DC conductivities in holographic matter.

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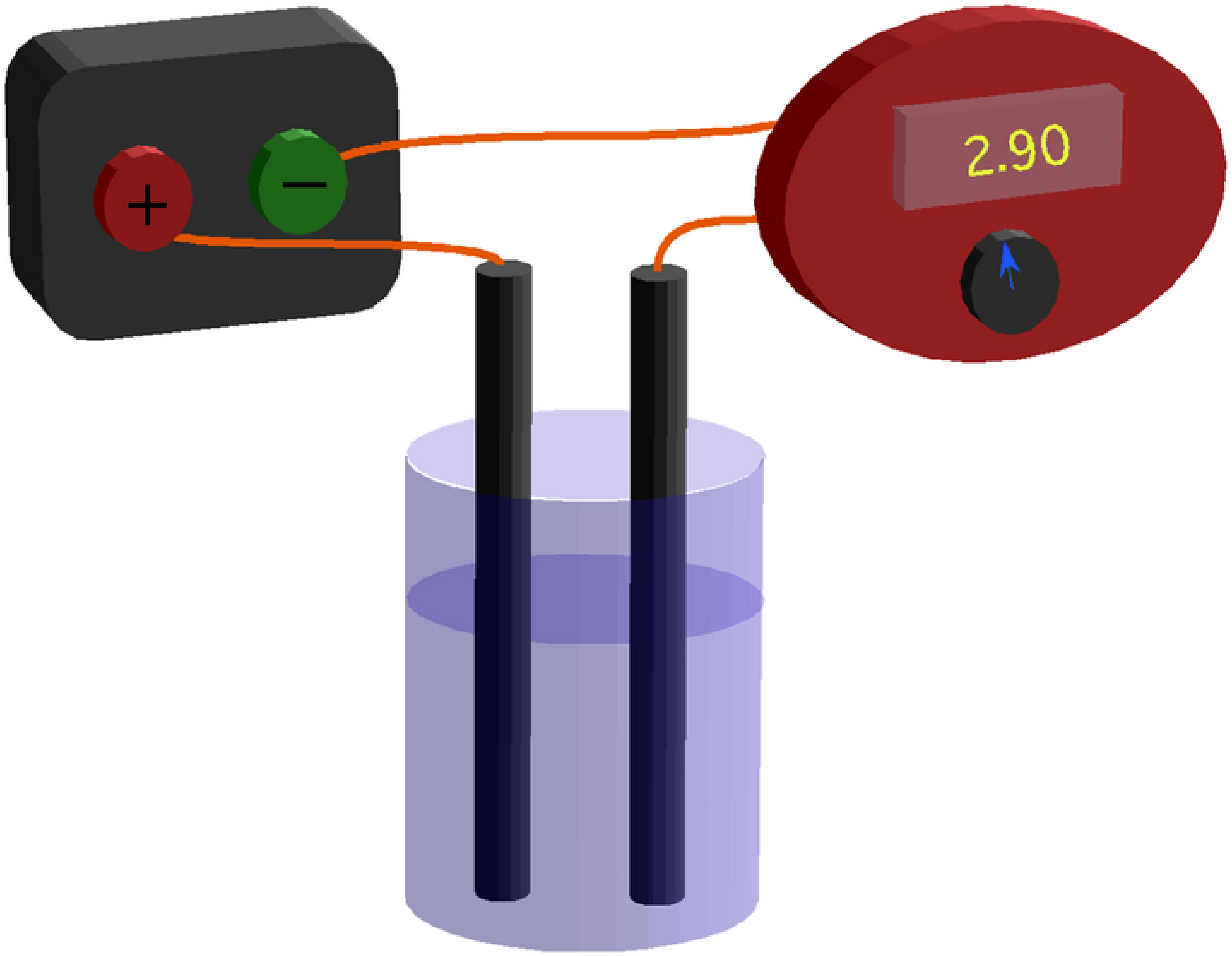
DC conductivities in holographic matter.

based on: arXiv:0705.3870 (AK, Andy O'Bannon)
arXiv:0708.1994 (Andy O'Bannon)

