Vortex reconnection

classical and superfluid turbulence compared and contrasted

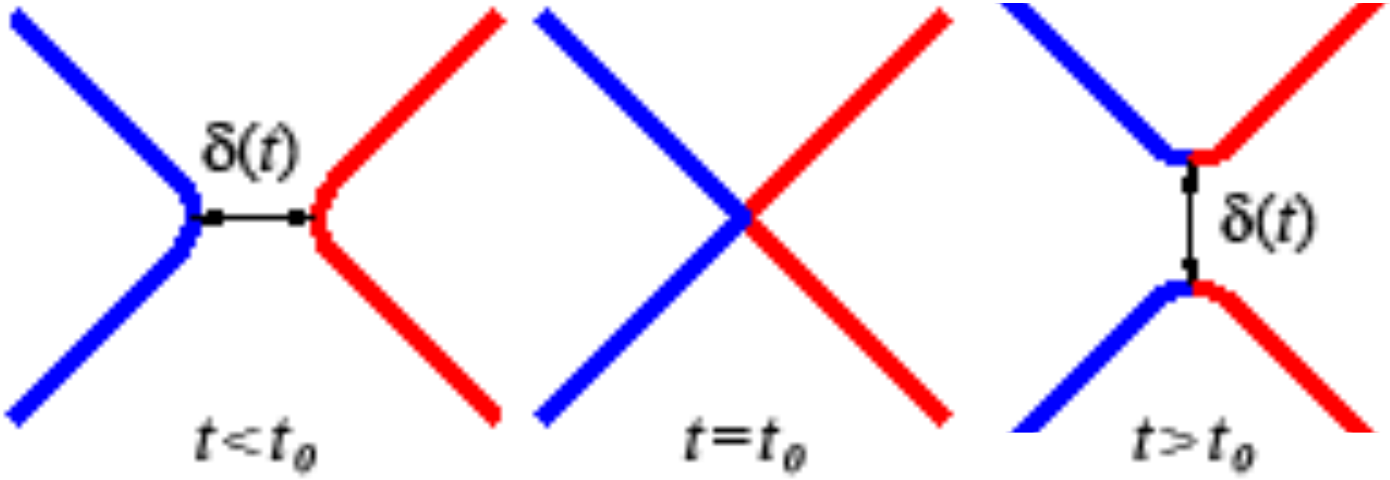
K.R. Sreenivasan

4 September 2009

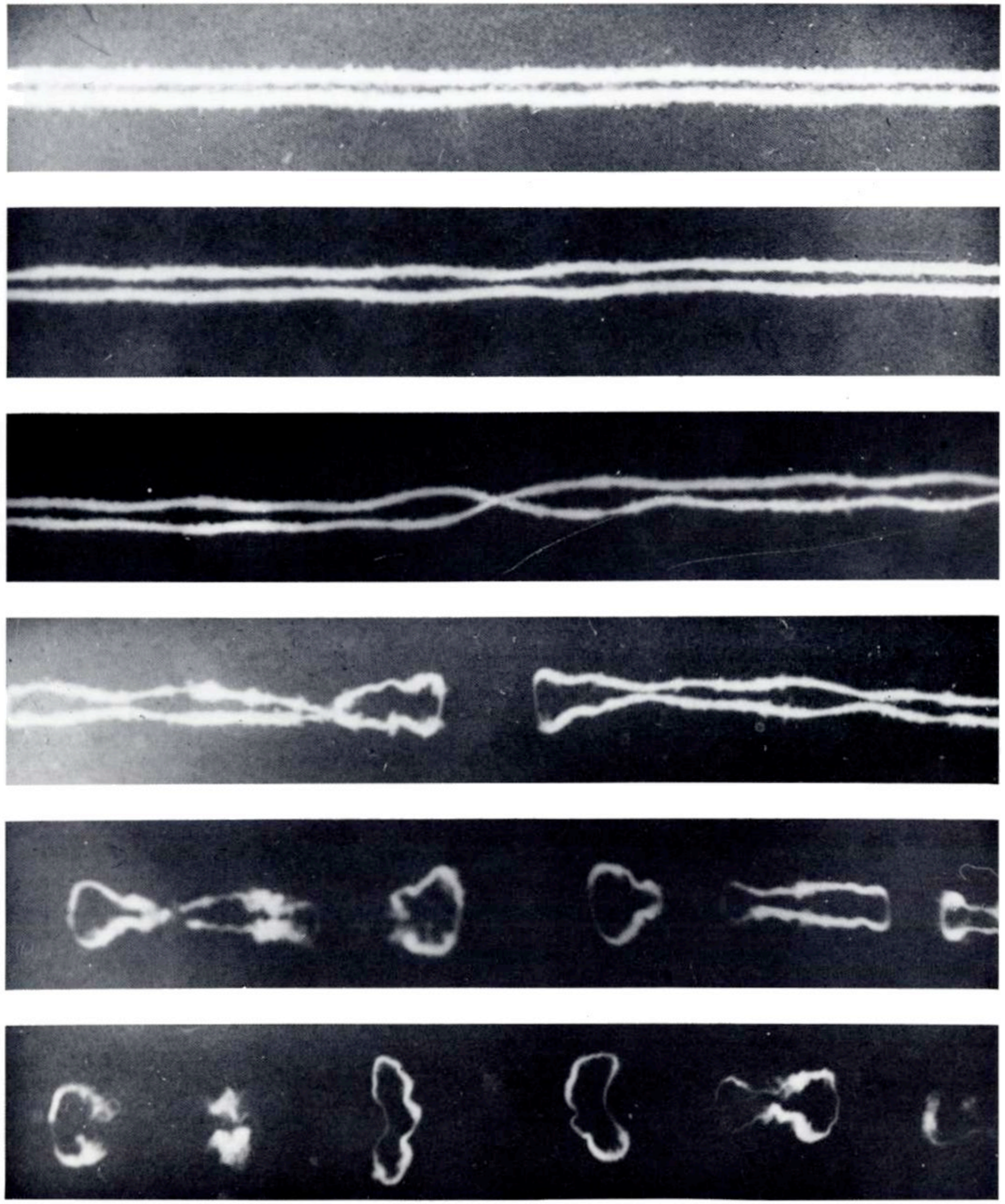
Fluid-Gravity
Correspondence

Arnold Sommerfeld Centre, LMU, Munich

Two vortices of opposite sign, which are attracted to each other, collide, splice parts of one to parts of the other, and move away from each other in a different direction.



Reconnecting vortex lines at the moment of reconnection, t_0, and before and after t_o.



16. **Instability of a pair of trailing vortices.** The vortex rail of a B-47 aircraft was photographed directly overhead it intervals of 15 s after its passage. The vortex cores are nade visible by condensation of moisture. They slowly ecede and draw together in a symmetrical nearly sinusoidal pattern until they connect to form a train of vortex rings. The wake then quickly disintegrates. This is commonly called Crow instability after the researcher who explained its early stages analytically. *Crow 1970, courtesy of Meteorology Research Inc.*

From Van Dyke’s Album

Selected references on reconnection dynamics in classical fluids

S. Crow, Stability theory for a pair of trailing vortices. *AIAA J.* **8**, 2172-2179 (1970)

T. Fohl & J.S. Turner, Colliding vortex rings. *Phys. Fluids* **18**, 433-36 (1975)

Y. Oshima & S. Asaka, Interaction of two vortex rings along parallel axes in air. *J. Phys. Soc. Jpn.* **42**, 708-13 (1977)

T. Kambe, A class of exact solutions of two-dimensional viscous flow. *J. Phys. Soc. Jpn.* **32**, 834 (1983)

E.D. Siggia, Collapse and amplification of a vortex filament. *Phys. Fluids* **28**, 794-805 (1985)

E.D. Siggia & A. Pumir, Incipient singularities in the Navier-Stokes equation. *Phys. Rev. Lett.* **55**, 1749-1752 (1985)

W.T. Ashurst & D.I. Meiron, Numerical study of vortex reconnection. *Phys. Rev. Lett.* **58**, 1632-1635 (1987)

S. Kida & M. Takaoka, Bridging in vortex reconnection. *Phys. Fluids* **30**, 2911-2924 (1987)

P.R. Schatzle, An experimental investigation of fusion of vortex rings. Ph.D. thesis, GALCIT, California Institute of Technology (1987)

Y. Oshima & N. Izutsu, Cross-linking of two vortex rings. *Phys. Fluids* **31**, 2401 (1988)

S. Kida & M. Takaoka, Reconnection of vortex tubes*. Fluid Dyn. Res.* **3**, 257-261 (1988)

A. Pumir & R.M. Kerr, Numerical simulation of interacting vortex tubes. *Phys. Rev. Lett.* **58**, 1636-39 (1988)

S. Kida, N. Takaoka & F. Hussain, Reconnection of two vortex rings. *Phys. Fluids A* **1**, 630-632 (1989)

R.M. Kerr & F. Hussain, Simulation of vortex reconnection. *Physica D* **37**, 474-484 (1989)

M.V. Melander & F. Hussain, Cross-linking of two antiparallel vortex tubes. *Phys. Fluids* **1**, 633-636 (1989)

N.J. Zabusky & M.V. Melander, Three-dimensional vortex tube reconnection: morphology for orthogonally-offset tubes. *Physica D* **37**, 555-562 (1989)

P.G. Saffman, A model of vortex reconnection. *J. Fluid Mech.* **212**, 295-402*A*

S. Kida, M. Takaoka & F. Hussain, Collision of two vortex rings. *J. Fluid Mech.* **230**, 583-646 (1991)

N.J. Zabusky, O.N. Baratov, R.B. Peltz, M. Gao, D. Silver & S.P. Cooper, Emergence of coherent patterns of vortex stretching during reconnection: a scattering paradigm. *Phys. Rev. Lett.* **62**, 2469-2471 (1991)

O.N. Boratov, R.B. Pelz & N.J. Zabusky, Reconnection in orthogonally interacting vortex tubes: direct numerical simulations and quantifications. *Phys. Fluids A* **4**, 581-605 (1992)

A. Pumir & E.D. Siggia, Finite-time singularities in the axisymmetric three-dimensional Euler equations. *Phys. Rev. Lett.* **68**, 1511-1513 (1992)

M.J. Shelly, D.I. Meiron & S.A. Orszag, Dynamical aspects of vortex reconnection of perturbed anti-parallel vortex tubes.

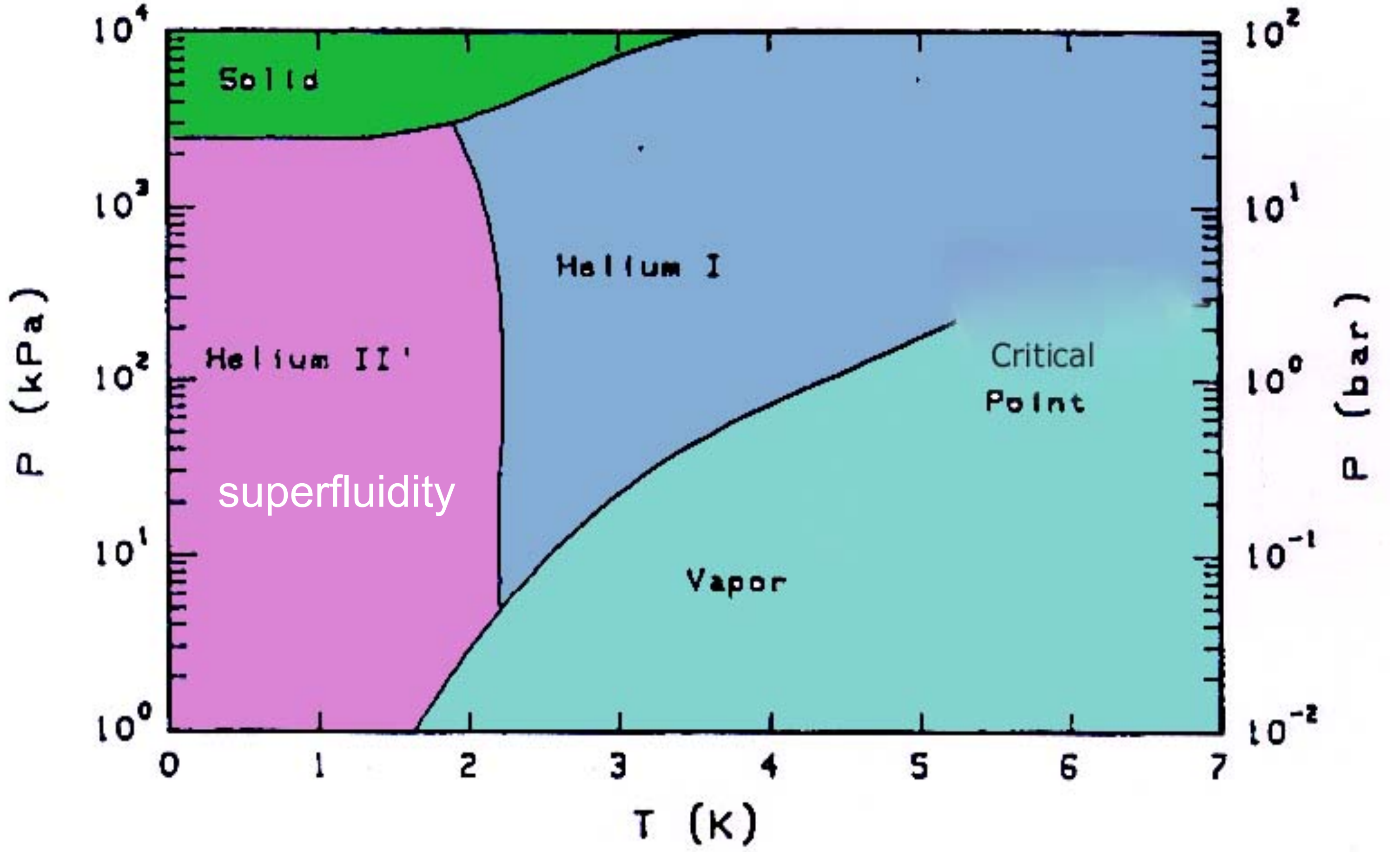
Viscous effects, core dynamics

(superfluid helium II: no viscosity, diameter ~ few angstrom)

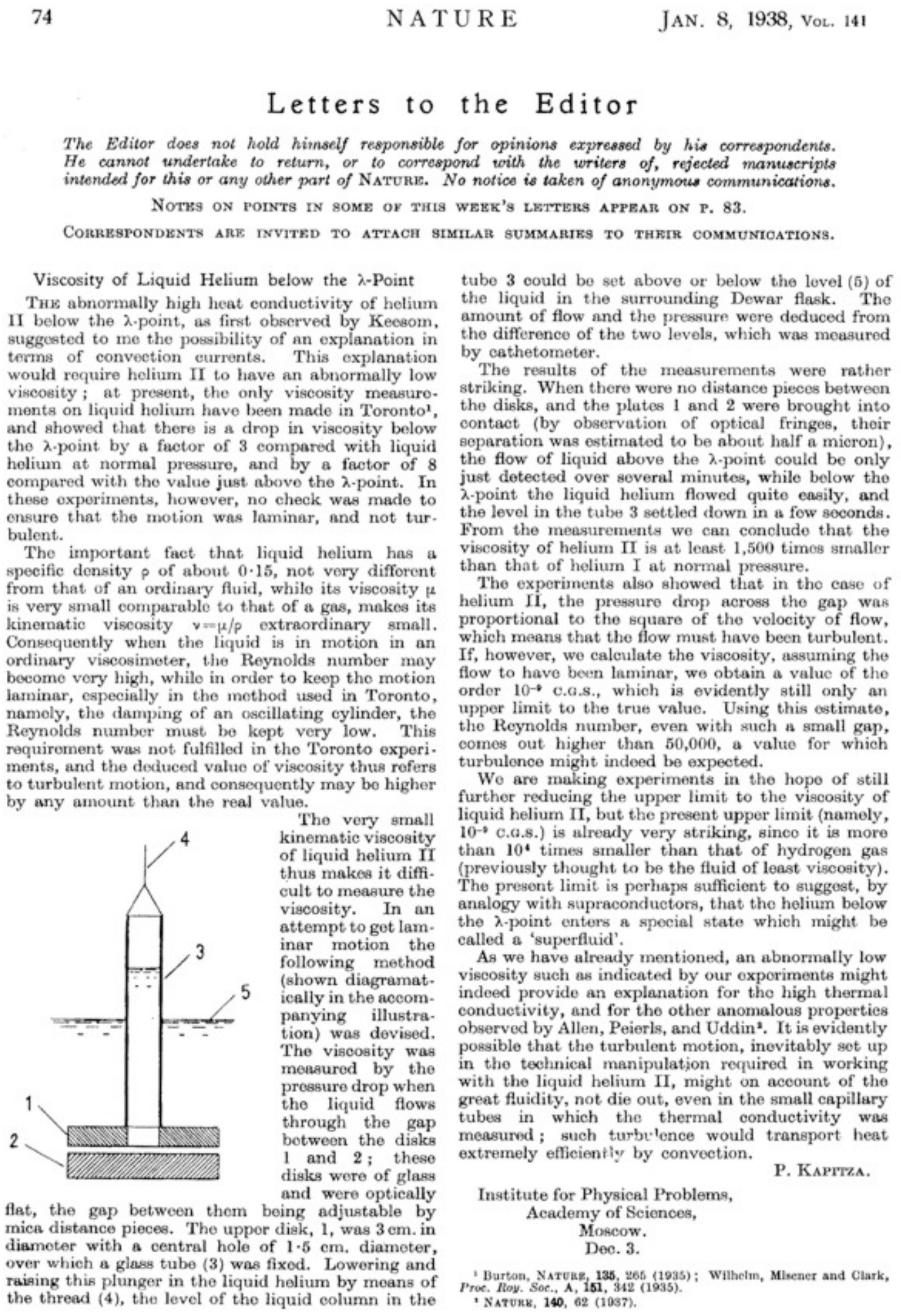
Some references on simulation of vortex dynamics and vortex reconnection in the superfluid state

