$\mathcal{R}^2$ Inflation from Scale Invariant Supergravity and Anomaly Free Superstrings with Fluxes

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arXiv: 1409.7076

Joint ERC workshop „Superfields, Selfcompletion and Strings&Gravity
Munich, 25. October 2014

Outline:

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I) Introduction

There are many attempts to realize inflation and De Sitter vacua from supergravity and superstrings, including no-go theorems and possible ways to bypass them.

(Dvali, Tye (1998), Kachru, Kallosh, Linde, Maldacena, McAllister, Trivedi (2003), Hertzberg, Kachru, Taylor, Tegmark (2007), Silverstein, Westphal (2008), Haack, Kallosh, Krause, Linde, D.L., Zagermann (2008), Danielson, Haque, Shiu, Van Riet (2009), Hebecker, Kraus, D.L. Steinfurt, Weigand (2011), Blumenhagen, Plauschinn, Herschmann (2014), Grimm (2014), Marchesano, Shiu, Uranga (2014), Palti, Weigand (2014), Hassler, D.L., Massai (2014); reviews: Baumann, McAllister (2014), Kallosh, Linde, Westphal (2014), ..)

In particular there is also several recent work to embed the $\mathcal{R} + \mathcal{R}^2$ Starobinsky model into supergravity.

(Ferrara, Kehagias, Porrati (2013), Ferrara, Kallosh, Linde, Porrati (2013), Ferrara, Kehagias, Riotto (2013), Kallosh, Linde (2013), Ellis, Nanopoulos, Olive (2013), Ellis, Garcia, Nanopoulos, Olive (2014), Lahanas, Tamvakis (2014), Antoniadis, Dudas, Ferrara, Sagnotti (2014), Ferrara, Kallosh, Van Proyen (2013), Ferrara, Porrati (2014), Ferrara, Kehagias(2014), Ceresole, Dall'Agata, Ferrara, Trigiante, Van Proyen (2014))

Our work is motivated by the observation that

the pure $\mathcal{R}^2$ theory is the only scale invariant theory without ghosts.

Consider the following gravitational action:

$$S = \frac{1}{2\kappa_4} \int d^4x \sqrt{|g|} \Big(\xi \mathcal{R} + \frac{\mathcal{R}^2}{8\mu^2} \Big)$$

Breaks scale invariance

Scale invariant

This theory is conformally equivalent to conventional Einstein gravity with an extra scalar degree of freedom.

(Starobinsky, 1980)

Scale invariant part $\Rightarrow$ cosmological constant with de Sitter vacuum.

Non-scale invariant part $\Rightarrow$ inflationary potential with flat Minkowski vacuum.

In order to be realistic one has to couple gravity to matter.

Assume that the classical theory is scale invariant $\mathcal{R}^2$ gravity coupled to scale invariant matter (SM except Higgs mass is scale invariant).

Scale violating terms like $\xi\mathcal{R}$ term or fermion masses are classically absent.

They will be induced by quantum effects:

- anomalous $U(1)_R$ symmetries and anomaly cancelation.
- related to FI D-terms in N=1 supergravity.
- are computable in string theory.

II) The $\mathcal{R}^2$ model plus conformally coupled matter

Minimal scale invariant theory:

$$S = \int d^4x \sqrt{|g|} \frac{1}{2} \left(\frac{\mathcal{R}^2}{8\mu^2} \right) \quad \longrightarrow \quad S = \int d^4x \sqrt{|g|} \frac{1}{2} \left[2t\mathcal{R} - 8\mu^2 t^2 \right]$$

t .... Lagrange multiplier: $t = \frac{1}{8\mu^2} \mathcal{R}$

Conformal rescaling of metric:

$$g_{\mu\nu} \longrightarrow g_{\mu\nu} (2t)^{-1}, \quad \alpha\phi = \log(2t) \text{ with } \alpha = \sqrt{\tfrac{2}{3}}$$

Jordan frame action → Einstein frame action:

$$S_E = \int d^4x \sqrt{|g|} \, \frac{1}{2} \left[\mathcal{R} - \partial_\mu \phi \partial^\mu \phi - 2\mu^2 \right]$$

Cosmological constant: $\Lambda = \mu^2$ → de Sitter space.

(This proves that the pure $\mathcal{R}^2$ theory is ghost free!)