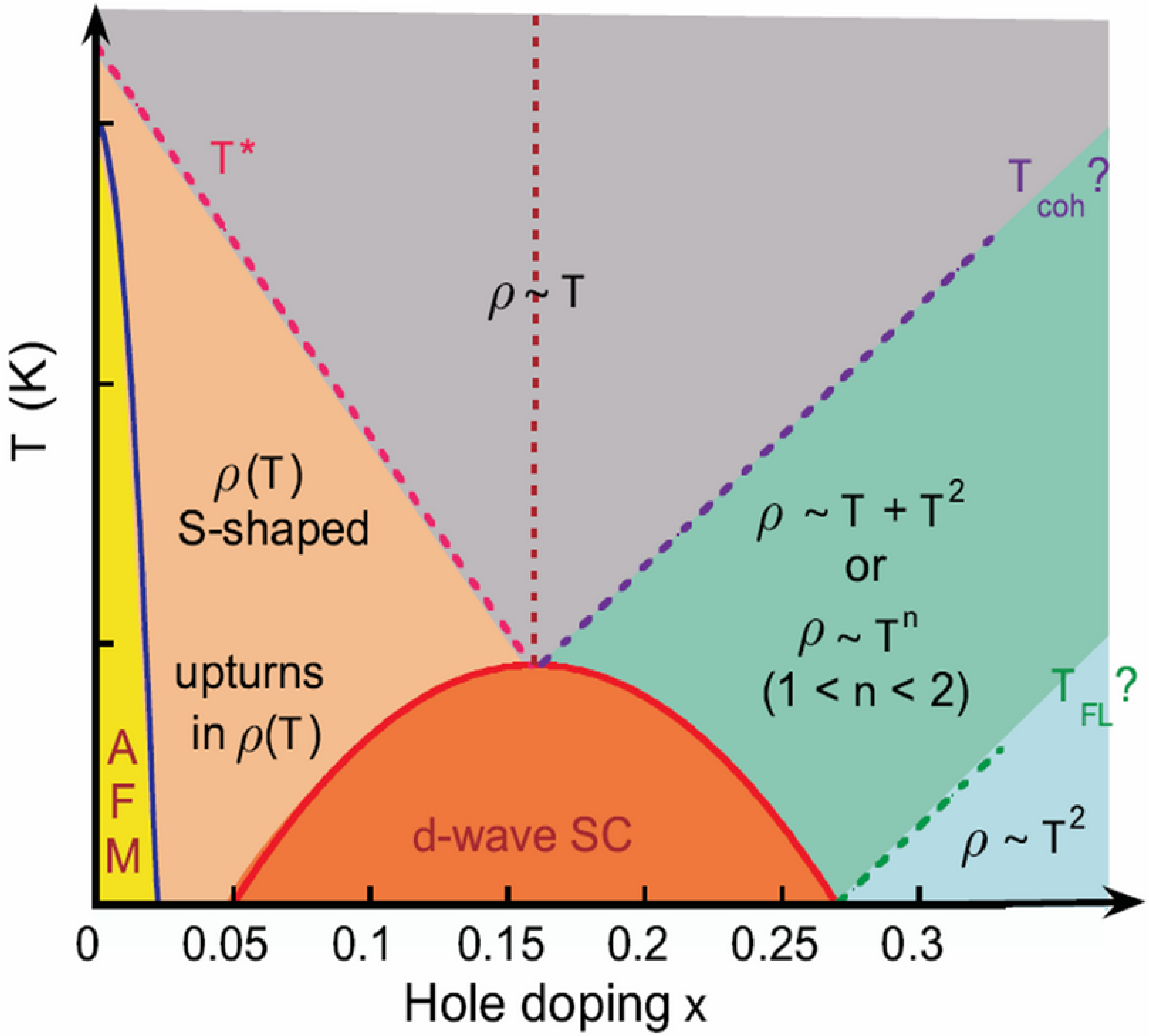
with focus on ongoing work with Shivaji Sondhi.

DC Conductivity/Resistivity



one of the most basic transport properties of any matter/fluid

Strange Metal / QCP



(Hussey; Sachdev)

Common belief:

Understanding linear (in T) resistivity of strange metal major step towards theory of high Tc super-conductors.

Sign of Quantum Criticality?

