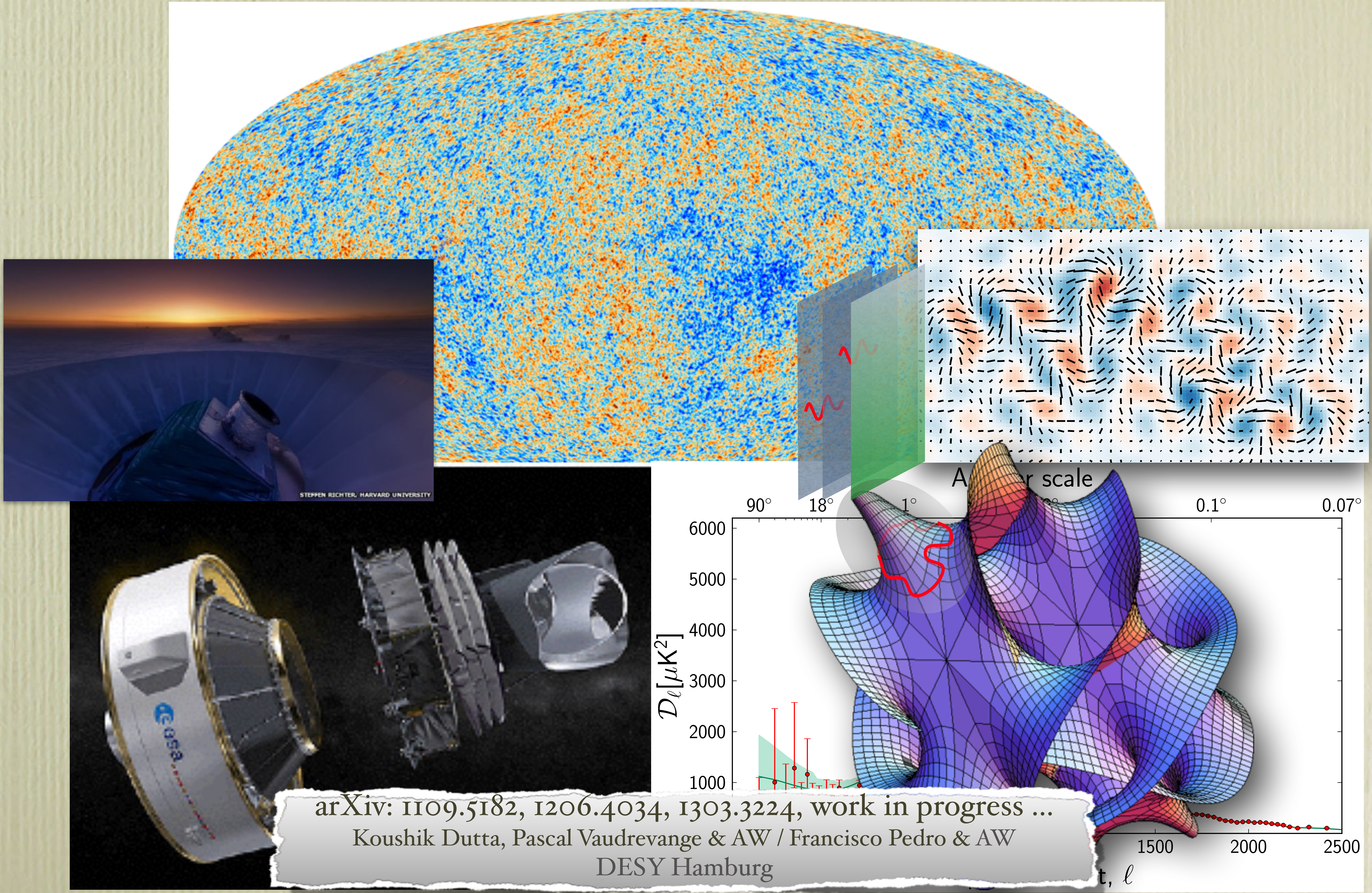
The Scale of Inflation in the Landscape

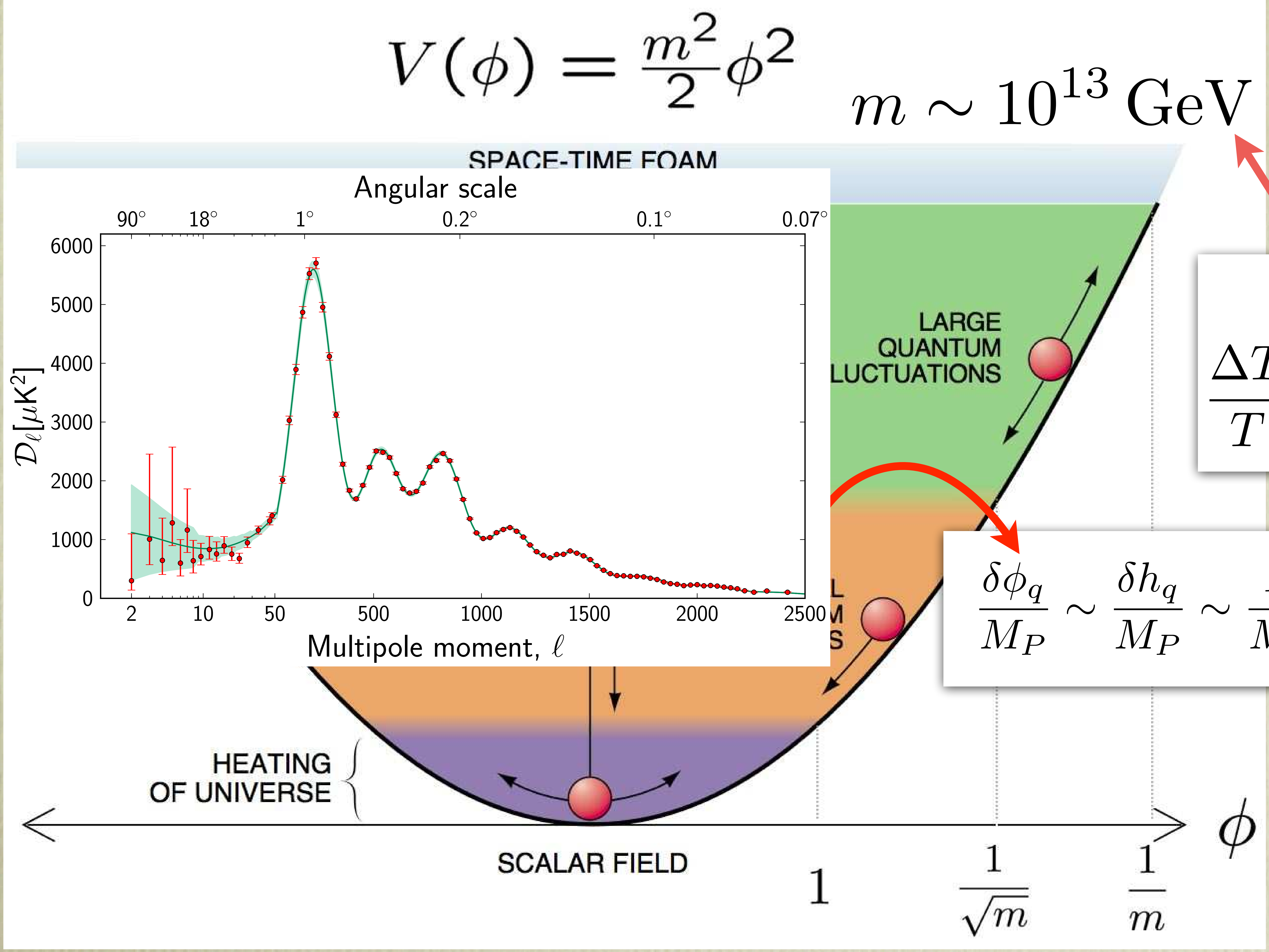


arXiv: 1109.5182, 1206.4034, 1303.3224, work in progress ...

Koushik Dutta, Pascal Vaudrevange & AW / Francisco Pedro & AW
DESY Hamburg

slow-roll inflation ...

[Linde ’82]



$$V(\phi) = \frac{m^2}{2}\phi^2$$

$$m \sim 10^{13}\,\mathrm{GeV}$$

CMB:

$$\frac{\Delta T}{T} \sim 10^{-5}$$

$$\frac{\delta\phi_q}{M_P} \sim \frac{\delta h_q}{M_P} \sim \frac{H}{M_P} \sim 10^{-5}$$

[picture from lecture notes: Linde ’07]

a speculative recipe ...

Let us start by making a set of assumptions about the landscape ...

these assumptions are not taken as proven to be true for the whole of the landscape

yet they have certain support/evidence from some corners of the landscape

given the set of assumptions, we can try to figure out the consequences - valid for a landscape conforming to these assumptions

premises / assumptions ...

- large-field inflation needs shift symmetry to control UV corrections:

$$\mathcal{O}_6 \sim V(\phi)\frac{\phi^2}{M_{\rm P}^2} \quad \Rightarrow \quad m_\phi^2 \sim H^2 \ , \ \eta \sim 1$$

➡(i) shift symmetries only from p-form gauge fields of string theory

- scalar fields with shift symmetry in string compactifications:

➡(ii) axions - field range is limited to < M_P

premises / assumptions ...

