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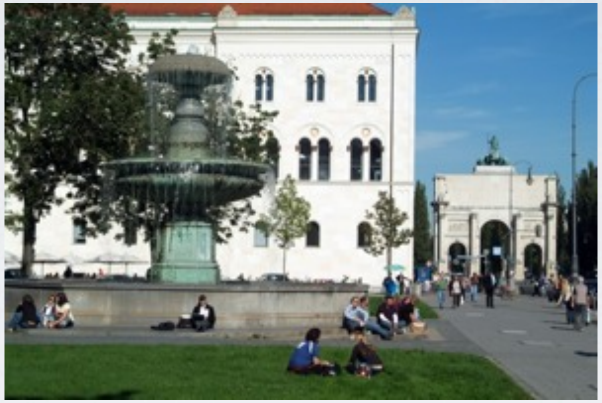

# Supersymmetric Completions and Generalizations of R+R² Gravity


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My talk is based on recent work with R. Kallosh, A. Linde, M. Porrati, A. van Proeyen and it describes a class of "minimal models" for inflation which seem to be supported by recent data from Planck satellite experiment.

These models are inspired by the supersymmetric extension of the $R + R^2$ theories of gravity (called Starobinsky model) and their generalization where higher curvature terms $R^4 + \ldots$ are added. These may be considered as particular cases of the so called $f(R)$ gravity

In these theories the "inflaton" is identified with the "scalaron", a pure gravitational degree of freedom which is actually "dual" to a standard "canonical" scalar field with a "potential"

The emergence of a new degree of freedom come from the observation that, at the quadratic level, (K. Stelle) if we add to the Einstein term terms quadratic in the scalar curvature $R$ and Weyl tensor $W_{\alpha\beta\gamma\delta}$

$$\mathcal{L} = \gamma R_E + \alpha R^2 + \beta W_{\alpha\beta\gamma\delta}^2$$

the spectrum of the theory contains, other than the massless graviton, two extra states, a spin 2 ghost with $m^2 = \frac{-\gamma}{2\beta}$ $(\beta < 0)$ and a spinless (non-ghost) with $m^2 = \frac{\gamma}{12\alpha}$ $(\alpha > 0)$

Pioneering work on $R+R^2$ theories

in supergravity dates back to late 70's

S.F. B. Zumino (NPB 134, 1978, 301)

S.F. Grisaru, van Nieuwenhuizen (NPB 138, 1978, 430)

De Wit, van Holten (NPB 155, 1979, 530)

De Wit, van Holten, van Proeyen (NPB 167, 1980) 186

De Wit, S.F. (Phys Lett B, 1979, 317)

The above refs are for linearized result

where the spectrum was found.

At the non-linear level N=1 theories

with their standard supergravity dual

were found by

S. Cecotti, Phys. Lett B ~~190~~ (1987) 86

S. Cecotti, S.F., M. Porrati, S. Sabharwal (NPB 306, 1988, 160

Today these models have become

fashionable as models for the inflaton

potential in cosmology

Basic observation:

The massive excitations of $R + R^2$ (super)-gravity theories exhaust the off-shell degrees of freedom of the gauge-excitations when they are counted as massive representations. (De Wit, S.F.)

Pure gravity:

$g_{\mu\nu}$ (10 degrees of freedom) − (4 diffeom.) = 6 off-shell degrees of freedom

$$6 = \underbrace{1}_{R^2} + \underbrace{5}_{W^2}$$

scalaron    spin 2 ghost

Similar counting works in supersymmetry

$N=1$ supergravity (minimal formulation)

Off-shell 6 12 6 (12+12)
graviton gravitino auxiliary fields

On-shell $N=1$ $R+R^2$ supergravity

$W^2$ spin 2 ghost $2, 2(\frac{3}{2}), 1$ $8+8$

spin 1 physical $1, 2(\frac{1}{2}), 0$ $4+4$

$R^2$ (new minimal)

2 chiral multiplets $2(\frac{1}{2}, 2(0))$ $4+4$

(old minimal)

(De Wit, van Holten; S.F. van Proeyen)

$N=2$ supergravity ("new" minimal)

