Old recipes for the near-equilibrium holography

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Workshop on the Fluid-Gravity Correspondence

Munich, September 2-7, 2009

Outline

Brief overview of holographic (gauge/gravity) duality approach

Finite temperature, holography and black hole physics

Transport in strongly coupled gauge theories from black hole physics

First- and second-order hydrodynamics and dual gravity

Photon/dilepton emission rates from dual gravity

Quantum liquids and holography

Other approaches

Over the last several years, holographic (gauge/gravity duality) methods were used to study strongly coupled gauge theories at finite temperature and density

These studies were motivated by the heavy-ion collision programs at RHIC and LHC (ALICE, ATLAS) and the necessity to understand hot and dense nuclear matter in the regime of intermediate coupling $\alpha_s(T_{\text{RHIC}}) \sim O(1)$

As a result, we now have a better understanding of thermodynamics and especially kinetics (transport) of strongly coupled gauge theories

Of course, these calculations are done for theoretical models such as N=4 SYM and its cousins (including non-conformal theories etc).

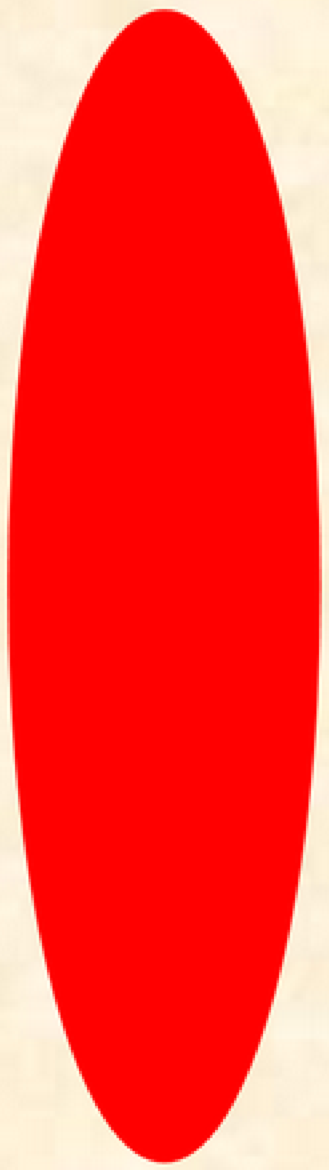
We don't know quantities such as $\frac{\eta}{s}\left(\frac{\Lambda_{\text{QCD}}}{T}\right)$ for QCD

Heavy ion collision experiments at RHIC (2000-current) and LHC (2009-??) create hot and dense nuclear matter known as the "quark-gluon plasma"

(note: qualitative difference between p-p and Au-Au collisions)

Evolution of the plasma "fireball" is described by relativistic fluid dynamics (relativistic Navier-Stokes equations)

Need to know



thermodynamics (equation of state)
kinetics (first- and second-order transport coefficients)
in the regime of intermediate coupling strength:

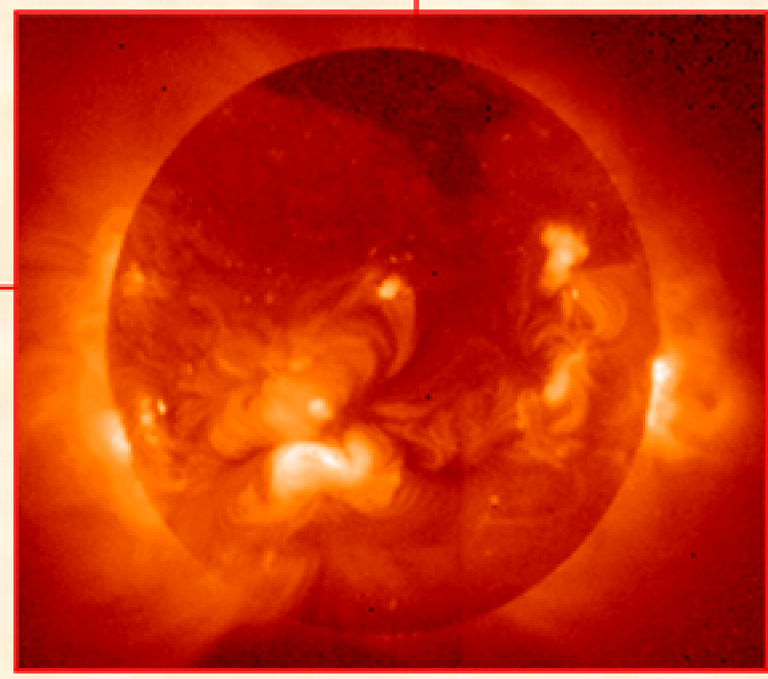
$$\alpha_s(T_{\mathrm{RHIC}}) \sim O(1)$$

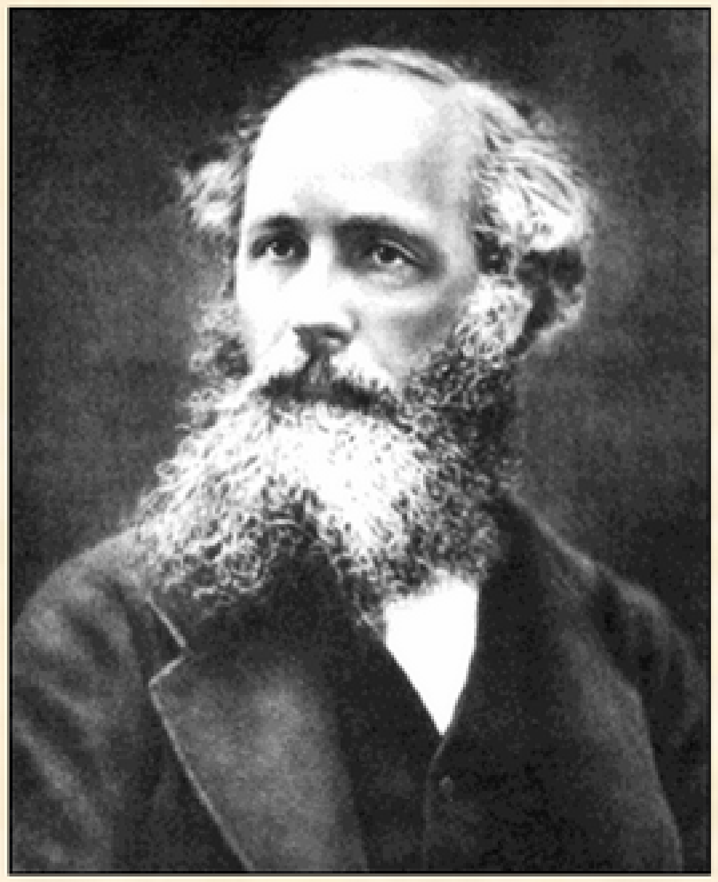
initial conditions (initial energy density profile)

thermalization time (start of hydro evolution)

freeze-out conditions (end of hydro evolution)

Quantum field theories at finite temperature/density





Equilibrium

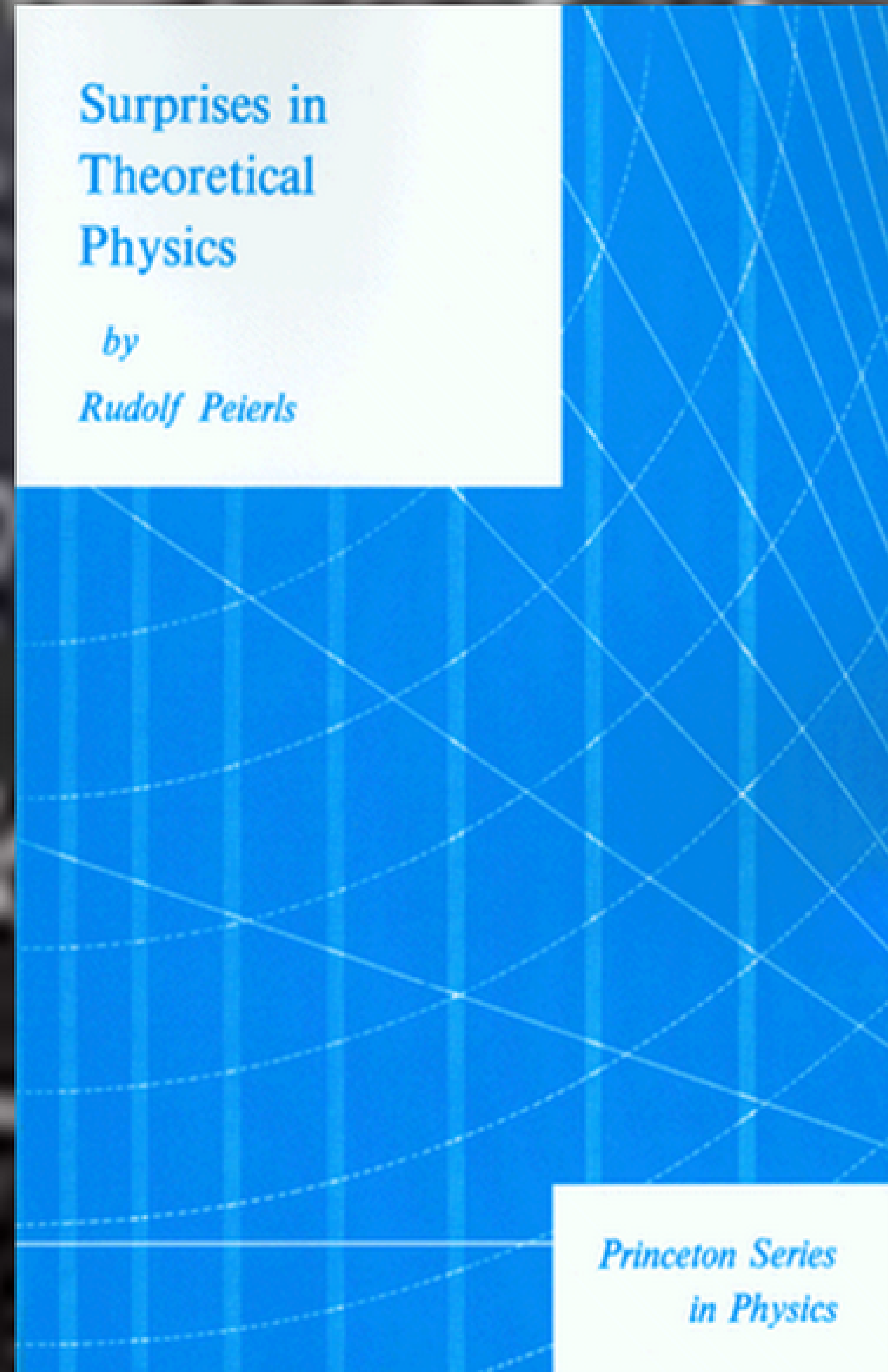
entropy
equation of state
....

- perturbative — pQCD
- non-perturbative — Lattice

Near-equilibrium

transport coefficients
emission rates
....

- perturbative — kinetic theory
- non-perturbative — ????



"For the viscosity…expansion was developed by Bogolyubov in 1946 and this remained the standard reference for many years. Evidently the many people who quoted Bogolyubov expansion had never looked in detail at more than the first two terms of this expansion. It was then one of the major surprises in theoretical physics when Dorfman and Cohen showed in 1965 that this expansion did not exist. The point is not that it diverges, the usual hazard of series expansion, but that its individual terms, beyond a certain order, are infinite.

Hydrodynamics: fundamental d.o.f. = densities of conserved charges

Need to add constitutive relations!

Example: charge diffusion

Conservation law

$$\partial_t j^0 + \partial_i j^i = 0$$

Constitutive relation

[Fick's law (1855)]

$$j_i = -D\, \partial_i j^0 + O[(\nabla j^0)^2, \nabla^2 j^0]$$

Diffusion equation

$$\partial_t j^0 = D \nabla^2 j^0$$

Dispersion relation

$$\omega = -i\, D\, q^2 + \cdots$$

Expansion parameters: $\omega \ll T\,, \quad q \ll T$

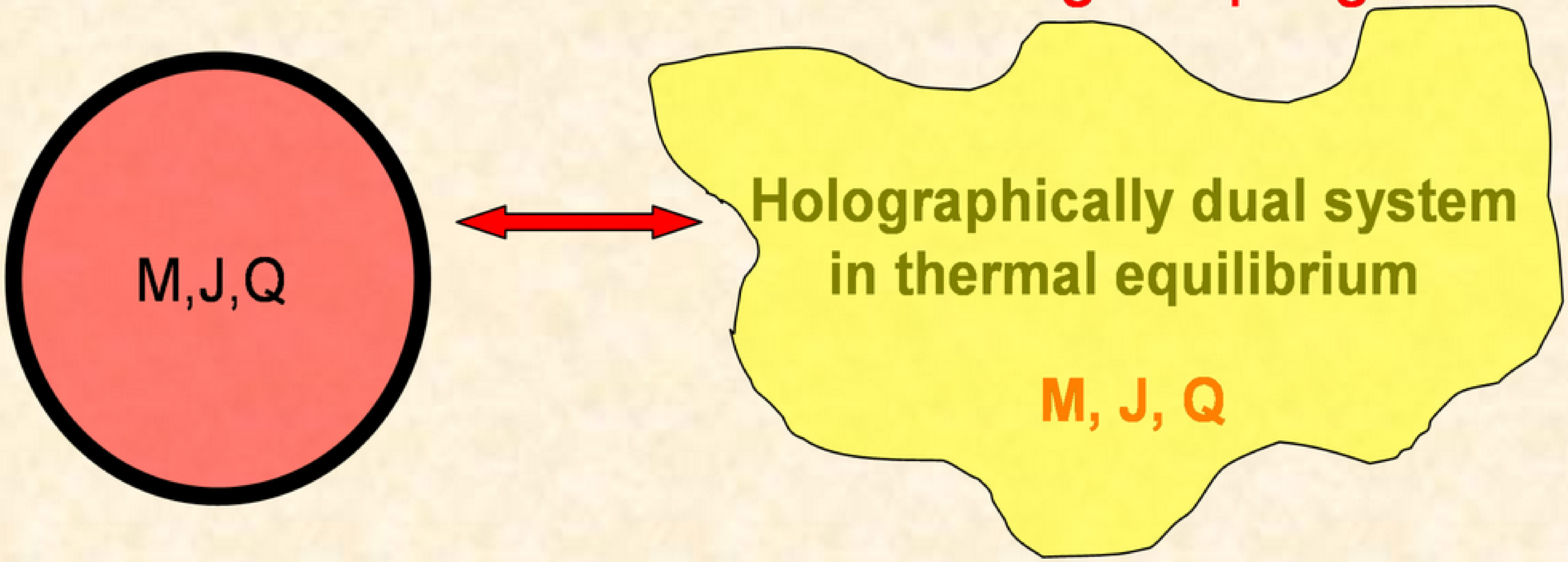
First-order transport (kinetic) coefficients

Shear viscosity	η
Bulk viscosity	ζ
Charge diffusion constant	D_Q
Supercharge diffusion constant	D_s
Thermal conductivity	κ_T
Electrical conductivity	σ

* Expect Einstein relations such as $\frac{\sigma}{e^2 \Xi} = D_{U(1)}$ to hold

10-dim gravity

4-dim gauge theory – large N, strong coupling



T_{Hawking} $S_{\text{Bekenstein-Hawking}}$ $\Longleftrightarrow$ T S

Gravitational fluctuations $\Longleftrightarrow$ Deviations from equilibrium

$g_{\mu\nu}^{(0)} + h_{\mu\nu}$ $\Longleftrightarrow$????

