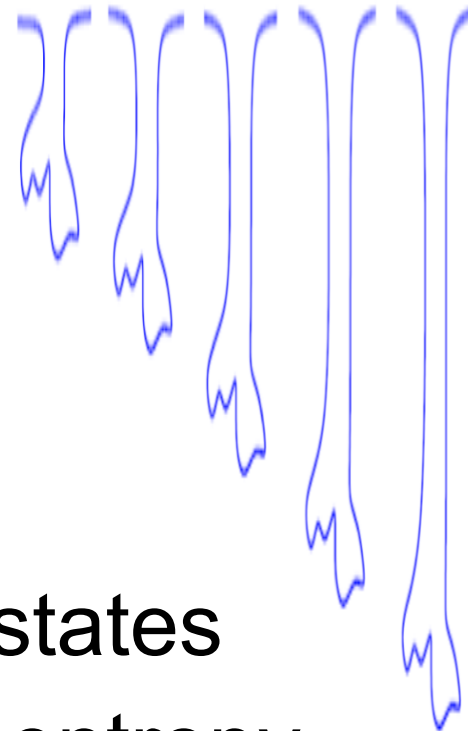
Bena, Heidmann, Turton

- Everybody & their brother & SYK & JT
- **AdS₂** - no finite-energy excitations
Maldacena, Michelson, Strominger
- backreaction of particle in AdS₂ either
 - destroys UV (work instead with *near-AdS₂*)
 - destroys IR → singularity
- Singularities in String Theory and AdS-CFT solved by **string and brane dynamics** involving **extra dimensions** 20 years of examples

Quantum Gravity in AdS_2

Bena, Heidmann, Turton

- Typical microstate geometries have long AdS_2 throat
- Limit when length $\rightarrow \infty$
- Disconnect from AdS_3
- Solutions above \rightarrow asymptotically- AdS_2
Bena, Heidmann, Turton
- Same entropy as microstates
- If superstrata count BH entropy, so do these solutions !