- K.W. Schwarz, Phys. Rev. B **38,** 2398 (1988)
- J. Koplik & H. Levine, Phys. Rev. Lett. **71**, 1375 (1993)
- J. Koplik & H. Levine, Phys. Rev. Lett. **76**, 4745 (1996)
- C.F. Barenghi et al. Phys. Fluids **9**, 2631 (1997)
- D.C. Samuels & C.F. Barenghi, Phys. Rev. Lett. **81,** 4381 (1998)
- M. Tsubota, T. Araki & S.K. Nemirovskii, Phys. Rev. B **62**, 11751 (2000)
- C.F. Barenghi & D.C. Samuels, Phys. Rev. Lett. **89**, 015601 (2002)
- D.R. Poole, H. Scoffield, C.F. Barenghi & D.C. Samuels, J. Low. Temp. Phys. **132**, 97 (2003)
- C.F. Barenghi & D.C. Samuels, J. Low Temp. Phys. **136**, 281 (2004)
- D. Kivotides, Phys. Rev. Lett. **96**, 175301 (2006)
- S.K. Nemirovskii, Phys. Rev. Lett. **96**, 015301 (2006)
- K. Morris, J. Koplik & D.W.I. Rouson, Phys. Rev. Lett. **101**, 015301 (2008)
- S.Z. Alamri, A.J. Youdh & C.F. Barenghi, Phys. Rev. Lett. **101**, 215302 (2008)

Phase diagram of helium



The superfluid flow without friction (like a perfect fluid) and has spontaneously generated thin vortex structures (resembling the ideal vortex lines)



Letters to the Editor

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NOTES ON POINTS IN SOME OF THIS WEEK'S LETTERS APPEAR ON P. 83.

CORRESPONDENTS ARE INVITED TO ATTACH SIMILAR SUMMARIES TO THEIR COMMUNICATIONS.

Viscosity of Liquid Helium below the λ-Point

THE abnormally high heat conductivity of helium II below the λ-point, as first observed by Keesom, suggested to me the possibility of an explanation in terms of convection currents. This explanation would require helium II to have an abnormally low viscosity; at present, the only viscosity measurements on liquid helium have been made in Toronto[1], and showed that there is a drop in viscosity below the λ-point by a factor of 3 compared with liquid helium at normal pressure, and by a factor of 8 compared with the value just above the λ-point. In these experiments, however, no check was made to ensure that the motion was laminar, and not turbulent.

The important fact that liquid helium has a specific density ρ of about 0·15, not very different from that of an ordinary fluid, while its viscosity μ is very small comparable to that of a gas, makes its kinematic viscosity $\nu=\mu/\rho$ extraordinary small. Consequently when the liquid is in motion in an ordinary viscosimeter, the Reynolds number may become very high, while in order to keep the motion laminar, especially in the method used in Toronto, namely, the damping of an oscillating cylinder, the Reynolds number must be kept very low. This requirement was not fulfilled in the Toronto experiments, and the deduced value of viscosity thus refers to turbulent motion, and consequently may be higher by any amount than the real value.

The very small kinematic viscosity of liquid helium II thus makes it difficult to measure the viscosity. In an attempt to get laminar motion the following method (shown diagramatically in the accompanying illustration) was devised. The viscosity was measured by the pressure drop when the liquid flows through the gap between the disks 1 and 2; these disks were of glass and were optically flat, the gap between them being adjustable by mica distance pieces. The upper disk, 1, was 3 cm. in diameter with a central hole of 1·5 cm. diameter, over which a glass tube (3) was fixed. Lowering and raising this plunger in the liquid helium by means of the thread (4), the level of the liquid column in the tube 3 could be set above or below the level (5) of the liquid in the surrounding Dewar flask. The amount of flow and the pressure were deduced from the difference of the two levels, which was measured by cathetometer.

The results of the measurements were rather striking. When there were no distance pieces between the disks, and the plates 1 and 2 were brought into contact (by observation of optical fringes, their separation was estimated to be about half a micron), the flow of liquid above the λ-point could be only just detected over several minutes, while below the λ-point the liquid helium flowed quite easily, and the level in the tube 3 settled down in a few seconds. From the measurements we can conclude that the viscosity of helium II is at least 1,500 times smaller than that of helium I at normal pressure.

The experiments also showed that in the case of helium II, the pressure drop across the gap was proportional to the square of the velocity of flow, which means that the flow must have been turbulent. If, however, we calculate the viscosity, assuming the flow to have been laminar, we obtain a value of the order 10^{-9} C.G.S., which is evidently still only an upper limit to the true value. Using this estimate, the Reynolds number, even with such a small gap, comes out higher than 50,000, a value for which turbulence might indeed be expected.

We are making experiments in the hope of still further reducing the upper limit to the viscosity of liquid helium II, but the present upper limit (namely, 10^{-9} C.G.S.) is already very striking, since it is more than 10^4 times smaller than that of hydrogen gas (previously thought to be the fluid of least viscosity). The present limit is perhaps sufficient to suggest, by analogy with supraconductors, that the helium below the λ-point enters a special state which might be called a 'superfluid'.

As we have already mentioned, an abnormally low viscosity such as indicated by our experiments might indeed provide an explanation for the high thermal conductivity, and for the other anomalous properties observed by Allen, Peierls, and Uddin[2]. It is evidently possible that the turbulent motion, inevitably set up in the technical manipulation required in working with the liquid helium II, might on account of the great fluidity, not die out, even in the small capillary tubes in which the thermal conductivity was measured; such turbulence would transport heat extremely efficiently by convection.

P. KAPITZA.

Institute for Physical Problems,
Academy of Sciences,
Moscow.
Dec. 3.

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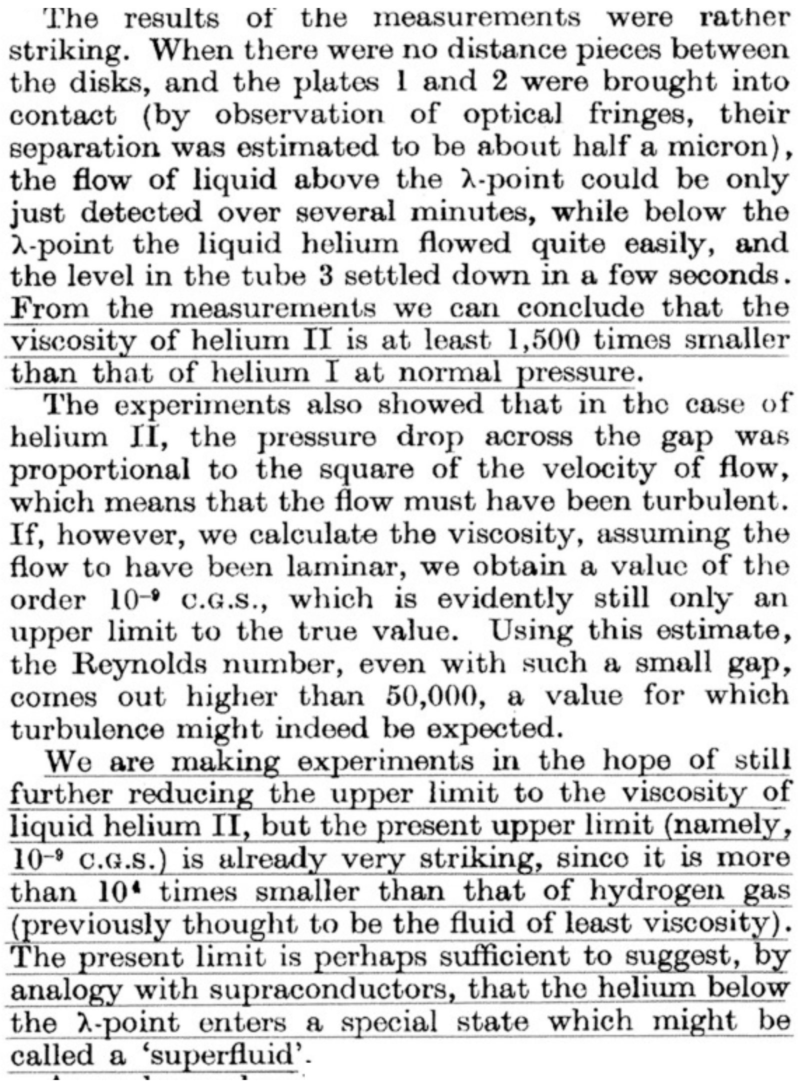
[1] Burton, NATURE, **135**, 265 (1935); Wilhelm, Misener and Clark, *Proc. Roy. Soc.*, A, **151**, 342 (1935).

[2] NATURE, **140**, 62 (1937).



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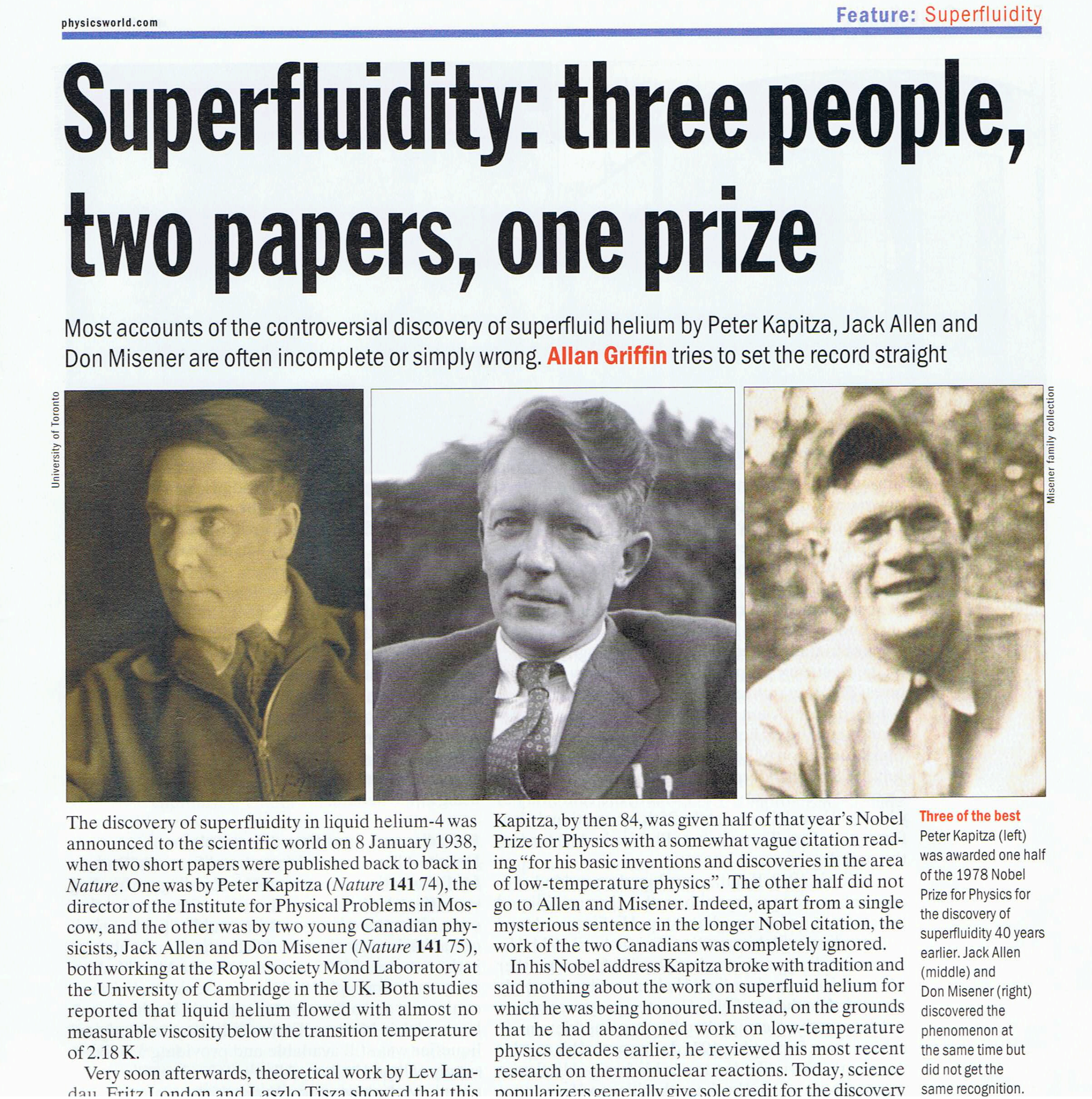
Nobel Prize in 1978

"The choice of the theme of my Nobel Lecture presents some difficulty for me."

Physics World: volume 21, no.8, pp.27-30 (2008)



Superfluidity: three people, two papers, one prize

Most accounts of the controversial discovery of superfluid helium by Peter Kapitza, Jack Allen and Don Misener are often incomplete or simply wrong. **Allan Griffin** tries to set the record straight

University of Toronto

Misener family collection

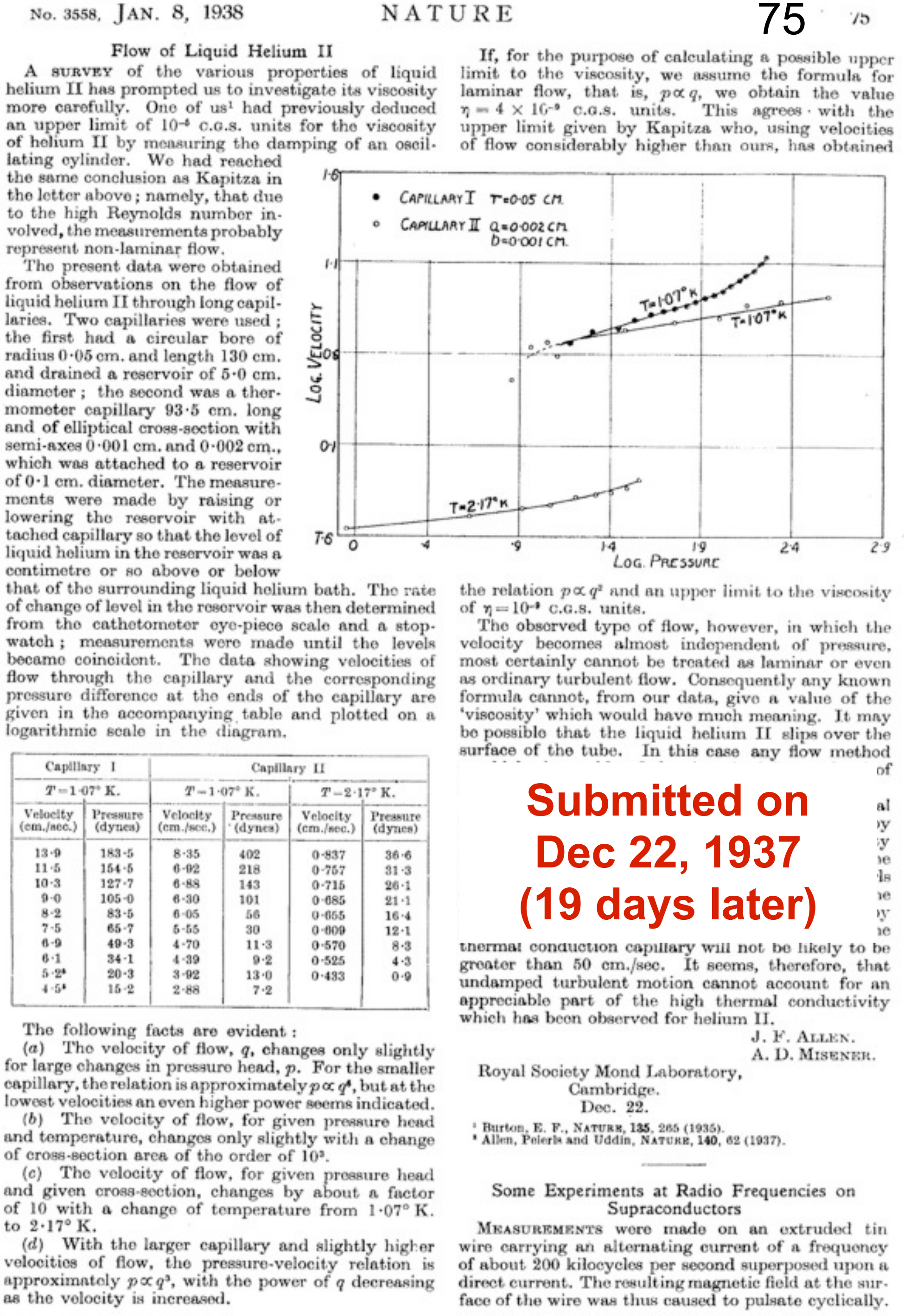
Three of the best Peter Kapitza (left) was awarded one half of the 1978 Nobel Prize for Physics for the discovery of superfluidity 40 years earlier. Jack Allen (middle) and Don Misener (right) discovered the phenomenon at the same time but did not get the same recognition.