Structure of gravity gauge fields

$(g_{\mu\nu}, \psi_\mu, B_\mu) + \underset{\text{auxiliaries}}{- - - - -}$

$40+40 = (24+24)+(8+8)+(8+8)$

$W_{eyl}^2 \to 2 \; 4(\tfrac{3}{2}) \; (5+1)(1), \; 4(\tfrac{1}{2}), 0$

$24+24$ massive spin 2 ghost

$R^2 \to 2\,(1, 4(\tfrac{1}{2}), 5(0))$ massive spin 1 physical

Standard $N=2$ supergravity theory

a quaternionic manifold of $n_q = 2$

with Higgsed phase $(U(1) \times U(1))$

Recent work (cosmology and post Planck)

S. Ketov

S. Ketov, Starobinsky

Kallosh, Linde

Ellis, Nanopoulos, Olive

Farakos, Kehagias, Riotto

S.F., Kallosh, Linde, Porrati

S.F., Kallosh, Van Proeyen

S.F., Kehagias, Porrati

Standard supergravity dual of $R + R^2$

theories.

The last particle, the scalaron, can be studied at the full non-linear level by considering the $R^2$ term added to the Einstein-Hilbert action

$$\mathcal{L} = -\frac{1}{2} R\sqrt{g} + \alpha R^2 \sqrt{g}$$

and rewriting in the equivalent form

$$\frac{1}{\sqrt{g}} \mathcal{L}_{\sigma,\Lambda} = -\frac{1}{2} R + \frac{1}{2}\sigma(\Lambda - R) + \alpha \Lambda^2$$

Now we observe that we have a Jordan frame function $(1+\sigma)$

$$-\frac{1}{2}(1+\sigma) R + \frac{1}{2}\sigma\Lambda + \alpha\Lambda^2$$

By going to the Einstein frame by using the identity

$$-e' R(e')(1+\sigma) = e\left(-R(e) - \frac{3}{2}\left(\partial_\mu \lg(1+\sigma)\right)^2\right)$$

$$(e'_{a\mu} = e_{a\mu}(1+\sigma)^{-1/2})$$

and integrating over $\Lambda$ we obtain

$$\frac{1}{\sqrt{g}}\mathcal{L} = -\frac{1}{2}R - \frac{3}{4}\left(\partial_\mu \log(1+\sigma)\right)^2 - \frac{1}{16\alpha}\frac{\sigma^2}{(1+\sigma)^2}$$