- population of the many vacua:
  - ➡(iii) only known mechanism:
    CdL or HM tunneling, combined with eternal inflation
- basic structure of the landscape of vacua
  - ➡(iv-1) exponentially many vacua in high-dimensional moduli space
  - ➡(iv-2) neighbouring vacua typically have large differences in vacuum energy:
    - small-c.c. vacua have neighbours with large c.c.
    - & need population from high-c.c. for anthropics to work
  - ➡(iv-3) eternal inflation

premises / assumptions ...

- eternal inflation

➡there is global-local duality for:

- causal patch measure

[Bousso, Freivogel & Yang ’06]
[Freivogel, Sekino, Susskind & Yeh ’06]

- scale factor time measure

[de Simone, Guth, Linde,
Noorbala, Salem & Vilenkin ’08]

- light-cone time cutoff measure

[Bousso ’09]
[Bousso & Yang ’09]
[Bousso, Freivogel, Leichenauer
& Rosenhaus ’10]

progenitor: *longest-lived* dS vacuum
seeds all other vacua
$V_{inf} \ll V_{progen.} \ll 1$ **, still very high !**

consequences of (i) & (ii) - axion monodromy

- EM Stueckelberg gauge symmetry:

$$S_{EM} = \int d^4x\sqrt{-g}\,\{F_{MN}F^{MN} - \rho^2(A_M + \partial_M C)^2 + \ldots\}$$

$$A_M \;\to\; A + \partial_M \Lambda_0 \qquad \Rightarrow \qquad C \to C - \Lambda_0$$

- string theory contains analogous gauge symmetries for NSNS and RR axions - e.g. IIA:

$$\begin{aligned} H &= dB\,, \\ F_0 &= Q_0\,, \\ \tilde{F}_2 &= dC_1 + F_0 B\,, \\ \tilde{F}_4 &= dC_3 + C_1 \wedge H_3 + \frac{1}{2} F_0 B \wedge B \end{aligned} \qquad \Rightarrow \qquad \begin{aligned} \delta B &= d\Lambda_1\,, \\ \delta C_1 &= -F_0 \Lambda_1\,, \\ \delta C_3 &= -F_0 \Lambda_1 \wedge B \end{aligned}$$

- type IIB similar

axion monodromy

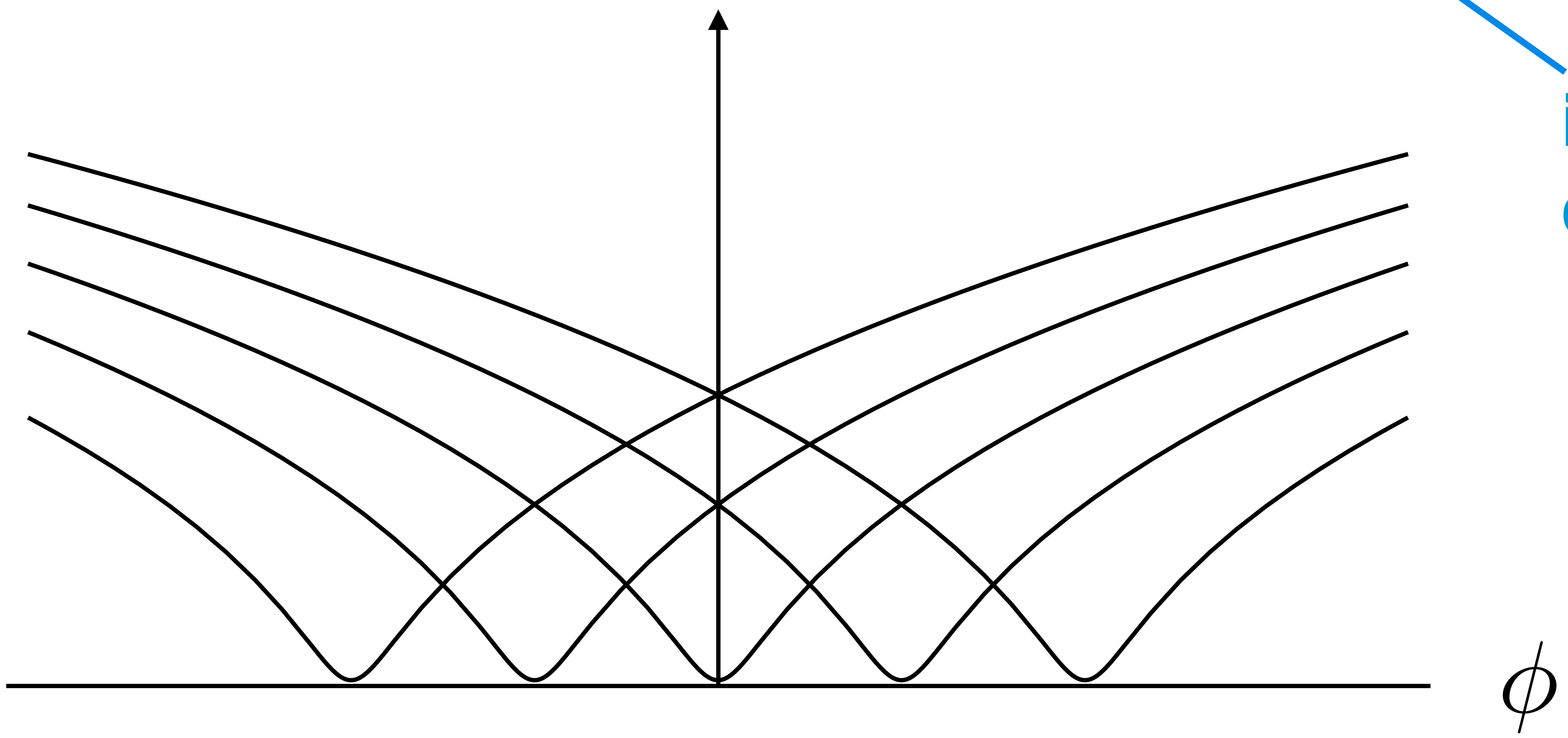
- p-form axions get **non-periodic** potentials from coupling to **branes** or **fluxes/field-strengths**

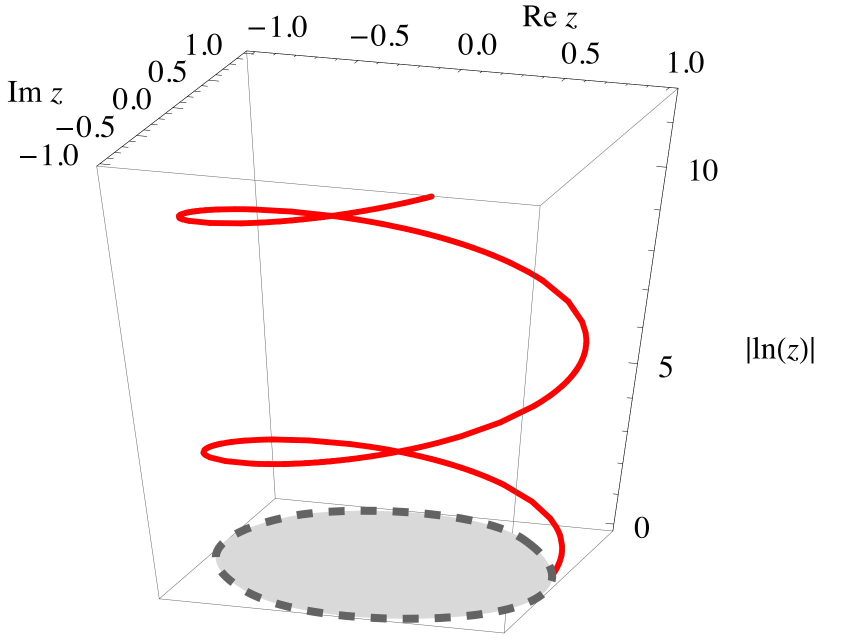
- produces periodically spaces set of multiple branches of large-field potentials:

$$V(\phi,\chi) = \mu^{4-p}\phi^p + \Lambda^4(\chi)\ \cos\left(\frac{\phi}{2\pi f}\right) + U_{mod.}(\chi)$$

instanton corrections

moduli potential





large-field string inflation ...