The discovery of superfluidity in liquid helium-4 was announced to the scientific world on 8 January 1938, when two short papers were published back to back in *Nature*. One was by Peter Kapitza (*Nature* **141** 74), the director of the Institute for Physical Problems in Moscow, and the other was by two young Canadian physicists, Jack Allen and Don Misener (*Nature* **141** 75), both working at the Royal Society Mond Laboratory at the University of Cambridge in the UK. Both studies reported that liquid helium flowed with almost no measurable viscosity below the transition temperature of 2.18 K.

Very soon afterwards, theoretical work by Lev Landau, Fritz London and Laszlo Tisza showed that this

Kapitza, by then 84, was given half of that year's Nobel Prize for Physics with a somewhat vague citation reading "for his basic inventions and discoveries in the area of low-temperature physics". The other half did not go to Allen and Misener. Indeed, apart from a single mysterious sentence in the longer Nobel citation, the work of the two Canadians was completely ignored.

In his Nobel address Kapitza broke with tradition and said nothing about the work on superfluid helium for which he was being honoured. Instead, on the grounds that he had abandoned work on low-temperature physics decades earlier, he reviewed his most recent research on thermonuclear reactions. Today, science popularizers generally give sole credit for the discovery



Flow of Liquid Helium II

A SURVEY of the various properties of liquid helium II has prompted us to investigate its viscosity more carefully. One of us[1] had previously deduced an upper limit of 10^{-6} c.g.s. units for the viscosity of helium II by measuring the damping of an oscillating cylinder. We had reached the same conclusion as Kapitza in the letter above; namely, that due to the high Reynolds number involved, the measurements probably represent non-laminar flow.

The present data were obtained from observations on the flow of liquid helium II through long capillaries. Two capillaries were used; the first had a circular bore of radius 0·05 cm. and length 130 cm. and drained a reservoir of 5·0 cm. diameter; the second was a thermometer capillary 93·5 cm. long and of elliptical cross-section with semi-axes 0·001 cm. and 0·002 cm., which was attached to a reservoir of 0·1 cm. diameter. The measurements were made by raising or lowering the reservoir with attached capillary so that the level of liquid helium in the reservoir was a centimetre or so above or below that of the surrounding liquid helium bath. The rate of change of level in the reservoir was then determined from the cathetometer eye-piece scale and a stop-watch; measurements were made until the levels became coincident. The data showing velocities of flow through the capillary and the corresponding pressure difference at the ends of the capillary are given in the accompanying table and plotted on a logarithmic scale in the diagram.

Capillary I		Capillary II			
$T=1{\cdot}07°$ K.		$T=1{\cdot}07°$ K.		$T=2{\cdot}17°$ K.	
Velocity (cm./sec.)	Pressure (dynes)	Velocity (cm./sec.)	Pressure (dynes)	Velocity (cm./sec.)	Pressure (dynes)
13·9	183·5	8·35	402	0·837	36·6
11·5	154·5	6·92	218	0·757	31·3
10·3	127·7	6·88	143	0·715	26·1
9·0	105·0	6·30	101	0·685	21·1
8·2	83·5	6·05	56	0·655	16·4
7·5	65·7	5·55	30	0·609	12·1
6·9	49·3	4·70	11·3	0·570	8·3
6·1	34·1	4·39	9·2	0·525	4·3
5·2[2]	20·3	3·92	13·0	0·433	0·9
4·5[2]	15·2	2·88	7·2		

The following facts are evident:

(*a*) The velocity of flow, q, changes only slightly for large changes in pressure head, p. For the smaller capillary, the relation is approximately $p \propto q^4$, but at the lowest velocities an even higher power seems indicated.

(*b*) The velocity of flow, for given pressure head and temperature, changes only slightly with a change of cross-section area of the order of 10^3.

(*c*) The velocity of flow, for given pressure head and given cross-section, changes by about a factor of 10 with a change of temperature from 1·07° K. to 2·17° K.

(*d*) With the larger capillary and slightly higher velocities of flow, the pressure-velocity relation is approximately $p \propto q^3$, with the power of q decreasing as the velocity is increased.

If, for the purpose of calculating a possible upper limit to the viscosity, we assume the formula for laminar flow, that is, $p \propto q$, we obtain the value $\eta = 4 \times 10^{-9}$ c.g.s. units. This agrees with the upper limit given by Kapitza who, using velocities of flow considerably higher than ours, has obtained the relation $p \propto q^2$ and an upper limit to the viscosity of $\eta = 10^{-9}$ c.g.s. units.

The observed type of flow, however, in which the velocity becomes almost independent of pressure, most certainly cannot be treated as laminar or even as ordinary turbulent flow. Consequently any known formula cannot, from our data, give a value of the 'viscosity' which would have much meaning. It may be possible that the liquid helium II slips over the surface of the tube. In this case any flow method

Submitted on Dec 22, 1937 (19 days later)

thermal conduction capillary will not be likely to be greater than 50 cm./sec. It seems, therefore, that undamped turbulent motion cannot account for an appreciable part of the high thermal conductivity which has been observed for helium II.

J. F. ALLEN.
A. D. MISENER.

Royal Society Mond Laboratory,
Cambridge.
Dec. 22.

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[1] Burton, E. F., NATURE, **135**, 265 (1935).

[2] Allen, Peierls and Uddin, NATURE, **140**, 62 (1937).

Some Experiments at Radio Frequencies on Supraconductors

MEASUREMENTS were made on an extruded tin wire carrying an alternating current of a frequency of about 200 kilocycles per second superposed upon a direct current. The resulting magnetic field at the surface of the wire was thus caused to pulsate cyclically.

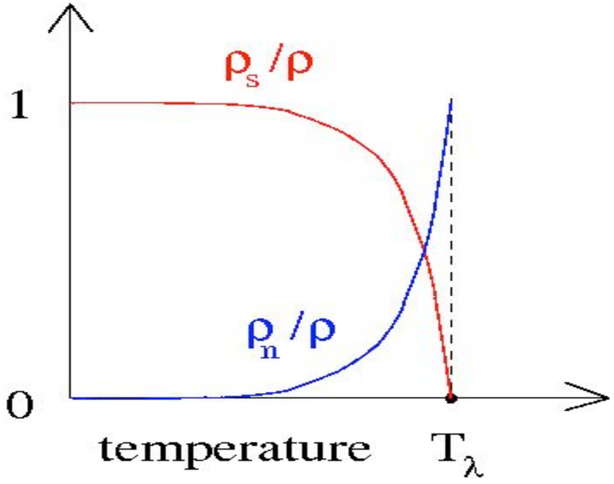
"A marked change in the viscosity takes place at 2.19 K, the temperature of transition of helium I to helium II."

J.O. Wilhelm, A.D. Misener & A.R. Clark, *Proc. Roy. Soc.* **151**, 342-347 (1935)

"It is not enough to make a discovery: one must also evaluate its significance for the development of science. But even that's not enough: a scientist must proceed from the essence of the discovery to produce others. It is only after this that he can consider that the discovery belongs to him."

P.L. Kapitza, as quoted by Andronikashvili, in "Reflections on Helium", AIP Press (1980)

Phenomenological model for He II



Superfluid: density ρ_s, velocity v_s, no viscosity, no entropy, Euler fluid

Normal fluid: density ρ_n, velocity v_n, carries viscosity and entropy, Navier Stokes fluid

"coexisting but non-interacting and interpenetrating"

F. London 1900-1954

L. Tisza 1907-2009

L.D. Landau 1908-1968

1962 Nobel Prize

Landau’s two-fluid model for He II

Starts with the quantum ground state, with “elementary excitations” with a particular spectrum (phonons and rotons).

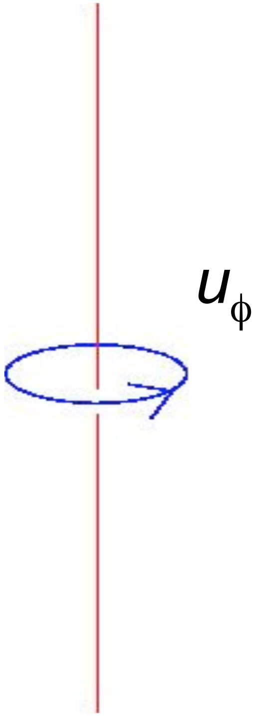
Landau: “We particularly emphasize that there is no division of the real particles of the liquid into “superfluid” and “normal” ones…which is a no more than a manner of expression…”

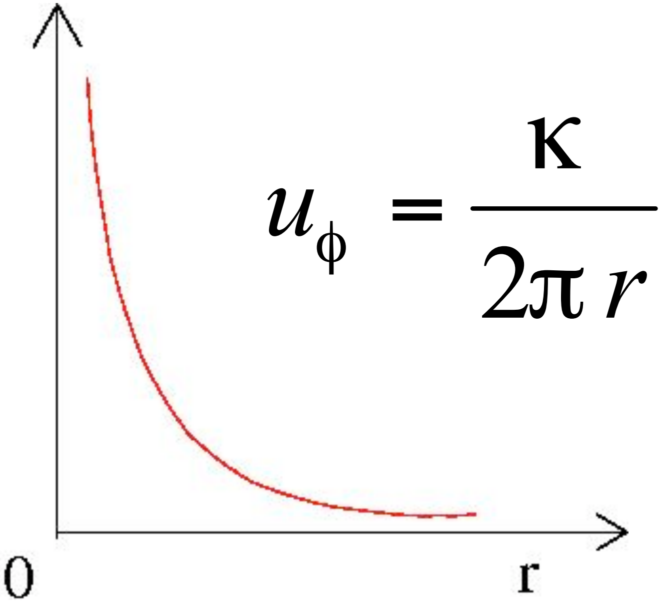
Landau’s picture was incomplete, and was later augmented by other publications. The present understanding is that the helium atoms indeed undergo Bose condensation and the superfluid velocity is the gradient of the phase of the condensate wavefunction. But the condensate is not the superfluid. Only 10% of the fluid is the condensate at 0 K, where 100% of the fluid is superfluid.

N.N. Bogoliubov
J. Phys. USSR 11, **23** (1956)
Bose condensation and its role

P.C. Hohenberg & P.C. Martin
Annals of Physics, **34**, 291 (1965)
full critique and microscopic theory

quantized vortices in helium II





$$u_\phi = \frac{\kappa}{2\pi r}$$

Onsager
1903-1976

Wave function: $\psi = \psi_0 \exp(i\varphi(r))$, $\psi_0 \rightarrow 0$ as $r \rightarrow 0$ and $\rightarrow 1$ as $r \rightarrow \infty$

Velocity is the gradient of $\varphi(r)$. The increment of its gradient over any closed path must be a multiple of 2π, for the wave function to remain single valued.

"Thus, the well-known invariant called hydrodynamic circulation is quantized; the quantum of circulation is h/m."

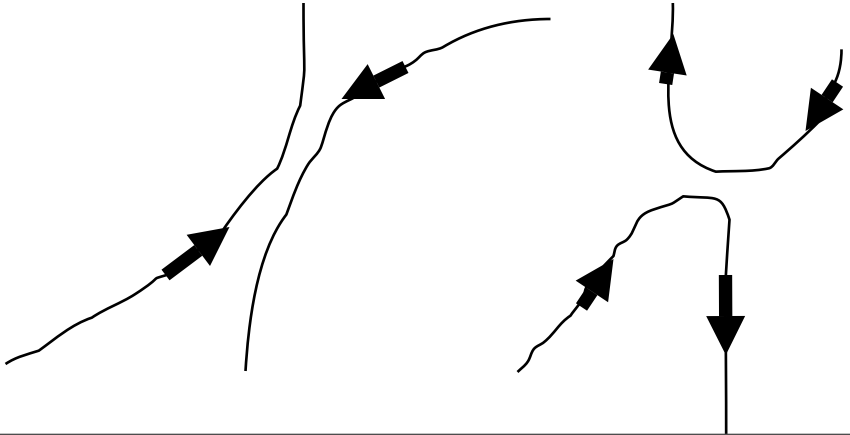
Onsager (1949)

Except for a few angstroms from the center of the core, the laws obeyed are those of classical hydrodynamics [e.g., Biot-Savart].

R.P. Feynman: 1918-1988

Prog. Low Temp. Phys. 1, 17 (1955)

If … two oppositely directed sections of [vortex] line approach closely, … the lines (which are under tension) may snap together and join connections a new way …



←

High-intensity vortex structures in homogeneous and isotropic turbulence (Vincenti & Meneguzzi 1991)

Vortex tangles ("**superfluid turbulence**") by Tsubota, Araki & Nemirovskii 2000); pioneering simulations by K.W. Schwarz (1985)

Microscopic details of reconnection were explored by J. Koplik and H. Levine, *Phys. Rev. Lett.* **71**, 1375 (1993), by solving the nonlinear Schrödinger equation with quadratic nonlinearity – which is a good model for the wavefunction in BEC.

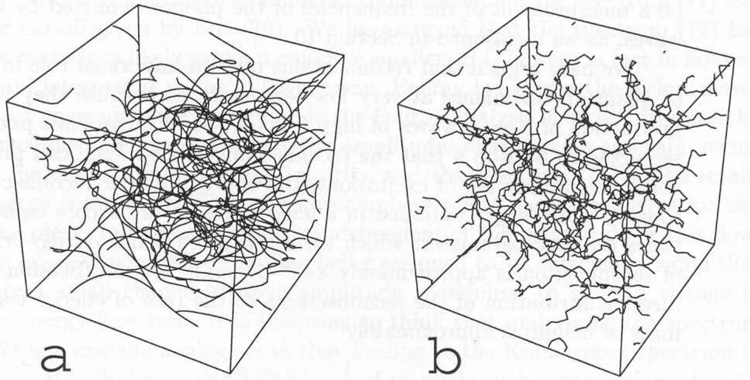
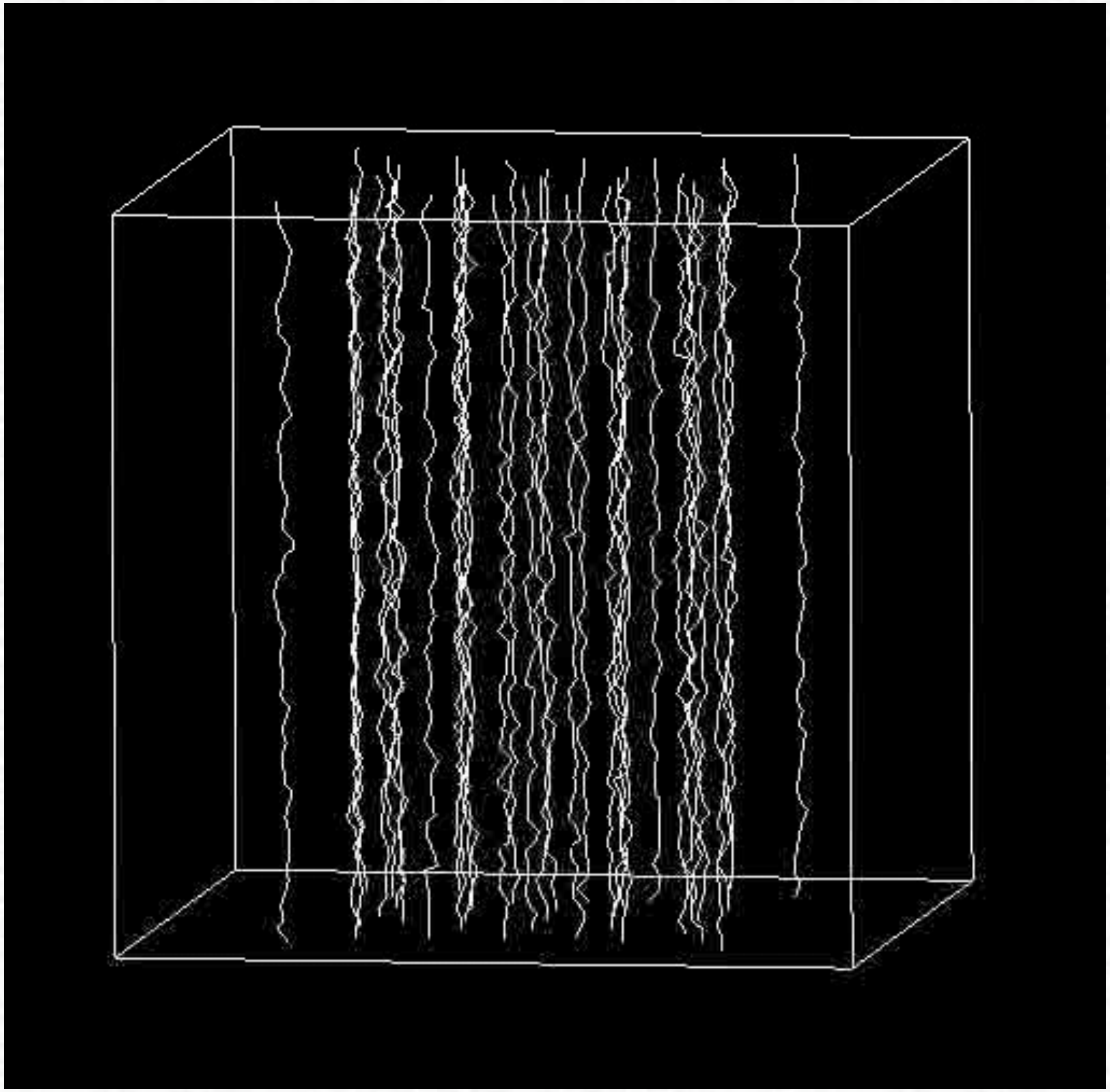