Breaking of scale invariance:

Now consider Jordan frame action:

$$S = \frac{1}{2}\int d^4x\sqrt{|g|}\left(\mathcal{R} + \frac{\mathcal{R}^2}{8\mu^2}\right) \rightarrow \; S = \int d^4x\sqrt{|g|}\frac{1}{2}\left[(1+2t)\mathcal{R} - 8\mu^2t^2\right]$$

Conformal rescaling: $g_{\mu\nu} \longrightarrow g_{\mu\nu}\,(2t+1)^{-1}$

Einstein frame: $$S = \int d^4x\sqrt{g}\left[\frac{1}{2}\mathcal{R} - 3\frac{\partial_\mu t\partial^\mu t}{(1+2t)^2} - \mu^2\frac{(2t)^2}{(1+2t)^2}\right]$$

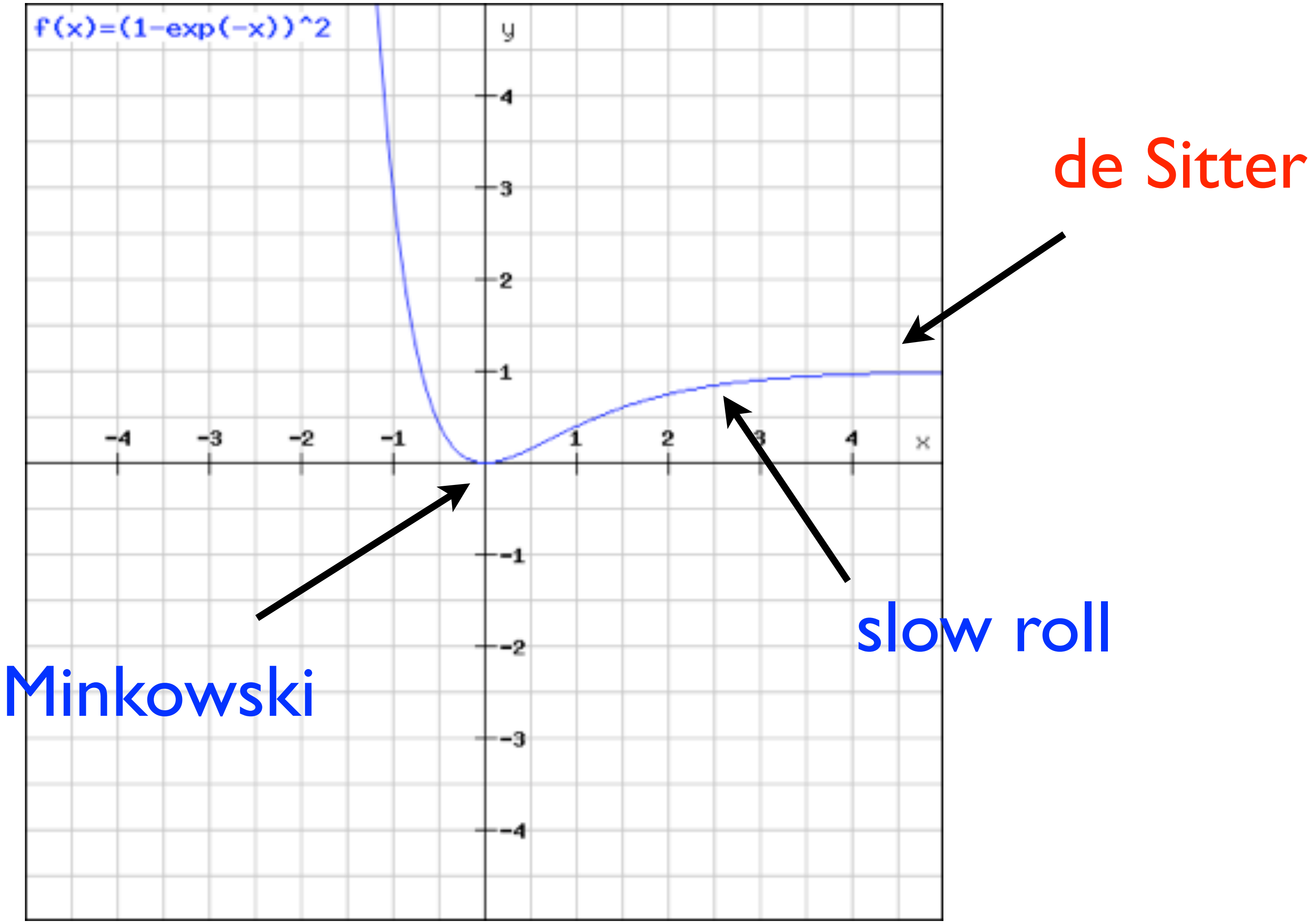
$$\alpha\phi = \log(1+2t) \text{ with } \alpha = \sqrt{\frac{2}{3}}$$

$$S = \int d^4x\sqrt{g}\,\frac{1}{2}\left[\mathcal{R} - \partial_\mu\phi\partial^\mu\phi - 2V(\phi)\right],$$

Starobinsky potential: $V(\phi) = \mu^2\left(1 - e^{-\alpha\phi}\right)^2$

(Starobinsky, 1980)

$$V(\phi) = \mu^2 \left(1 - e^{-\alpha\phi} \right)^2$$



(The value r for the tensor modes is generically suppressed in Starobinsky inflation.)