- Fibre inflation ($r < 0.01$) Cicoli, Burgess & Quevedo

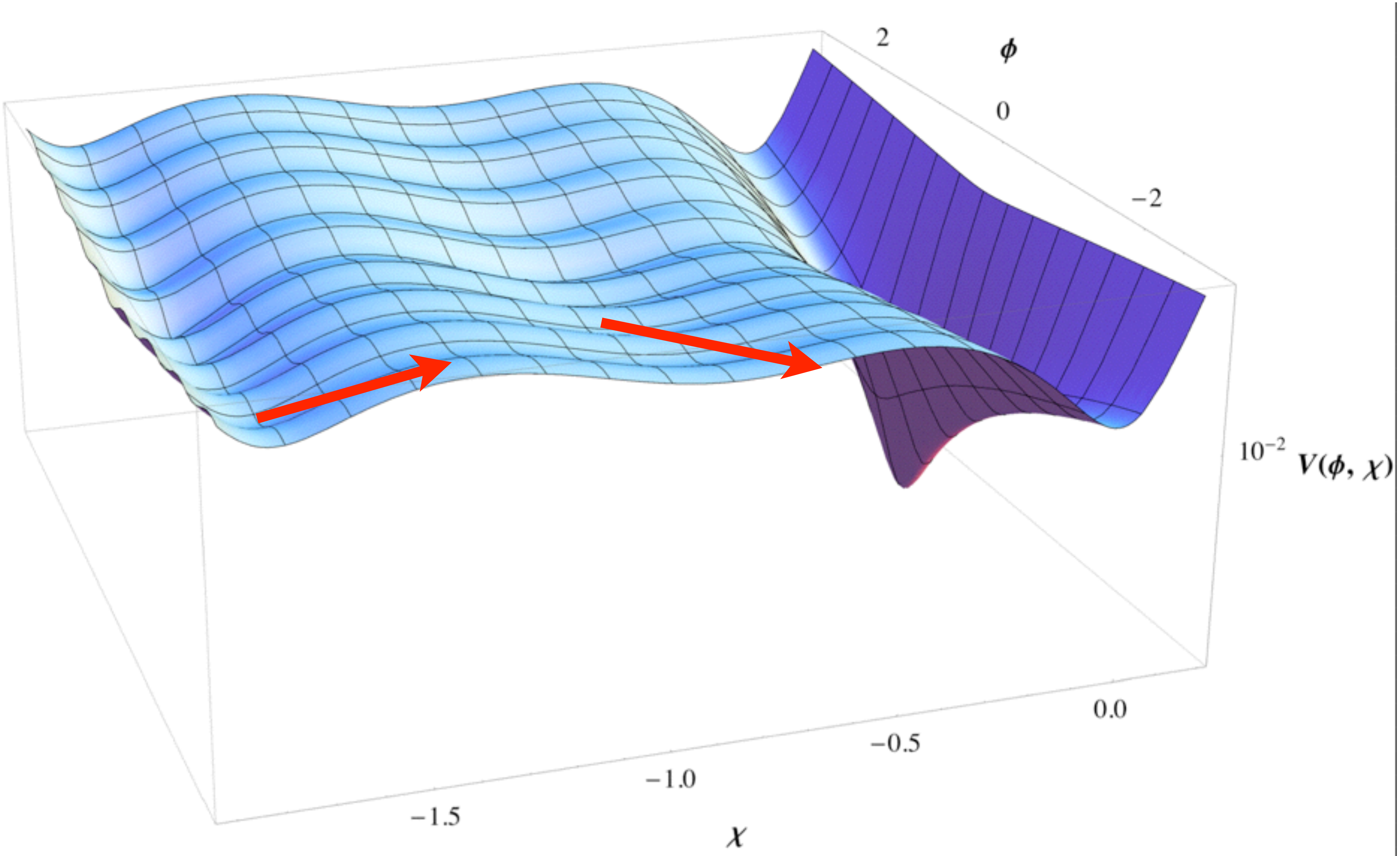
$r \sim 0.001$

- Single-Axion inflation with $f > M_P$ Grimm; Blumenhagen & Plauschinn;
- 2-Axion inflation Kim, Nilles & Peloso; Berg, Pajer & Sjors; Kappl, Krippendorf & Nilles; Long, McAllister & McGuirk;Tye & Wong; Ben-Dayan, Pedro & AW; Gao, Li & Shukla ...
- N-flation Dimopoulos, Kachru, McGreevy, Wacker; Easther & McAllister; Grimm; Cicoli, Dutta & Maharana; Choi, Kim & Yun; Bachlechner, Dias, Frazer & McAllister

$r \sim 0.1$

- axion monodromy Silverstein & AW; McAllister, Silverstein & AW; Flauger, McAllister, Pajer, AW & Xu; Dong, Horn, Silverstein & AW; Shlaer; Palti & Weigand; Marchesano, Shiu & Uranga; Blumenhagen & Plauschinn; Hebecker, Kraus & Witkowski; Ibanez & Valenzuela; Kaloper, Lawrence & Sorbo; McAllister, Silverstein, AW & Wrase; Franco, Galloni, Retolaza & Uranga; Blumenhagen, Herschmann & Plauschinn;

consequences of (i) , (iii), (iv-2) & (vi-3)



local measures: longest-lived dS - still generically of ***very high-scale*** c.c. - is progenitor to all small-c.c. dS vacua

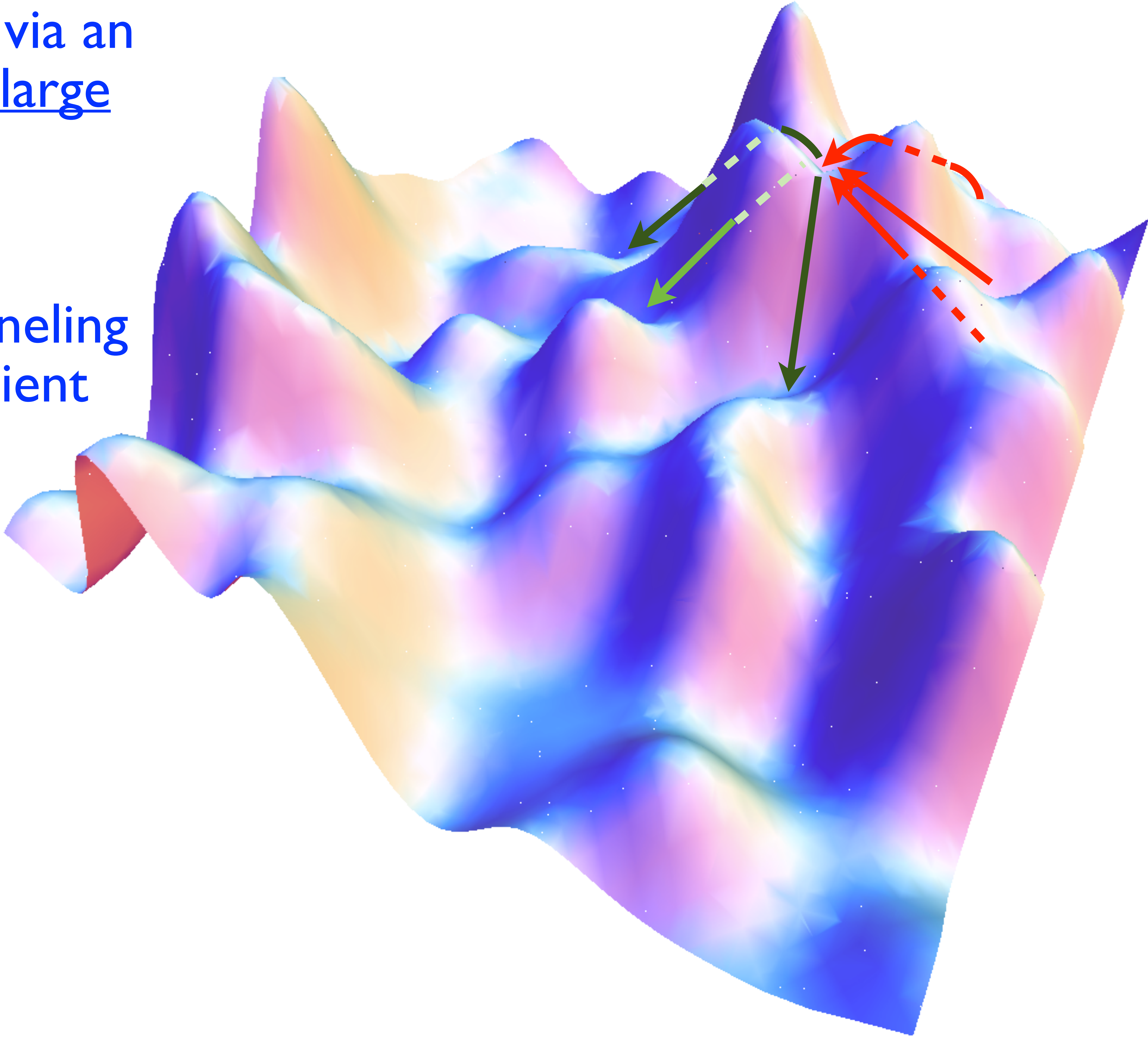
consequences of (iii) & (vi)

➡population of <u>sufficiently many</u> small-c.c. vacua must go via an <u>intermediate very large c.c</u> vacuum