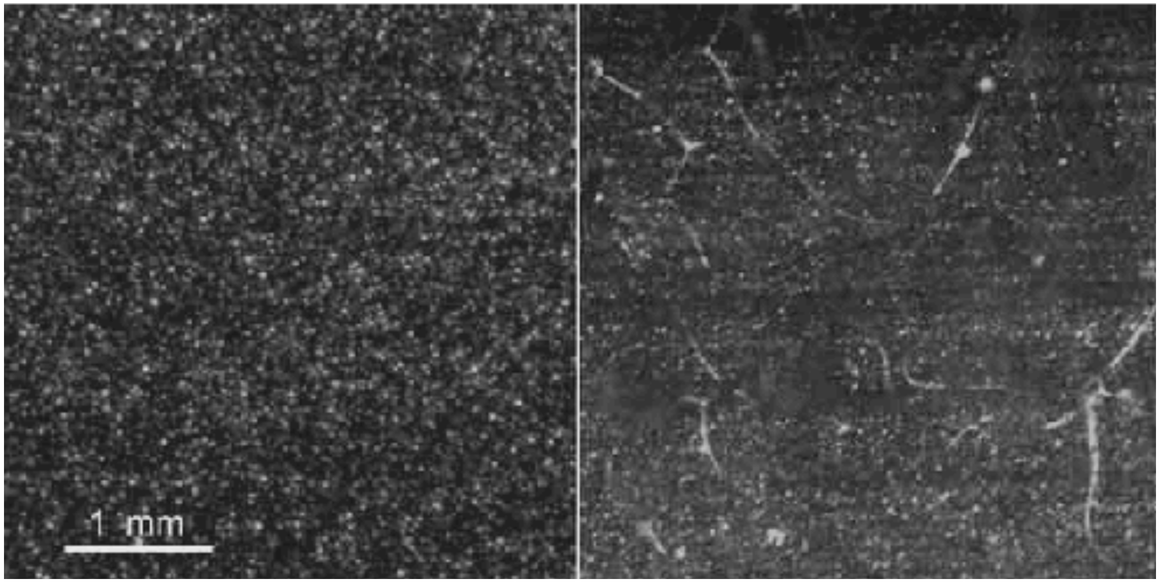
Fig. 10. Vortex tangles at (a) $T = 1.6$K and (b) $T = 0$K. From Tsubota *et al*[50].

Nature of superfluid turbulence, its connection to classical turbulence



Carlo Barenghi and colleagues

50 years on…



~50 mK above T_λ **~50 mK below T_λ**

The left panel shows a suspension of hydrogen particles just above the transition temperature. The right panel shows similar hydrogen particles after the fluid was cooled below the lambda point. Some particles have collected along filaments, while other are randomly distributed as before. Fewer free particles are apparent on the right only because the light intensity was reduced to highlight the brighter filaments in the image. Volume fraction $\cong 3X10^{-5}$.

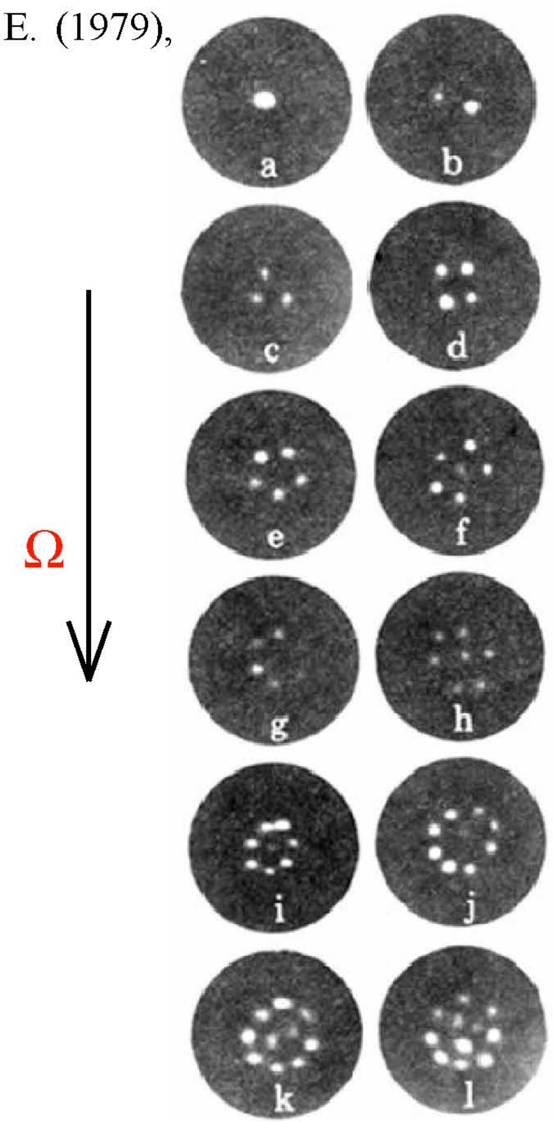
G.P. Bewley, D.P. Lathrop & KRS, Nature 441, 558 (2006)

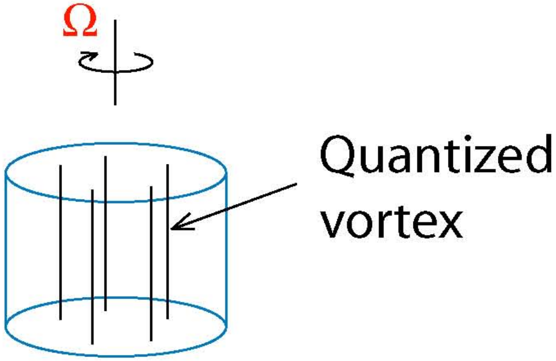
Previous observation of quantized vortices

Yarmchuk, E.J., Gordon, M.J.V. and Packard, R.E. (1979),
Phys. Rev. Lett. **43**, 214-217.

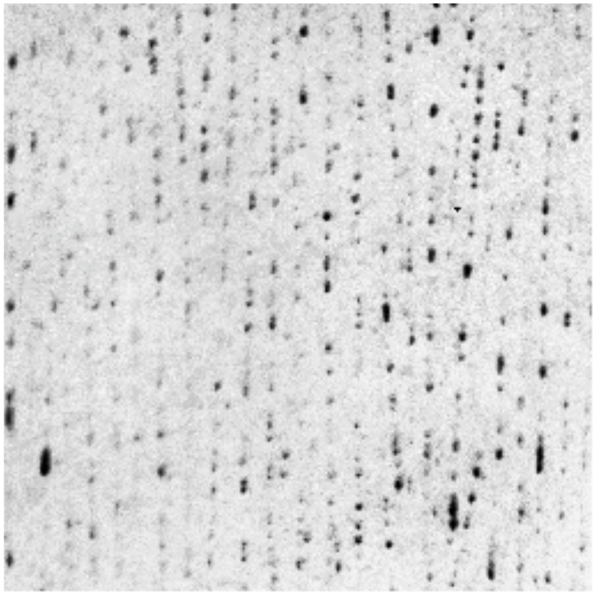
technique not suitable for visualizing tangled vortices

indirectly inferred by Hall & Vinen, *Proc. Roy. Soc.* **A238,** 204 (1956)

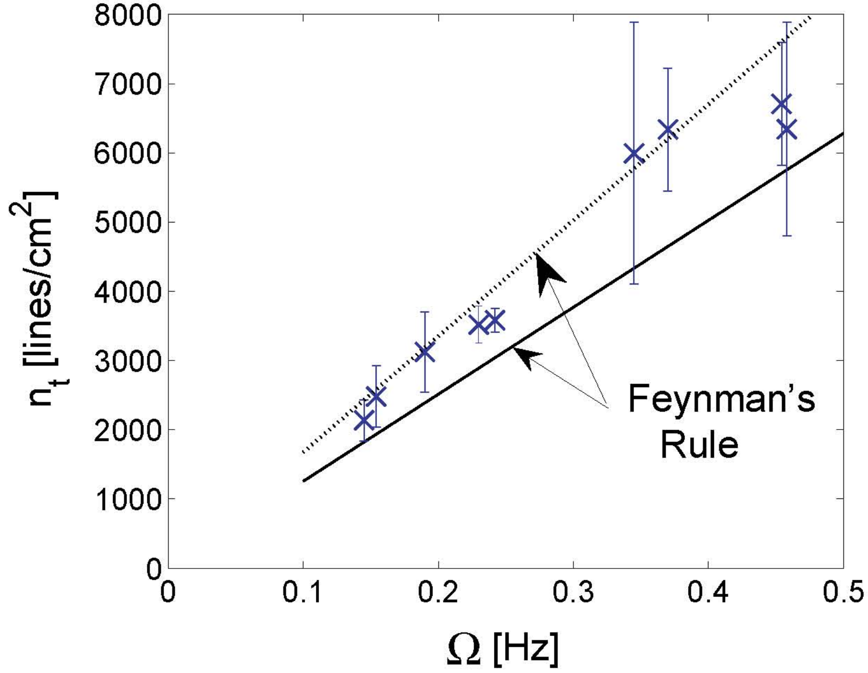




Number of vortices

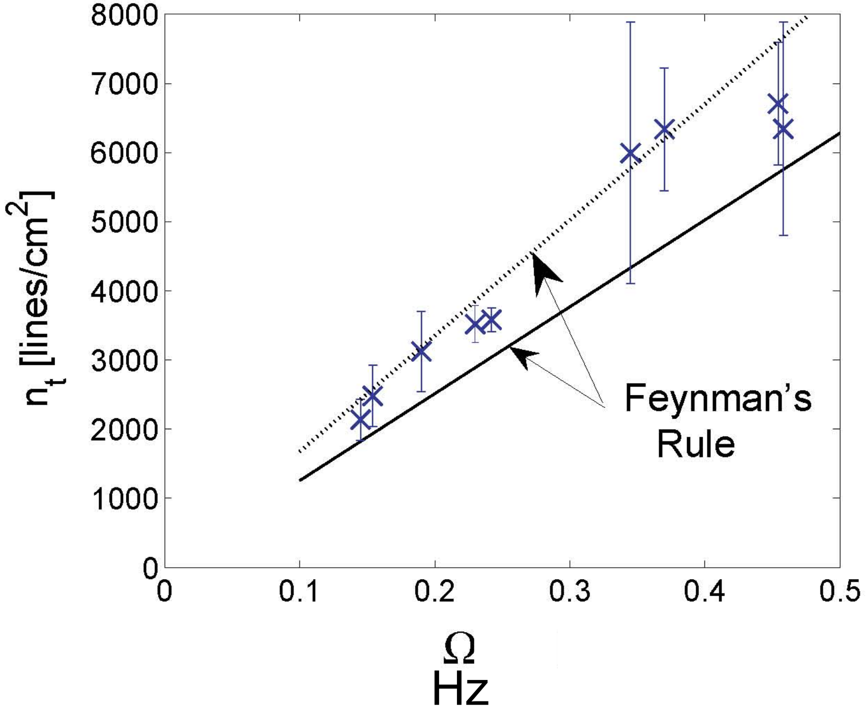


c

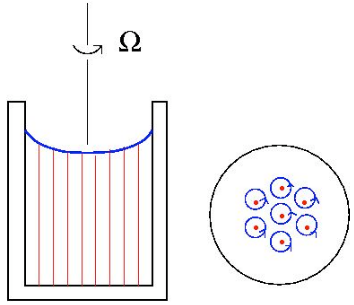


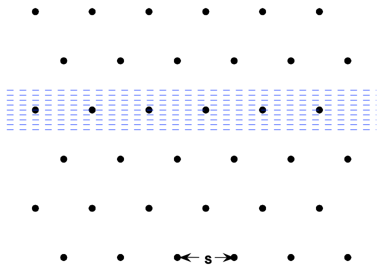
The left panel shows an example of particles arranged along vertical lines when the system is rotating steadily about the vertical axis. The spacing of lines is remarkably uniform, although there are occasional distortions of the lattice and possible points of intersection. Their number follows Feynman's rule pretty well.

Lattice density

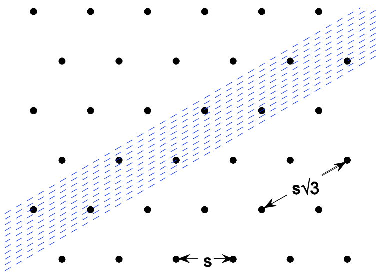


Feynman's rule
$n_t \cong 2000\Omega$ (in rad/s)





$n_o \approx 2000\Omega$ lines/cm^2



What kind of particles?

Requirements on particle properties

- Must be small enough to follow the flow with fidelity (i.e., must respond to the smallest scales of the flow with fast response); in particular, must have the same density as the fluid (e.g., Maxey & Riley, *Phys. Fluids* **26**, 883 (1983))
- Must be large enough to be imaged with ‘usable’ illumination and detection equipment
- Must not cluster

In liquid helium

- Because of small apparatus and large Reynolds numbers, small scales are smaller than in water, demands on fidelity are higher; in particular, helium I has a density of 1/8 that of water
- Very small particles cannot be imaged
- Mutual attraction of particles and clustering cannot be suppressed by using surfactants as in water.

Particles that have worked

Nearly neutral particles of frozen mixtures of helium and hydrogen.