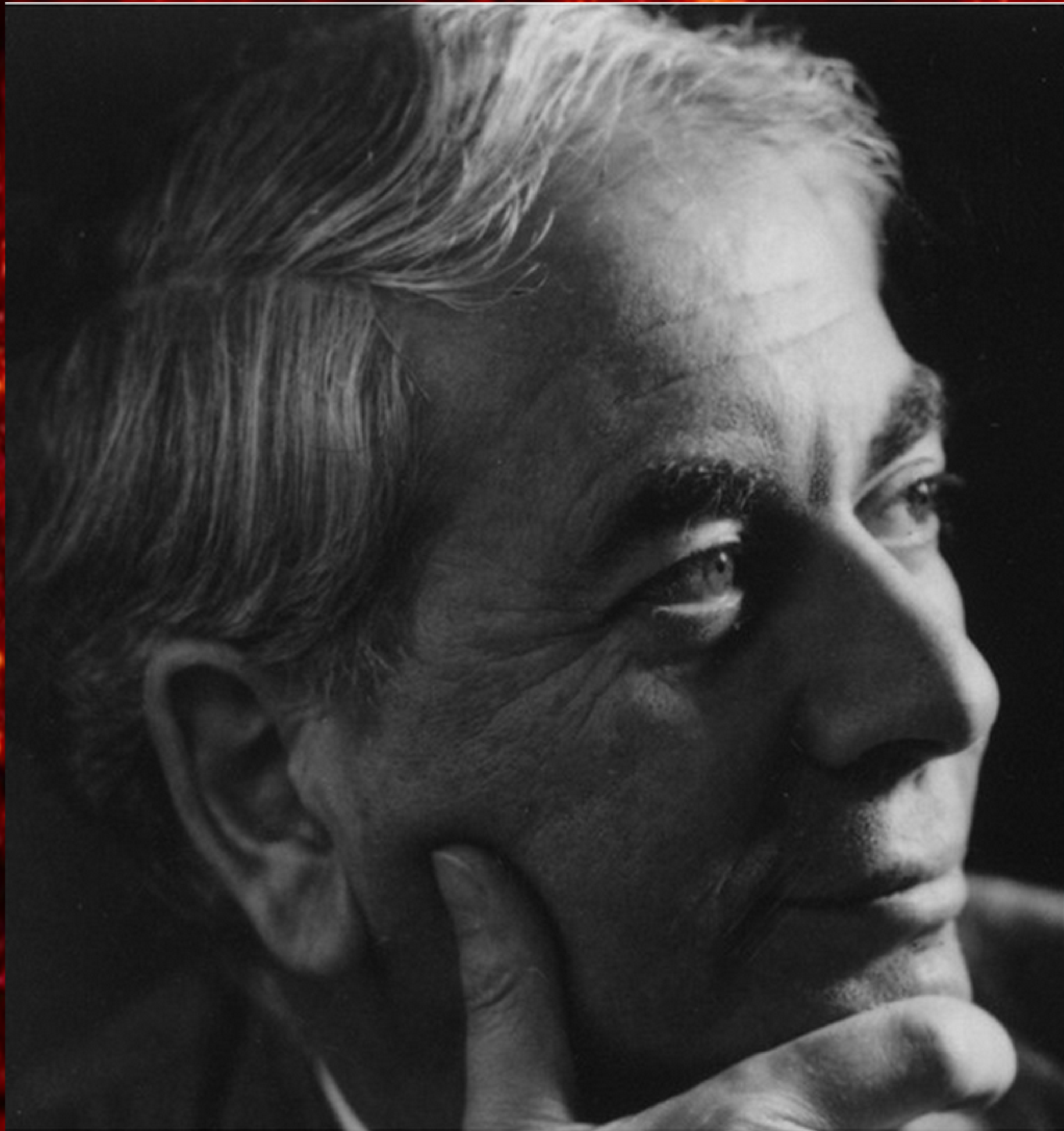
$$j_i = -D\partial_i j^0 + \cdots$$

$"\Box" \, h_{\mu\nu} = 0$ and B.C. $\Longleftrightarrow$

$$\partial_t j^0 + \partial_i j^i = 0$$

$$\partial_t j^0 = D\nabla^2 j^0$$

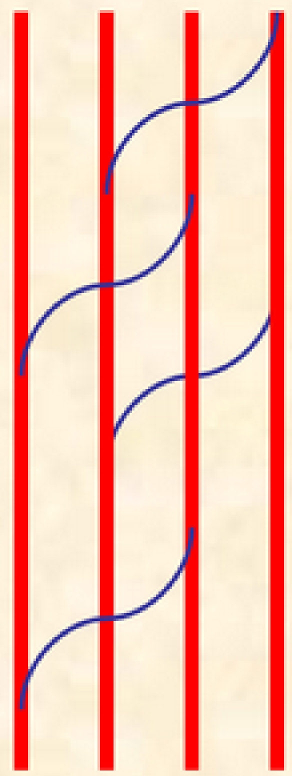
Quasinormal spectrum $\Longleftrightarrow$ $\omega = -iDq^2 + \cdots$



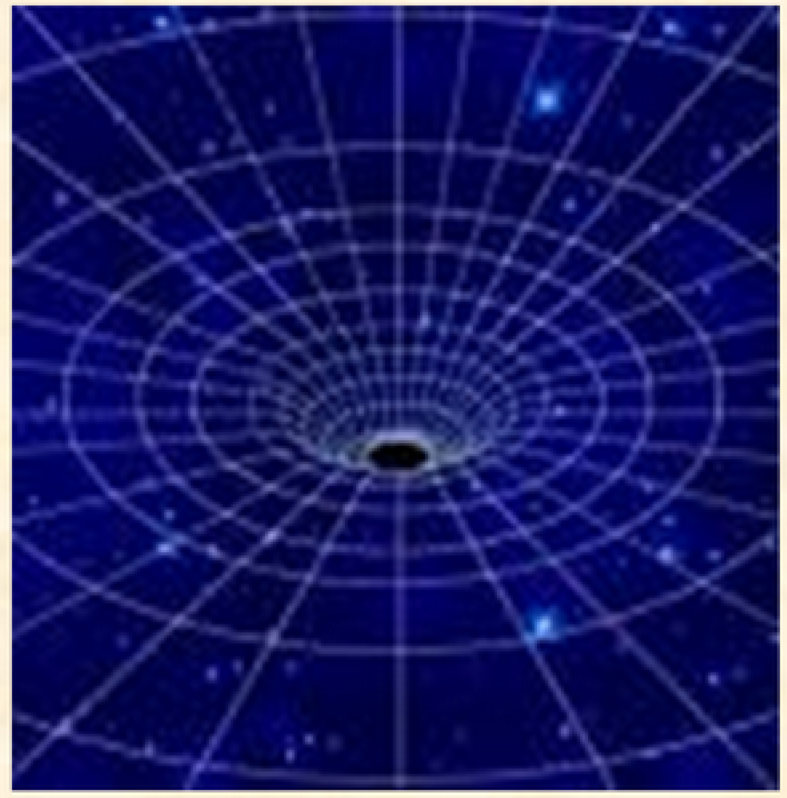
Dennis W. Sciama (1926-1999)

P.Candelas & D.Sciama, “Irreversible thermodynamics of black holes”, PRL,38(1977) 1732

From brane dynamics to AdS/CFT correspondence



$\Longleftrightarrow$



Open strings picture:
dynamics of N_c coincident D3 branes
at low energy is described by

Closed strings picture:
dynamics of N_c coincident D3 branes
at low energy is described by

$\mathcal{N}=4$ supersymmetric
$SU(N_c)$ YM theory in 4 dim

$\Longleftrightarrow$

conjectured
exact equivalence

type IIB superstring theory
on $AdS_5 \times S^5$backgrond

Maldacena (1997); Gubser, Klebanov, Polyakov (1998); Witten (1998)

AdS/CFT correspondence

$\mathcal{N}=4$ supersymmetric $SU(N_c)$ YM theory in 4 dim $\Longleftrightarrow$ type IIB superstring theory on $AdS_5 \times S^5$ backgrond

conjectured exact equivalence

Latest test: Janik'08

$$Z_{\mathrm{SYM}}[J] = \langle e^{-\int J\,\mathcal{O}\,d^4x} \rangle_{\mathrm{SYM}} = Z_{\mathrm{string}}[J]$$

Generating functional for correlation functions of gauge-invariant operators $\Longleftrightarrow$ String partition function

$\langle \mathcal{O}\ \mathcal{O}\ \cdots \mathcal{O} \rangle$

In particular

$$Z_{\mathrm{SYM}}[J] = Z_{\mathrm{string}}[J] \simeq e^{-S_{\mathrm{grav}}[J]}$$

$\lambda \equiv g_{YM}^2 N_c \gg 1$

$N_c \gg 1$

Classical gravity action serves as a generating functional for the gauge theory correlators

AdS/CFT correspondence: the role of J

$$Z_{\mathsf{SYM}}[J] = \langle e^{-\int J\,\mathcal{O}\,d^4x} \rangle_{\mathsf{SYM}} \simeq e^{-S_{\mathsf{grav}}[J]}$$

For a given operator $\mathcal{O}$, identify the source field J, e.g. $T^{\mu\nu} \Longleftrightarrow h_{\mu\nu}$

$$e^{-S_{\mathsf{grav},M}[\phi_{\mathsf{BG}}+\delta\phi]} = Z[J = \delta\phi\big|_{\partial M}]$$

$\delta\phi$ satisfies linearized supergravity e.o.m. with b.c. $\delta\phi \to \delta\phi_0 \equiv J$

The recipe:

To compute correlators of $\mathcal{O}$, one needs to solve the bulk supergravity e.o.m. for $\delta\phi$ and compute the on-shell action as a functional of the b.c. $\delta\phi_0 \equiv J$

Warning: e.o.m. for different bulk fields may be coupled: need self-consistent solution

Then, taking functional derivatives of $e^{-S_{\mathsf{grav}}[J]}$ gives $\langle \mathcal{O}\,\mathcal{O} \rangle$

Holography at finite temperature and density

$$\langle \mathcal{O} \rangle = \frac{\mathrm{tr}\rho\mathcal{O}}{\mathrm{tr}\rho} \qquad H \to T^{00} \to T^{\mu\nu} \to h_{\mu\nu}$$

$$\rho = e^{-\beta H + \mu Q} \qquad Q \to J^0 \to J^\mu \to A_\mu$$

Nonzero expectation values of energy and charge density translate into nontrivial background values of the metric (above extremality)=horizon and electric potential = CHARGED BLACK HOLE (with flat horizon)

$$ds^2 = -F(u)\,dt^2 + G(u)\left(dx^2 + dy^2 + dz^2\right) + H(u)\,du^2$$

$T = T_H$ temperature of the dual gauge theory

$$A_0 = P(u)$$

$\mu = P(\text{boundary}) - P(\text{horizon})$ chemical potential of the dual theory

Computing transport coefficients from "first principles"

Fluctuation-dissipation theory
(Callen, Welton, Green, Kubo)

Kubo formulae allows one to calculate transport coefficients from microscopic models

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d^3x e^{i\omega t} \langle \left[T_{xy}(t,x), T_{xy}(0,0) \right] \rangle$$

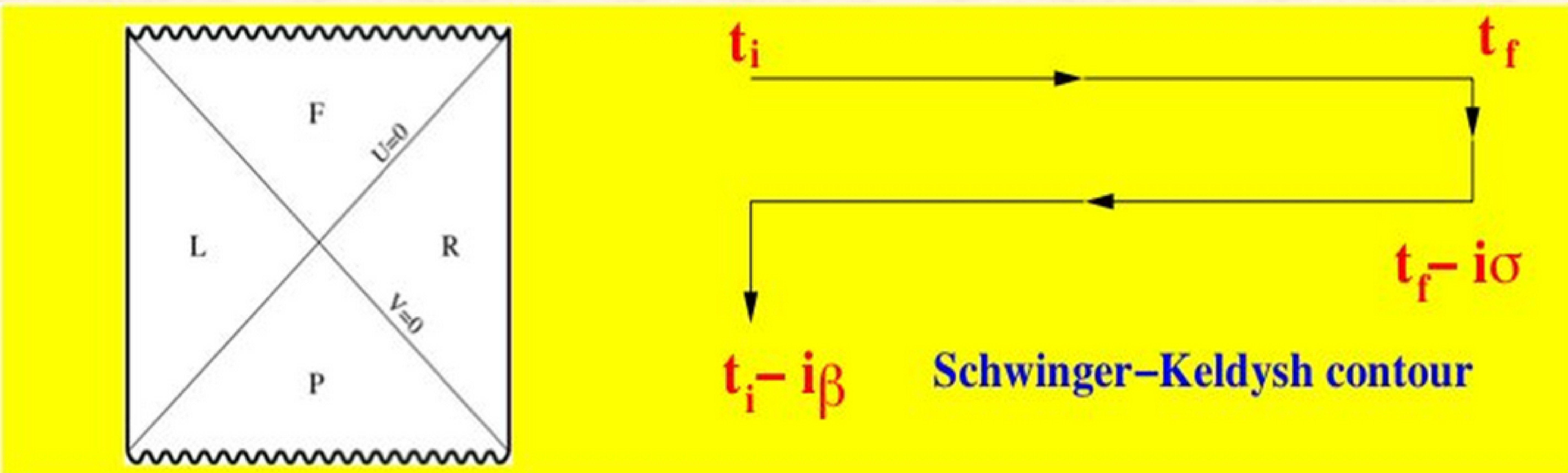
In the regime described by a gravity dual the correlator can be computed using the gauge theory/gravity duality

Computing real-time correlation functions from gravity

To extract transport coefficients and spectral functions from dual gravity, we need a recipe for computing Minkowski space correlators in AdS/CFT

The recipe of [D.T.Son & A.S., 2001] and [C.Herzog & D.T.Son, 2002] relates real-time correlators in field theory to Penrose diagram of black hole in dual gravity

Quasinormal spectrum of dual gravity = poles of the retarded correlators in 4d theory
[D.T.Son & A.S., 2001]



Computing transport coefficients from dual gravity

Assuming validity of the gauge/gravity duality,
all transport coefficients are completely determined
by the lowest frequencies
in quasinormal spectra of the dual gravitational background

(D.Son, A.S., hep-th/0205051, P.Kovtun, A.S., hep-th/0506184)

This determines kinetics in the regime of a thermal theory
where the dual gravity description is applicable

Transport coefficients and quasiparticle spectra can also be
obtained from thermal spectral functions $\chi = -2\,\mathrm{Im}\, G^R(\omega, q)$

Example: R-current correlator in $4d\ \mathcal{N}=4$ SYM

in the limit $N_c \to \infty\,, \quad g^2_{YM} N_c \to \infty$

Zero temperature: $< J_i(x)\, J_i(y) > \sim \dfrac{N_c^2}{|x-y|^6}$

$$G_E(k) = \frac{N_c^2 k_E^2}{32\pi^2} \ln k_E^2$$

$$G^{\text{ret}}(k) = \frac{N_c^2 k^2}{32\pi^2} \left(\ln |k^2| - i\,\pi\theta\,(-k^2)\ \text{sgn}\,\omega \right) \qquad k^2 = -\omega^2 + q^2$$