Calculating Conductivities:

"Microscopic Approach": Kubo Formula

$$\sigma = \frac{1}{6} \lim_{\omega\to 0} \frac{1}{\omega} \int d^4x \, e^{i\omega t} \, \langle [j_i^{\rm EM}(t,\mathbf{x}), \; j_i^{\rm EM}(0)] \rangle_{\rm eq}$$

Fluctuations of the Equilibrium system determine linearized response to external source.

standard matching relation in EFT.

Shortcomings of Kubo approach:

Application of Kubo Formula requires great care at Quantum Critical Point: (Damle and Sachev)

Correlator $C(\omega)$ is really $C(\omega/T)$ in a scale invariant theory at finite temperature.

Calculating C at T=0 automatically gives $C(\infty)$.

Kubo Formula for DC conductivity requires $C(0)$.

Of course correct limit can be taken if C is known at finite temperature and frequency.

Shortcomings of Kubo approach:

Unfortunately even for $\omega/T=0$ Kubo formula produces "wrong" result. (Greene and Sondhi)

At zero T, correlator $C(\omega)$ is really $C(\omega/E^{1/2})$ in a scale invariant theory at finite temperature.

Calculating C at E=0 automatically gives $C(\infty)$.

Kubo Formula for DC conductivity requires $C(0)$.

In order to extract correct DC conductivity of a quantum critical point we need to work at finite E! Limits do not commute.

Missing: explicit example where this can be demonstrated.

Example: (Greene and Sondhi)

Scaling: $$\sigma(\delta, E) = E^{\frac{d-2}{z+1}} \Sigma\left(\frac{\delta}{E^{\nu(z+1)}}\right)$$

e.g.: d=3 (spatial dimensions)
z=1 (relativistic theory, time and space scale the same)

$$j = \sigma E \propto E^{3/2}$$ Invisible in linearized response!

Prediction: $\sigma=0$ at $E=0=T$, $\omega \to 0$,
but $\sigma \sim E^{1/2}$ at $\omega=0=T$ but E finite.

Another Example: (Greene and Sondhi)

Scaling: $$\sigma(\delta, E) = E^{\frac{d-2}{z+1}} \Sigma\left(\frac{\delta}{E^{\nu(z+1)}}\right)$$

e.g.: d=2 (spatial dimensions)
z=1 (relativistic theory, time and space scale the same)

$$j = \sigma E \propto E$$

σ just a number. Linearized response?

Prediction: NO! The number σ at E=0=T, ω→0, is still different from σ at ω=0=T but E finite. Linearized response gives the former, experiment the latter!

Calculating Conductivities:

"Macroscopic Approach": Ohm's Law

(expectation value,
AdS/CFT: normalizable)

$$\langle J^x \rangle = \sigma E$$

(external field,
AdS/CFT: non-normalizable)

Since we are forced to work at finite electric field in any case, we may as well extract conductivity directly from a 1-pt function! No need to calculate 2-pt functions.

Problem: Loss Rates.

Ward Identities in translationally invariant system:

$$\partial^\mu \langle T_{\mu\nu} \rangle = F_{\nu\rho} \langle J^\rho \rangle$$

In particular (Work-Energy-Theorem):

$$\partial_t \langle T^t{}_t \rangle = -E \langle J^x \rangle \quad = \text{const. !}$$

Without Dissipation no stationary solution.
DC conductivity ill-defined!

Sondhi and Greene on Loss Rates:

$$j \propto E^{power}$$

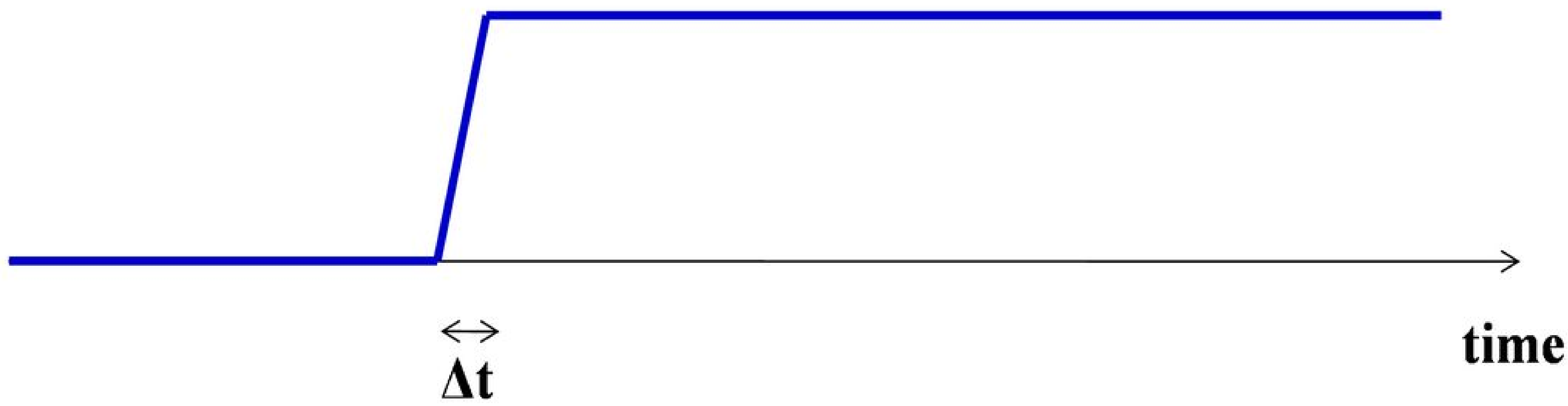
$$ohmic\ loss \propto j \cdot E \propto E^{power+1}$$

even without dissipation for small E (that is for time scales less than $1/E^{1/2}$) loss rate negligible.

Even without dissipation DC conductivity at QCP is well defined and calculable, albeit not in linearized response!

How can one verify this picture?

Look at time dependent E:



expect:

$t<0$: no current

$0<t<\Delta t$: current ramps up

$\Delta t<t<1/E^{1/2}$: stationary state

$1/E^{1/2}<t$: backreaction kicks in

Roadmap for the rest of the talk:

- Introduce dissipation. For QCP we don't need it, but the framework we have allows us to calculate DC conductivities at any temperature and carrier density.
- Use Ohm's law in AdS/CFT to calculate conductivity. Take T to zero. What do we get?
- Study Ohm's law with a full time-dependent E-field in 2+1 dimensions.

Adding Dissipation.

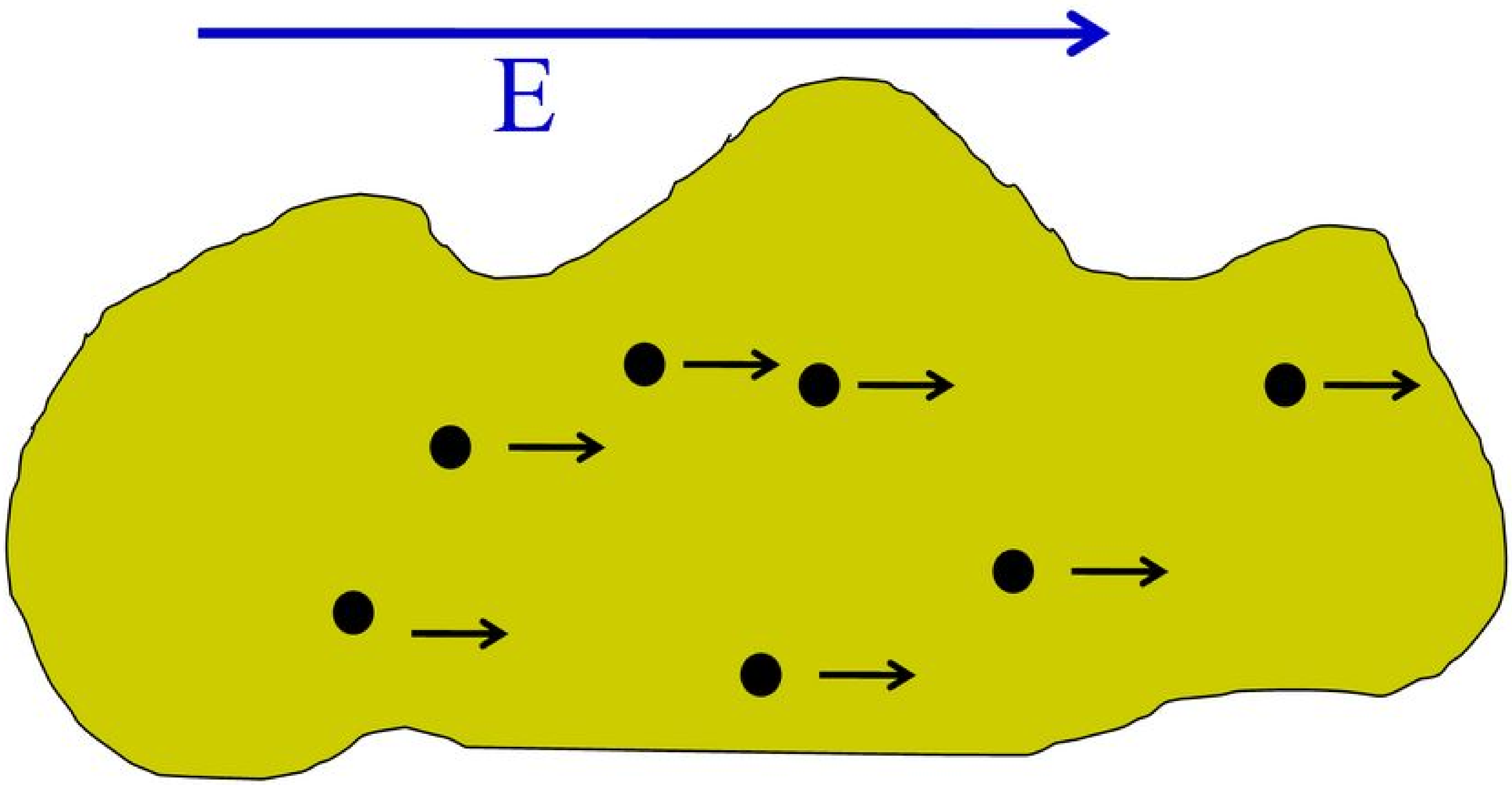
Add dissipation.