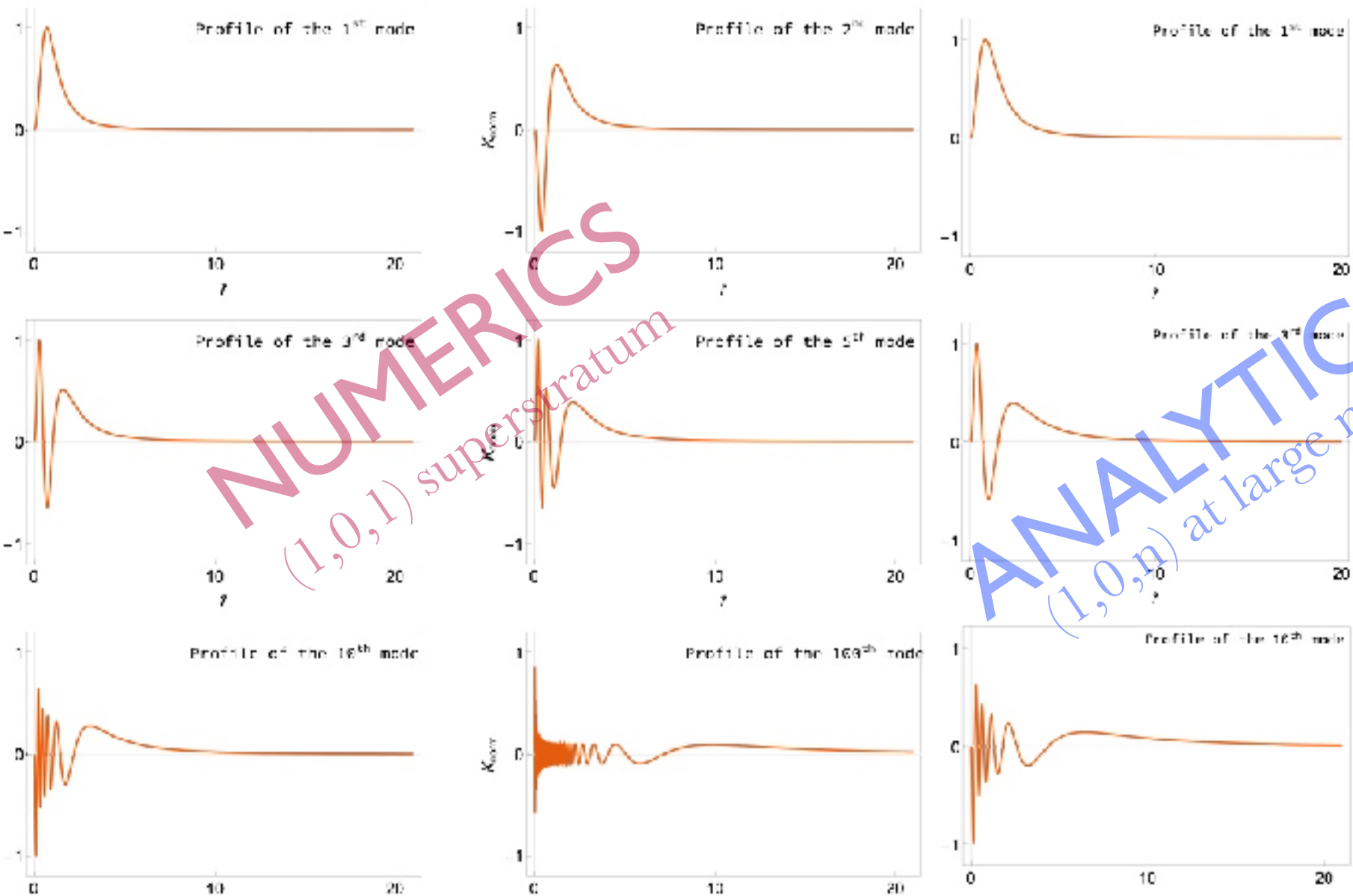
Quantum Gravity in AdS_2

Bena, Heidmann, Turton

- Lots of geometries with AdS_2 UV and IR cap
- BPS ground states of CFT_1 dual to AdS_2
- finite-energy time-dependent excitations \rightarrow
Paulos
- CFT_1 has no conf.-invariant ground state !!!
- Empty Poincaré AdS_2 not dual to any ground state of CFT_1 (similar to Poincaré AdS_3)
- All CFT_1 ground states break conf. symmetry
- Tower of finite-energy excitations above each and every one of them
- Microstates of AdS_2 non-extremal black hole

Castro, Grumiller, Larsen, McNees





- Claims that CFT_1 does not have such excitations - looking at the wrong ground state

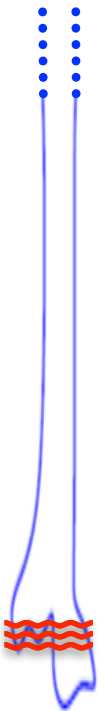


AdS₂/CFT₁

2 options:

Bena, Heidmann, Turton

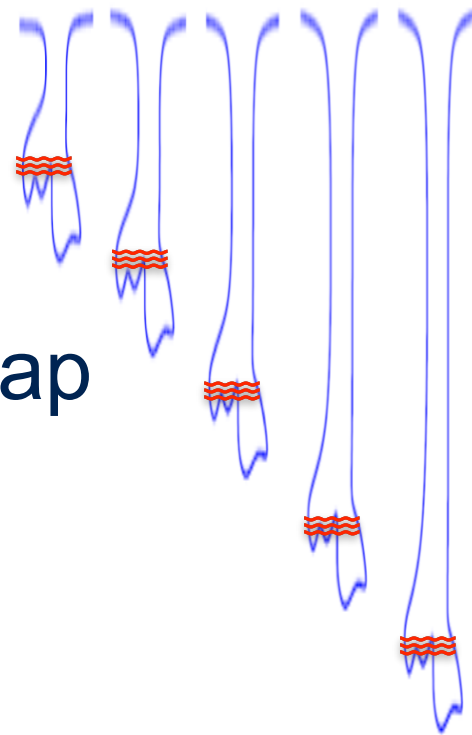
- **Keep AdS₂ UV.** Work in String Theory
 - **Kosher** holography
 - **All CFT₁** ground states break conf. sym. **IR cap**
 - Excitations, non-trivial dynamics, entropy
- **Destroy AdS₂ UV.** Toy models (**SYK, J-T**)
 - **לא כשר**: irrelevant ops, **IR** to **UV** flows
 - No **CFT₁** dynamics. Only nAdS (Singleton-like)
 - Conf. sym. **preserved in IR** →
Nothing to do with **AdS₂/CFT₁** in String Theory
- **AdS₂ holography is NOT subtle !**
 - Crystal clear if done in String Theory and incorrect assumptions discarded - **Mind the Cap !!!**



Gluing back to AdS_3

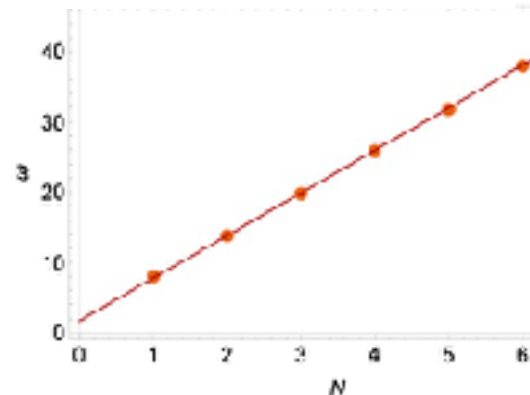
Bena, Heidmann, Turton

- Longer throats with decreasing J
- AdS_3 mass gap depends on length
- smallest gap = $2 J/N_1 N_5$
- precisely matches smallest CFT_2 gap
- CFT_1 finite-energy excitations \rightarrow CFT_2 excitations above gap
- Wightman functions also match