Finite temperature: $G^{\text{ret}}(\omega, q)$

$$G^{\text{ret}}(\omega, 0) = \frac{N_c^2 T^2}{8} \left\{ \frac{i\omega}{2\pi T} + \frac{\omega^2}{4\pi^2 T^2} \left[\psi\left(\frac{(1-i)\omega}{4\pi T} \right) + \psi\left(-\frac{(1+i)\omega}{4\pi T} \right) \right] \right\}$$

Poles of G^{ret} = quasinormal spectrum of dual gravity background

(D.Son, A.S., hep-th/0205051, P.Kovtun, A.S., hep-th/0506184)

Example: stress-energy tensor correlator in $4d\ \mathcal{N}=4$ SYM

in the limit $N_c \to \infty\,, \quad g^2_{YM} N_c \to \infty$

Zero temperature, Euclid: $$G_E(k) = \frac{N_c^2 k_E^4}{32\pi^2} \ln k_E^2$$

Finite temperature, Mink:

$$< T_{tt}(-\omega,-q), T_{tt}(\omega,q) >^{\text{ret}} = \frac{3N_c^2\pi^2 T^4 q^2}{2(\omega^2 - q^2/3 + i\omega q^2/3\pi T)} + \cdots$$

(in the limit $\omega/T \ll 1\,,\ \ q/T \ll 1$)

The pole
(or the lowest quasinormal freq.)

$$\omega = \pm\frac{1}{\sqrt{3}} q - \frac{i}{6\pi T} q^2 + \frac{3 - 2\ln 2}{24\pi^2\sqrt{3}T^2} q^3 + \cdots$$

Compare with hydro: $$\omega = \pm v_s q - \frac{i}{2sT}\left(\zeta + \frac{4}{3}\eta\right) q^2 + \cdots$$

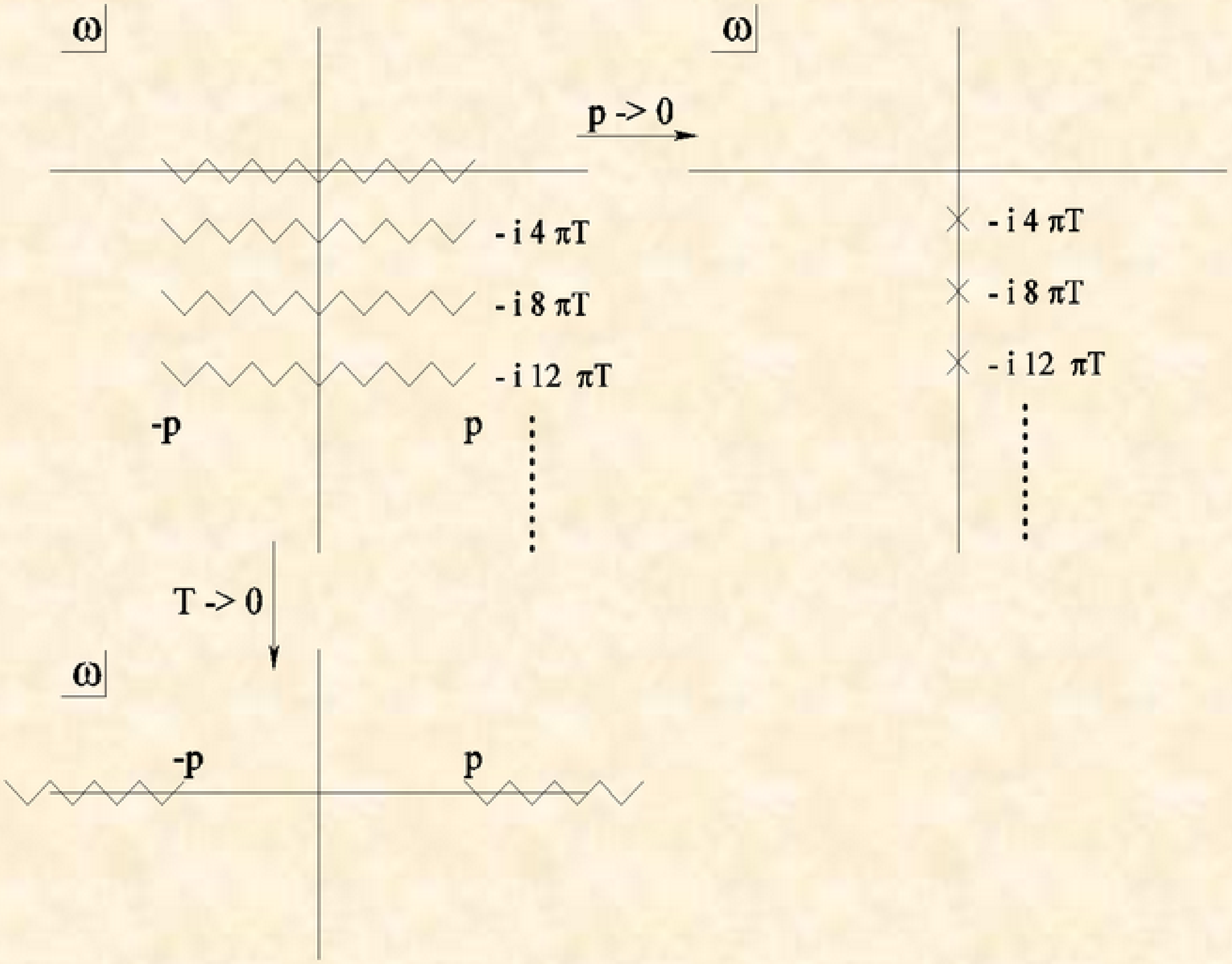
In CFT: $v_s = \dfrac{1}{\sqrt{3}}, \quad \zeta = 0$

$$\Rightarrow \quad \eta = \pi N_c^2 T^3/8$$

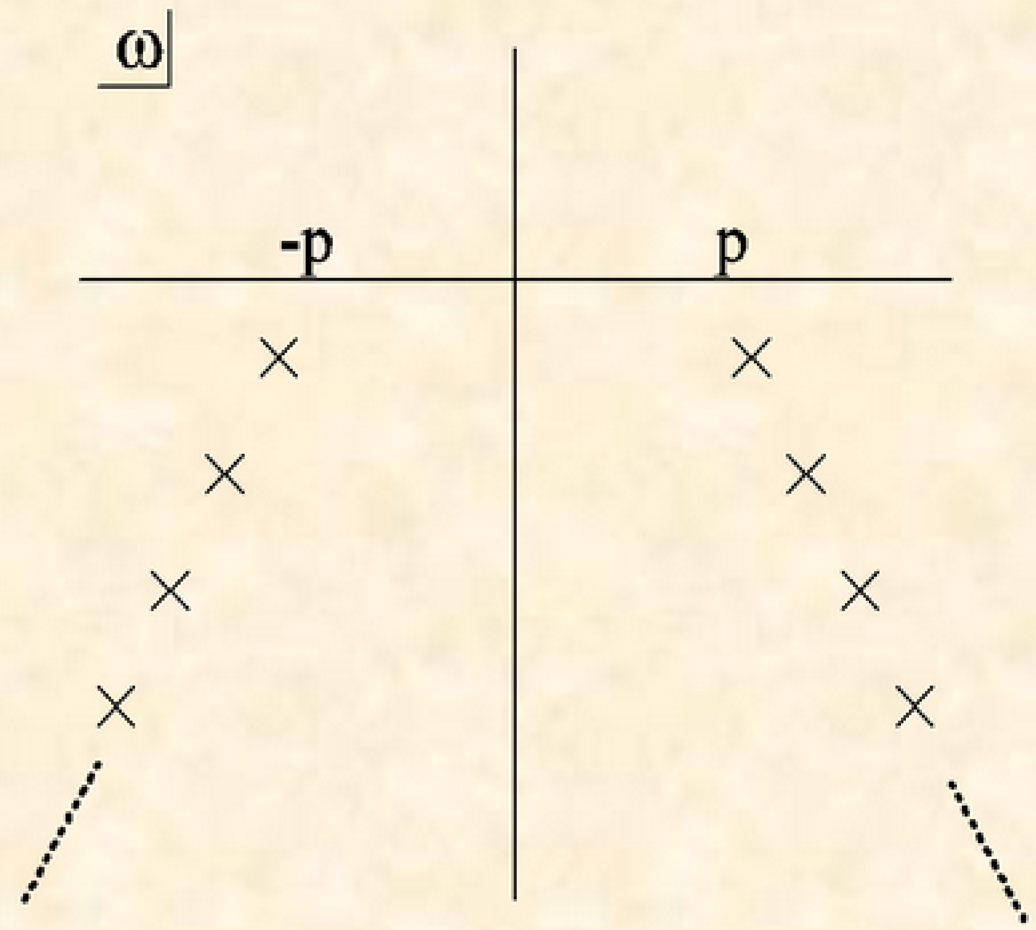
Also, $s = \pi^2 N_c^2 T^3/2$ (Gubser, Klebanov, Peet, 1996)