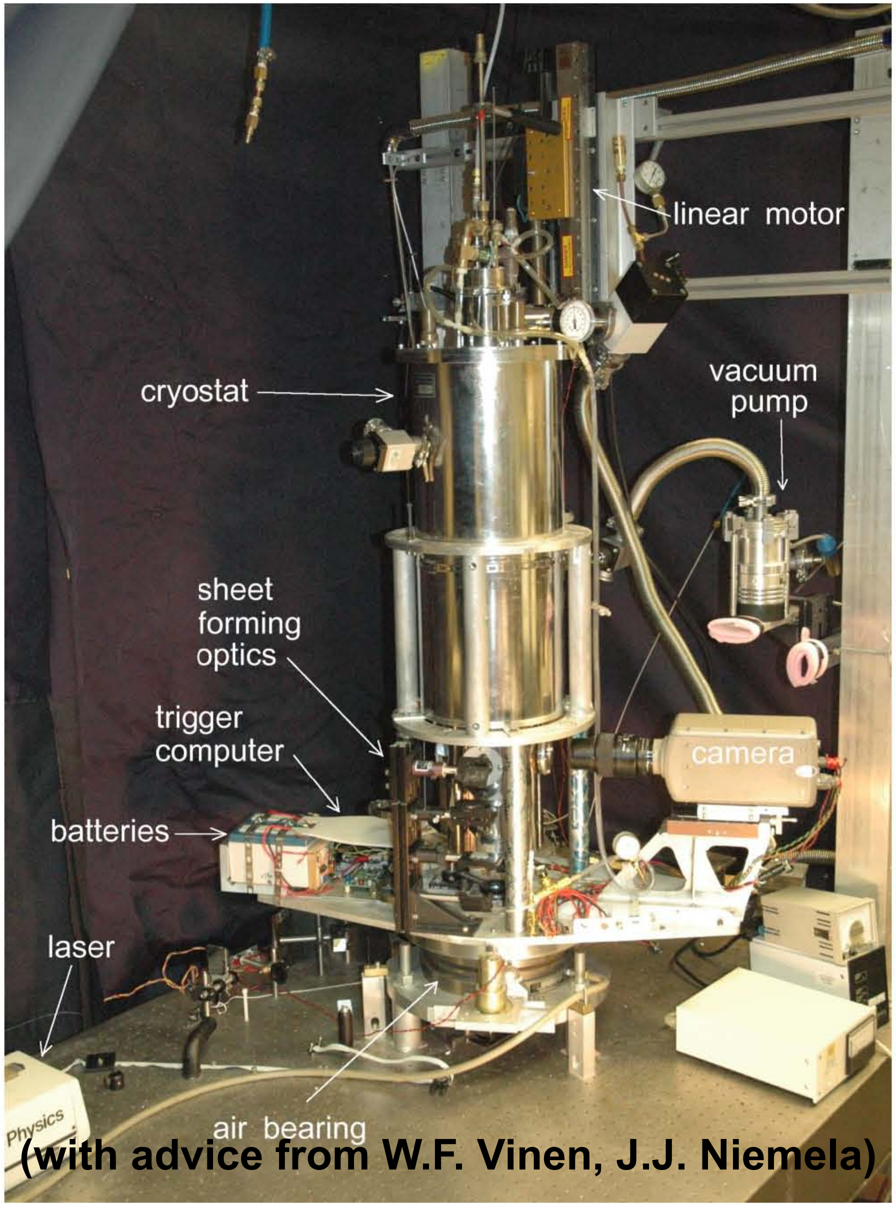
Bewley, Lathrop & KRS, *Nature* **441**, 558 (2006); *Experiments in Fluids* **44**, 887 (2008); Paoletti, Fiorito, KRS & Lathrop, *J. Phys. Soc. Jpn* 77, 80702 (2008)


Greg Bewley

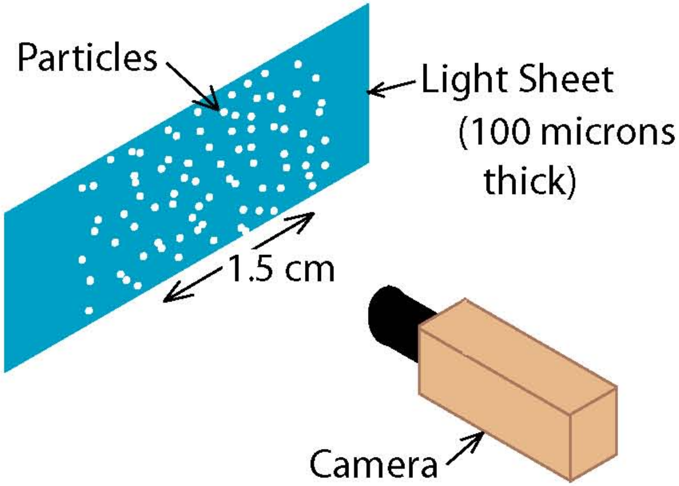

Matt Paoletti

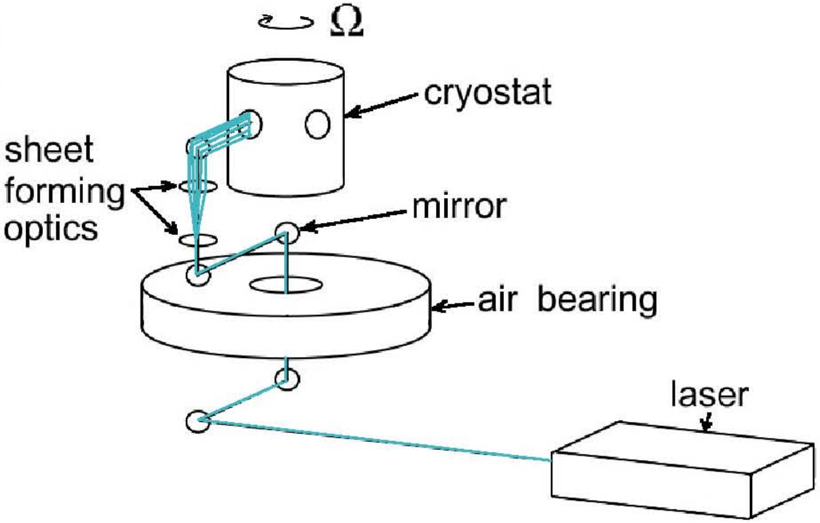

Dan Lathrop

Apparatus

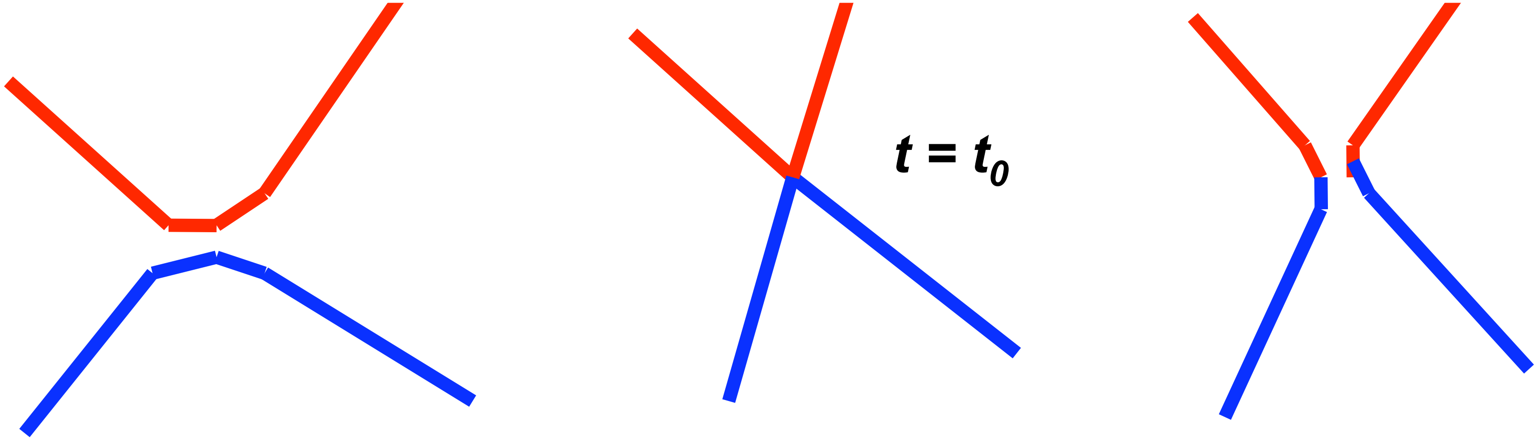


(with advice from W.F. Vinen, J.J. Niemela)



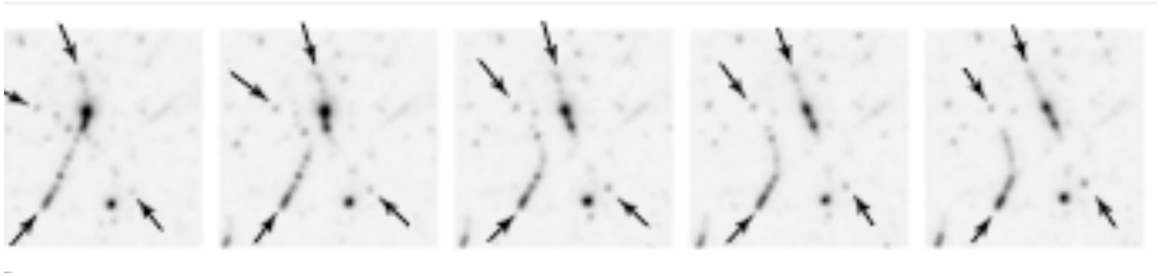


Laser Beam routing



Schematic of cores of reconnecting vortices before and and after reconnection at $t > t_o$.

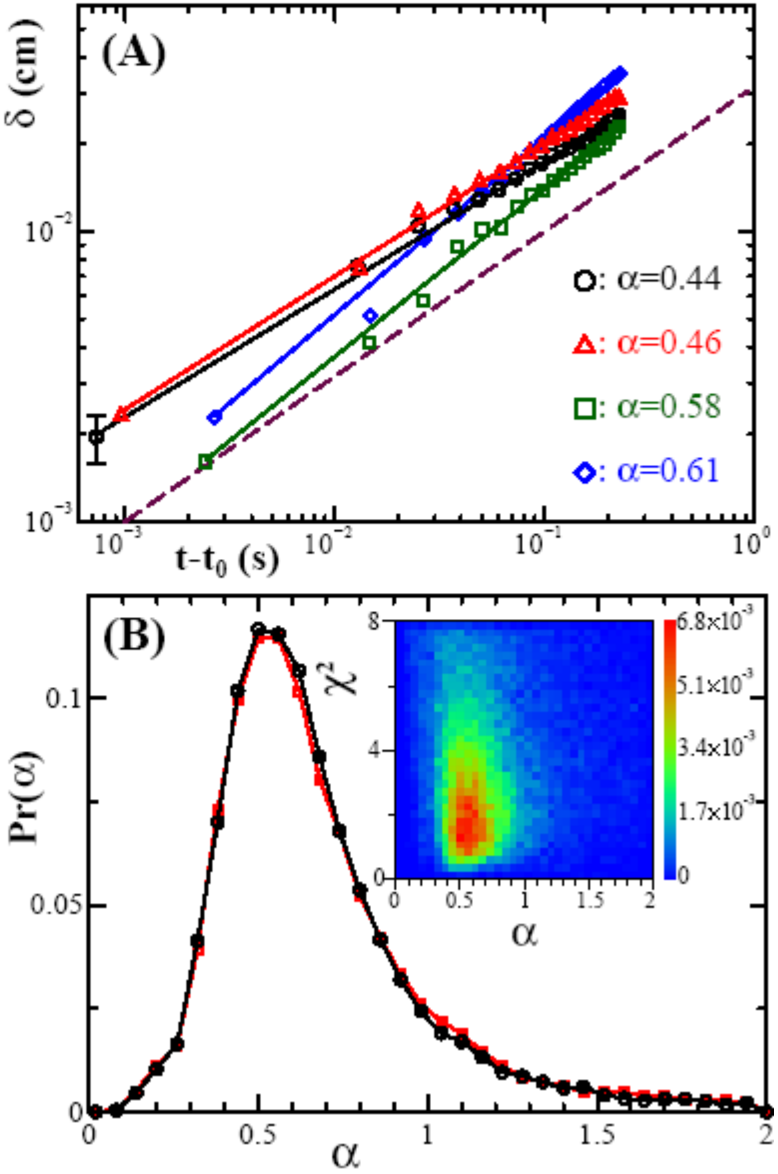
reconnection movie 2.avi



Images of hydrogen particles suspended in liquid helium, taken at 50 *ms* intervals, for $t > t_0$. Some particles are trapped on quantized vortex cores, while others are randomly distributed in the fluid. Before reconnection, particles drift collectively with the background flow. Subsequent frames show reconnection as the sudden motion of a group of particles.

Bewley, Poaletti, KRS & Lathrop, *PNAS* **105**, 13707 (2008)

Define delta

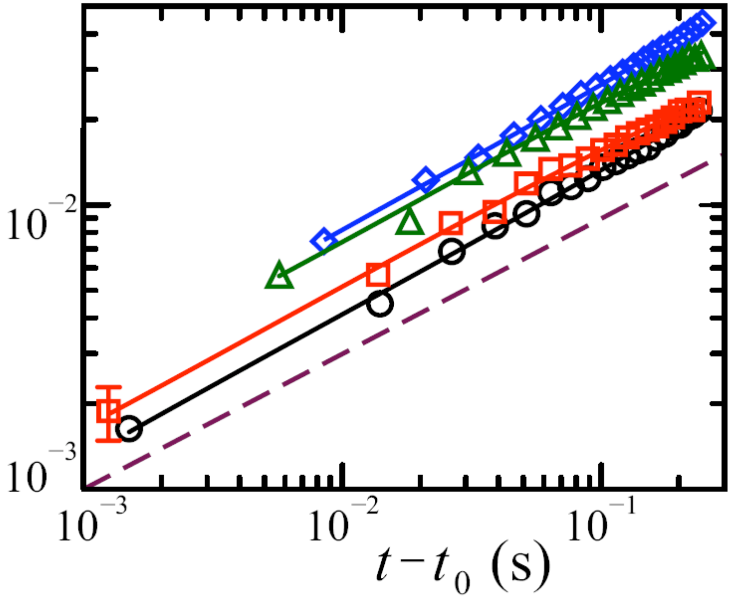


$$\delta(t) = A\kappa(t-t_0)^{\alpha}$$

dimensional analysis: $\alpha = 1/2$

Alternatively:

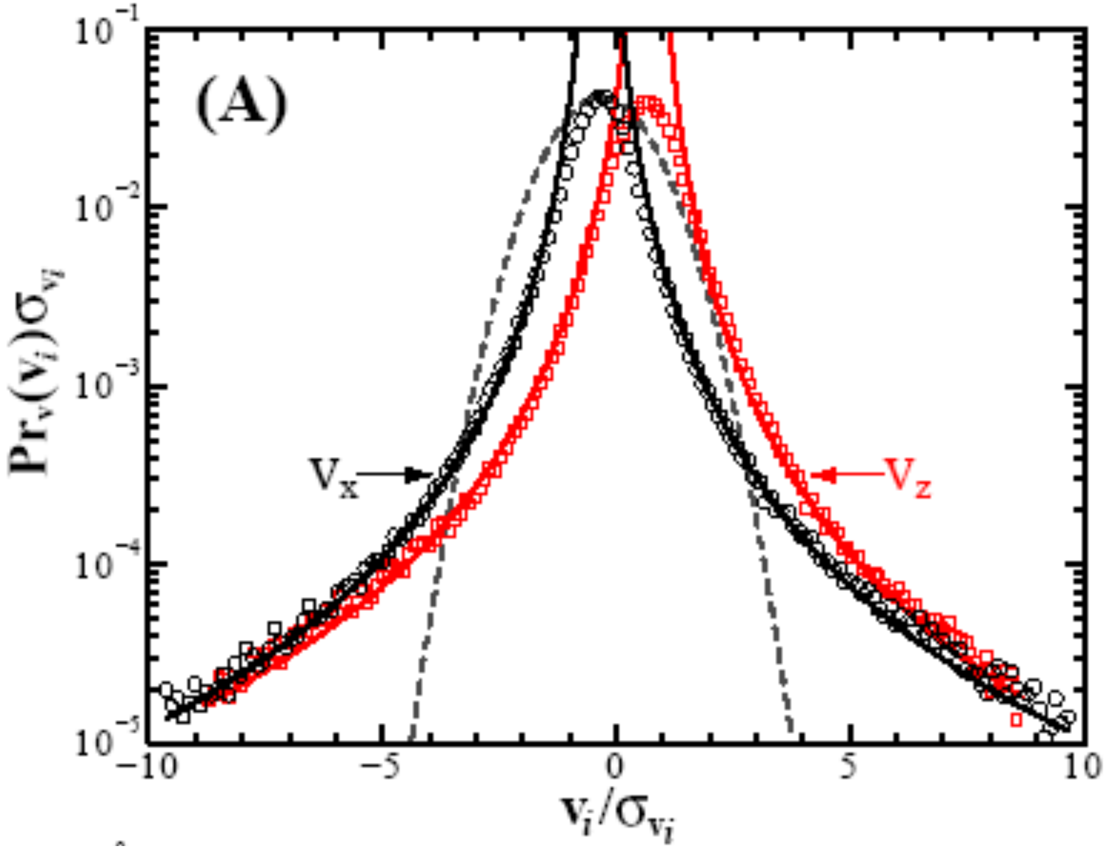
$$\delta(t) = A\kappa(t-t_0)^{1/2}[1+c(t-t_0)]$$



Paoletti, Fisher, KRS & Lathrop, *Phys. Rev. Lett.* (2008)

Reconnection statistics are time reversible?