Raju, Shrivastava

$$\lambda_{fuzz} \equiv \lim_{\gamma \rightarrow \infty} \frac{\log |G(\omega, \gamma)|}{\gamma} = \lambda_{BH} + \frac{\pi(3n-1)J}{32 N_1 N_5 n^{3/2}}$$



Connection with T-branes

Bena, Blåbäck, Savelli, Zoccarato

$$F^{(0,2)} = 0,$$

$$\bar{\partial}_{\bar{A}} \Phi = 0,$$

$$\omega \wedge F_2 = [\Phi, \Phi^\dagger]$$

$$A_x = \frac{1}{2}(\Phi_1 + i\Phi_2)$$

$$A_y = \frac{1}{2}(\Phi_3 + i\Phi_4)$$

$$\Phi = \frac{1}{2}(\Phi_5 - i\Phi_6)$$

Constant worldvolume fields T-dualize

$$F^{(0,2)} = -i[A_x, A_y] = 0 \iff \begin{cases} [\Phi_1, \Phi_3] = [\Phi_2, \Phi_4] \\ [\Phi_1, \Phi_4] = [\Phi_3, \Phi_2] \end{cases}$$

$$\bar{\partial}_{\bar{A}_x} \Phi = 0 = -i[A_{\bar{x}}, \Phi] = 0 \iff \begin{cases} [\Phi_1, \Phi_5] = [\Phi_2, \Phi_6] \\ [\Phi_1, \Phi_6] = [\Phi_5, \Phi_2] \end{cases}$$

$$\bar{\partial}_{\bar{A}_y} \Phi = 0 = -i[A_{\bar{y}}, \Phi] = 0 \iff \begin{cases} [\Phi_3, \Phi_5] = [\Phi_4, \Phi_6] \\ [\Phi_3, \Phi_6] = [\Phi_5, \Phi_4] \end{cases}$$

$$\omega \wedge F_2 - [\Phi, \Phi^\dagger] = [A_x, A_{\bar{x}}] + [A_y, A_{\bar{y}}] - [\Phi, \Phi^\dagger] \iff [\Phi_1, \Phi_2] + [\Phi_3, \Phi_4] + [\Phi_5, \Phi_6]$$

Connection with T-branes

Bena, Blåbäck, Savelli, Zoccarato

Solutions with infinite matrices:

$$D_i = \bigotimes_{j=1}^6 ((1 - \delta_{ij})\mathbb{I}_M + \delta_{ij}D) , X_i = \bigotimes_{j=1}^6 ((1 - \delta_{ij})\mathbb{I}_M + \delta_{ij}X) \quad [D, X] = i\mathbb{I}_M$$

$$\begin{aligned} \Phi_1 &= D_1 - \frac{1}{2}\sqrt{\alpha_1}X_1X_3 \\ \Phi_2 &= D_2 - \frac{1}{2}\sqrt{\alpha_1}X_1X_4 - \frac{1}{2}\sqrt{\alpha_2}X_1X_2 \\ \Phi_3 &= D_3 - \frac{1}{2}\sqrt{\alpha_3}X_3X_5 \\ \Phi_4 &= D_4 - \frac{1}{2}\sqrt{\alpha_3}X_3X_6 + \frac{1}{2}\sqrt{\alpha_2}X_3X_1 \\ \Phi_5 &= D_5 + \frac{1}{2}\sqrt{\alpha_2}X_1X_5 \\ \Phi_6 &= D_6 + \frac{1}{2}\sqrt{\alpha_2}X_2X_5 + \frac{1}{2}\sqrt{\alpha_3}X_5X_6 \end{aligned}$$


D0	-	-	-	-	-	-
D4	×	×	×	×	-	-
D4	×	×	-	-	×	×
D4	-	-	×	×	×	×

D0 description of 4D susy BH

Only AdS₂ no AdS₃ !

Could give **first microscopic counting** for such black holes

The take-home story

- Huge number of BH microstate geometries
 - Fns. of 2 variables: $\infty \times \infty$ dim. moduli space
 - Smooth solutions, low curvature, no horizon
 - Topology and fluxes prevent collapse
 - Black hole entropy
 - Mass gaps, Wightman fns. match typical CFT states
- AdS₂ holography is easy in String Theory
 - IR cap \Rightarrow nontrivial dynamics \Rightarrow CFT₁ NOT topological
 - No conformally-invariant ground state !
 - BH with horizon not dual to any pure CFT state
 - Toy models need to be improved - Mind the Cap !
- T-branes :  description BH with only AdS₂ (no AdS₃)
- Extension: extremal non-BPS and non-extremal