by defining $(1+\sigma) = -C = +\exp\sqrt{\frac{2}{3}}\varphi$

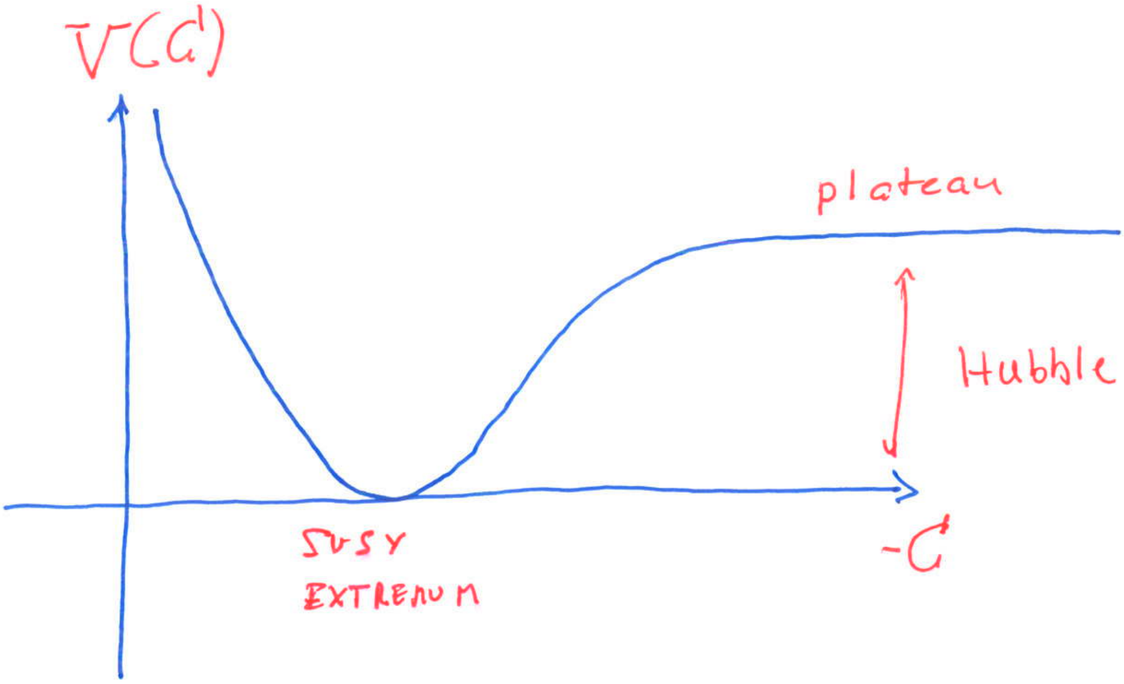
we end up with the scalar potential

$$\frac{1}{\sqrt{g}}\mathcal{L} = -\frac{1}{2}R - \frac{1}{2}(\partial\varphi)^2 - \frac{1}{16\alpha}\left(1 - \exp -\sqrt{\tfrac{2}{3}}\varphi\right)^2$$

which has a plateau (De Sitter c.c. $= \frac{1}{16\alpha}$)

with a slow roll toward $\varphi = 0$

(Minkowski vacuum).



It is clear that the supersymmetric extension of the $R+R^2$ theory requires an "off-shell" formulation of the theory. The minimal set of auxiliary fields is "six bosonic degrees of freedom" since the count of graviton and gravitino degrees of freedom (taking into account of local supersymmetry and diffeomorphism) is

graviton $= 10-4=6$ gravitino $= 16-4=12$

so off-shell $n_f - n_b = 6$

The two candidates are

(old minimal) $\underset{(2)}{u=S+iP}$, $\underset{(4)}{A_\mu}$ dof

(new minimal) $\underset{(3)}{B_{\mu\nu}}$, $\underset{(3)}{A_\mu}$ dof

(with gauge invariances $\delta B_{\mu\nu}=\partial_\mu \xi_\nu - \partial_\nu \xi_\mu$, $\delta A_\mu = \partial_\mu \xi$)

In particular this implies that matter must have a conserved current.

The linearized analysis for supersymmetric $R + R^2$ theory was performed in 1978 (S.F., Grisaru, van Nieuwenhuizen) and at the non-linear level by Cecotti (1987) for the old minimal set of auxiliary fields. For the "new minimal" set it was performed by (Cecotti, SF, Porrati, Sabharwal) both at linear and non linear level.

The outcome was that the supersymmetric extension of the "Scalaron" mode required in both formulation an equal number of degrees of freedom (4b, 4f) but differently distributed:

Old Minimal: two chiral multiplets with the same mass $m^2 = \frac{\gamma}{12\alpha}$ $(\frac{1}{2}, 0^+, 0^-)$

New minimal: a massive Vector multiplet with $m^2 = \frac{\gamma}{12\alpha}$ $(1, 2(\frac{1}{2}), 0)$

If one add the superWeyl square term the spectrum is completed, in both cases, by a ghost spin 2 massive multiplet

$$\left(2, \tfrac{3}{2}, \tfrac{3}{2}, 1\right) \quad \text{with} \quad m^2 = \frac{-\gamma}{2\beta}$$

It is now instructive to understand the source of these degrees of freedom -

All fermionic degrees of freedom come from the high order gravitino eqs. of motion. The spin $\frac{3}{2}$ spectrum (one massless and two ghost massive) is the same in the two theories.

What is changing is the number of scalars: (3 + scalaron) in old minimal + a spin 1 ghost - scalaron in new minimal + a spin 1 ghost (to complete the spin 2 ghost multiplet) + a spin 1 physical.