Matter couplings:

We introduce n conformally coupled scalar fields Φ_i :

$$S = \int d^4x \sqrt{g} \frac{1}{2} \left(\frac{\mathcal{R}^2}{8\mu^2} - \frac{1}{6}\Phi_i^2 \mathcal{R} - \partial_\mu \Phi_i \partial^\mu \Phi_i - 2V_c(\Phi_i) + \ldots \right)$$

non-canonical, scale invariant Einstein term

Φ_i have conformal weight $w_\Phi = 1$.

Quartic scalar potential: $V_c = \lambda_{ijkl}\ \Phi_i \Phi_j \Phi_k \Phi_l$

Introduce Lagrange multiplier field t :

$$S = \int d^4x \sqrt{g} \frac{1}{2} \left(\left[2t - \frac{1}{6}\Phi_i^2 \right] \mathcal{R} - \partial_\mu \Phi_i \partial^\mu \Phi_i - 2V_c(\Phi_i) - 8\mu^2\, t^2 \right)$$

Conformal rescaling: $$g_{\mu\nu} \longrightarrow g_{\mu\nu}\, e^{-\log(2t-\frac{1}{6}\Phi_i^2)}$$

$$\alpha\phi = \log\left(2t - \frac{1}{6}\Phi_i^2\right) \qquad \alpha = \sqrt{\frac{2}{3}}$$

Action in Einstein frame:

$$S_E = \int d^4x\sqrt{|g|}\,\frac{1}{2}\left[\mathcal{R} - \partial_\mu\phi\partial^\mu\phi - e^{-\alpha\phi}\partial_\mu\Phi_i\partial^\mu\Phi_i - 2V(\phi,\Phi_i)\right]$$

$$V(\phi,\Phi_i) = e^{-2\alpha\phi}V_c(\Phi_i) + \mu^2\left(1 + \frac{e^{-\alpha\phi}\Phi_i^2}{6}\right)^2$$

- The action is invariant under the following scaling symmetry:

$$\alpha\phi \to \alpha\phi + 2\sigma\,, \qquad \Phi_i \to e^{\sigma}\Phi_i\,, \qquad g_{\mu\nu} \to g_{\mu\nu}$$

- The n+1 scalar fields parametrize the following manifold:

$$\mathcal{M}(\phi, \Phi_i) = \mathcal{H}^{n+1} \equiv \frac{SO(1,1+n)}{SO(1+n)}$$

- The vacuum is de Sitter space time:

$$\Lambda = \mu^2\,, \quad \phi = \text{const.}\,, \quad \Phi_i = 0\,, \quad m_i^2 = \frac{2}{3}\mu^2$$

Matter couplings with a scale violating term:

The additional $\mathcal{R}$ term breaks scale invariance.

Shift: $2t \to y = 2t + 1$

$$\alpha\phi = \log\left(2t + 1 - \frac{1}{6}\Phi_i^2\right) = \log\left(2y - \frac{1}{6}\Phi_i^2\right)$$

$$V_{\mu^2} = \mu^2\left(1 + \frac{e^{-\alpha\phi}\Phi_i^2}{6}\right)^2 \longrightarrow V_{\mu^2} = \mu^2\left(1 - e^{-\alpha\phi} + \frac{e^{-\alpha\phi}\Phi_i^2}{6}\right)^2$$

(i) $\phi \gg 0$ Scale invariant de Sitter phase

(ii) $\phi \sim 0$ Flat Minkowski vacuum

Unlike the pure $\mathcal{R}^2$ case the Minkowski vacuum can be degenerate with flat directions: $e^{-\alpha\phi} - 1 = \frac{e^{-\alpha\phi}\Phi_i^2}{6}$

Remark: $\mu^2 < 0$: anti-de Sitter space time.

Theory with negative $\mu^{-2}\mathcal{R}^2$ is also consistent.