Analytic structure of the correlators

$g^2N = 0$



$g^2N = \infty$

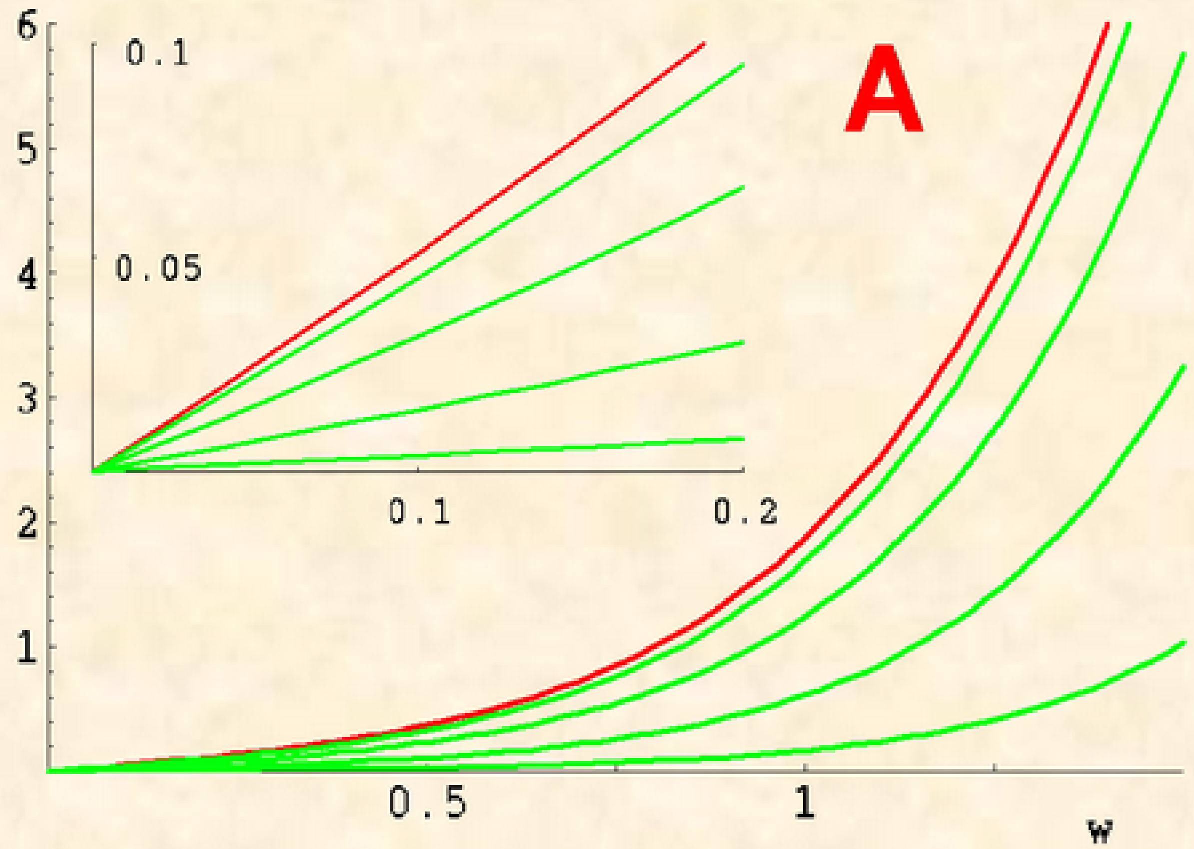


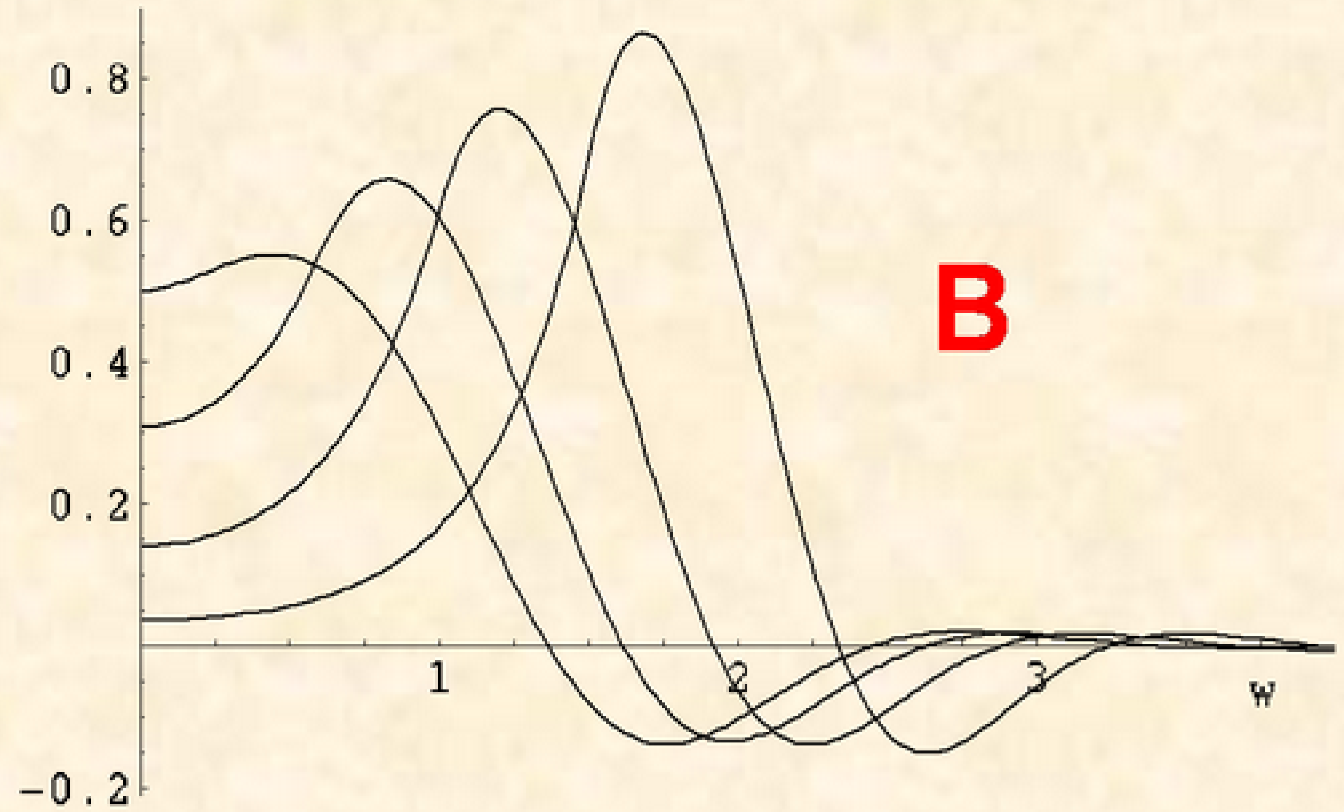
Strong coupling: A.S., hep-th/0207133

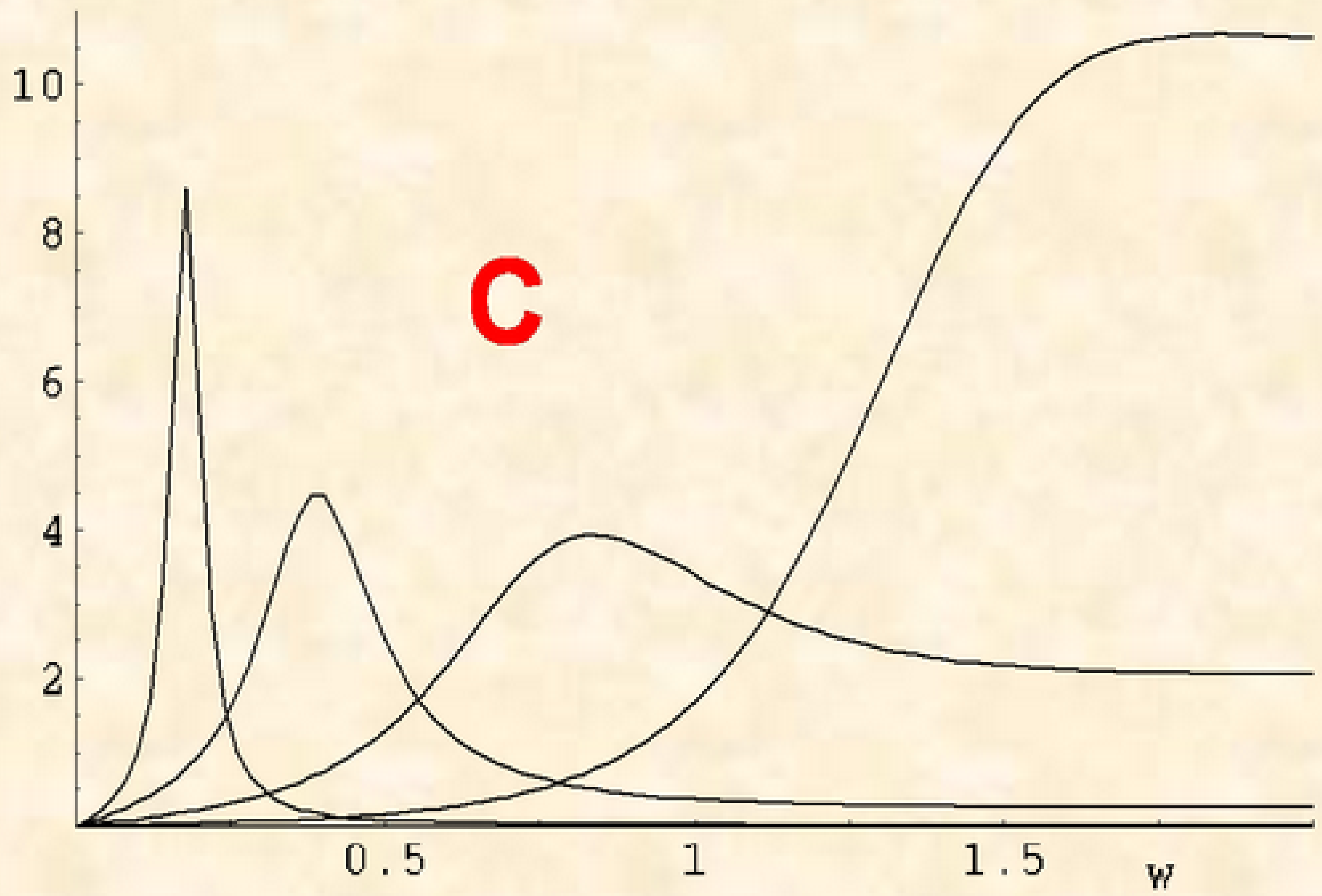
Weak coupling: S. Hartnoll and P. Kumar, hep-th/0508092

Spectral function and quasiparticles

$$\chi_{\mu\nu,\alpha\beta}(k) = \int d^4x\, e^{-ikx} \langle \left[T_{\mu\nu}(x) T_{\alpha\beta}(0) \right] \rangle = -2\, \mathrm{Im}\, G^R_{\mu\nu,\alpha\beta}(\omega, q)$$







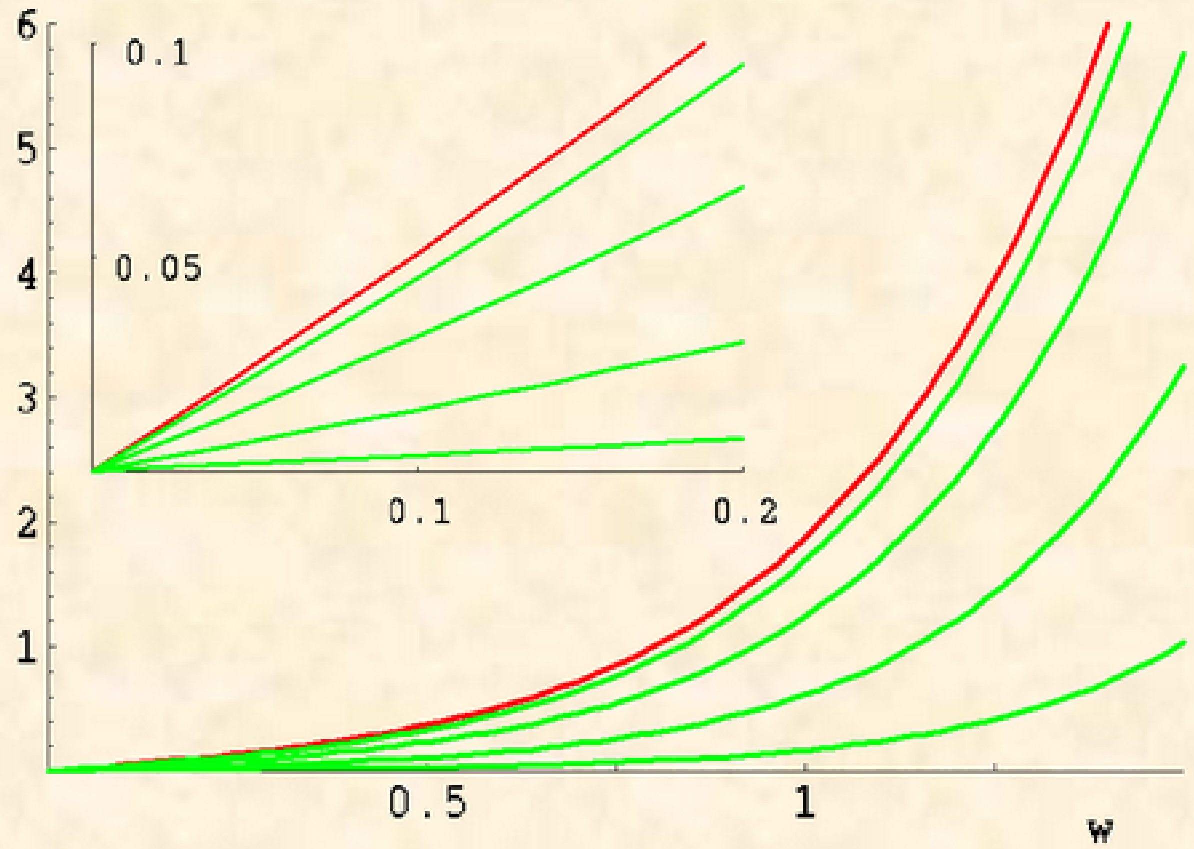
A: scalar channel

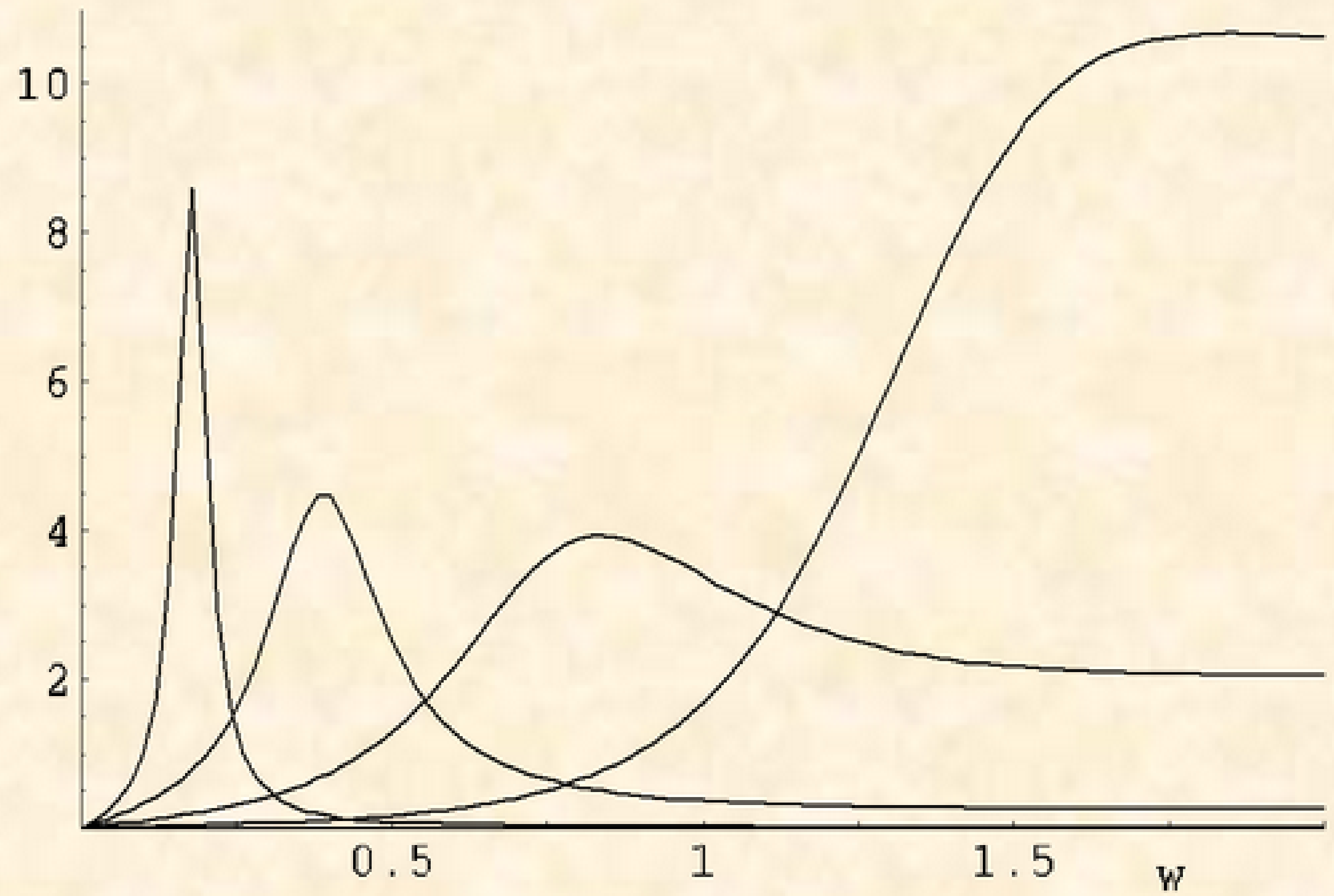
B: scalar channel - thermal part

C: sound channel

Spectral sum rules for the QGP

$$\chi_{\mu\nu,\alpha\beta}(k) = \int d^4x\, e^{-ikx} \langle \left[T_{\mu\nu}(x) T_{\alpha\beta}(0)\right] \rangle = -2\,\mathrm{Im}\, G^R_{\mu\nu,\alpha\beta}(\omega, q)$$





$$\frac{2}{5}\epsilon = \frac{1}{\pi}\int \frac{d\omega}{\omega}\left[\chi_{xy,xy}(\omega) - \chi^{T=0}_{xy,xy}(\omega)\right]$$

In N=4 SYM at ANY coupling

P.Romatschke, D.Son, 0903.3946 [hep-ph]

First-order transport coefficients in N = 4 SYM

in the limit $N_c \to \infty \,, \quad g^2_{YM} N_c \to \infty$

Shear viscosity $\quad \eta = \frac{\pi}{8} N_c^2 T^3 \left[1 + O\left(\frac{1}{(g^2 N_c)^{3/2}}, \frac{1}{N_c^2} \right) \right]$

Bulk viscosity $\quad \zeta = 0$

for non-conformal theories see Buchel et al; G.D.Moore et al Gubser et al.

Charge diffusion constant $\quad D_R = \frac{1}{2\pi T} + \cdots$

Supercharge diffusion constant $\quad D_s = \frac{2\sqrt{2}}{9\pi T}$

NEW! (G.Policastro, 2008)

Thermal conductivity $\quad \frac{\kappa_T \; \mu^2}{\eta \; T} = 8\pi^2 + \cdots$

Electrical conductivity $\quad \sigma = e^2 \frac{N_c^2 T}{16 \pi} + \cdots$

Second-order hydrodynamics in $4d\ \mathcal{N}=4$ SYM

Second-order conformal hydrodynamics can be systematically constructed

Using AdS/CFT, all new transport coefficients for N=4 SYM can be computed

$$\eta = \frac{\pi}{8} N_c^2 T^3, \quad \tau_\Pi = \frac{2 - \ln 2}{2\pi T}, \quad \kappa = \frac{N_c^2 T^2}{8}$$

$$\lambda_1 = \frac{\eta}{2\pi T}, \quad \lambda_2 = -\frac{\eta \ln 2}{\pi T}, \quad \lambda_3 = 0$$

Here we also used results from: S.Bhattacharyya,V.Hubeny,S.Minwalla,M.Rangamani, 0712.2456 [hep-th]

Generalized to CFT in D dim: Haack & Yarom, 0806.4602 [hep-th];

Finite ‘t Hooft coupling correction computed: Buchel and Paulos, 0806.0788 [hep-th]

Question: does this affect RHIC numerics?