↕

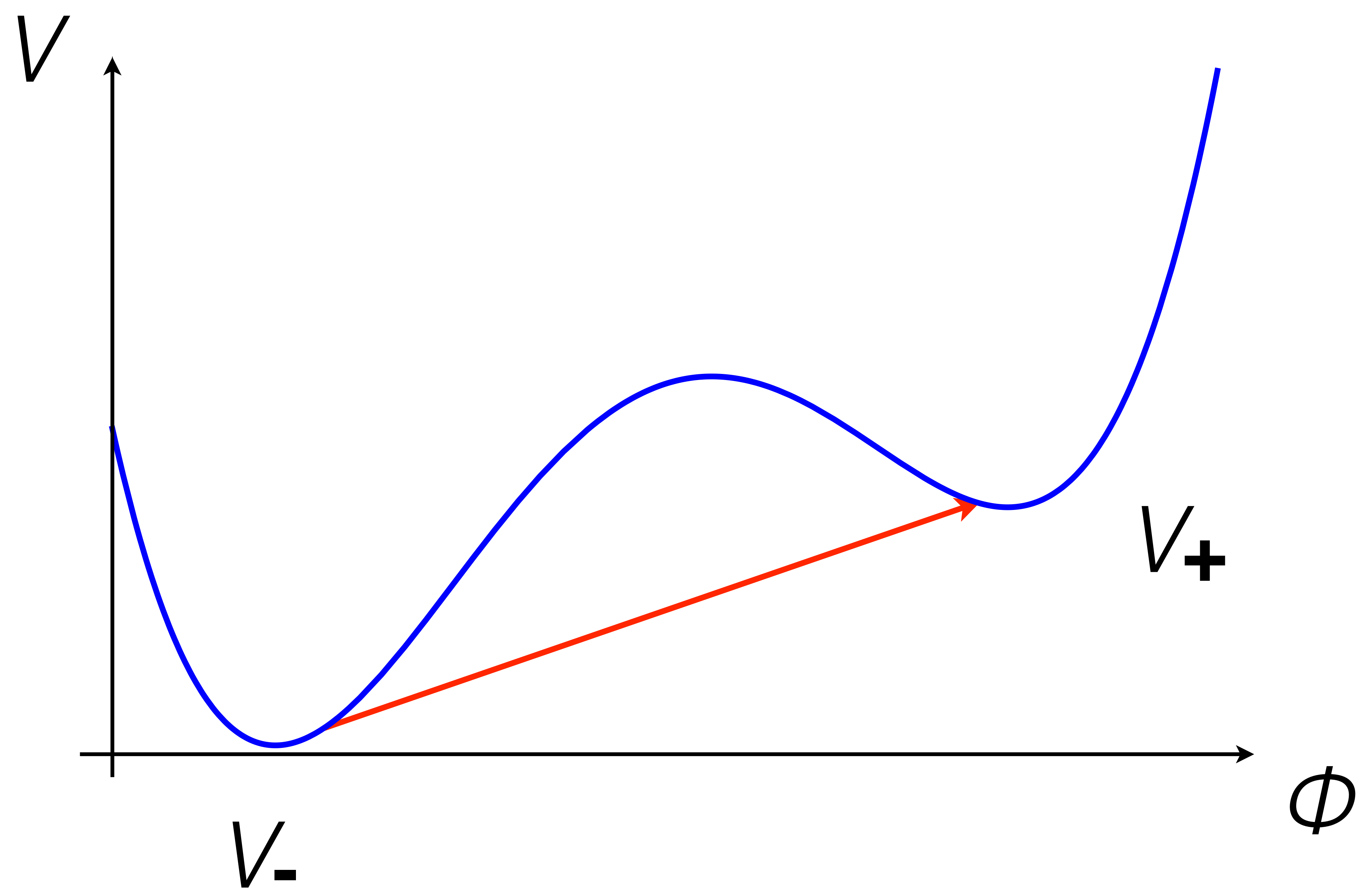
because down tunneling is much more efficient

➡maintained by all measures free of obvious paradoxa



consequences of (iii) tunneling ...

➥ up tunneling very expensive & undemocratic



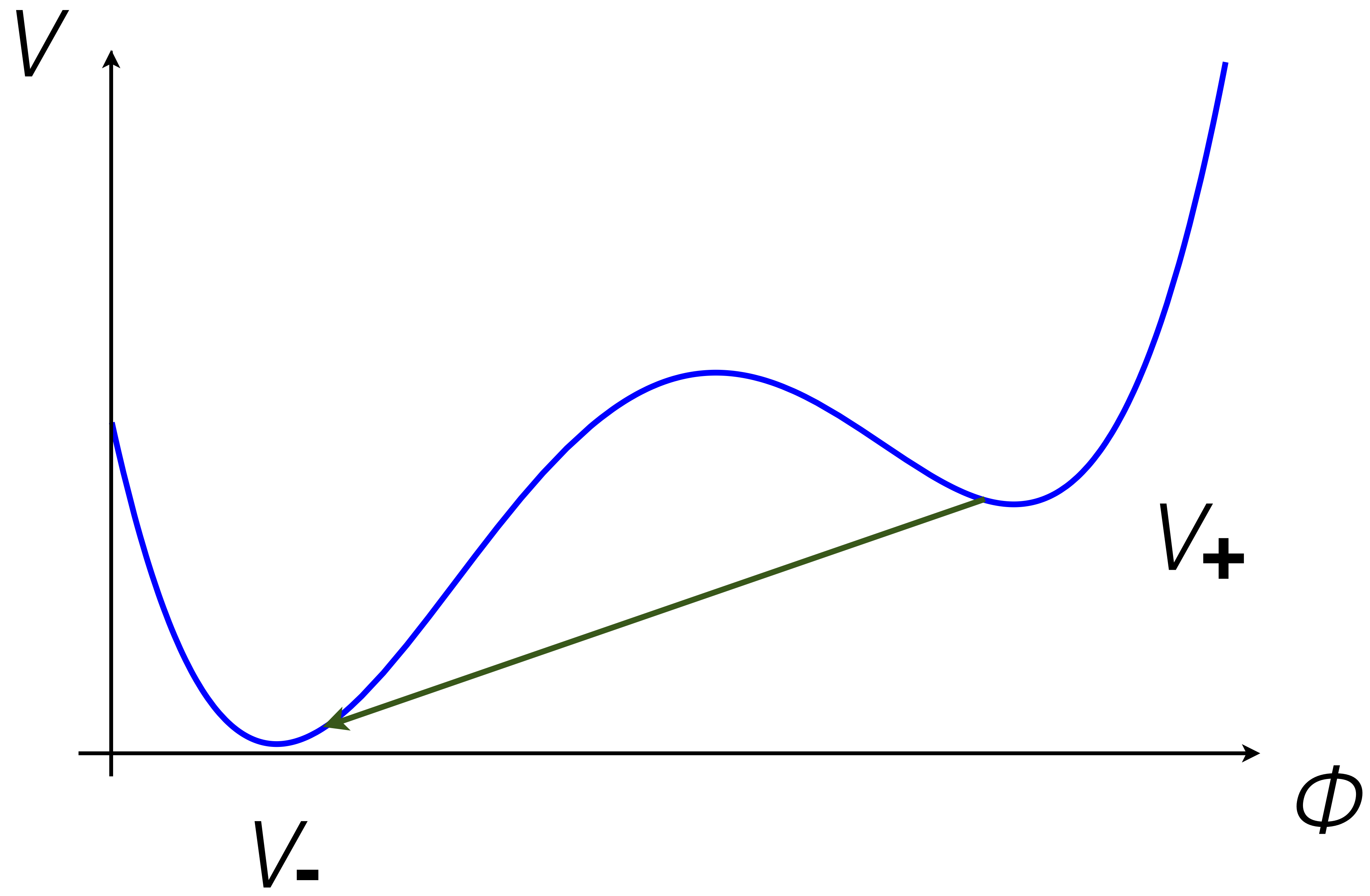
$$\Gamma_{V_+} \sim e^{-\left(\frac{1}{V_-} - \frac{1}{V_+}\right)}$$

➥ ratio of up tunneling rates into 2 different higher dS vacua

$$\frac{\Gamma_{V'_+}}{\Gamma_{V_+}} \sim e^{-\frac{1}{V_+}} \quad , \quad V'_+ > V_+$$

consequences of (iii) tunneling ...

➥ down tunneling less expensive & democratic



$$\Gamma_{V_-} \sim e^{-\frac{1}{V_+}+S_E(\phi)}$$

$$\frac{\Gamma_{V_+}}{\Gamma_{V_-}} \sim e^{-\frac{1}{V_-}} \ll e^{-\frac{1}{V_+}} \ll 1$$

consequences of (iii) tunneling ...