Typically dissipation requires breaking of translational invariance:

Disorder

perturbatively small random disorder potential has been introduced in AdS/CFT by Hartnoll and Herzog.

Dissipation without disorder:



Phononbath.
Energy Density ~ N^2
charge neutral.

Charge carriers.
Energy Density ~ N

dP/dt =Phonondrag=-dP/dt

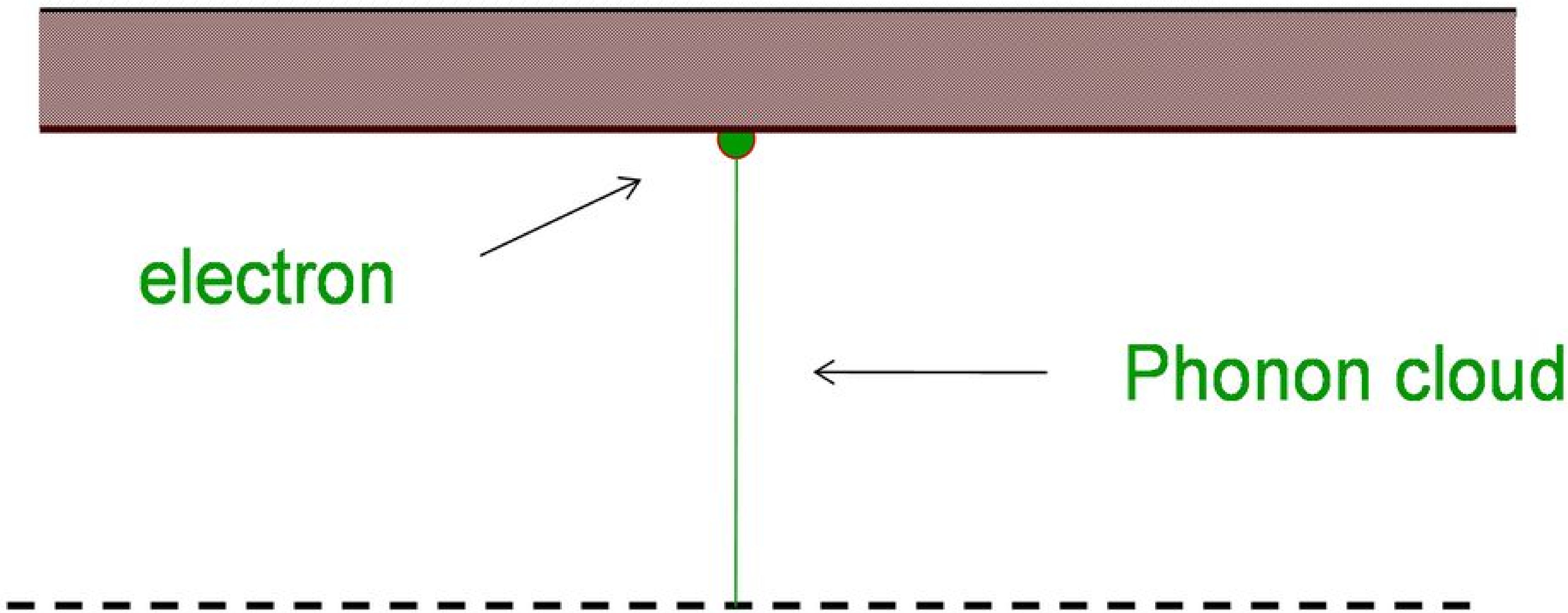
Phonondrag ~ N: backreation on phononbath negligible up to times of order N