Supersymmetric sound mode ("phonino") in $4d\ \mathcal{N}=4$ SYM

Conserved charge $\Longrightarrow$ Hydrodynamic mode (infinitely slowly relaxing fluctuation of the charge density) $\Longrightarrow$ Hydro pole in the retarded correlator of the charge density

$$\partial_\mu T^{\mu\nu} = 0 \qquad T^{\mu\nu}_{equib} + \delta T^{\mu\nu} \qquad \langle T_{\mu\nu}(-k)\, T_{\rho\sigma}(k)\rangle$$

Sound wave pole: $\omega = \pm v_s\, q - i\frac{2}{3sT}\left(\eta + \frac{3}{4}\zeta\right) q^2 + \cdots \qquad v_s = \sqrt{\frac{\partial P}{\partial \epsilon}}$

$$\partial_\mu S^\mu_\alpha = 0 \qquad S^\mu_\alpha + \delta S^\mu_\alpha \qquad \langle \bar{S}^\mu_{\dot\alpha}(-k) S^\nu_\beta(k)\rangle$$

Supersound wave pole: $\omega = \pm v_{ss}\, q - i D_s q^2 + \cdots \qquad v_{SS} = \frac{P}{\epsilon}$

Lebedev & Smilga, 1988 (see also Kovtun & Yaffe, 2003)

Sound and supersymmetric sound in $4d\ \mathcal{N}=4$ SYM

In 4d CFT

$$\epsilon = 3P \qquad \zeta = 0 \qquad \Longrightarrow \qquad v_s = \sqrt{\frac{\partial P}{\partial \epsilon}} = \frac{1}{\sqrt{3}}, \qquad v_{SS} = \frac{P}{\epsilon} = \frac{1}{3}$$

Sound mode: $\omega = \pm \frac{q}{\sqrt{3}} - i \frac{2\eta}{3sT} q^2 + \cdots$

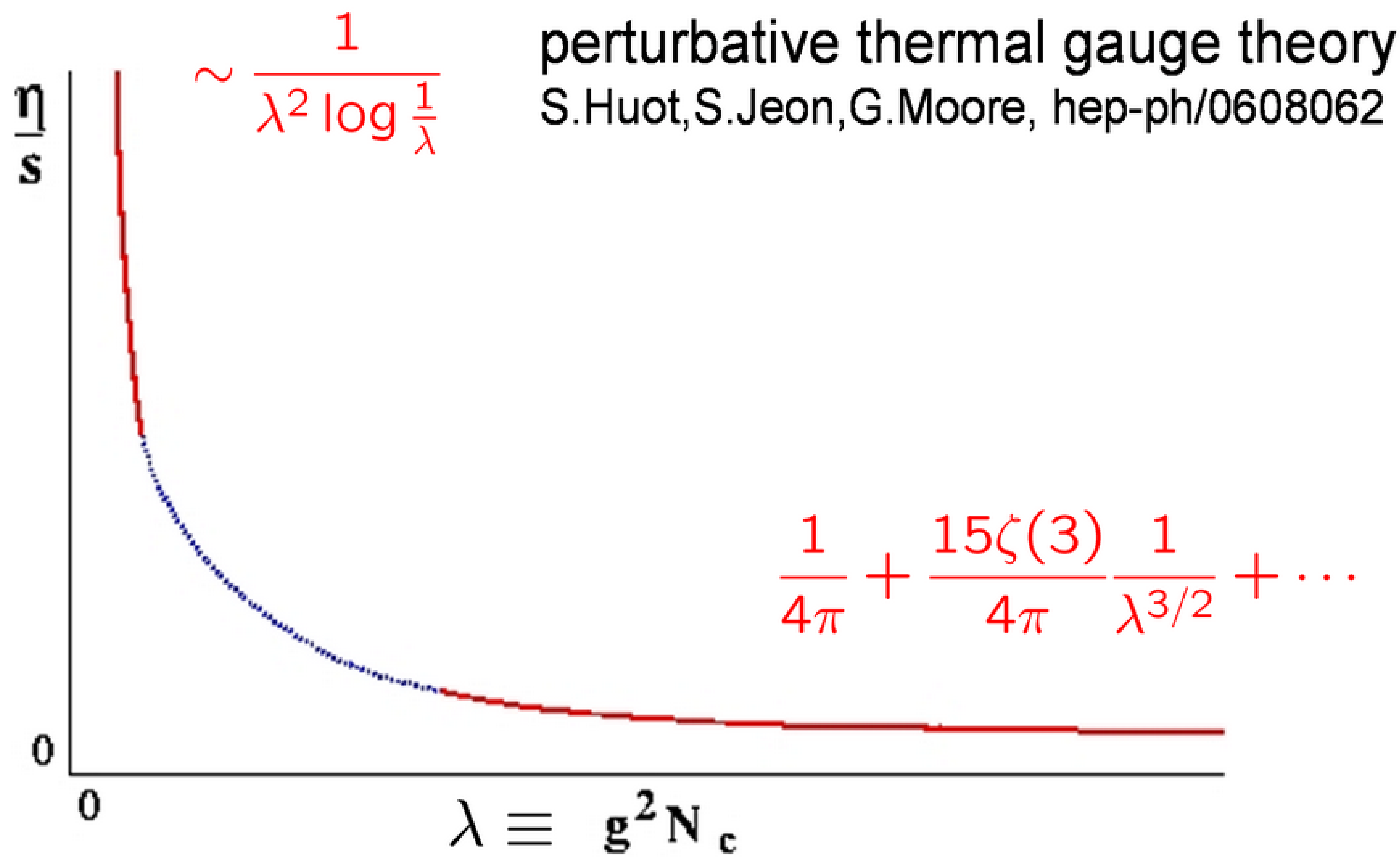
Supersound mode: $\omega = \pm \frac{q}{3} - i D_s q^2 + \cdots$

Quasinormal modes in dual gravity: G.Policastro, 2008

Graviton: $\omega = \pm \frac{q}{\sqrt{3}} - i \frac{1}{6\pi T} q^2 + \cdots \quad \Longrightarrow \quad \frac{\eta}{s} = \frac{1}{4\pi}$

Gravitino: $\omega = \pm \frac{q}{3} - i \frac{2\sqrt{2}}{9\pi T} q^2 + \cdots \quad \Longrightarrow \quad D_s = \frac{2\sqrt{2}}{9\pi T}$

Shear viscosity in $\mathcal{N} = 4$ SYM



Correction to $1/4\pi$: Buchel, Liu, A.S., hep-th/0406264

Buchel, 0805.2683 [hep-th]; Myers, Paulos, Sinha, 0806.2156 [hep-th]

Electrical conductivity in $\mathcal{N} = 4$ SYM

Weak coupling: $\sigma = 1.28349 \frac{e^2 (N_c^2 - 1) T}{\lambda^2 \left[\ln \lambda^{-1/2} + O(1)\right]}$

$\lambda \ll 1$

Strong coupling: $\sigma = \frac{e^2 N_c^2 T}{16 \pi} + O\Big(\frac{1}{\lambda^{3/2}}\Big)$

$\lambda \gg 1$

* Charge susceptibility can be computed independently: $\Xi = \frac{N_c^2 T^2}{8}$

D.T.Son, A.S., hep-th/0601157

Einstein relation holds: $\frac{\sigma}{e^2 \Xi} = D_{U(1)} = \frac{1}{2\pi T}$

Universality of η/s

Theorem:

For a thermal gauge theory, the ratio of shear viscosity to entropy density is equal to $1/4\pi$
in the regime described by a dual gravity theory

(e.g. at $g^2_{YM} N_c = \infty, N_c = \infty$ in $\mathcal{N} = 4$ SYM)

Remarks:

- Extended to non-zero chemical potential:

  Benincasa, Buchel, Naryshkin, hep-th/0610145

- Extended to models with fundamental fermions in the limit $N_f/N_c \ll 1$

  Mateos, Myers, Thomson, hep-th/0610184

- *String/Gravity dual to QCD is currently unknown*

Universality of shear viscosity in the regime described by gravity duals

$$ds^2 = f(w)\left(dx^2 + dy^2\right) + g_{\mu\nu}(w)dw^\mu dw^\nu$$

$$\left.\begin{aligned} \eta &= \lim_{\omega\to 0} \frac{1}{2\omega}\int dt\, dx e^{i\omega t}\langle\left[T_{xy}(t,x), T_{xy}(0,0)\right]\rangle \\ \sigma_{abs} &= -\frac{16\pi G}{\omega}\, \mathrm{Im}\, G^R(\omega) \\ &= \frac{8\pi G}{\omega}\int dt\, dx e^{i\omega t}\langle\left[T_{xy}(t,x), T_{xy}(0,0)\right]\rangle \end{aligned}\right\} \quad \eta = \frac{\sigma_{abs}(0)}{16\pi G}$$

Graviton's component h_y^x obeys equation for a minimally coupled massless scalar. But then $\sigma_{abs}(0) = A_H$.

Since the entropy (density) is $s = A_H/4G$ we get

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Three roads to universality of η/s

- **The absorption argument**

  D. Son, P. Kovtun, A.S., hep-th/0405231

- **Direct computation of the correlator in Kubo formula from AdS/CFT** A.Buchel, hep-th/0408095

- **"Membrane paradigm" general formula for diffusion coefficient + interpretation as lowest quasinormal frequency = pole of the shear mode correlator + Buchel-Liu theorem**

  P. Kovtun, D.Son, A.S., hep-th/0309213, A.S., 0806.3797 [hep-th], P.Kovtun, A.S., hep-th/0506184, A.Buchel, J.Liu, hep-th/0311175

Shear viscosity - (volume) entropy density ratio from gauge-string duality

In ALL theories (in the limit where dual gravity valid) : $\frac{1}{4\pi} + \text{corrections}$

In particular, in N=4 SYM: $\frac{1}{4\pi} + \frac{15\zeta(3)}{4\pi}\frac{1}{\lambda^{3/2}} + \cdots$

Other higher-derivative gravity actions

$$S = \int d^D x \sqrt{-g}\left(R - 2\Lambda + c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \cdots\right)$$

Y.Kats and P.Petrov: 0712.0743 [hep-th]

M.Brigante, H.Liu, R.C.Myers, S.Shenker and S.Yaida: 0802.3318 [hep-th], 0712.0805 [hep-th].

R.Myers,M.Paulos, A.Sinha: 0903.2834 [hep-th] (and ref. therein – many other papers)

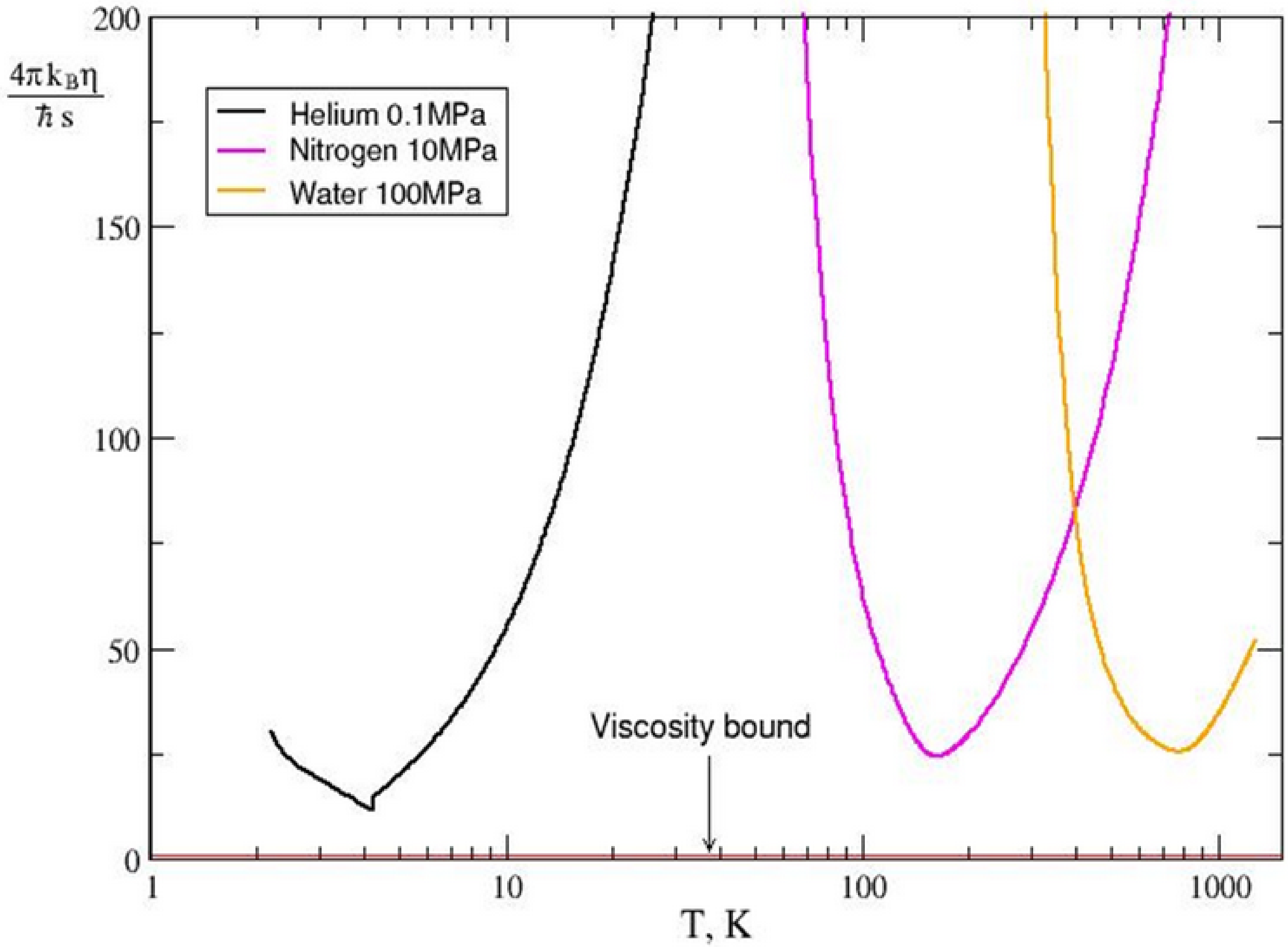
$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - 8c_1 + \ldots\right)$$

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - \frac{1}{2N}\right)$$ for superconformal Sp(N) gauge theory in d=4

Also: The species problem: T.Cohen, hep-th/0702136; A. Dolbado, F.Llanes-Estrada: hep-th/0703132