# Supercurvatures:

## Old minimal:

two chiral superfields $\mathcal{R}$ , $W_{\alpha\beta\gamma}$

(curvature scalar + Weyl)

$$\mathcal{R} = (S - iP) + \ldots \theta^2 (R + i\, \partial \cdot A)$$

$$W_{\alpha\beta\gamma} = \ldots \theta^\delta W_{\alpha\beta\gamma\delta} + \theta_\alpha F_{\beta\gamma}(A) + \ldots$$

$E_{\alpha\dot\alpha}$ (Einstein real multiplet)

(not relevant because $E^2_{\alpha\dot\alpha}$ is related to the others by super Gauss-Bonnet)

$$\alpha\, \mathcal{R}\bar{\mathcal{R}}\Big|_D \longrightarrow \alpha \left[ (\partial_\mu S)^2 + (\partial_\mu P)^2 - (\partial \cdot A)^2 + R^2 \right]$$

$$\beta\, W^2_{\alpha\beta\gamma}\Big|_F \longrightarrow (W_{\alpha\beta\gamma\delta})^2 + F^2_{\mu\nu}(A)$$

$$\gamma\, \mathcal{L}_E \longrightarrow \gamma \left[ \frac{1}{2} R - \frac{1}{3}(S^2 + P^2 - A_\mu^2) \right]$$

# Supercurvatures

## New Minimal

two chiral superfields: $W_\alpha$, $W_{\alpha\beta\gamma}$

as before but now the scalar curvature

is a chiral spinor superfield -

The Einstein multiplet is linear and real

but not relevant for our discussion

The basic object are now the gauge

auxiliary vector field $A_\mu$ and the

$B_{\mu\nu}$ field. The latter must enter

in the combination

$$V_\mu = \epsilon_{\mu\nu\rho\sigma} \partial^\nu B^{\rho\sigma} \quad (\partial^\mu V_\mu = 0)$$

because of gauge invariance).

The presence of these fields in the

curvatures is as follows:

$$W_\alpha = \ldots \theta^\beta F_{\alpha\beta}(A+2V) + \theta_\alpha R + \ldots$$

$$W_{\alpha\beta\gamma} = \ldots \theta_{(\gamma} F_{\alpha\beta)}(A-V) + \theta^\delta W_{\alpha\beta\gamma\delta} + \ldots$$

$$\mathcal{L}_E = \gamma\left[\mathcal{L}_{SUGRA} + (2A+V)\cdot V\right]$$

$$\mathcal{L}_{R^2} = \alpha\left[F_{\mu\nu}^2 (A+2V) + R^2\right]$$

$$\mathcal{L}_{Weyl^2} = \beta\left[F_{\mu\nu}^2 (A-V) + W_{\alpha\beta\gamma\delta}^2\right]$$

By defining $A_1 = A-V$, $A_2 = A+2V$

the term in $\mathcal{L}_E$ becomes $\frac{1}{3}(A_2^2 - A_1^2)$

so if we assign positive square mass to both one is a ghost and the other is physical

$$\left(2, \tfrac{3}{2}, \tfrac{3}{2}, 1\right)$$

$A_2$ $A_1$

scalaron

$$\left(1, \tfrac{1}{2}, \tfrac{1}{2}, 0\right)$$

Other models address the inflationary cosmology with no-scale structure of certain class of theories.

(Ellis, Olive, Nanopoulos).

Recent attempts ~~to~~ use integrable systems to describe inflationary cosmology (Fre, Sorin) (Fré, Sorin, Sagnotti).

Other approaches use brane supersymmetry ~~breaking~~ to obtain potentials with inflationary behaviour

(Dudas, Kitazawa, Sagnotti)

# General $R+R^2$ theories in old minimal:

(S.F., Kallosh, van Proeyen)

From the linearized analysis we know that this theory must be a standard supergravity with two chiral multiplets

We start with (conformal compensator notation)

$$\mathcal{L} = S_0\bar{S}_0\Big|_D - h\left(\frac{R}{S_0},\frac{\bar{R}}{\bar{S}_0}\right)S_0\bar{S}_0\Big|_D + W(R/S_0)S_0^3\Big|_F$$