In the following we will extend the $\mathcal{R} \oplus \mathcal{R}^2$ theories to N=1 supergravity plus conformally coupled matter fields.

Supersymmetry will give additional restrictions.

F-term potential: Stable anti-de Sitter, Minkowski or unstable de Sitter.

F+D term potential: Stable de Sitter.

Scale violating terms: will be associated to special gaugings, related to U(1) anomalies and the anomaly cancellation mechanism $\Rightarrow$ inflation.

III) Supergravity extension of the $\mathcal{R}^2$ theory

(Cecotti (1987), Cecotti, Ferrara, Porrati, Sabharwal (1988), see also:Rinaldi, Cognola, Vanzo, Zerbini (2014), ...)

3.1 Minimal version, F-term potential

We will use the old minimal N=1 Sugra formalism with chiral superfields.

Complex fields: $\psi^I = (T, z^i)\,, \quad T = t + ib\,.$

$I = 1, \ldots, n+1 \quad (n = 1, \ldots, n)$

n can also be zero!

Scale invariant Jordan frame action:

$$S = \int d^4x \sqrt{|g|}\ \left[\frac{1}{2} Y \left(\mathcal{R} + \frac{2}{3} A_\mu A^\mu\right) - J_\mu A^\mu + 3 Y_{I\bar{J}}\, \partial_\mu \psi^I \partial^\mu \bar{\psi}^{\bar{J}} - V_c(T, z^i)\right]$$

where

$$Y = T + \bar{T} - |z^i|^2\,, \quad Y_I = \frac{\partial Y}{\partial \psi^I}\,, \quad Y_{\bar{I}} = \frac{\partial Y}{\partial \bar{\psi}^{\bar{I}}}\,, \quad Y_{I\bar{J}} = \frac{\partial Y}{\partial \psi^I \partial \bar{\psi}^{\bar{J}}}$$

Auxiliary vector field: $A_\mu = \frac{3}{2}\frac{J_\mu}{Y} = \frac{3}{2} i \frac{Y_{\bar{I}}\partial_\mu \bar{\psi}^{\bar{I}} - Y_I \partial_\mu \psi^I}{Y}$

Current: $J_\mu = i(Y_{\bar{I}}\partial_\mu \bar{\psi}^{\bar{I}} - Y_I \partial_\mu \psi^I)$

Lagrange multiplier Y has no kinetic term in Jordan frame.

Equation of motion: ($\phi = \alpha^{-1}\log Y$: no scale modulus)

$$\mathcal{R} = \frac{\partial}{\partial t} V_c(T, z^i) \quad \rightarrow \quad T + \bar{T} = f(\mathcal{R}, b, z^i)$$

For scale invariant $\mathcal{R}^2$ gravity: $V_c \sim t^2$

Einstein frame: standard Cremmer et al.

N=1 Sugra action:

- scalar potential: $V_c = Y^2 V_E$

$$V_E = e^K \left\{ (W_I + K_I W)\, K^{I\bar{J}}\, (\bar{W}_{\bar{J}} + K_{\bar{J}}\bar{W}) - 3W^2 \right\}$$

- Kähler potential: no-scale supergravity model $\mathbb{C}P^{1,n}$

$$K = -3\log\left(T + \bar{T} - |z^i|^2\right) \longrightarrow \quad \mathcal{M}(T, z^i) = \frac{SU(1, 1+n)}{U(1) \times SU(1+n)}$$

In the Einstein frame the scaling symmetry acts as:

$$T \to e^{2\sigma} T, \qquad z^i \to e^{\sigma} z^i$$

The theory is scale invariant (and equivalent to the $\mathcal{R}^2$ gravity) if the function G is scale invariant:

$$\mathbf{G} \equiv K + \log|W|^2$$ (So W has scaling weight 3.)

This leaves three possible choices for the superpotential:

(i) $W = c\,T^{3/2}$ (ii) $W = c_{ijk}\, z^i z^j z^k$ (iii) $W = c_k\, z^k\, T$

(i) $W = cT^{3/2}$ anti-de Sitter realization

$\Rightarrow$ $\mathcal{R}^2$ theory with negative cosmological constant.

