# Non-commutative IIA and IIB geometries from $Q$-branes and their intersection 

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## Spacetime geometry "seen" by point particles

- general relativity: spacetime = smooth manifold



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- moduli stabilization
- effective cosmological constant
- spontaneous SUSY breaking


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## NS 5-brane

- brane charged under the Kalb-Ramond field $B$

|  | uncompact |  |  |  | compact on torus $y^{i} \sim y^{i}+2 \pi$ |  |  |  |  |  |  |
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- vanishing $B$-field and dilaton
- geometric background
- $A_{2}$ and $A_{3}$ components of one-form gauge field
- we choose gauge $A_{3}=0$
- remaining component $A_{2}$ ( $=B_{y^{1}, y^{2}}$ of NS 5-brane) is connected with $h$

$$
\partial_{y^{3}} A_{2}=\partial_{x^{3}} h
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- non-geometric background:
already considered by
(E.Lozano-Tellechea, T. Ortin, 2001) (J. de Boer, M. Sigemori, 2010)

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- simplifies calculations considerably


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intersecting NS5-branes, KK-monopoles, $Q$-branes and $R$-branes

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- intersecting branes via "harmonic superposition rules" (A.A. Tseytlin, 1996)


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(C. Kounnas, D. Lust, P.M. Petropoulos, D. Tsimpis, 2007)

|  | $x^{0}$ | $x^{1}$ | $x^{2}$ | $x^{3}$ | $y^{1}$ | $y^{2}$ | $y^{3}$ | $y^{4}$ | $y^{5}$ | $y^{6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NS5 | $\bigotimes$ | $\bigotimes$ | $\bigotimes$ |  | $\bigotimes$ |  | $\bigotimes$ |  | $\bigotimes$ |  |
| NS5 $^{\prime}$ | $\bigotimes$ | $\bigotimes$ | $\bigotimes$ |  | $\bigotimes$ |  |  | $\bigotimes$ |  | $\bigotimes$ |
| NS5" $^{\prime \prime}$ | $\bigotimes$ | $\bigotimes$ | $\bigotimes$ |  |  | $\bigotimes$ |  | $\bigotimes$ | $\bigotimes$ |  |
| NS5'"' $^{\prime \prime}$ | $\bigotimes$ | $\bigotimes$ | $\bigotimes$ |  |  | $\bigotimes$ | $\bigotimes$ |  |  | $\bigotimes$ |

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- $h(r)=H x^{3}$ according to "harmonic superposition rules"

$$
H_{y^{2}, y^{4}, y^{6}}=H_{y^{2}, y^{5}, y^{3}}=H_{y^{1}, y^{6}, y^{3}}=H_{y^{1}, y^{5}, y^{4}}=H
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- in near horizon limit $x^{3} \rightarrow 0$ we get $\operatorname{AdS}_{4} \times \mathrm{T}^{6}$


## 4 Q-branes (IIA)

- T-Duality along $y^{1}, y^{2}, y^{3}$ and $y^{4}$ (isometries)

|  | $x^{0}$ | $x^{1}$ | $x^{2}$ | $x^{3}$ | $y^{1}$ | $y^{2}$ | $y^{3}$ | $y^{4}$ | $y^{5}$ | $y^{6}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\bigotimes$ | $\bigotimes$ | $\bigotimes$ |  | $\bigotimes$ |  | $\bigotimes$ |  | $\bigotimes$ |  |
|  | $\bigotimes$ | $\bigotimes$ | $\bigotimes$ |  | $\bigotimes$ |  |  | $\bigotimes$ |  | $\bigotimes$ |
|  | $\bigotimes$ | $\bigotimes$ | $\bigotimes$ |  |  | $\bigotimes$ |  | $\bigotimes$ | $\bigotimes$ |  |
|  | $\bigotimes$ | $\bigotimes$ | $\bigotimes$ |  |  | $\bigotimes$ | $\bigotimes$ |  |  | $\bigotimes$ |

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| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $Q$ | $\bigotimes$ | $\bigotimes$ | $\bigotimes$ |  | $\bigotimes$ | $\bullet$ | $\bigotimes$ | $\bullet$ | $\bigotimes$ |  |
| $Q^{\prime}$ | $\bigotimes$ | $\bigotimes$ | $\bigotimes$ |  | $\bigotimes$ | $\bullet$ | $\bullet$ | $\bigotimes$ |  | $\bigotimes$ |
| $Q^{\prime \prime}$ | $\bigotimes$ | $\bigotimes$ | $\bigotimes$ |  | $\bullet$ | $\bigotimes$ | $\bullet$ | $\bigotimes$ | $\bigotimes$ |  |
| $Q^{\prime \prime \prime}$ | $\bigotimes$ | $\bigotimes$ | $\bigotimes$ |  | $\bullet$ | $\bigotimes$ | $\bigotimes$ | $\bullet$ |  | $\bigotimes$ |