This is the most general $R+R^2$ theory which only involve the gravitational curvature. Cecotti minimal model is obtained, by taking $S_0\bar{S}_0 h\left(\frac{R}{S_0},\frac{\bar{R}}{\bar{S}_0}\right) = R\bar{R}$ and $W(R/S_0)=0$

The dual theory is obtained by introducing two lagrange multipliers (chiral multiplets): $\Lambda$, $A$

$$\mathcal{L}_{DUAL} = S_0\bar{S}_0\Big|_D + \Lambda\left(\frac{R}{S_0} - A\right)S_0^3\Big|_F - h(A,\bar{A})S_0\bar{S}_0\Big|_D$$

(Cecotti, Phys.Lett. 190B (1987))

$$+ W(A)S_0^3\Big|_F$$

$\frac{\delta \mathcal{L}}{\delta \Lambda} = 0$ gives back the gravitational theory

Using the scalar trick we instead use the identity

$$\Lambda S_0 \Sigma \bar{S}_0 + h.c.\Big|_F = (\Lambda + \bar{\Lambda})S_0\bar{S}_0\Big|_D$$

and then rewrite the standard supergravity expression

$$\mathcal{L} = S_0\bar{S}_0\left(1 + \Lambda + \bar{\Lambda} - h(A,\bar{A})\right)\Big|_D + \Lambda A S_0^3 + W(A)S_0^3\Big|_F$$

This is the most general theory which embeds in supergravity $R + R^2$. Note that if we set $h(A,\bar{A}) = 0$ the $A$ multiplet is no longer dynamical and we obtain

The lagrangian $\mathcal{L}_D$ has a hidden symmetry that allows to remove $W(A)$.

In fact by writing $W(A) = A\,g(A) + \lambda$ ($\lambda$ constant) the $F$ term becomes

$$\left(-A(\Lambda - g(A)) + \lambda\right) S_0^3 \Big|_F$$

and then we note that the Lagrangian is invariant under the following transformation

$$\Lambda \to \Lambda + f(A)$$
$$g(A) \to g(A) + f(A)$$
$$h(A,\bar{A}) \to h(A,\bar{A}) + f(A) + \bar{f}(\bar{A}).$$

Taking $g(A) = -f(A)$ one gets the action

$$\mathcal{L} = S_0 \bar{S}_0 \left(1 + \Lambda + \bar{\Lambda} - h(A,\bar{A})\right)\Big|_D - \left(\Lambda A - \lambda\right) S_0^3 \Big|_F$$

$h(A,\bar{A})_{A\bar{A}} = 0$ is the condition for the $A$ field not to be dynamical. In such a case $\tilde{W}(\Lambda) = -A^2 g'(A)\Big|_{(A g(A))' = \Lambda} + \lambda$

is the superpotential for the chiral superfield $\Lambda$ which is the Legendre transform of $W(A)$ - This is the theory whose potential has been studied by Ketov.

However this theory, having only one chiral multiplet violates the linearized analysis of $R+R^2$ theory. It is in fact an $R$ theory with one more degree of freedom. Looking at the linearized analysis the chiral multiplet which becomes dynamical is the one with $S+iP$ as $\theta=0$ component which is not the scalaron multiplet.

According to the linearized analysis the partner of the $R+R^2$ "scalaron" is in fact the scalar $\partial^\mu A_\mu$ ($A_\mu$ is the Auxiliary vector of Einstein Supergravity

# $R + R^2$ theory in new minimal supergravity

Pure sugra (old minimal)

$$\mathcal{L} = -S_0 \bar{S}_0 \big|_D$$

rewrite as follows

$$\mathcal{L}(S_0, U, L_0) = -S_0 \bar{S}_0 e^{U} \big|_D + L_0 U \big|_D$$

$$(\Sigma L = \bar{\Sigma} L = 0)$$

$$\frac{\delta \mathcal{L}}{\delta L} \to U = \Sigma + \bar{\Sigma} \longrightarrow -S_0' \bar{S}_0' \big|_D$$

$$(S_0' = S_0 e^{\Sigma})$$

Dual (new minimal

$$\frac{\delta \mathcal{L}}{\delta U} = 0 \quad L_0 = S_0 \bar{S}_0 e^{U} \to \mathcal{L}(L_0) = L_0 \ln L_0 / S_0 \bar{S}_0$$