➥ down tunneling less expensive & democratic

$$S_E(\phi) \sim \int d\xi a^3(\xi) V(\phi)$$
$$\sim S_E^{(0)}(\phi)\ \left[1 + \mathcal{O}\left(\frac{V_-}{V_+}\right)\right]$$

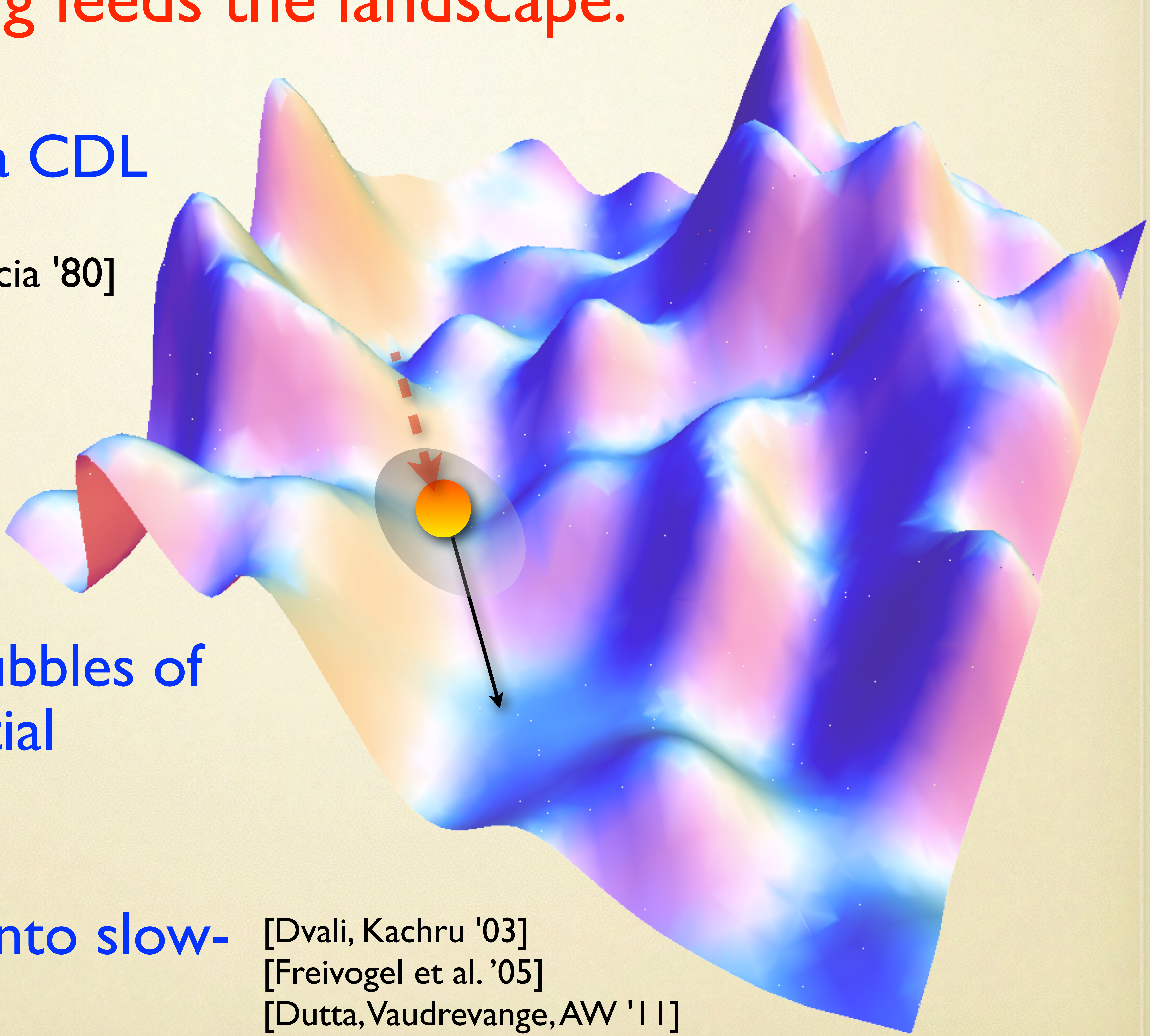
- independent from small *V*-

- can ***average*** over barrier height

➥ averaged ratio of down tunneling rates into 2 lower dS vacua

$$\frac{\Gamma^{av.}_{V'_-}}{\Gamma^{av.}_{V_-}} \sim 1 \quad , \quad V_+ \gg V_- \ , \ V'_-$$

(iii) Tunneling feeds the landscape:



- ➡proceeds via CDL instanton
  [Coleman, De Luccia '80]
- ➡nucleates bubbles of negative spatial curvature
- ➡drops field into slow-roll *always* !

[Dvali, Kachru '03]
[Freivogel et al. '05]
[Dutta, Vaudrevange, AW '11]

consequences of (i) , (ii) , (iii) & (iv)

[AW '12]

➥ successful anthropic explanation of present-day small c.c. requires efficient population of a very large # of small-c.c. vacua

➥ large-c.c. vacuum is effective progenitor of most inflationary valleys with exit into small c.c. vacua - because down tunneling is efficient

➥ down tunneling populates small-c.c. vacua & valleys democratically

➥ negative curvature inside CDL bubbles removes initial condition problem for subsequent slow-roll

➥ a *universal* bias seems to appear: no bias ... small-field and large-field regimes appear to be seeded democratically (on the level of exponential bias)

consequences of (i) , (ii) , (iii) & (iv)

[AW '12]

➥ consequence:

if the measure choice decouples & tunneling treats small-field and large-field regimes approximately neutral ...

distribution of field-range is fully determined by number frequency of inflationary solutions

➥ 'valley' statistics determines *r* , as vacuum statistics (anthropically) determines late-time c.c. ! This is in principle a string theory question ...

➥ accessible via random matrix theory ...

[Susskind '04; Douglas '04; Denef & Douglas '04; Aazami & Easther '05; Marsh, McAllister & Wrase '11; Chen, Shiu, Sumitomo & Tye '11; Bachlechner, Marsh, McAllister & Wrase '12; Marsh, McAllister, Pajer & Wrase '13; Bachlechner '13; ...]
[Battefeld, Battefeld & Schulz '12]

valley statistics ...

[AW '12]

- The landscape '*Drake equations*' of tensor modes

$$\Rightarrow \quad \frac{N_{\Delta\phi > M_P}}{N_{\Delta\phi < M_P}} \lesssim \frac{\beta_{h^{1,1}_- > 0}\cdot\langle h^{1,1}_-\rangle\cdot\beta_{V^{\frac{1}{4}} > 10^{16}\mathrm{GeV}}}{\beta_{flat\ saddle}\cdot\left(1-\beta_{V^{\frac{1}{4}} > 10^{16}\mathrm{GeV}}\right)} \quad \times\ \frac{N_{CY}\,\xcancel{N_{cr.}\,\beta_{dS-vac.}}}{N_{CY}\,\xcancel{N_{cr.}\,\beta_{dS-vac.}}}$$

??

$$\Rightarrow \quad \frac{N_{r > 0.01}}{N_{r < 0.01}} \sim \frac{N_{\Delta\phi > M_P}}{N_{\Delta\phi < M_P}}$$

$$\lesssim \frac{\beta_{h^{1,1}_- > 0}\cdot\langle h^{1,1}_-\rangle\cdot\beta_{V^{\frac{1}{4}} > 10^{16}\mathrm{GeV}}}{\beta_{flat\ saddle}\cdot\left(1-\beta_{V^{\frac{1}{4}} > 10^{16}\mathrm{GeV}}\right)}$$

[AW ’12]

➥ we know:

$\beta_{h^{1,1}_{-}>0} < 1$ not all CYs support the topology for axion monodromy

$h^{1,1}_{-} \lesssim \mathcal{O}(100)$ at least if # of CYs finite

$\beta_{V^{\frac{1}{4}}>10^{16}\mathrm{GeV}}$ likely to be non-exponential in V

➥ we need:

$$\beta_{flat\ saddle}$$

random supergravity ...

[Marsh, McAllister & Wrase '11]

[Chen, Shiu, Sumitomo & Tye '11]

➥ approximate dS landscape from CY flux compactifications (e.g. KKLT, LVS, Kähler uplifting ...) by a random supergravity:

➥ random K and W generate scalar potential:

$$V = e^K \left(F_a \bar{F}^a - 3|W|^2 \right)$$

➥ Hessian of critical points approximated by sum of random matrices:

$$\mathcal{H} = \underbrace{\mathcal{H}_{SUSY}, + \mathcal{H}_{K^{(3)}}}_{Wishart+Wishart} + \underbrace{\mathcal{H}_{pure} + \mathcal{H}_{K^{(4)}}}_{Wigner} + \mathcal{H}_{shift}.$$

random supergravity ...

➥ if we draw from random distribution:

$$K_{a\bar{b}}\,,\; K_{a\bar{b}c}\,,\; K_{a\bar{b}c\bar{d}}\,,\; W_a\,,\; W_{ab}$$