A viscosity bound conjecture

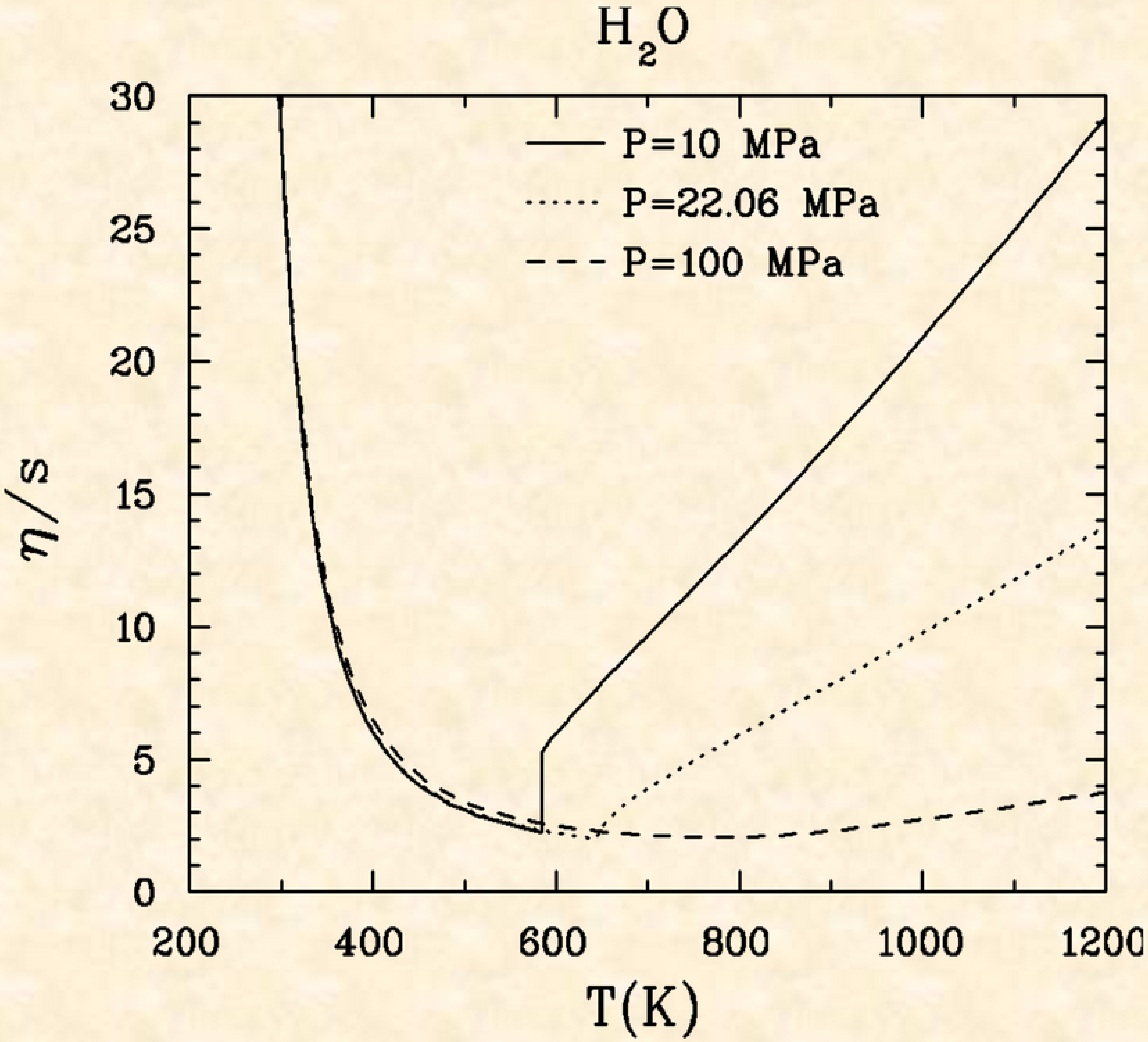
$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B} \approx 6.08 \cdot 10^{-13} K \cdot s$$



Minimum of $\frac{\eta}{s}$ in units of $\frac{\hbar}{4\pi k_B}$

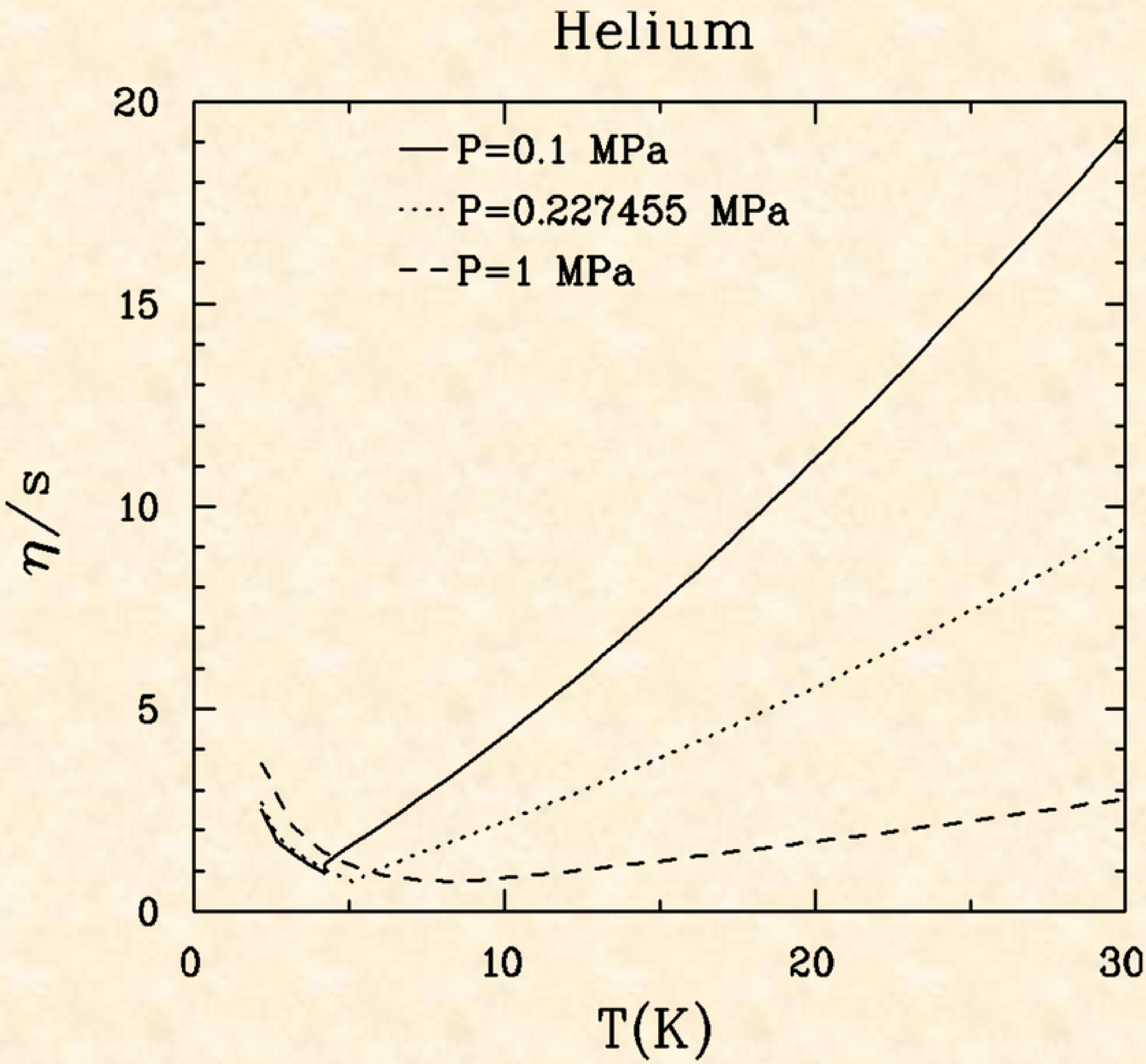
Xe	84
Kr	57
CO_2	32
H_2O	25
C_2H_5OH	22
Ne	17
He	8.8

P.Kovtun, D.Son, A.S., hep-th/0309213, hep-th/0405231



$$(\eta/s)_{\text{min}} \sim 25 \quad \text{in units of } \frac{\hbar}{4\pi k_B}$$

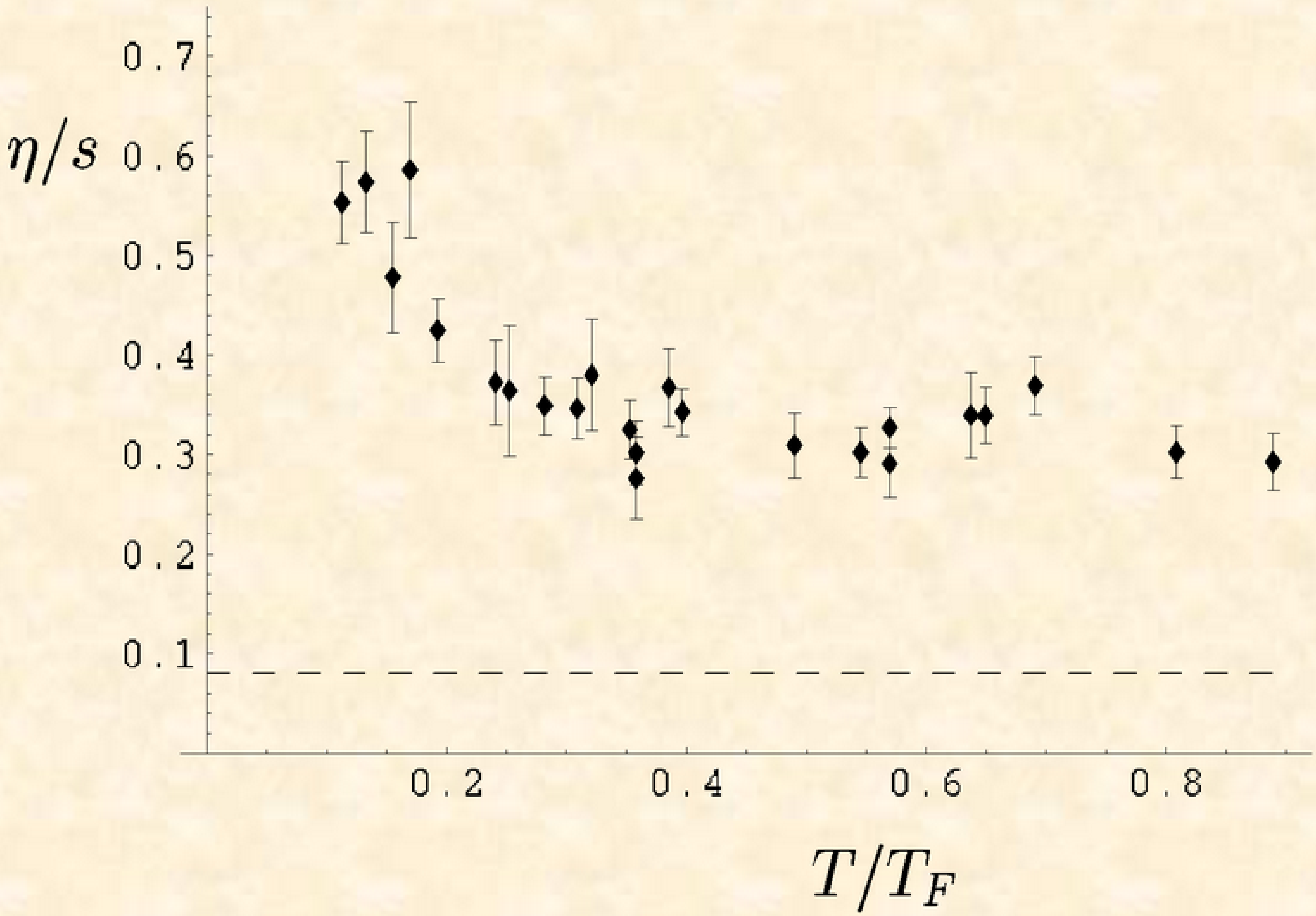
Chernai, Kapusta, McLerran, nucl-th/0604032



$(\eta/s)_{\text{min}} \sim 8.8$ in units of $\frac{\hbar}{4\pi k_B}$

Chernai, Kapusta, McLerran, nucl-th/0604032

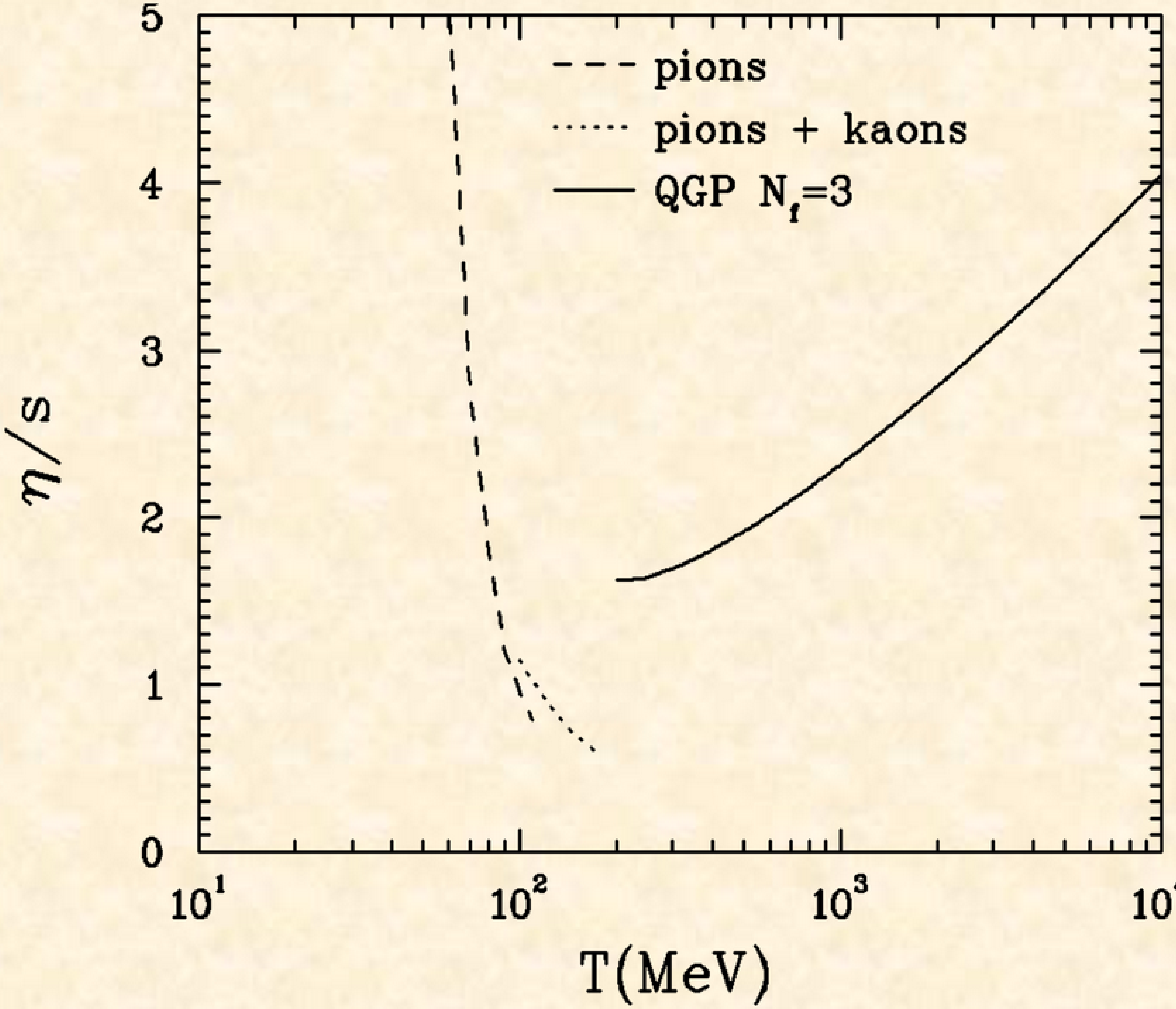
Viscosity-entropy ratio of a trapped Fermi gas



$\eta/s \sim 4.2$ in units of $\dfrac{\hbar}{4\pi k_B}$ T.Schafer, cond-mat/0701251

(based on experimental results by Duke U. group, J.E.Thomas et al., 2005-06)