Dual (new minimal) coupled to matter

$$\mathcal{L} = -S_0 \bar{S}_0 \Phi(S, e^{gV} \bar{S}) \big|_D \longrightarrow \mathcal{L}_D = L_0 \ln L_0 / S_0 \bar{S}_0 + \frac{1}{3} L_0 f(S, e^{gV} \bar{S})$$

$$f = -3 \log \Phi$$

# $R + R^2$ THEORY (CFPS)

$$\mathcal{L} = L_0 V_R \Big|_D + \alpha W_\alpha^2 (V_R) \Big|_F \quad V_R = \ln \frac{L_0}{S_0 \bar{S}_0}$$

$$\mathcal{L}_{DUAL} = L_0 V_R \Big|_D + \alpha W_\alpha^2 (V) \Big|_F + L'(V - V_R)$$

By plugging $L_0 = S_0 \bar{S}_0 e^{V + \Lambda + \bar{\Lambda}}$ we get

$$\mathcal{L}_{DUAL} = e^{gV + \Lambda + \bar{\Lambda}} (gV + \Lambda + \bar{\Lambda}) S_0 \bar{S}_0 \Big|_D + W_\alpha^2 (V) \Big|_F$$

$$\left(\alpha = \frac{1}{2g^2}\right)$$

Massive vector $gV + \Lambda + \bar{\Lambda}$

(Stueckelberg ~~su~~ supersymmetric version)

$$\underset{m=0}{\left(\cancel{2}1, \tfrac{1}{2}\right) + \left(\tfrac{1}{2}, 0^-, 0^+\right)} \rightarrow \underset{m \neq 0}{\left(1, \tfrac{1}{2}, \tfrac{1}{2}\, 0^+\right)}$$

$0^- \rightarrow$ would be Goldstone field

$0^+ \rightarrow$ Higgs field

In our version the Higgs field is the scalaron dual the $R+R^2$ $\mathcal{J}$ function with arbitrary $\mathcal{J}(C)$. In terms of the canonically normalized $\varphi$ the lagrangian is

$$\mathcal{L} = \ldots - \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{1}{2} g^2 D(\varphi)^2$$

with $D(C(\varphi)) = D(\varphi)$

by noticing that $\left(\frac{d\varphi}{dC}\right)^2 = -\mathcal{J}''(C) = D'(C) = \frac{dD}{d\varphi}\frac{d\varphi}{dC}$

we have $\left|\frac{dD}{d\varphi}\right| = \sqrt{-D'(C)}$ , $\frac{dD}{d\varphi} = -\frac{d\varphi}{dC}$

therefore $C(\varphi) = -\int d\varphi \left(\frac{dD}{d\varphi}\right)^{-1}$

For flat space we have two solutions

1) $\mathcal{J}(C) = -\frac{m^2}{2} C$ (chaotic inflation, Linde 1983)

the other is

2) $\mathcal{J}(C) = \exp C$ (runaway potential)

A class of attractors for these models exist (Linde)

# $R^N$ corrections in Minimal Supergravity

$R^4 \to$ UNIQUE $\quad \dfrac{W_\alpha^2 \bar{W}_\alpha^2}{L_0^2} \to \xi_4 g^4 D^4 C^2$

$C \to$ scalar of the Vector multiplet (scalaron)

$R^N \to$ (BORN-INFELD TYPE) (obtained for $p=0$)

$$\left(\frac{1}{L^2} \to V^2\right) \alpha_{k\ell p} \frac{W_\alpha^2 \bar{W}_\alpha^2}{L_0^2} \sum \left(\frac{\bar{W}^2}{L_0^2}\right)^k \sum \left(\frac{W^2}{L_0^2}\right)^\ell \left(\frac{D^\alpha W_\alpha}{L_0}\right)^p$$

It produces "potential" corrections as

$$\mathcal{L}_N \sim \xi_N g^N D^N C^{N-2} \qquad 4+2k+2\ell+p = N \geq 4$$

it can be shown that these corrections are identical to the pure gravity $R^N$ corrections to the scalaron potential coming from the Lagrangian