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| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $Q$ | $\otimes$ | $\otimes$ | $\otimes$ |  | $\otimes$ | - | $\otimes$ | $\bullet$ | $\otimes$ |  |
| $Q^{\prime}$ | $\otimes$ | $\otimes$ | $\otimes$ |  | Q | $\bullet$ | - | Q |  | $\otimes$ |
| $Q^{\prime \prime}$ | $\otimes$ | Q | $\otimes$ |  | - | Q | $\bullet$ | Q | Q |  |
| $Q^{\prime \prime}$ | $\otimes$ | Q | $\otimes$ |  | - | $\otimes$ | $\otimes$ | $\bullet$ |  | $\otimes$ |

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| $Q^{\prime \prime \prime}$ | $\bigotimes$ | $\bigotimes$ | $\bigotimes$ |  | $\bullet$ | $\bigotimes$ | $\bigotimes$ | $\bullet$ |  | $\bigotimes$ |

- non-geometric configuration
- near horizon limit with $x=1+Q^{2}\left(\left(y^{5}\right)^{2}+\left(y^{6}\right)^{2}\right)$

$$
\begin{aligned}
& d s_{4 Q \mathrm{int}}=\frac{1}{x} \sum_{i=1}^{4}\left(d y^{i}\right)^{2}+\sum_{j=5,6}\left(d y^{j}\right)^{2} \\
& -B_{24}=B_{13}=\frac{Q y^{6}}{x} \quad B_{14}=B_{23}=\frac{Q y^{5}}{x}
\end{aligned}
$$

## Field redefinition leads to

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- $\tilde{G}$ and $\beta$ have the same form as $g$ and $B$ of 4 NS 5 -branes
- globally well defined representation
- in near horizon limit: flat torus with four $Q$-fluxes

$$
Q_{6}^{24}=-Q_{6}^{13}=-Q_{5}^{14}=-Q_{5}^{23}=Q,
$$

and IIA superpotential

$$
W_{Q}=Q_{6}^{24} S T_{1} T_{2}+Q_{5}^{23} T_{1} T_{2} U_{1}+Q_{5}^{14} T_{1} T_{2} U_{2}+Q_{6}^{13} T_{1} T_{2} U_{3}
$$

## 1 H-flux, $1 Q$-flux and $2 f$-fluxes (IIA)

- T-Duality along $y^{1}$ and $y^{3}$

|  | $x^{0}$ | $x^{1}$ | $x^{2}$ | $x^{3}$ | $y^{1}$ | $y^{2}$ | $y^{3}$ | $y^{4}$ | $y^{5}$ | $y^{6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{NS} \mathrm{5}^{\prime}$ | $\bigotimes$ | $\bigotimes$ | $\bigotimes$ |  | $\bigotimes$ |  | $\bigotimes$ |  | $\bigotimes$ |  |
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| $Q^{\prime \prime}$ | $\bigotimes$ | $\bigotimes$ | $\bigotimes$ |  |  | $\bigotimes$ |  | $\bigotimes$ | $\bigotimes$ |  |
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| $\mathrm{NS} \mathrm{5}^{\prime}$ | $\bigotimes$ | $\bigotimes$ | $\bigotimes$ |  | $\bigotimes$ |  | $\bigotimes$ |  | $\bigotimes$ |  |
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| $Q^{\prime \prime}$ | $\bigotimes$ | $\bigotimes$ | $\bigotimes$ |  | $\bullet$ | $\bigotimes$ | $\bullet$ | $\bigotimes$ | $\bigotimes$ |  |
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| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{NS} \mathrm{5}^{5}$ | $\bigotimes$ | $\bigotimes$ | $\bigotimes$ |  | $\bigotimes$ |  | $\bigotimes$ |  | $\bigotimes$ |  |
| $\mathrm{KK}^{\prime}$ | $\bigotimes$ | $\bigotimes$ | $\bigotimes$ |  | $\bigotimes$ |  | $\bullet$ | $\bigotimes$ |  | $\bigotimes$ |
| $Q^{\prime \prime}$ | $\bigotimes$ | $\bigotimes$ | $\bigotimes$ |  | $\bullet$ | $\bigotimes$ | $\bullet$ | $\bigotimes$ | $\bigotimes$ |  |
| $\mathrm{KK}^{\prime \prime \prime}$ | $\bigotimes$ | $\bigotimes$ | $\bigotimes$ |  | $\bullet$ | $\bigotimes$ | $\bigotimes$ |  |  | $\bigotimes$ |

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does not give globally well defined $\tilde{G}$ and $\beta$

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- non-geometric background
- BUT: field redefinition

$$
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$$

does not give globally well defined $\tilde{G}$ and $\beta$
We need a more general field redefinition with the corresponding fluxes and superpotentials!

## Conclusions

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> When you are curious about $Q$ - and $R$-branes, you can have a look at arXiv:1303.1413 (F. Hasier, D. Lüst, 2013)