$$V_c \sim -t^2 \qquad \Rightarrow \qquad \frac{\mathcal{R}^2}{\mu^2} \quad \text{with} \quad \mu^2 < 0$$

$$V_F = -3m^2_{\frac{3}{2}} \,, \qquad \mu^2 = -3m^2_{\frac{3}{2}}$$

(ii) $W = h\, z^3$ flat Minkowski realization

$$V_F = 3h^2 \frac{|z|^4}{Y^2} = H^2 |\Phi|^4 \qquad \Phi = \frac{z}{\sqrt{Y}}$$

Vacuum: $\Phi = 0 \;\Rightarrow\; V_F = 0$ $\qquad H^2 = 3h^2$

(iii) $W = hzT$ de Sitter realization

$$V_F = \frac{|h|^2}{3Y^2}\{|T|^2 - (T+\bar{T})|z|^2\} = \frac{|h|}{3}\left\{\frac{b^2+t^2-2t|z|^2}{Y^2}\right\}$$

$$= \frac{|h|^2}{12}\{1+4B^2-2|\Phi|^2-3|\Phi|^4\} \qquad B = \frac{b}{Y}, \quad \Phi = \frac{z}{\sqrt{Y}}$$

Extremum: $B = \Phi = 0 \;\Rightarrow\; V_F = \dfrac{|h|^2}{12}$

$\Rightarrow$ $\mathcal{R}^2$ theory with positive cosmological constant.

However the z^i directions of the potential are unstable !

The F-part of the potential will never lead to a stable de Sitter vacuum!

In order to get rid off the instability we will include Fayet-Iliopoulos D terms.

3.2 De Sitter realization with D terms

(Binetruy, Dvali 1996), Binetruy, Dvali, Kallosh, Van Proeyen (2004), ..., Catino, Villadoro, Zwirner (2012), Li, Li, Nanopoulos (2014), ...)

$$V_D = \frac{g_\alpha^2}{2}\,(D^\alpha)^2 = \frac{g_\alpha^2}{2}\,\left(K_i\,(T^\alpha)_{\bar{\imath}j}\,z^j\right)^2 = \frac{g_\alpha^2}{2}\,\left(\frac{3\,\bar{z}^{\bar{\imath}}\,(T^\alpha)_{\bar{\imath}j}\,z^j}{Y}\right)^2$$

This potential is scale invariant in case the T-field, and also W, are singlets under the U(1) gauge symmetry.

However in the case, which we will consider in the following, the superpotential transforms non-trivially, and hence W will contribute to the D-term.

$\Rightarrow$ Fayet-Iliopoulos term of N=1 supergravity!

There are 3 possible gaugings that leave G invariant:

$$(a)\ U(1)_R\ :\ z \to e^{iw} z$$

Classical scale invariant De Sitter potential, however the U(1) gauge symmetries posses quantum anomalies.

$$(b)\ O(1,1)_d :\ \ z \to e^{\sigma} z\ \ \text{and}\ \ T \to e^{2\sigma} T$$

$$(c)\ R_t\ :\ T \to T + is$$

Quantum non-scale invariant inflationary potential, the U(1) gauge symmetry is anomaly free.

(a): $U(1)_R \longrightarrow D_R = \mathbf{G}_i (qz)^i = q\left(K_z z + \frac{W_z z}{W}\right)$

(ii) $W = h\, z^3$: $\quad (H^2 = 3h^2, \quad G^2 = 9\frac{g^2\, q^2}{2})$

$$V = V_F + V_D = H^2\, |\Phi|^4 + G^2\, \left(1 + |\Phi|^2\right)^2$$

Cosmological constant: $\mu^2 = G^2$

(iii) $W = h\, z\, T$: $\quad (H^2 = \frac{h^2}{12}, \quad G^2 = 3\, \frac{g^2 q^2}{2})$

$$V = V_F + V_D = \{H^2 + \frac{1}{3}G^2\} + 4H^2\, B^2 + (G^2 - H^2)\, \{2|\Phi|^2 + 3|\Phi|^4\}$$

Stable for $G^2 \geq H^2$

Cosmological constant: $\mu^2 = H^2 + \frac{1}{3}G^2$

Consistent realizations of no-scale de Sitter models!

3.3 Scale violating terms and inflation

$$U(1)_t \,:\quad T \to T + i\xi q w\,, \qquad Z \to e^{iqw} Z$$

Gauging of a non-anomalous axionic shift symmetry.

This is not an arbitrary choice but a necessary anomaly free condition, which must be valid at the quantum level.

$$U(1)_R \;\to U(1)_t$$

$$\mathcal{R}^2 \;\to \mathcal{R}^2 + \xi\mathcal{R}, \qquad \xi = \mathrm{Tr}\, Q_R$$

Q_R : charges with respect to anomalous $U(1)_R$

(ii) $W = h\, z^3$:

$$D_t = 3q\left(|\Phi|^2 + 1 - \frac{\xi}{Y}\right)$$

$$V = H^2|\Phi|^4 + G^2\left(|\Phi|^2 + 1 - \frac{\xi}{Y}\right)^2$$

$$(H^2 = 3h^2, \quad G^2 = 9\frac{g^2\, q^2}{2})$$

Inflationary, Starobinsky like potential.