$R + \xi_4 R^4 + \alpha R^2$ whose "dual" is

obtained from $\quad R + \sigma(R-\Lambda) + \xi_4 \Lambda^4 + \alpha \Lambda^2$

with $1+\sigma = -C$

$$V = D^2 + g\, J'(C)\, D + \sum_{N \geq 4} g^2 D^N (gC)^{N-2}$$

This is the same as in $f(R)$ gravity

$$\mathcal{L} = R + \sigma (R - \Lambda) + f(\Lambda)$$

$$\Lambda / _{1+\sigma} = D \qquad J'(C) = \left(+\frac{1}{C} + 1\right) = \frac{\sigma}{1+\sigma}$$

$$-C = 1+\sigma$$

Supersymmetric completion of $f(R)$ bosonic theory

The $R^4$ case (old minimal sugra)

(Cecotti)

$$\mathcal{R}\bar{\mathcal{R}}\,\Sigma(\mathcal{R})\bar{\Sigma}(\bar{\mathcal{R}})\Big|_D \supset R^4 + \ldots$$

$$\mathcal{R} = \ldots\ \theta^2 R \qquad \mathcal{R}\big|_{\theta=0} = S + iP = z^*$$

$$\Sigma(\mathcal{R}) = \mathcal{R} + \ldots$$

(Cremmer, Ferrara, Girardello, Van Proeyen)

More in general

$$\mathcal{R}^n\bar{\mathcal{R}}^m\,\Sigma(\mathcal{R})^p\bar{\Sigma}(\bar{\mathcal{R}})^q\Big|_D \supset R^{4+p+q-2} + \ldots$$

$(S_0 = 1)$

$(n, m, (p, q) \geq 0)$

or by introducing the compensator $S_0$

$$(S_0\bar{S}_0)^{-n-m-2p-2q}\,\mathcal{R}^n\bar{\mathcal{R}}^m\,\Sigma(\mathcal{R})^p\bar{\Sigma}(\bar{\mathcal{R}})^q\,S_0\bar{S}_0\Big|_D$$

We can now introduce two more lagrange multiplier chiral superfields

$$\mathcal{L} = \ldots + \Lambda\left(\frac{R}{S_0} - A\right) S_0^3\Big|_F + M\left(\frac{\Sigma(R)}{S_0^2} - B\right) S_0^3\Big|_F$$

$$+ \ldots$$

and using again the identity

$$M S_0 \Sigma(R)\Big|_F = \Sigma(\bar{M}\bar{S}_0 R)\Big|_F = \left(\bar{M}\frac{R}{S_0}\right) S_0 \bar{S}_0\Big|_D$$

and the fact that $A = R/S_0$ we finally obtain a "dual" superpotential term

$$W = -\Lambda A - MB$$

together with a D-term (Kahler potential

$$S_0\bar{S}_0\left(1 + \Lambda + \bar{\Lambda} + M\bar{A} + \bar{M}A + H(A,\bar{A},B,\bar{B})\right)\Big|_D$$

where

$$H(A,\bar{A},B,\bar{B}) = \sum_{\substack{n,m,p,q \geq 0 \\ n,m \geq 1}} a_{nmpq} A^n \bar{A}^m B^p \bar{B}^q \text{ the.}$$

$$H_4 = A\bar{A}B\bar{B} \quad \text{(lowest } R^4 \text{ term)}$$

$R^4$ conection.

New minimal (unique) $$\frac{W^2(L_0)\bar{W}^2(L_0)}{L_0^2}$$

Old minimal $H_{1,1}(A,\bar{A})\, B\bar{B}$ $\qquad R^4$

$$\left(H_{1,1}(A,\bar{A}) = c_{11} A\bar{A} + \dots\right)$$

$$H_{p,q}(A,\bar{A})\, B^p \bar{B}^q \qquad R^{2+p+q}$$

$$H_{0,0}(A,\bar{A}) \qquad R^2$$

Strikingly the old minimal requires two extra chiral multplets.