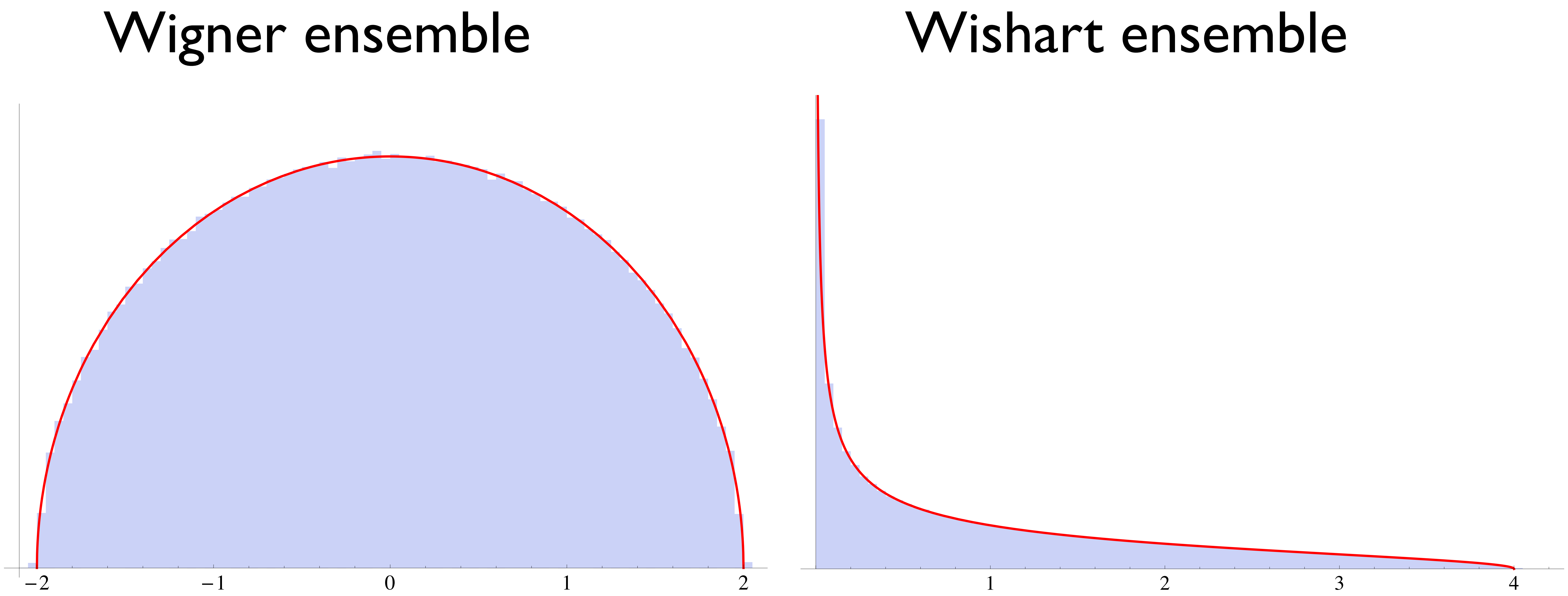
➥ Hessian approximated by sum of random matrices:

$$\mathcal{H} = \underbrace{\mathcal{H}_{SUSY} + \mathcal{H}_{K^{(3)}}}_{Wishart+Wishart} + \underbrace{\mathcal{H}_{pure} + \mathcal{H}_{K^{(4)}}}_{Wigner} + \mathcal{H}_{shift}.$$

➥ universality at large number of fields ensures independence from distribution choice

random supergravity ...

➥ eigenvalue spectra



Wigner ensemble

Wishart ensemble

➥ inflationary *<u>saddle</u>* points controlled by Wigner ensemble!!

➥ dS minima:

large eigenvalue fluctuation to positivity - exponentially suppressed (physics: 1D particles in attractive potential with mutual repulsion - squeezing all to one side strongly disfavored)

small field inflation in the landscape ...

[Pedro & AW '13]

➥ inflation: at least one eigenvalue *negative* - *less* eigenvalue repulsion, so small-field inflation should be ***more*** likely than minima

➥ can compute probability for *n* eigenvalues to be above a given value η:

$$dP(\lambda_1, ..., \lambda_{N_f}) = exp\left(-\frac{1}{\sigma^2}\sum_{i=1}^{N_f}\lambda_i^2\right)\prod_{i<j}(\lambda_i-\lambda_j)^2$$

$$P(\forall\lambda > -\eta) = \prod_{i=1}^{N_f}\int_{-\eta}^{\infty}\mathrm{d}\lambda_i \mathrm{d}P(\lambda_1, ..., \lambda_{N_f})$$

$$= \sum_{n=0}^{N_f}\frac{N_f!}{n!(N_f-n)!}\prod_{i=1}^{n}\int_{-\eta}^{\eta}\mathrm{d}\lambda_i\prod_{j>n}^{N_f}\int_{\eta}^{\infty}\mathrm{d}\lambda_j\mathrm{d}P$$

➥ Dean & Majumdar's result on exponential suppression of large eigenvalue fluctuation:

$$P(\forall\lambda > \xi) = e^{-2\Phi(\xi)N_f^2}$$

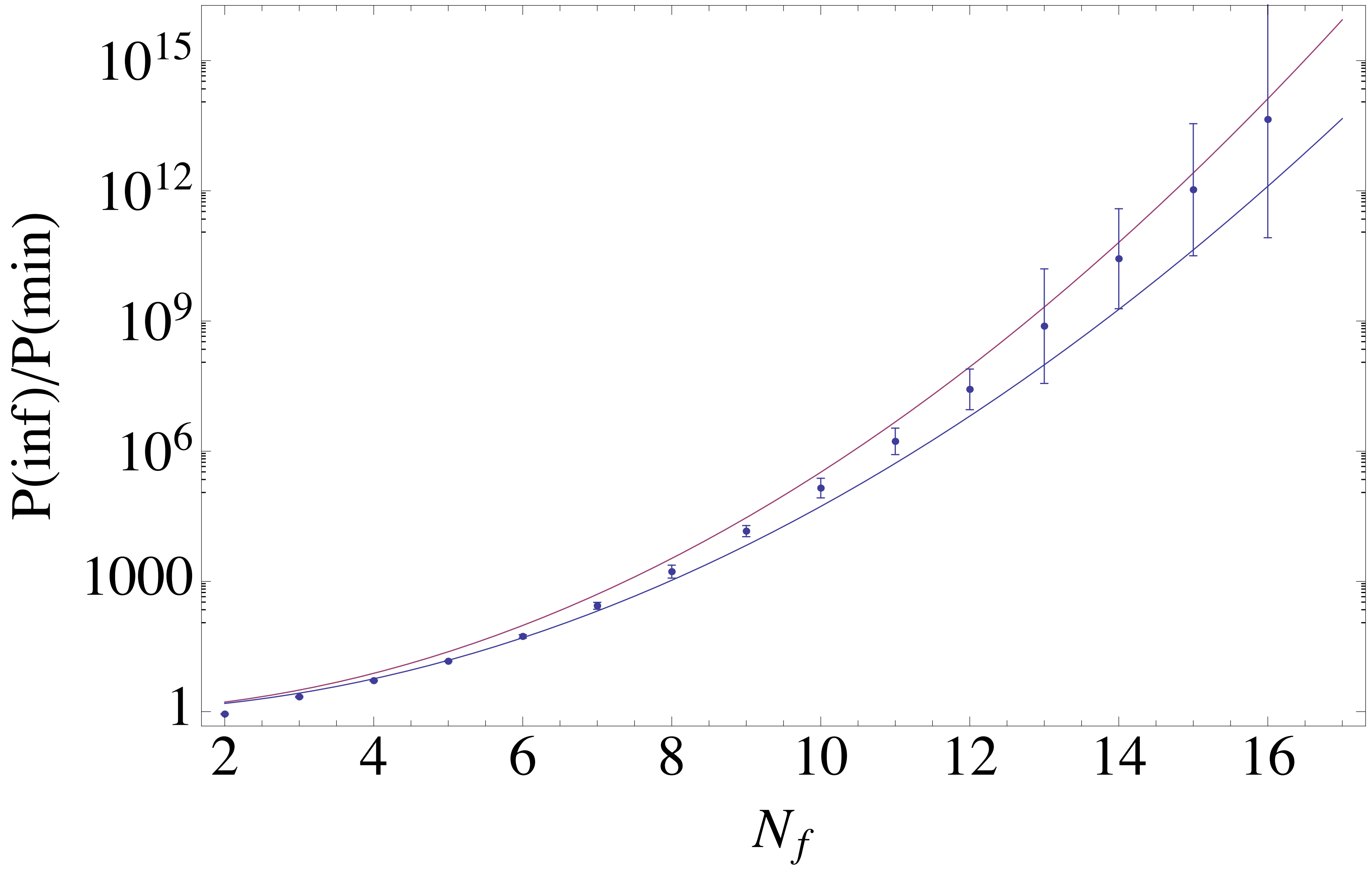
small field inflation in the landscape ...

[Pedro & AW ’13]

➥ can compute the ratio of flat inflationary saddle points vs minima from joint PDF of Wigner ensemble for N_f fields using Dean & Majumdar’s result:

$$\frac{P(inf)}{P(min)} = (e^{2\Delta c N_f^2} - 1)e^{2\widetilde{\Delta} c N_f^2} \sim e^{2\eta\Phi'(0)N_f^2} + \mathcal{O}(\eta^2)$$

inflationary saddle points are defined by N_f fields having mass eigenvalues between $[-\eta\ ,\eta]$ where $\eta < 0.1$ in terms of the typical mass scale of an F-term supergravity scalar potential $(m_{3/2})^2$



$$\Rightarrow \qquad \beta_{flat\ saddle} \gg 1$$

small field inflation in the landscape ...