IV) String induced $\mathcal{R}^2$ models

- Lagrange multiplier Y (no scale modulus ϕ) :

Volume of six-dimensional space.

- Superpotential & F-terms: Tree level, Internal fluxes.

- Gaugings & D-terms:

$U(1)_t$ gauging is not choice but a stringy necessity.

The anomaly free combination always exists.

Scale breaking term ξ (quantum anomaly coefficient) is computable from chiral spectrum.

(Ibanez, Quevedo (1999))

Prototype string compactification:

- Heterotic on $T^2 \times T^2 \times T^2/(\mathbb{Z}_2 \times Z_2)$ orbifold.
- Type IIB with D5/D5/D5/D9-branes.

$$S = \int d^4x\sqrt{|g|}\,\left[\frac{1}{2}\mathcal{R} - \frac{\partial_\mu S\,\partial^\mu \bar{S}}{(S+\bar{S})^2} - \frac{\partial_\mu T^A\,\partial^\mu \bar{T}^A}{(T^A+\bar{T}^A)^2} - \frac{\partial_\mu U^A\,\partial^\mu \bar{U}^A}{(U^A+\bar{U}^A)^2} + \ldots - V_F - V_D\right]$$

$T^A \quad (A=1,2,3):$ Kähler moduli

$S, U^A \quad (A=1,2,3):$ Dilaton, complex structure moduli

fixed by fluxes or non-perturbative effects

Conformal transformation of the metric $\Rightarrow$ Jordan frame:

$$\left[\frac{1}{2}\prod_A (T_A+\bar{T}_A)^{1/3}\left(\mathcal{R} + \frac{2}{3}A_\mu A^\mu\right) - J_\mu A^\mu\right] + \cdots - \prod_A (T_A+\bar{T}_A)^{2/3}(V_F+V_D)_E$$

There are no kinetic terms for the three T-fields.

The previous SUGRA case is obtained by

$$-\frac{\partial_\mu T^A\,\partial^\mu \bar{T}^A}{(T^A+\bar{T}^A)^2} \longrightarrow -3\,\frac{\partial_\mu T\,\partial^\mu \bar{T}}{(T+\bar{T})^2}\,, \quad \text{with} \quad K=-3\log(T+\bar{T})$$

Inclusion of matter fields:

$$K=-\log Y_1-\log Y_2-\log Y_3, \qquad Y_A=(T_A+\bar{T}_A)-|z^i_A|^2$$

Generic (tree level) superpotential:

$$W=d^{ABC}_{ijk}\,z^i_A\,z^j_B\,z^k_C=h\,z_1\,z_2\,z_3$$

There are three distinct $U(1)^A_R$, one for each plane.

They are all anomalous !

Gauging of three $U(1)^A_t \Rightarrow$ three scale violating D-terms.

3-field inflaton potential:

$$
\begin{aligned}
V &= h^2\Big(|\Phi_2\,\Phi_3|^2 + |\Phi_3\,\Phi_1|^2 + |\Phi_1\,\Phi_3|^2\Big) \\
&+ G^2\Big(|\Phi_1|^2 + |\Phi_2|^2 + |\Phi_3|^2 + 3 - \frac{\xi_1}{Y_1} - \frac{\xi_2}{Y_2} - \frac{\xi_3}{Y_3}\Big)^2
\end{aligned}
$$

Slow roll transition from de Sitter to Minkowski space.

IV) Summary

Systematic study of scale invariant (super)gravity coupled to matter:

- Scale invariant $\mathcal{R}^2$ gravity is ghost free and is equivalent to (anti) de Sitter in Einstein gravity
- Scale invariance is broken by quantum effects

  $\Rightarrow$ Einstein term in Jordan frame.

  $\Rightarrow$ Inflationary potential in Einstein frame.
- It is easy to get stable de Sitter and inflation in SUGRA after inclusion of D-term (against many prejudices).
- String theory is in disguise an $\mathcal{R}^2$ theory with broken scale invariance at the quantum level.
- How our description is compared to the $\mathcal{R}^2$ SUGRA theories of Ferrara, Porrati et. al, where one needs at least two chiral multiplets in the old minimal formalism?