Two fluid model of dissipation

Neutral background (N^2)	Charged (N)
water/fluid	**ions**
phononbath	**electrons**
N=4 SYM plasma	**N=2 fundamental hypermultiplets**

Stationary state with finite DC conductivity for times up to $t \sim N$

AdS/CFT realization: single electron



Tension times horizon radius

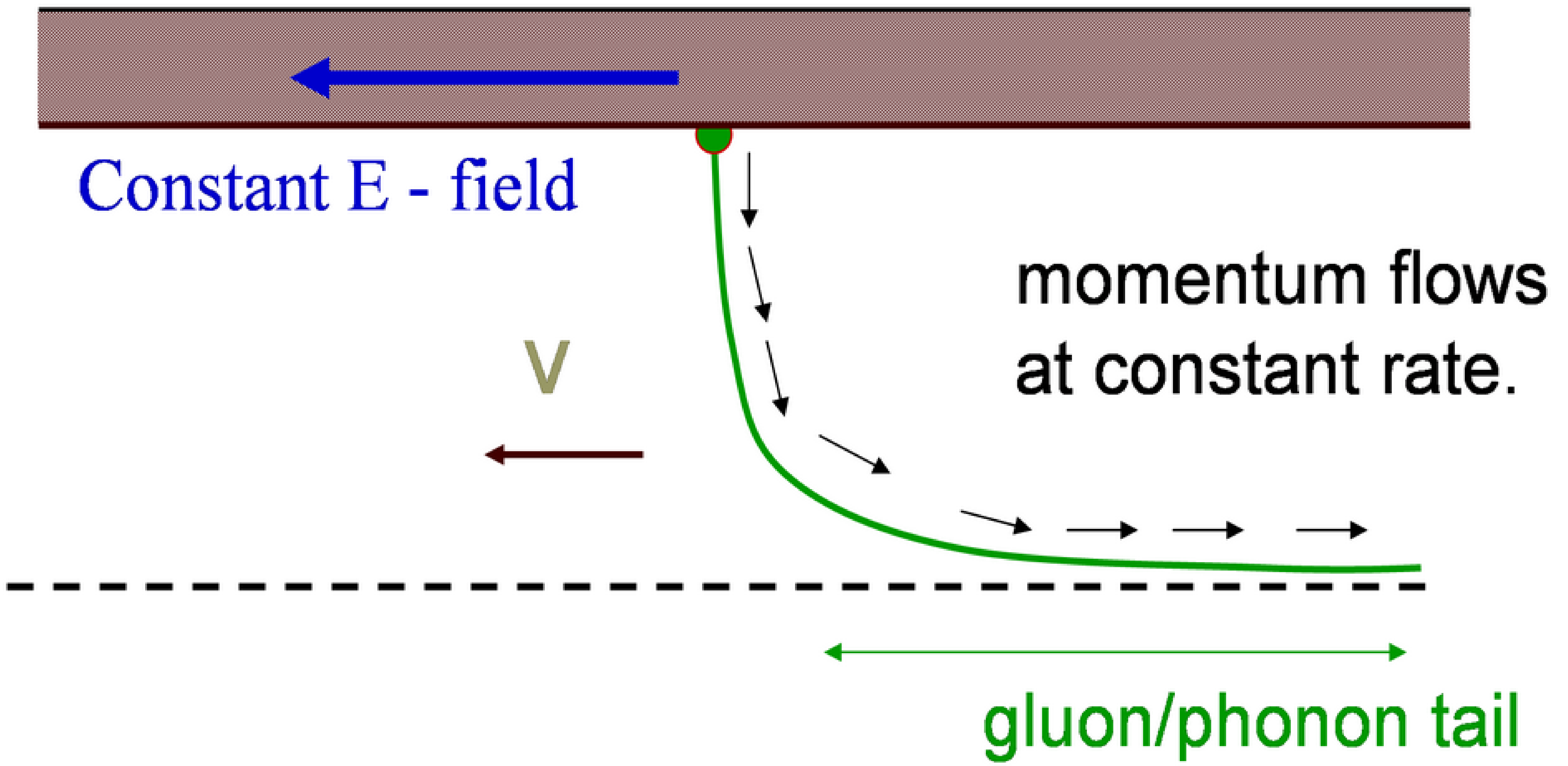
Tension times length of string.