[Pedro & AW '14]
[work in progress]

➥ however !! --- valley statistics alone is insufficient:

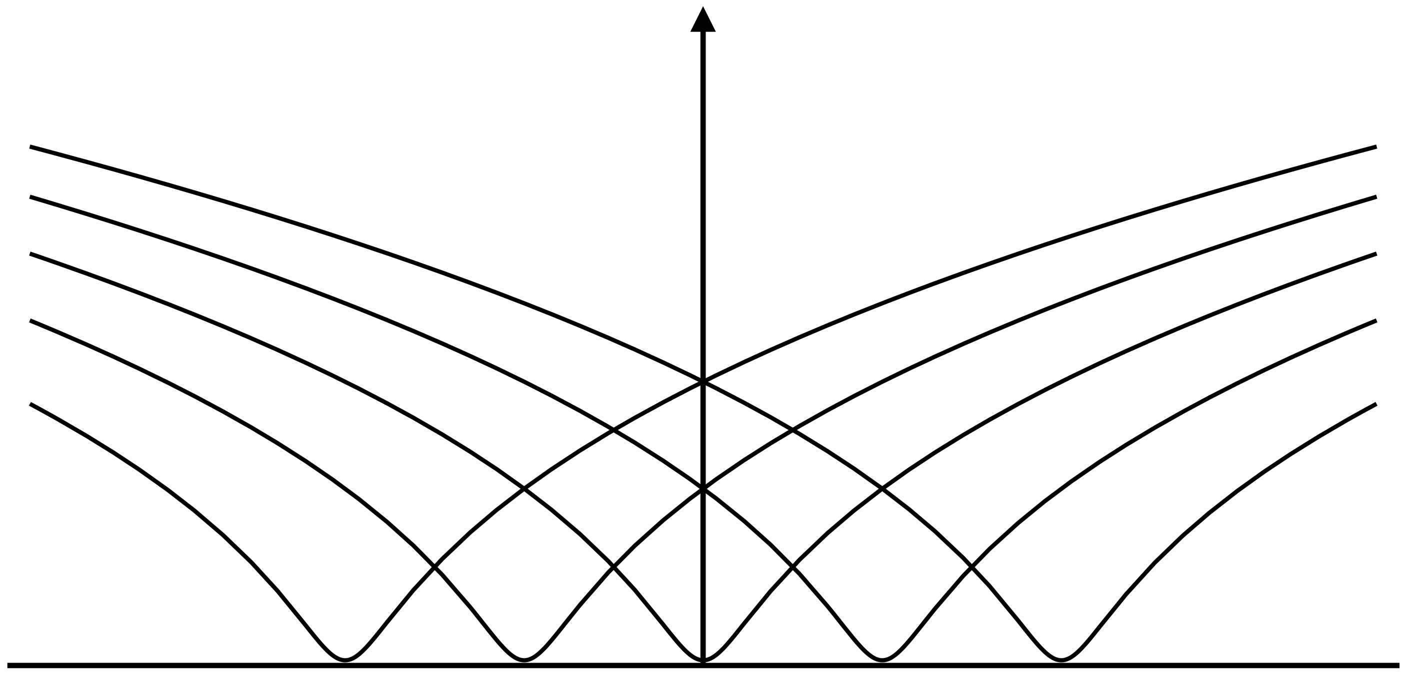
- We need a graceful exit into a dS minimum

$$\frac{N_{r\,>\,0.01}}{N_{r\,<\,0.01}} \quad \lesssim \quad \frac{\beta_{h_-^{1,1}>0}\cdot\langle h_-^{1,1}\rangle\cdot\beta_{V^{\frac{1}{4}}>10^{16}\mathrm{GeV}}}{\beta_{flat\ saddle}\cdot\left(1-\beta_{V^{\frac{1}{4}}>10^{16}\mathrm{GeV}}\right)} \quad \times \quad \frac{P_{exit}^{\Delta\phi>M_{\mathrm{P}}}}{P_{exit}^{\Delta\phi<M_{\mathrm{P}}}}$$

small field inflation in the landscape ...

[Pedro & AW '14]
[work in progress]

➥ <u>large-field models:</u> minimum built-in !



$$P_{exit}^{\Delta\phi > M_{\rm P}} \sim 1$$

➥ but for small-field models: close-by vacua usually AdS
➜ ΔN = ±1 changes of flux cause *O*(1) changes in vacuum energy

... viable dS minima are **distant**

➥ need to compute: ***P*****(distant minimum | flat crit. point)** :
the probability to get a positive Hessian far away from a flat saddle point ...

small field inflation in the landscape ...

[Dyson ’62]

- ➥ use Dyson Brownian Motion: eigenvalue relaxation ...

$$\mathcal{H}_{ab} \to \mathcal{H}_{ab} + \frac{\delta s}{\Lambda}\left(A_{ab} - \mathcal{H}_{ab}\right)$$

correlation length in field space

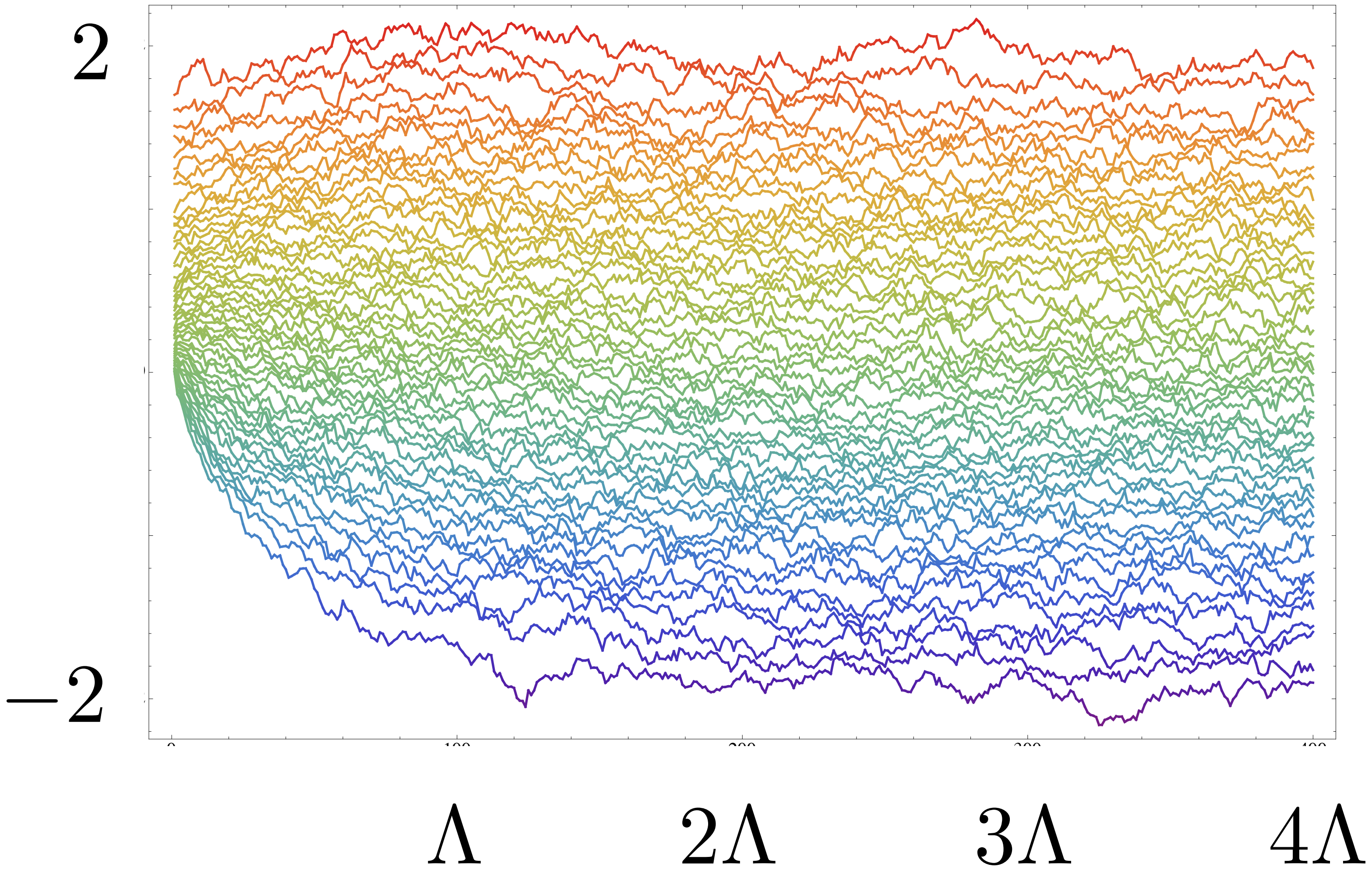
random, symmetric, zero-mean perturbation

[Marsh, McAllister, Pajer & Wrase ’13]

- ➥ e.g. start with a 50 x 50 Wishart matrix: all eigenvalues positive

eigenvalues relax in 1 corr. length toward Wigner distro

fluctuating toward a minimum *extremely* suppressed



small field inflation in the landscape ...

[Pedro & AW ’14]
[work in progress]

➥ use Dyson Brownian Motion:

... described in continuum limit by Fokker-Planck equation

➜ time-dependent probability distribution for Hessian

[Uhlenbeck & Ornstein ’30]

$$P_{exit} \sim e^{-\frac{1}{\sigma^2}\mathrm{Tr}(\mathcal{H}_{min.} - q\,\mathcal{H}_{inf.})^2} \quad , \quad q = e^{-\delta s/\Lambda} \quad , \quad \Lambda = \sigma^2 f$$

➥ from eigenvalue distributions $\rho(\lambda)$ of random matrix ensembles:

$$\mathrm{Tr}(A^2) = \sum_i (\lambda_i)^2 = N_f \langle \lambda_i^2 \rangle = N_f \int \mathrm{d}\lambda\, \rho(\lambda)\, \lambda^2$$

and: $\sigma^2 \sim \dfrac{1}{N_f}$

small field inflation in the landscape ...