QCD



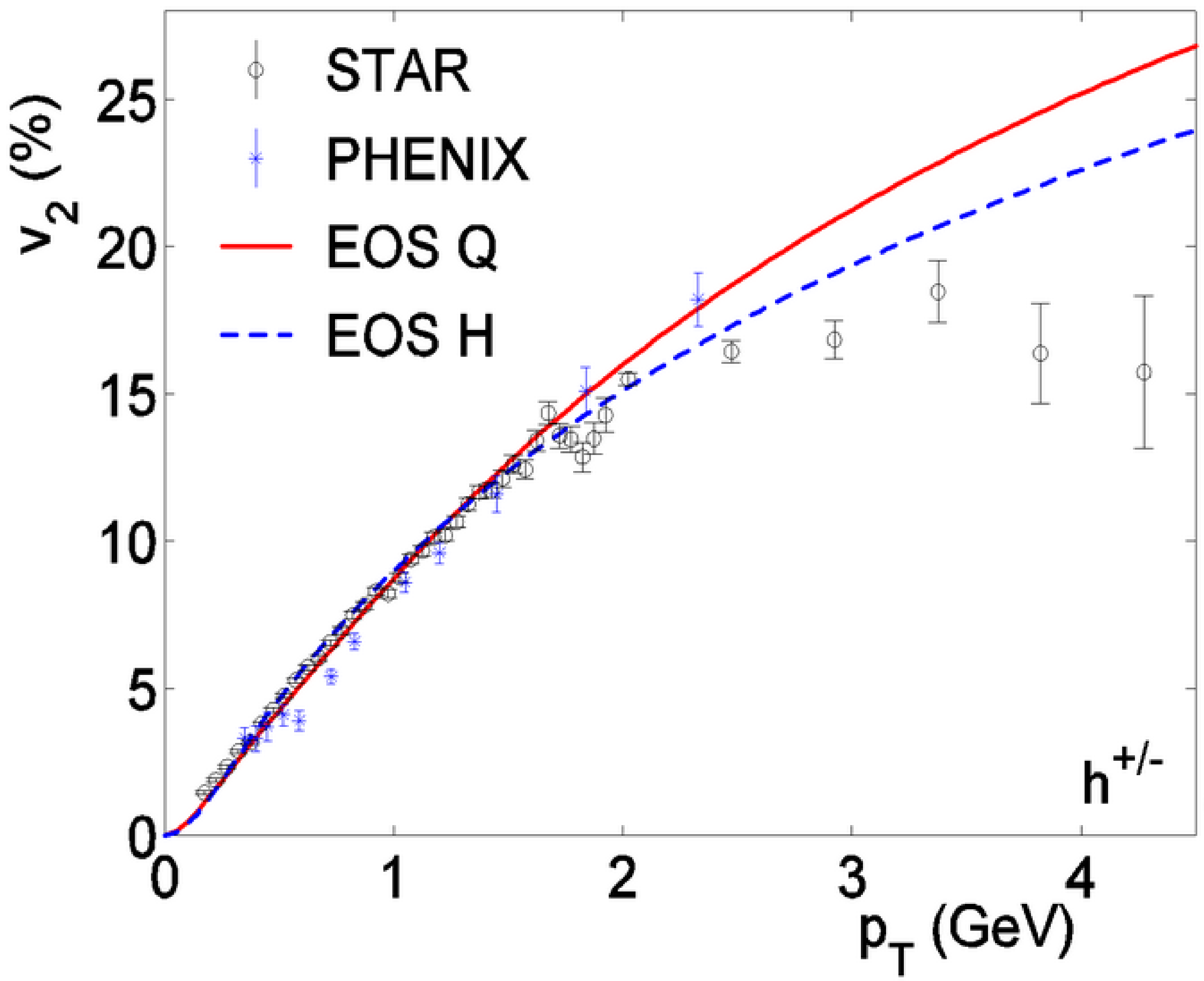
Chernai, Kapusta, McLerran, nucl-th/0604032

Viscosity "measurements" at RHIC

Viscosity is ONE of the parameters used in the hydro models describing the azimuthal anisotropy of particle distribution

$$\frac{d^2N^i}{dp_T d\phi} = N_0^i \left[1 + 2v_2^i(p_T)\, \cos 2\phi + \cdots\right]$$

$v_2^i(p_T)$ -elliptic flow for particle species "i"



Elliptic flow reproduced for

$$0 < \eta/s \leq 0.3$$

e.g. Baier, Romatschke, nucl-th/0610108

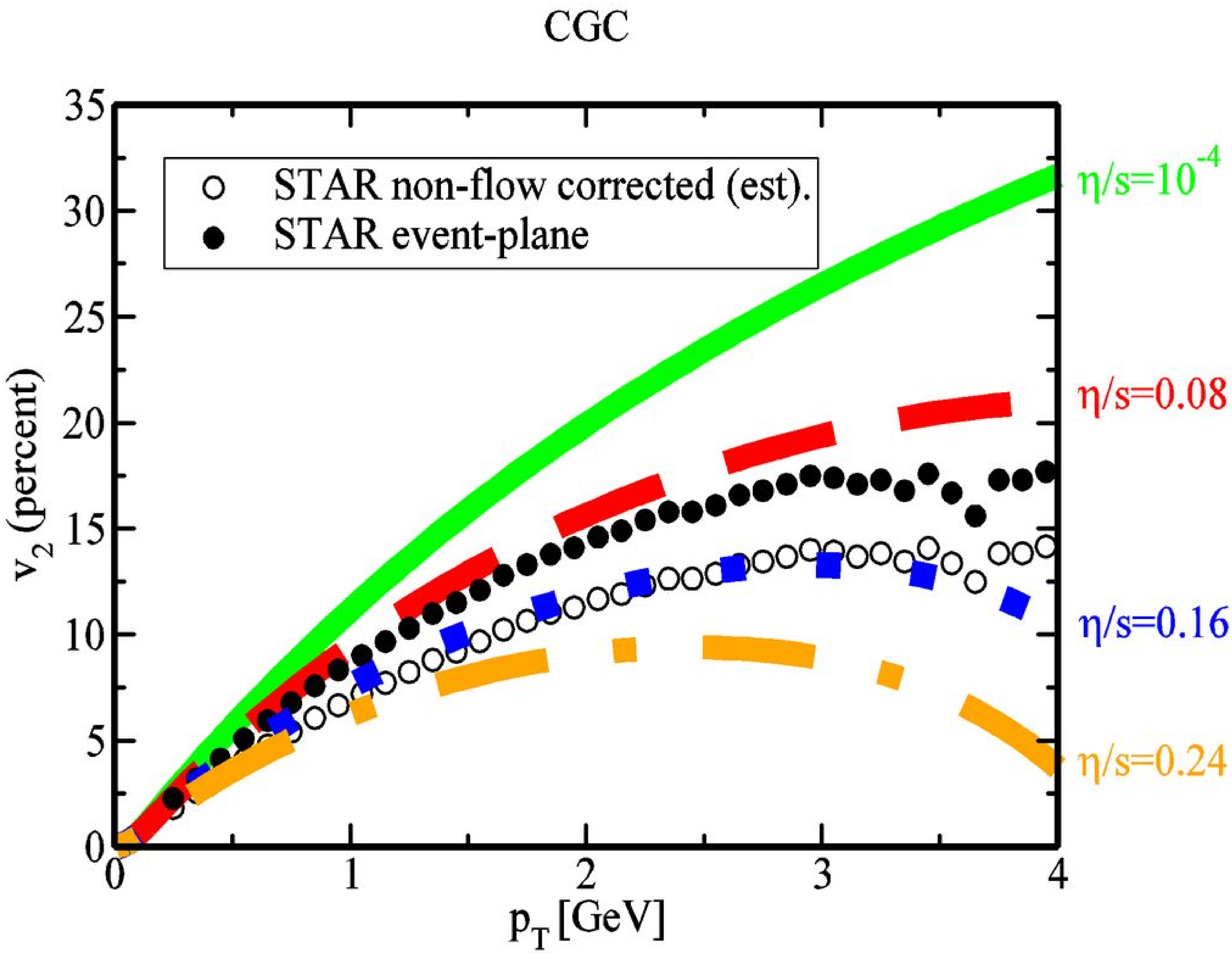
Perturbative QCD:

$$\eta/s\,(T_{\rm RHIC}) \approx 1.6 \sim 1.8$$

Chernai, Kapusta, McLerran, nucl-th/0604032

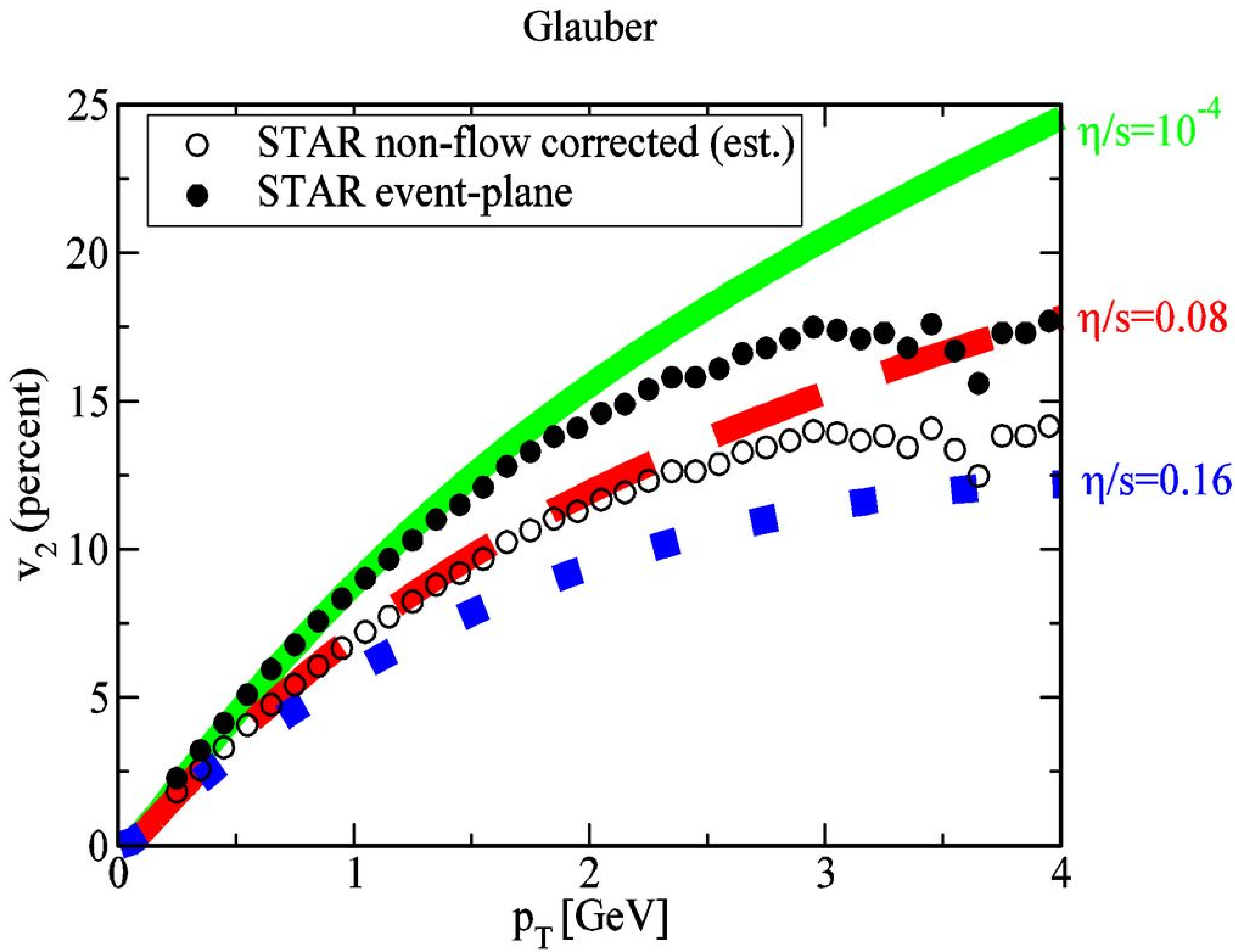
SYM: $\eta/s \approx 0.09 \sim 0.28$

Elliptic flow with color glass condensate initial conditions



Luzum and Romatschke, 0804.4015 [nuc-th]

Elliptic flow with Glauber initial conditions



Luzum and Romatschke, 0804.4015 [nuc-th]

Viscosity/entropy ratio in QCD: current status

Theories with gravity duals in the regime where the dual gravity description is valid

$$\frac{\eta}{s} = \frac{1}{4\pi} \approx 0.08$$

(universal limit)

Kovtun, Son & A.S; Buchel; Buchel & Liu, A.S

QCD: RHIC elliptic flow analysis suggests

$$0 < \frac{\eta}{s} < 0.2$$

QCD: (Indirect) LQCD simulations

H.Meyer, 0805.4567 [hep-th]

$$0.08 < \frac{\eta}{s} < 0.16$$

$$1.2\, T_c < T < 1.7\, T_c$$

Trapped strongly correlated cold alkali atoms

T.Schafer, 0808.0734 [nucl-th]

$$\left(\frac{\eta}{s}\right)_{\rm min} \approx 0.5$$

Liquid Helium-3

$$\left(\frac{\eta}{s}\right)_{\rm min} \approx 0.7$$

Shear viscosity at non-zero chemical potential

$\mathcal{N} = 4$ SYM

$q_i \in U(1)^3 \subset SO(6)_R$ $\Longleftrightarrow$ Reissner-Nordstrom-AdS black hole with three R charges

$Z = \mathrm{tr}\, e^{-\beta H + \mu_i q_i}$

(see e.g. Yaffe, Yamada, hep-th/0602074)

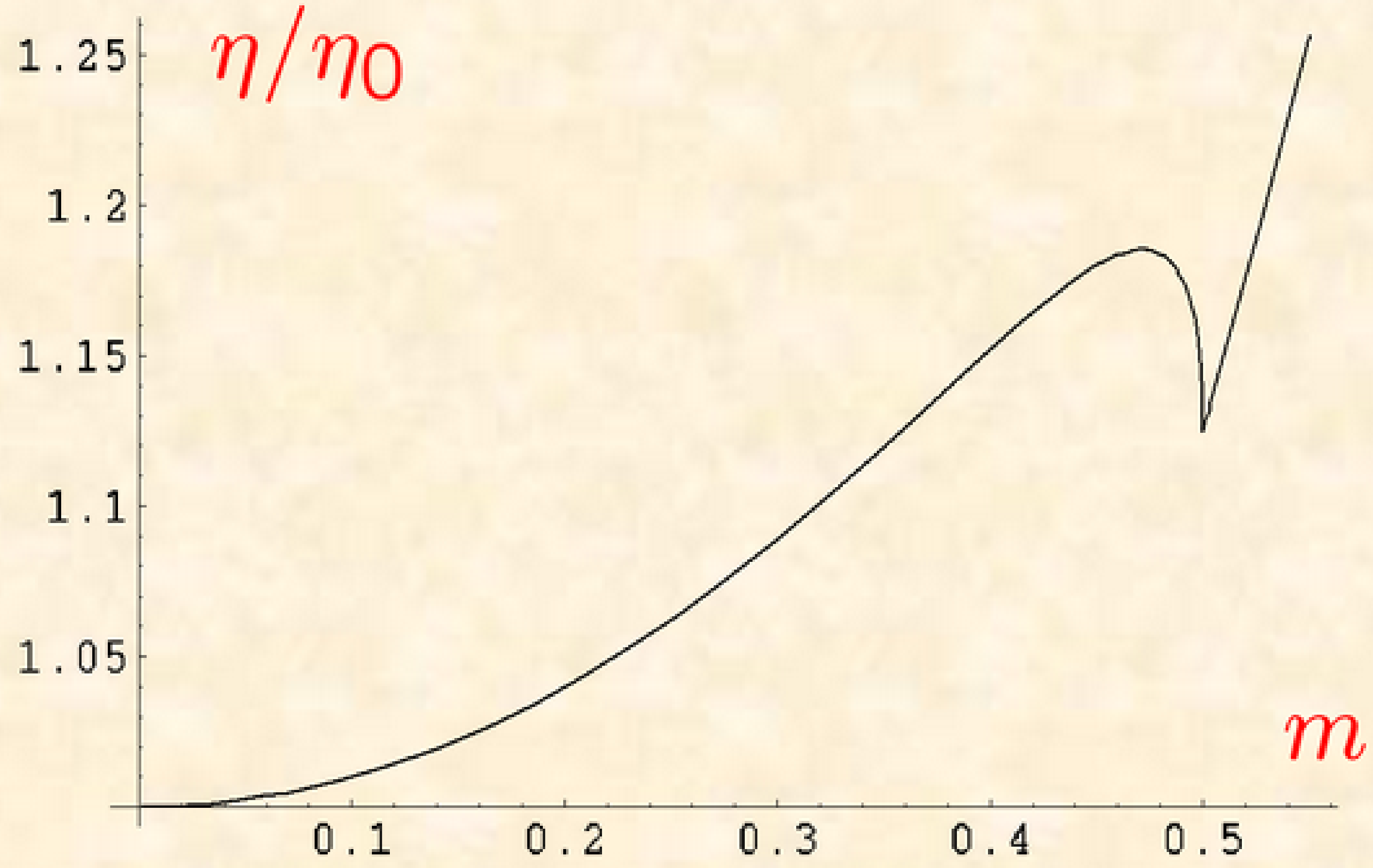
(Behrnd, Cvetic, Sabra, 1998)