Mass of Quasiparticle:

$$m_{QP} = m_e - \frac{\sqrt{\lambda}}{2} T$$

(HKKKY)

AdS/CFT realization: single electron



Phonondrag: $$\frac{dP}{dt} = -\frac{\sqrt{\lambda}}{2\pi}\frac{v}{\sqrt{1-v^2}}(\pi T)^2$$

(HKKKY, Gubser)

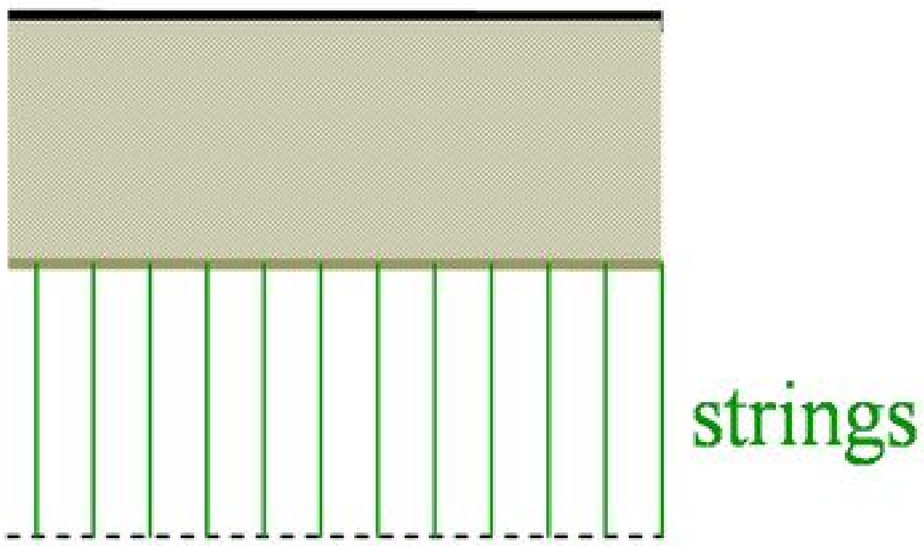
Universal Properties of Holographic Matter

For Dq probe in Dp background leading density dependent term is q independent.

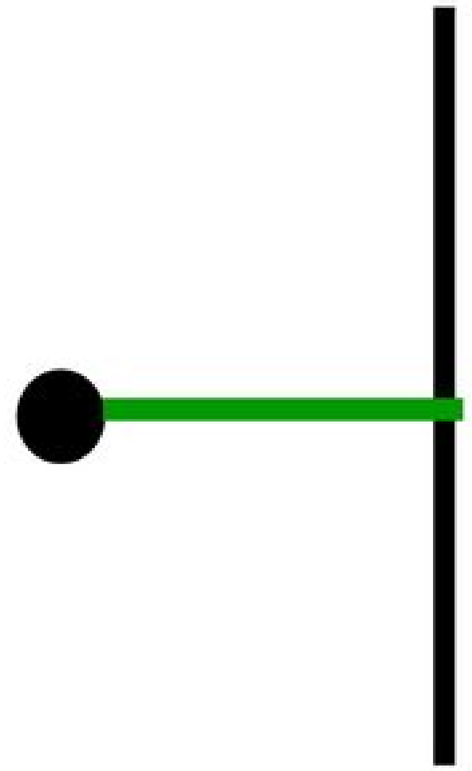
p	Free energy	Heat capacity	Resistivity
0	$T^{2/5}$	$T^{-3/5}$	no electric field possible
1	$T^{1/4}$	$T^{-3/4}$	$T^{3/2}$
2	$T^{2/3}$	$T^{-1/3}$	$T^{5/3}$
3	T	q-dependent	T^2
4	T^2	T	T^3