[Pedro & AW '14]
[work in progress]

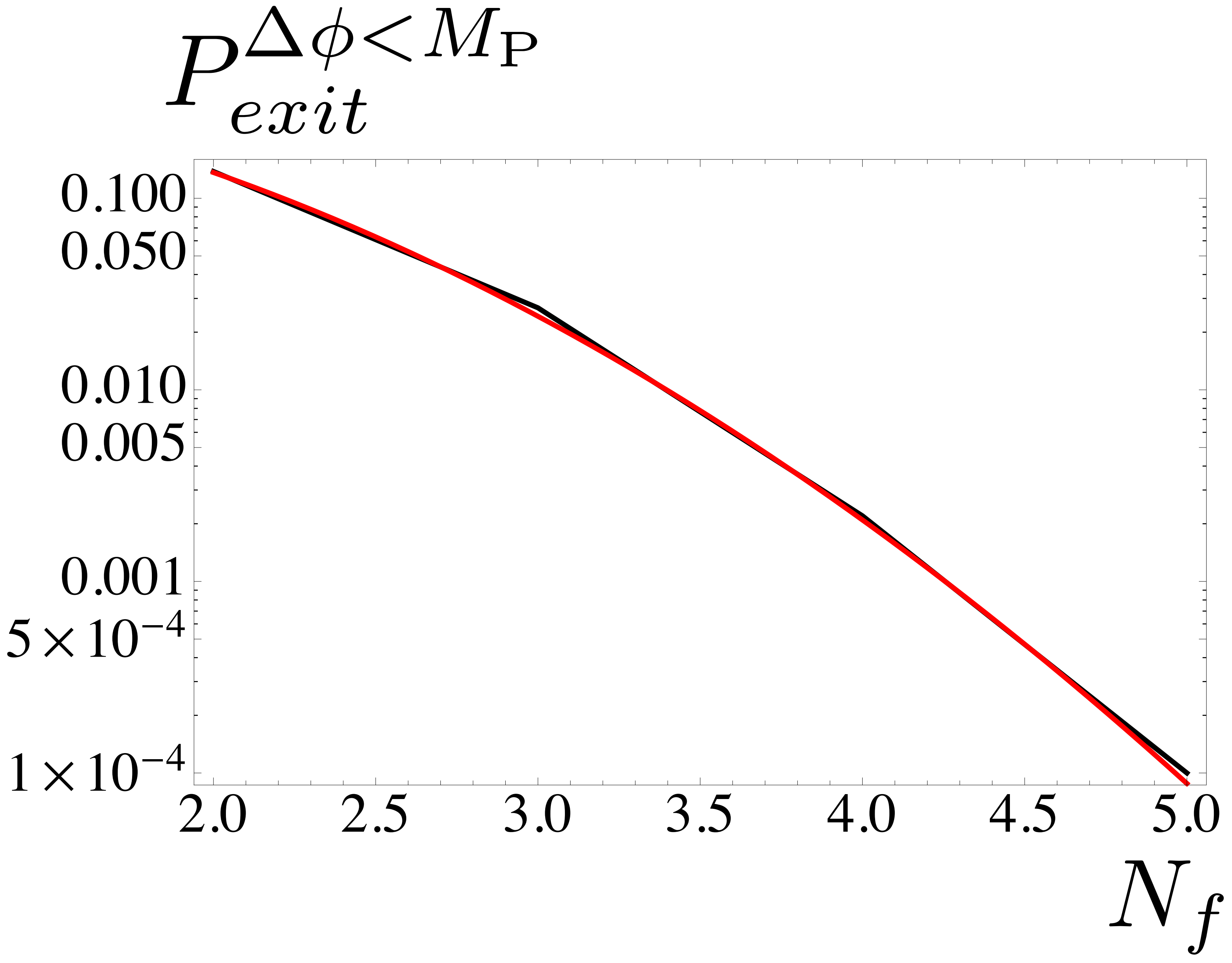
➥ so we get:

$$P_{exit}^{\Delta\phi<M_{\rm P}} \sim e^{-\frac{1}{\sigma^2}{\rm Tr}(\mathcal{H}_{min.}-q\,\mathcal{H}_{inf.})^2}$$

$$\sim e^{-c\,N_f^2} \quad , \quad q=e^{-\delta s/\Lambda}$$

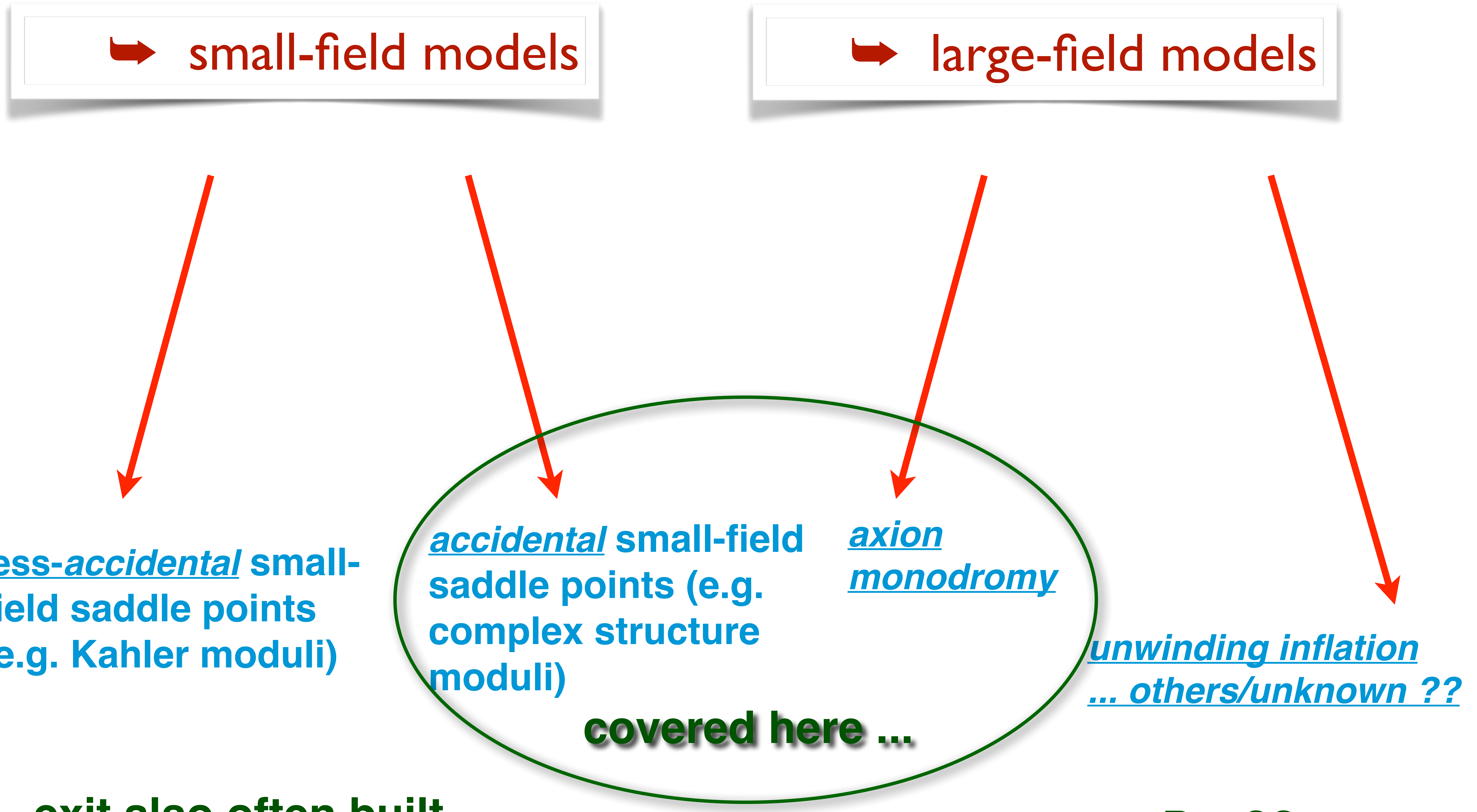
$$\frac{N_{r>0.01}}{N_{r<0.01}} \lesssim e^{-c_{s.p.}N_f^2} \times \frac{P_{exit}^{\Delta\phi>M_{\rm P}}}{P_{exit}^{\Delta\phi<M_{\rm P}}}$$

$$\sim e^{(c-c_{s.p.})N_f^2}$$



➥ can undo saddle point count!

where do we go from here ...



➥ small-field models

➥ large-field models

less-*accidental* small-field saddle points (e.g. Kahler moduli)

accidental small-field saddle points (e.g. complex structure moduli)

axion monodromy

unwinding inflation

... others/unknown ??

covered here ...

exit also often built-in ... P = ??

maybe
P_{large} / P_{small} = non-exp.

P = ??