Nearly homogeneous turbulence following a counterflow

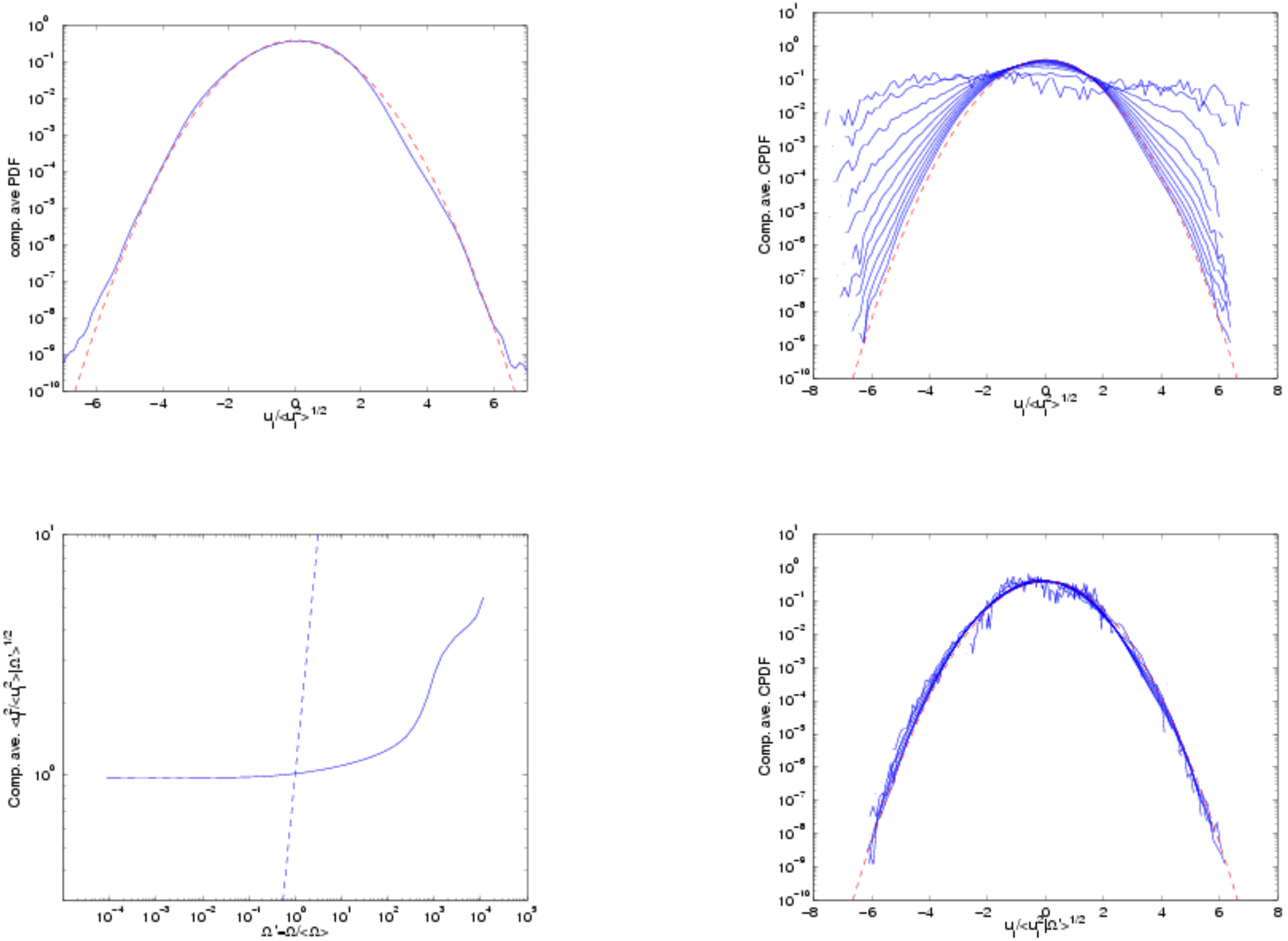


$$\mathrm{Pr}(v)\,dv = \mathrm{Pr}(t)\,dt$$
$$v = \kappa(t - t_0)^{-0.5}$$
$$\mathrm{Pr}(v) \sim |v|^{-3}$$

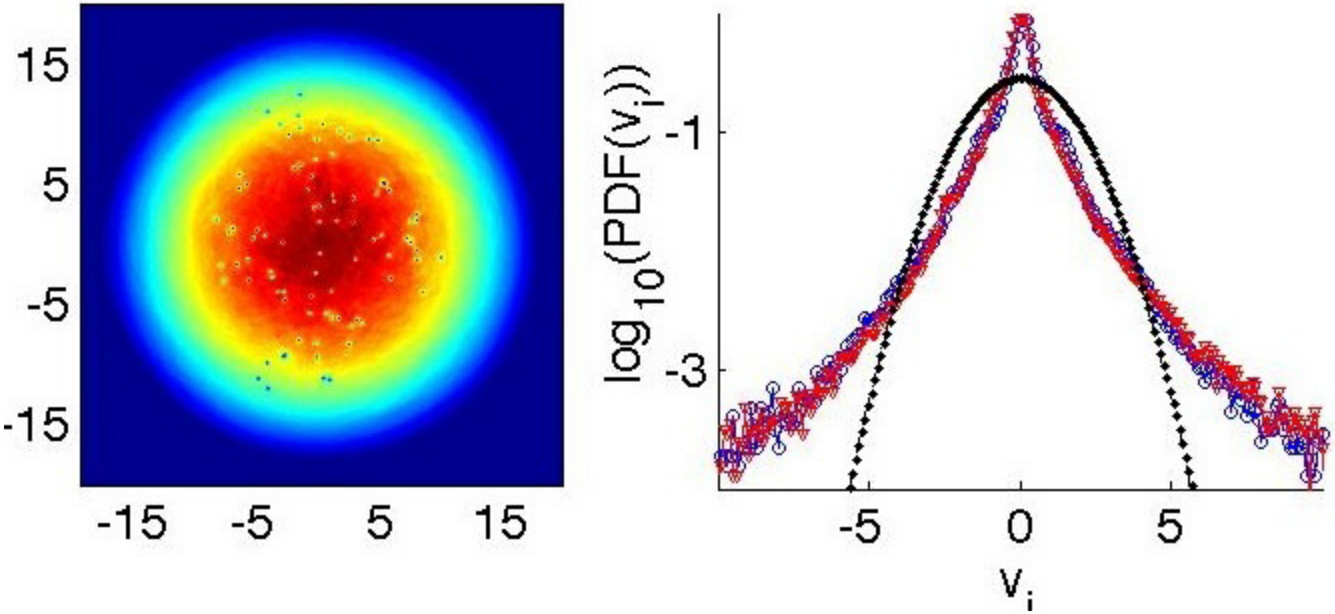
No instances (away from solid boundary) where power-law tails exist for velocity distributions in classical turbulence.

Even by conditioning velocity PDFs on intense vorticity in classical turbulence, one finds no sign of anything other than a Gaussian.

Paoletti, Fisher, KRS & Lathrop, *Phys. Rev. Lett.* (2008)



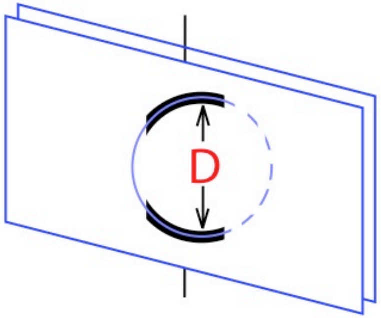
Velocity PDFs conditioned on strong vorticity

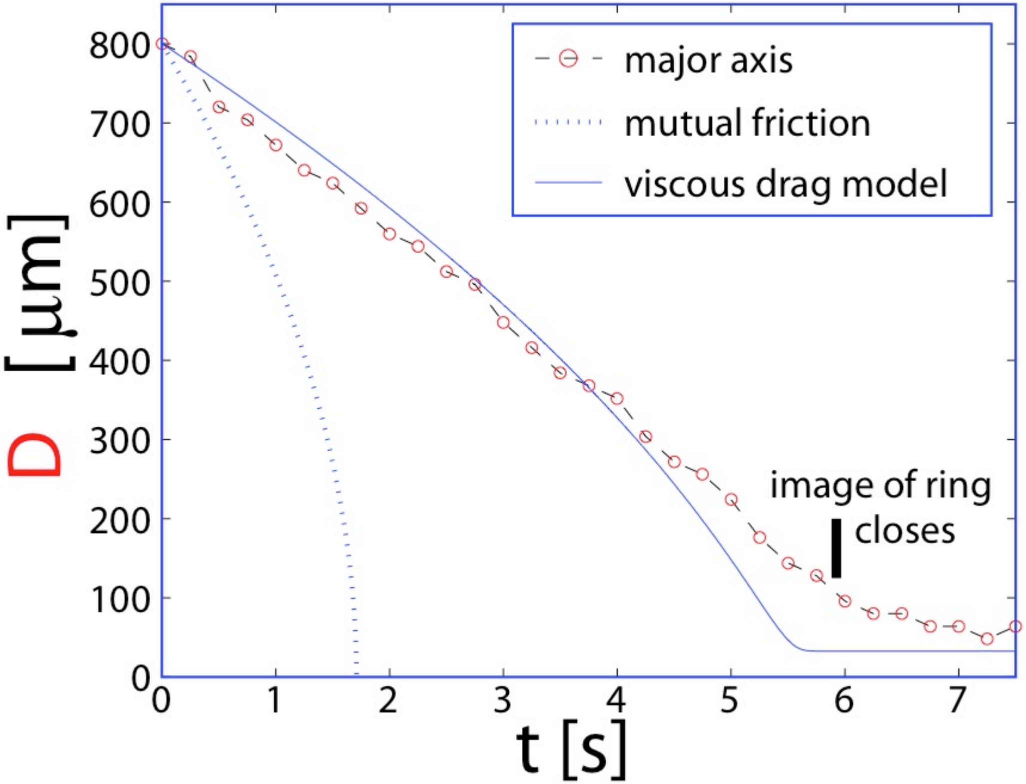


Density (left) and velocity PDFs (right) for a 2D BEC.
Left: 86 vortices (core radius 2.66) can be identified.
Right: Plots of log$_{10}$[PDF(v)]. Corresponding Gaussian PDF is displayed as well.

From “Non-classical velocity statistics in a turbulent atomic Bose-Einstein condensate” by Angela White, Nick P. Proukakis, and Carlo F. Barenghi (preprint, 2009)

T = 2.05 K





(estimated error in D = 16 μm)

Joe Vinen

Comparisons of classical and superfluid turbulence

Superfluid turbulence (helium II)

- Velocity distribution follows a power law
- Reconnections plays a crucial role
- Quantization of circulation imposes severe restrictions on the stretching of vortex line elements
- Dissipation mechanism is not well understood

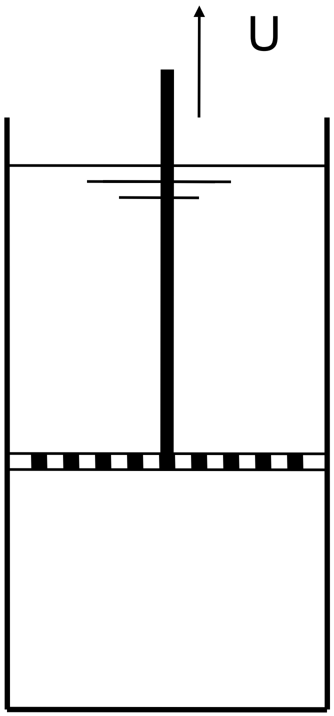
Classical turbulence (3D)

- Velocity distribution is nearly normal
- The role of reconnections is not clear
- Vortex stretching plays a key role in scale-to-scale energy transfer
- Energy dissipation occurs because of fluid viscosity

Yet…

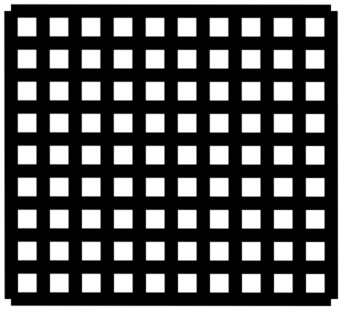
- **−5/3 slope in the spectral form is common**
- Decay law is the same as in classical turbulence
- The concept of eddy viscosity seems to apply in the decaying case

Classical turbulence behind pull-through grid

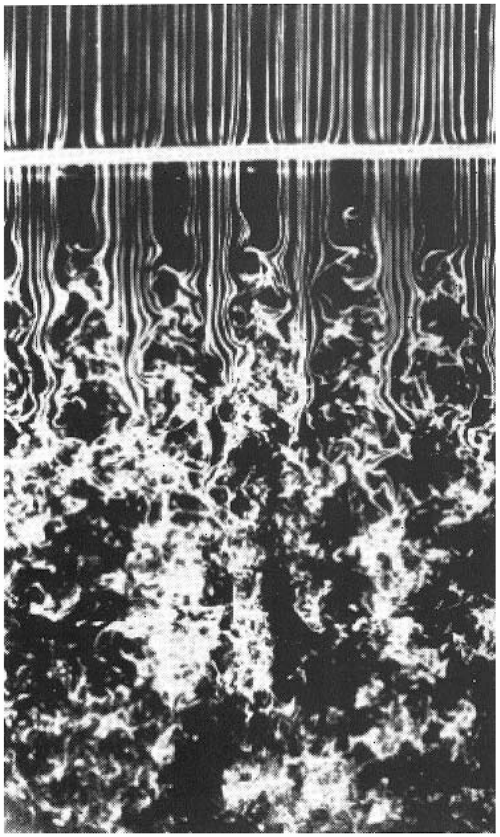


tank of water

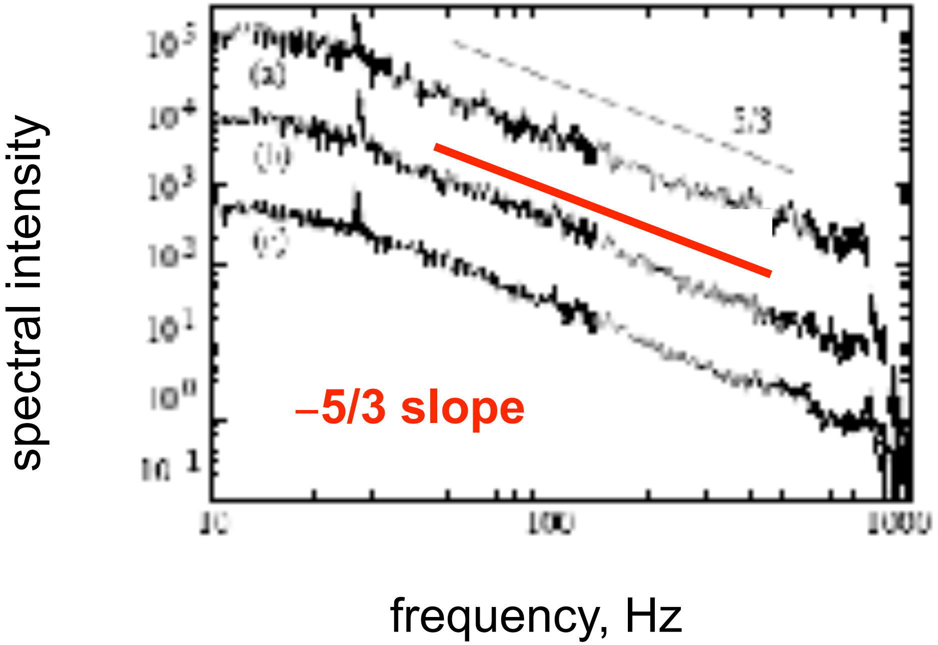
nearly isotropic
turbulence is
generated.



square grid of bars



grid turbulence in air:
reoriented; Corke & Nagib

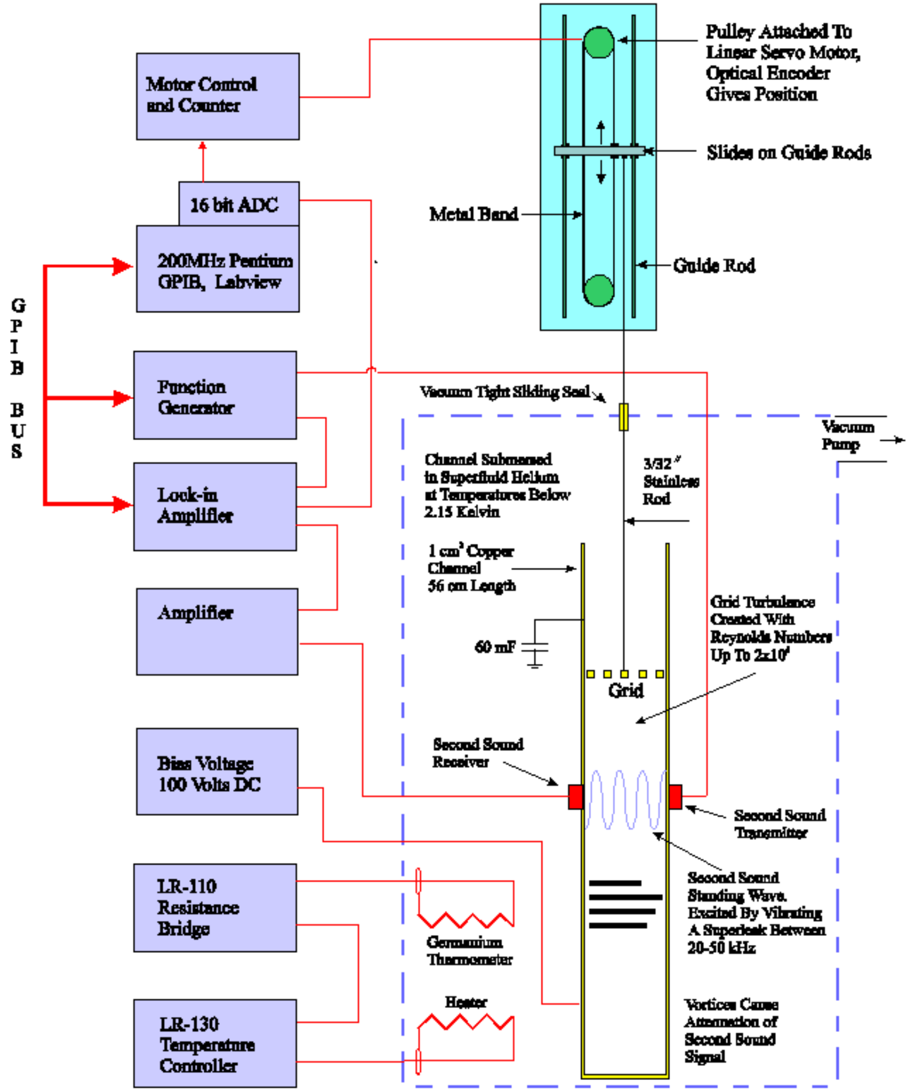


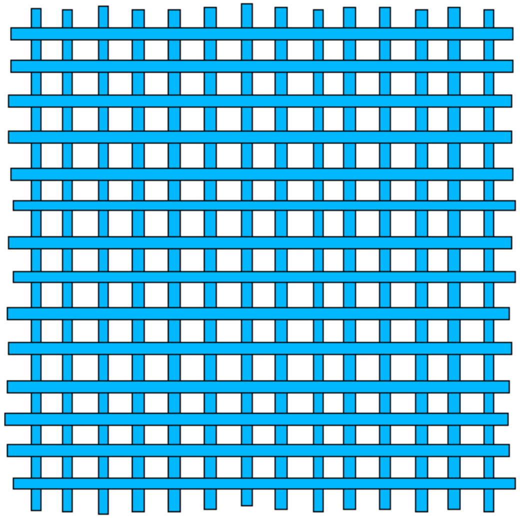
Superfluid turbulence in Karman flow:
J. Maurer & P. Tabeling, *Europhys. Lett.* **43**, 29 (1998)

In simulations:

- C. Nore, M. Abid & M.E. Brachet, *Phys. Rev. Lett.* **78**, 3896 (1997)
- T. Araki, M. Tsubota & S.K. Nemirovskii, *Phys. Rev. Lett.* **89**, 145301 (2002)
- M. Kobayashi & M. Tsubota, *Phys. Rev. Lett.* **94**, 065302 (2005)
- P.E. Roche *et al. Europhys. Lett.* **77**, 66002 (2007)

Obvious? Surprising?