We still have $\frac{\eta}{s} = \frac{1}{4\pi}$

J.Mas
D.Son, A.S.
O.Saremi
K.Maeda, M.Natsuume, T.Okamura

$$\eta = \pi N^2 T^3 \frac{m^2(1-\sqrt{1-4m^2}-m^2)^2}{(1-\sqrt{1-4m^2})^3}$$

$m \equiv \mu/2\pi T$



Photon emission from SYM plasma

Photons interacting with matter: $e\, J_\mu^{\mathrm{EM}}\, A^\mu$

To leading order in e

$$d\Gamma_\gamma = \frac{d^3k}{(2\pi)^3}\,\frac{e^2}{2|k|}\,\eta^{\mu\nu} C^<_{\mu\nu}(k^0 = |k|)$$

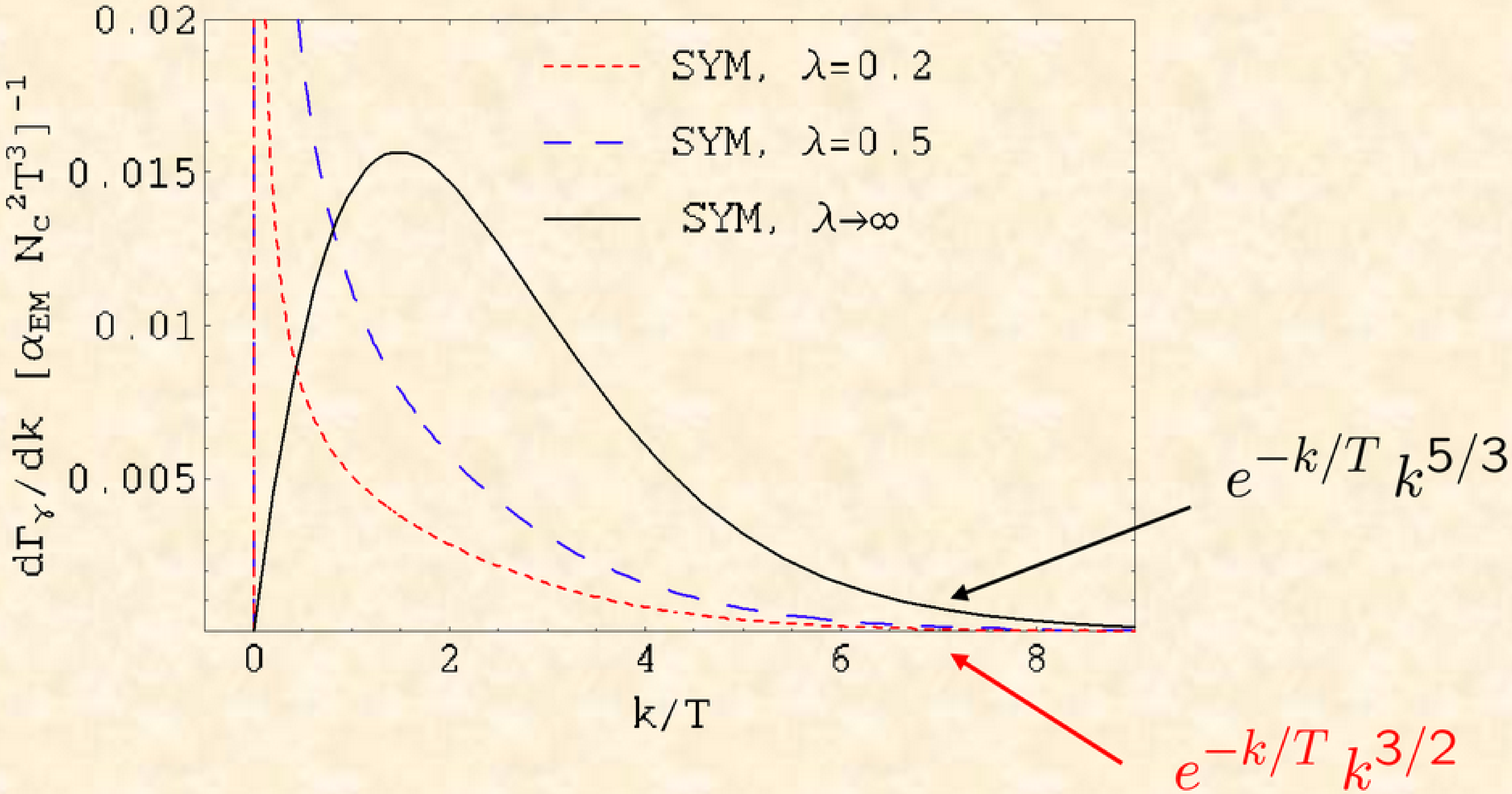
$$C^<_{\mu\nu} = \int d^4X e^{-iKX} \langle J_\mu^{\mathrm{EM}}(0) J_\nu^{\mathrm{EM}}(X) \rangle$$

Mimic J_μ^{EM} by gauging global R-symmetry $U(1) \subset SU(4)$

$$\mathcal{L} = \mathcal{L}_{\mathcal{N}=4\,\mathrm{SYM}} + e\, J_\mu^3\, A^\mu - \frac{1}{4} F_{\mu\nu}^2$$

Need only to compute correlators of the R-currents J_μ^3

Photoproduction rate in SYM



(Normalized) photon production rate in SYM for various values of ‘t Hooft coupling

$$\frac{d\Gamma_\gamma}{dk\,\alpha_{em}N_c^2T^3} = n_B(k)\left(\frac{k}{4\pi T}\right)^2 \left|\,{}_2F_1\left(1-\frac{(1+i)k}{4\pi T}, 1+\frac{(1-i)k}{4\pi T}; 1-\frac{ik}{2\pi T}; -1\right)\right|^{-2}$$

Now consider strongly interacting systems at finite density and LOW temperature

Probing quantum liquids with holography

Quantum liquid in p+1 dim	Low-energy elementary excitations	Specific heat at low T
Quantum Bose liquid	phonons	$\sim \; T^p$
Quantum Fermi liquid (Landau FLT)	fermionic quasiparticles + bosonic branch (zero sound)	$\sim \; T$

Departures from normal Fermi liquid occur in

- 3+1 and 2+1 –dimensional systems with strongly correlated electrons
- In 1+1 –dimensional systems for any strength of interaction (Luttinger liquid)

One can apply holography to study strongly coupled Fermi systems at low T

The simplest candidate with a known holographic description is

$SU(N_c)$ $\mathcal{N}=4$ SYM coupled to N_f $\mathcal{N}=2$ fundamental hypermultiplets

at finite temperature T and nonzero chemical potential associated with the "baryon number" density of the charge $U(1)_B \subset U(N_f)$

There are two dimensionless parameters: $\frac{n_q^{1/3}}{T}$ $\frac{M}{T}$

n_q is the baryon number density

M is the hypermultiplet mass

The holographic dual description in the limit $N_c \gg 1\,,\;\; g_{YM}^2 N_c \gg 1\,,\;\; N_f$ finite is given by the D3/D7 system, with D3 branes replaced by the AdS-Schwarzschild geometry and D7 branes embedded in it as probes.

Karch & Katz, hep-th/0205236

AdS-Schwarzschild black hole (brane) background

$$ds^2 = \frac{r^2}{R^2}\left[-\left(1-\frac{r_H^4}{r^4}\right)dt^2 + d\vec{x}^2\right] + \left(1-\frac{r_H^4}{r^4}\right)^{-1}\frac{R^2}{r^2}dr^2$$

D7 probe branes

$$S_{DBI} = -N_f T_{D7}\int d^8\xi\sqrt{-\mathrm{det}(g_{ab}+2\pi\alpha' F_{ab})}$$

The worldvolume U(1) field A_μ couples to the flavor current J^μ at the boundary

Nontrivial background value of A_0 corresponds to nontrivial expectation value of J^0

We would like to compute

- the specific heat at low $(T n_q^{-1/3} \ll 1)$ temperature
- the charge density correlator $G^R \sim \langle J^0(k)\, J^0(-k)\rangle$

★ The specific heat (in p+1 dimensions):

$$c_V = \mathcal{N}_q\, p \left(\frac{4\pi}{p+1}\right)^{2p+1} \frac{T^{2p}}{n_q} \left[1 + O(T n_q^{-\frac{1}{p}})\right]$$

(note the difference with Fermi $c_V \sim T$ and Bose $c_V \sim T^p$ systems)

★ The (retarded) charge density correlator $G^R \sim \langle J^0(k)\, J^0(-k)\rangle$ has a pole corresponding to a propagating mode (zero sound) - even at zero temperature

$$\omega = \pm \frac{q}{\sqrt{p}} - \frac{i\,\Gamma(\frac{1}{2})\, q^2}{n_q^{\frac{1}{p}} \Gamma(\frac{1}{2} - \frac{1}{2p}) \Gamma(\frac{1}{2p})} + O(q^3)$$

(note that this is NOT a superfluid phonon whose attenuation scales as q^{p+1})

New type of quantum liquid?

Other avenues of (related) research

Bulk viscosity for non-conformal theories (Buchel, Gubser,…)

Non-relativistic gravity duals (Son, McGreevy,…)

Gravity duals of theories with SSB (Kovtun, Herzog,…)

Bulk from the boundary (Janik,…)

Navier-Stokes equations and their generalization from gravity (Minwalla,…)

Quarks moving through plasma (Chesler, Yaffe, Gubser,…)

New directions

S. Hartnoll
"Lectures on holographic methods for condensed matter physics",
0903.3246 [hep-th]

C. Herzog
"Lectures on holographic superfluidity and superconductivity",
0904.1975 [hep-th]

M. Rangamani
"Gravity and hydrodynamics: Lectures on the fluid-gravity correspondence",
0905.4352 [hep-th]

THANK YOU

A hand-waving argument

$$\eta \sim \rho\, v\, l \;\sim \rho\, v^2\, \tau \sim n\, m\, v^2 \tau \;\sim n\, \epsilon\tau$$

$$s \sim n$$

Thus $\quad \dfrac{\eta}{s} \sim \epsilon\tau \geq \hbar$

Gravity duals fix the coefficient: $\quad \dfrac{\eta}{s} \geq \hbar/4\pi$

Energy density vs temperature for various gauge theories

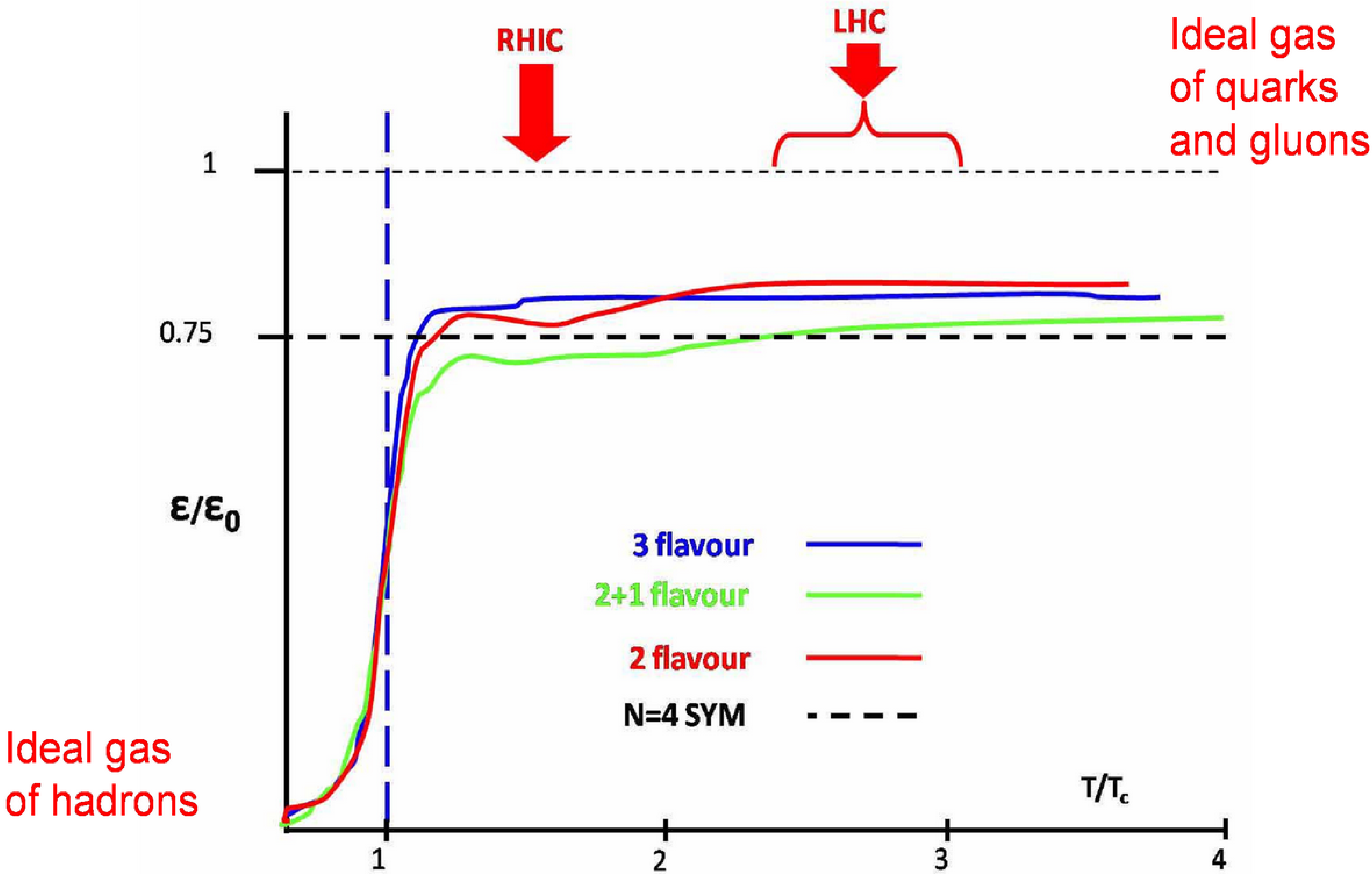


Figure: an artistic impression from Myers and Vazquez, 0804.2423 [hep-th]