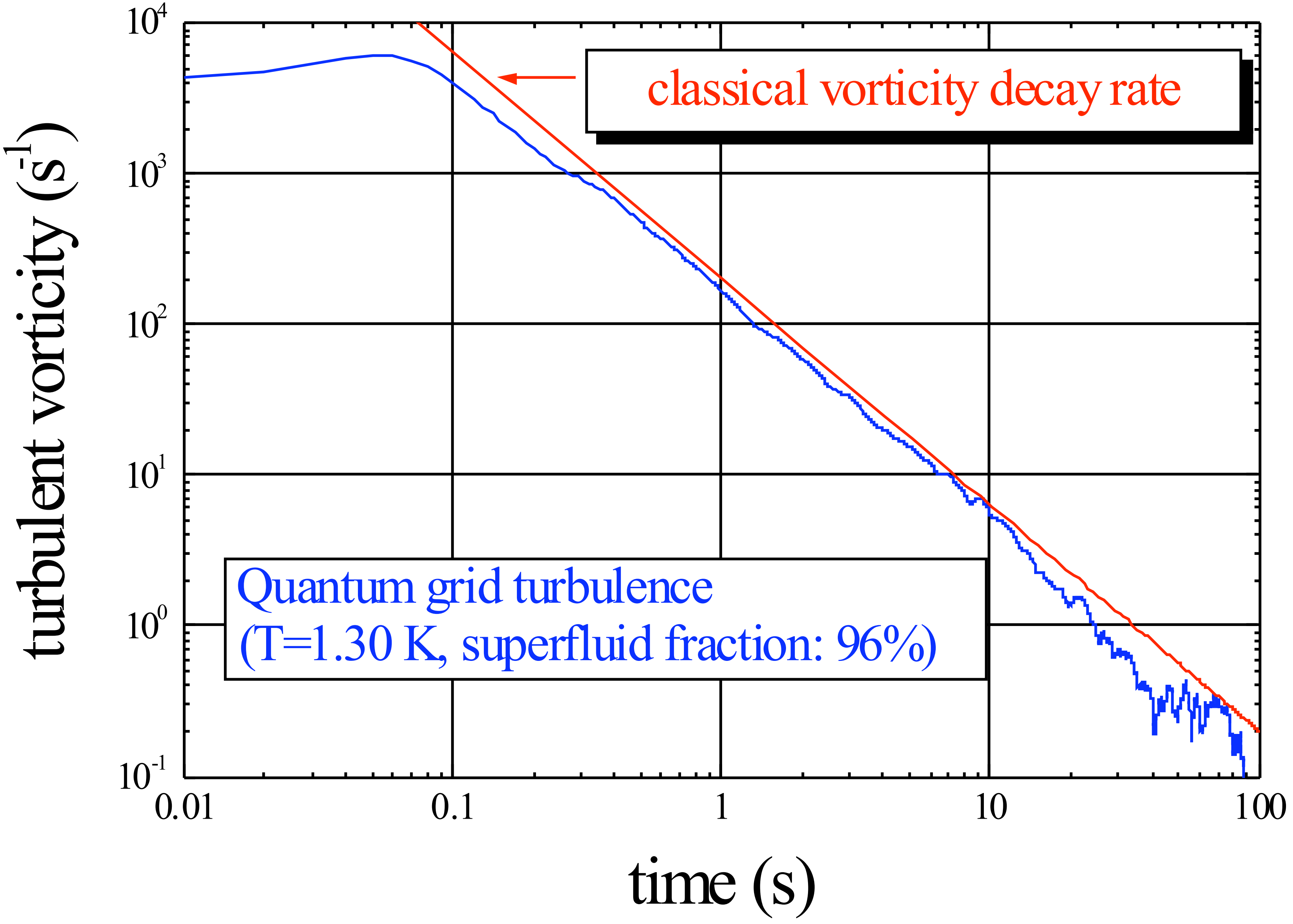


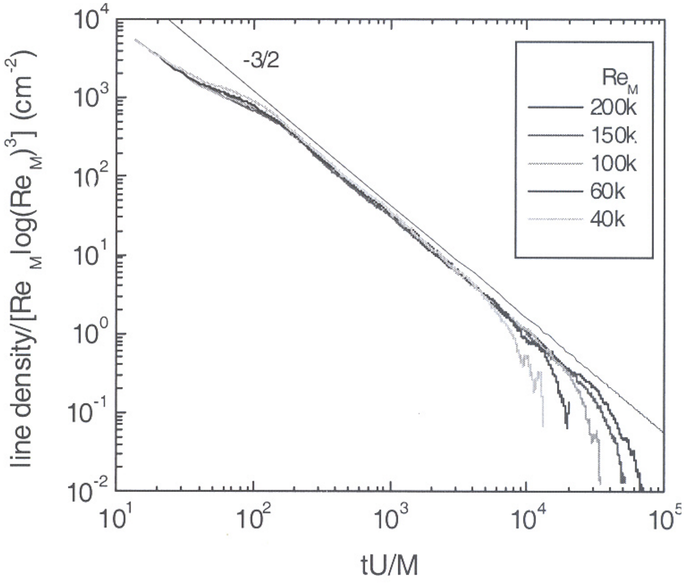


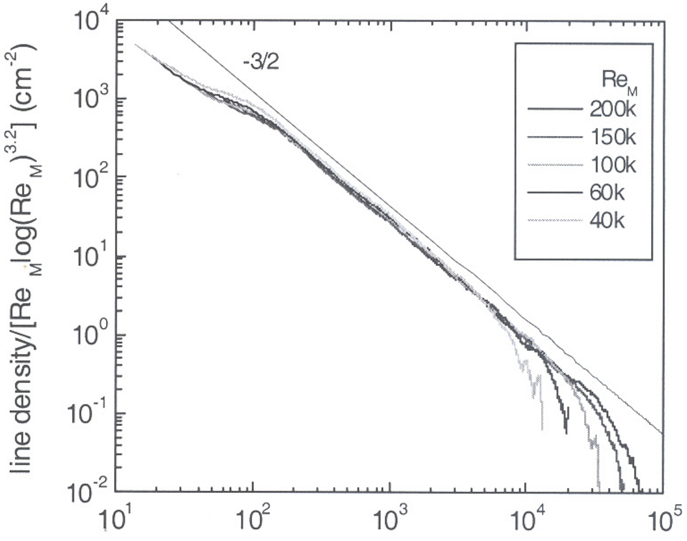
turbulence-generating grid

(as in Comte-Bellot & Corrsin)

Stalp, Niemela, Vinen, Donnelly, Skrbek, etc


classical vorticity decay rate
Quantum grid turbulence
(T=1.30 K, superfluid fraction: 96%)
turbulent vorticity (s^{-1})
10^4
10^3
10^2
10^1
10^0
10^{-1}
0.01
0.1
1
10
100
time (s)





Because superfluid vorticity decays as $t^{-3/2}$, just as does classical vorticity, and the observed prefactors are as expected, the notion arises that the two turbulence fields are coupled in a range of scales. This is the hypothesis of coupled vorticity (Barenghi, Donnelly, Niemela, KRS, Vinen, Volovik, etc)

Obviously different mechanisms operate on dissipative scales:

- Vinen, Phys. Rev. B 61, 1410 (2000)
- Vinen, Tsubota & Mitani, PRL, 91, 135301 (2003)
- L'vov, Nazarenko & Volovik, JETP Lett. 80, 546 (2004)
- Kozik & Svistunov, Phys. Rev. Lett. 92, 172505 (2005)
- L'vov, Nazarenko and Rudenko, Phys. Rev. B 76, 024520 (2007)
- Walmsley, Golov, Hall, Levchenko and Vinen, PRL 99, 265302 (2007)

Walmsley, Golov, Hall, Levchenko & Vinen

PRL **99,** 265302 (2007)

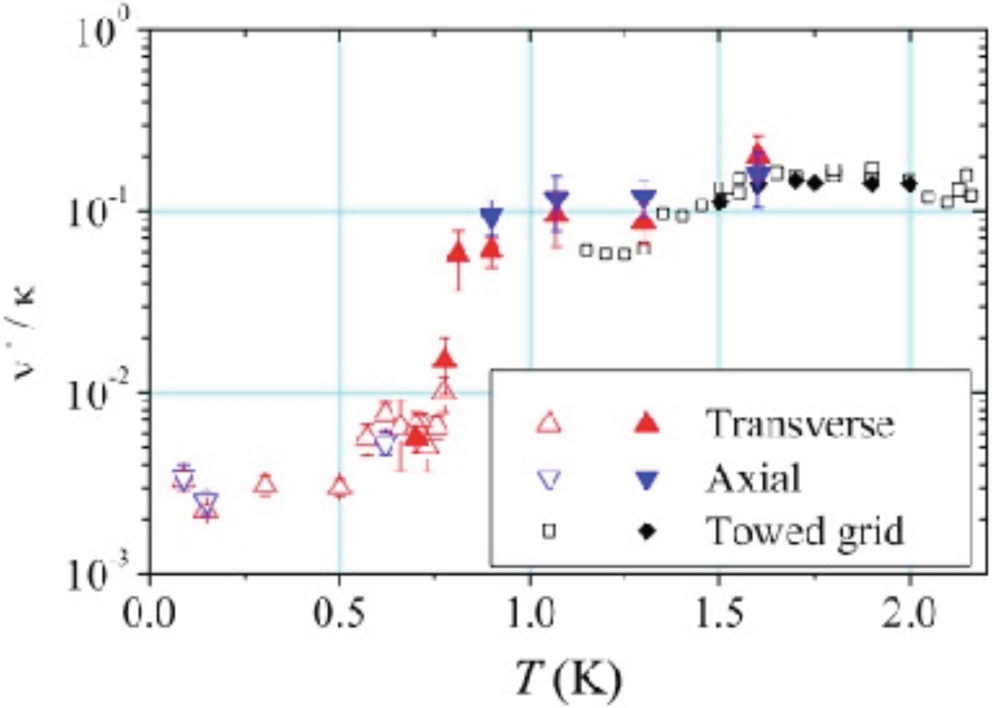


FIG. 5 (color online). The effective kinematic viscosity ν' after a spin down from $\Omega = 1.5$ rad/s measured in the transverse (△) and axial (▽) directions. Closed (open) triangles correspond to measurements with free ions (charged vortex rings). Error bars specify the uncertainty of fitting. Squares and diamonds: second sound measurements of grid turbulence [12,22].

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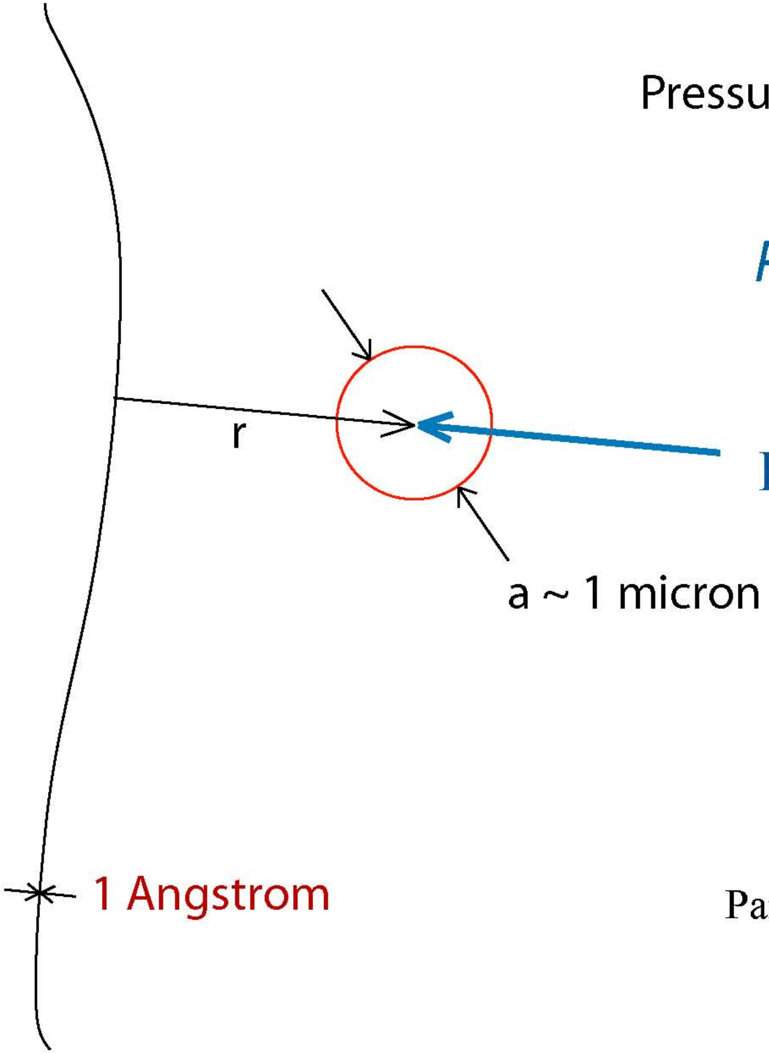
Only beginnings have been made to understand these aspects

Particle Trapping



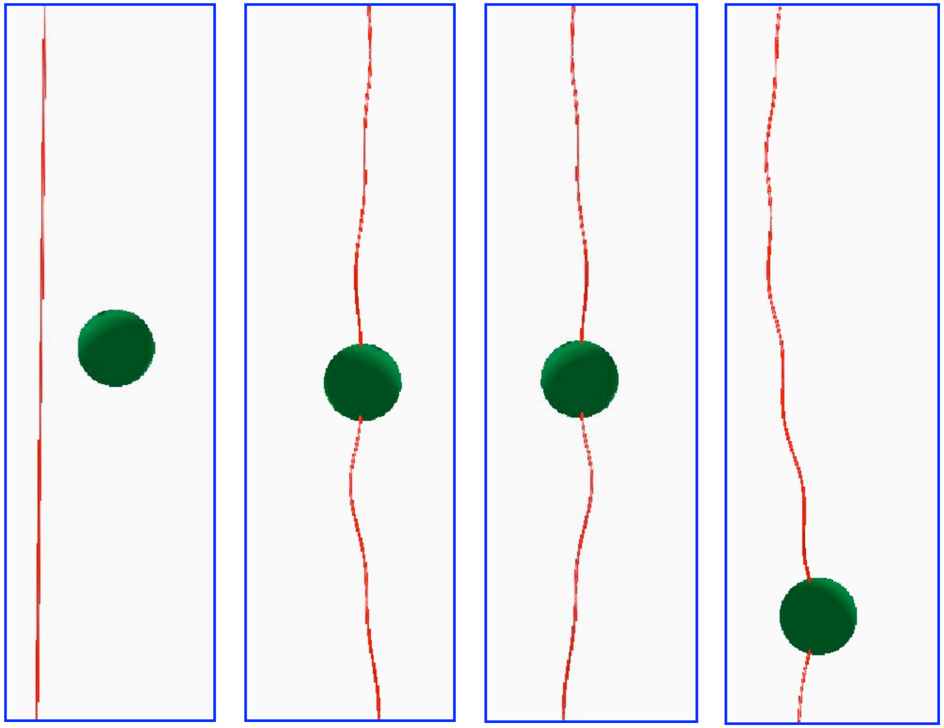
Pressure gradient:

$$P = \frac{-\rho_s \kappa^2}{8\pi^2 r^2}$$

$$\mathrm{F} = \frac{4}{3}\pi a^3 \nabla P \sim \frac{1}{r^3}$$

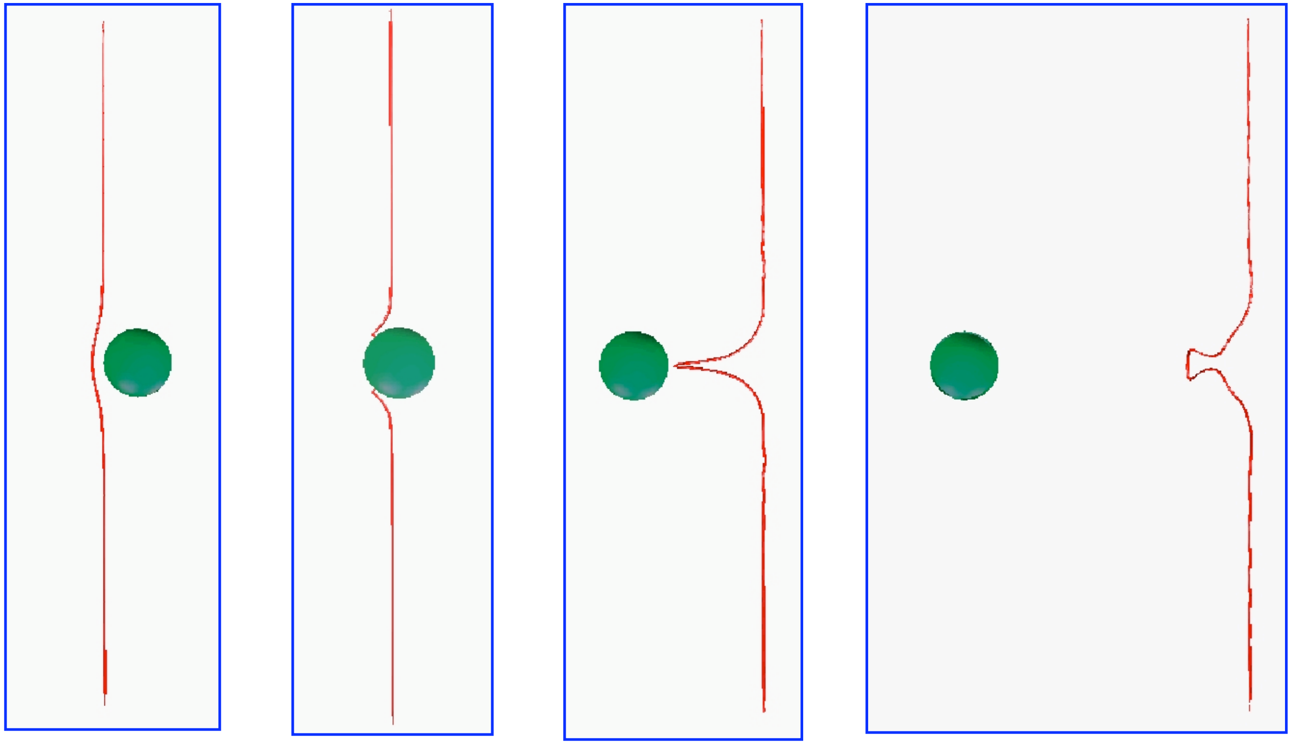
Parks, P.E. and Donnelly, R.J. (1966), *Phys. Rev. Lett.* **16**, 45–48.

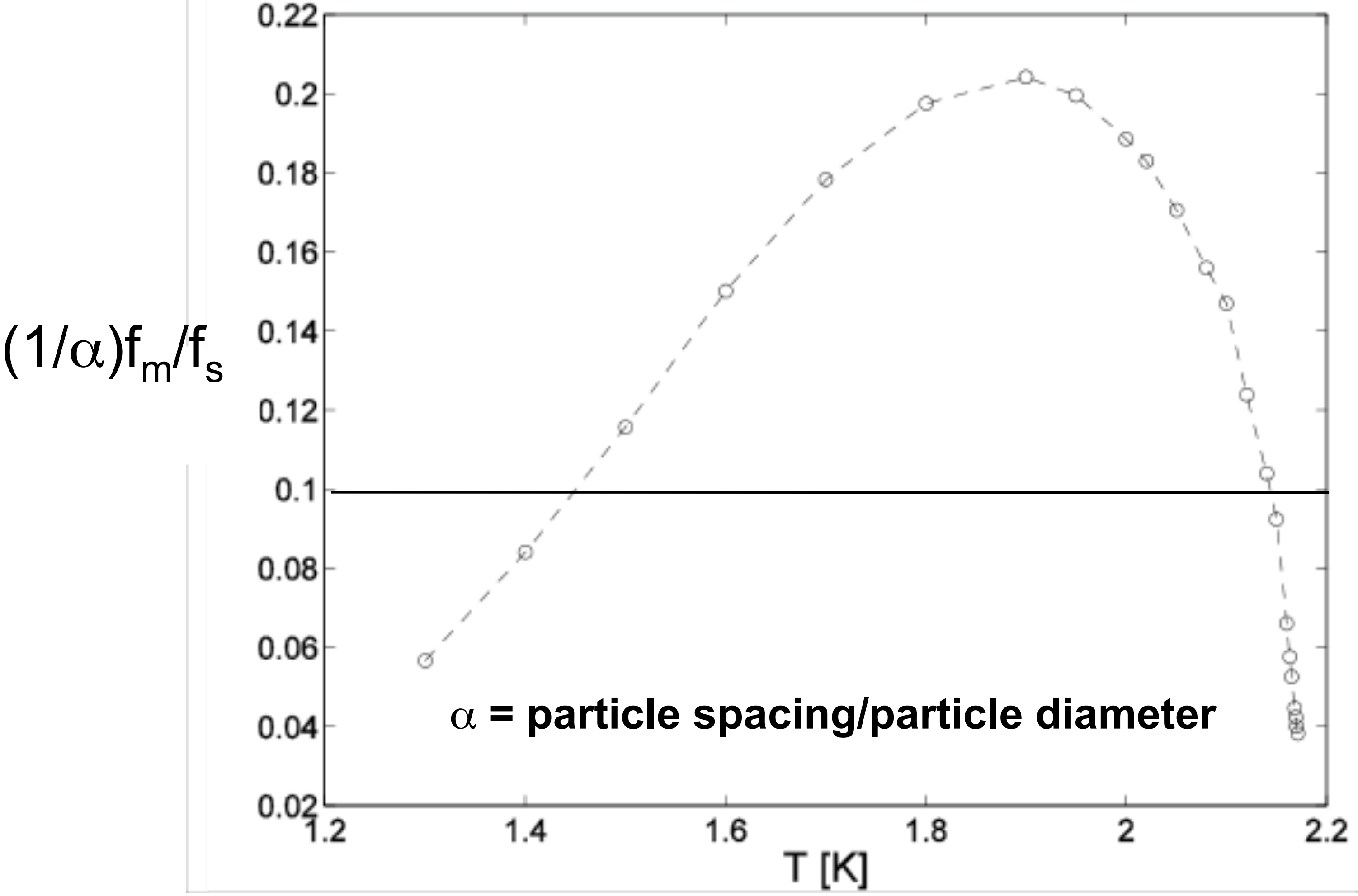
sphere is trapped by vortex



For a discussion of interaction between the fluid and particles in He II, see Sergeev, Barenghi & Kivotides, *Phys. Rev. B* **74**,184506 (2006); the simulations shown are by these authors.

sphere escapes vortex

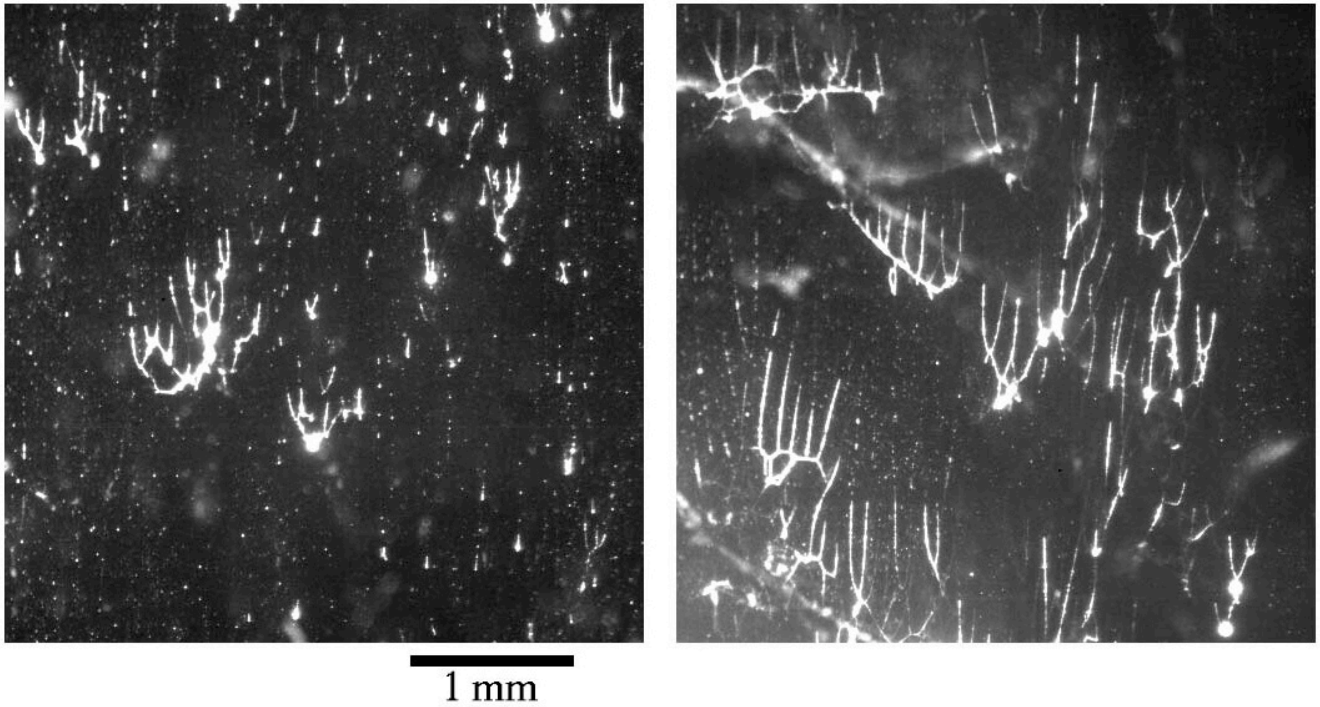


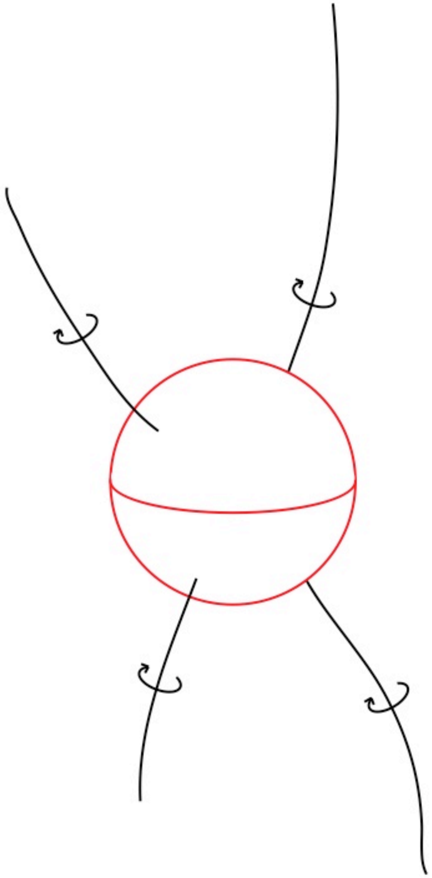


α = particle spacing/particle diameter

The ratio of the mutual friction force per unit length of a vortex to the drag on a particle trapped on the line. At about 2.17 K, the particle drag is equal to mutual friction if neighboring particles are about ten diameters apart.

Particles are not always passive tracers!





S.Z. Alamri, A.J. Youd & C.F. Barenghi

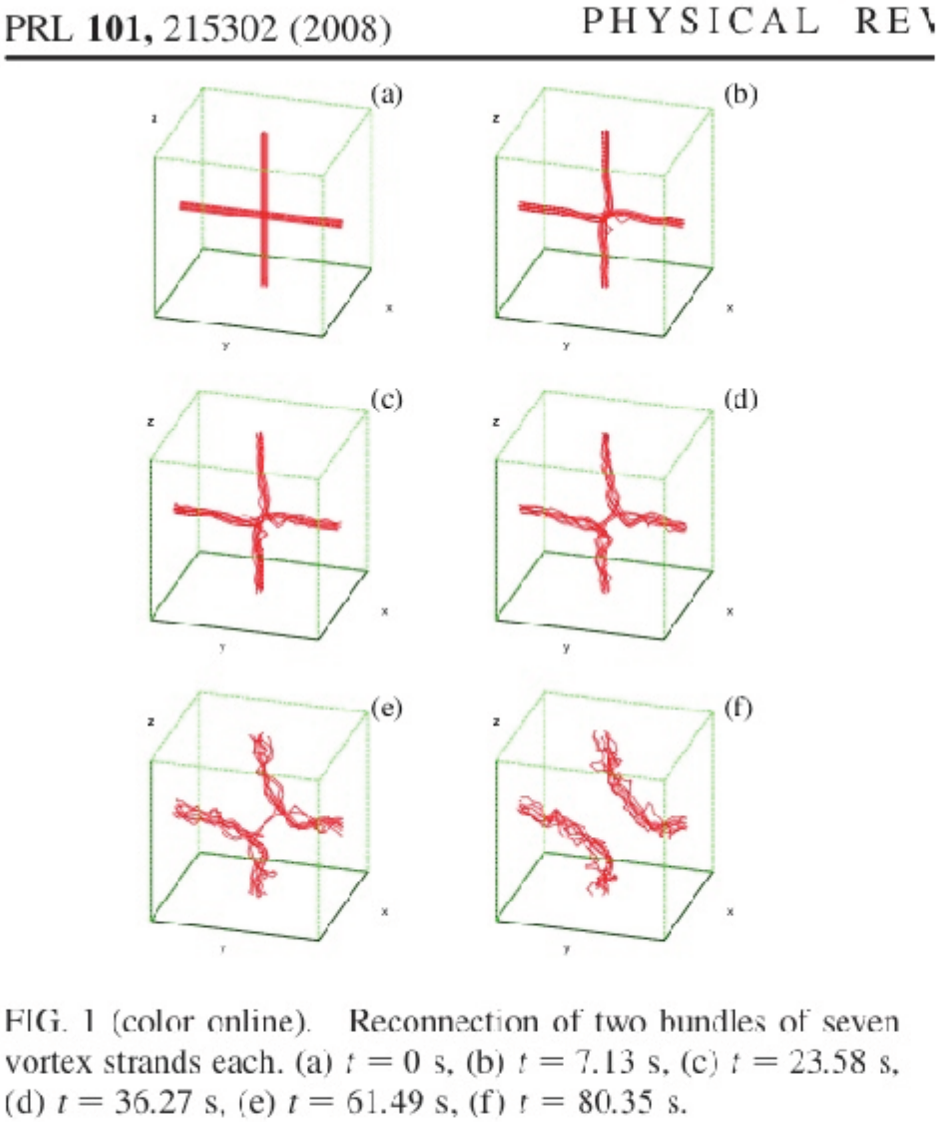

PRL **101,** 215302 (2008)   PHYSICAL REV

FIG. 1 (color online). Reconnection of two bundles of seven vortex strands each. (a) $t = 0$ s, (b) $t = 7.13$ s, (c) $t = 23.58$ s, (d) $t = 36.27$ s, (e) $t = 61.49$ s, (f) $t = 80.35$ s.

Brief chronological developments

1. 1950’s: Hall and Vinen (indirect inference)
2. Late 1980’s: Schwarz, Koplik, etc (vortex dynamics)
3. Mid-2000’s: Visualization, quantitative measurements

Thank you for your attention