(Karch, Kulaxizi, Parnachev)

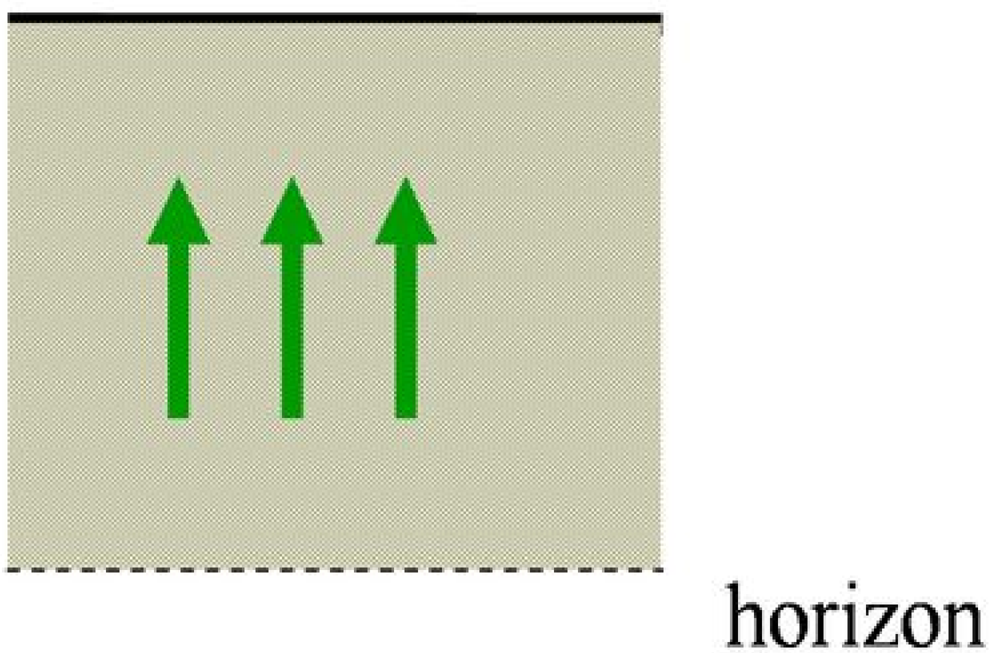
Finite density, no E-field.



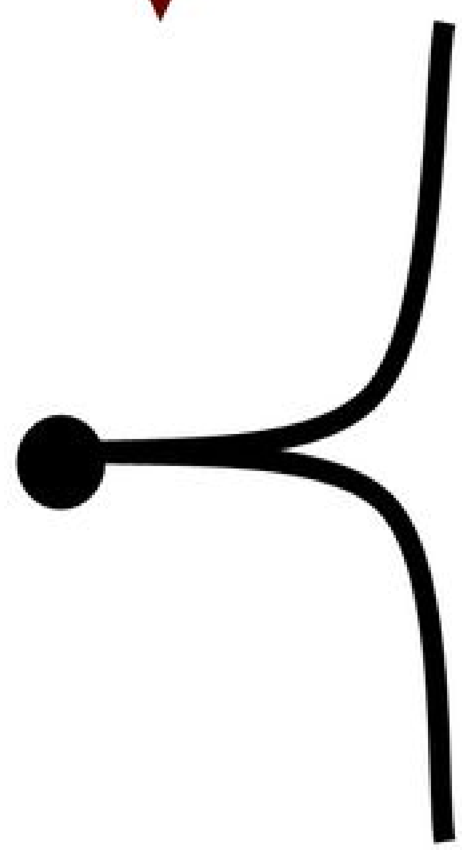
Minkowski embeding



Backreaction



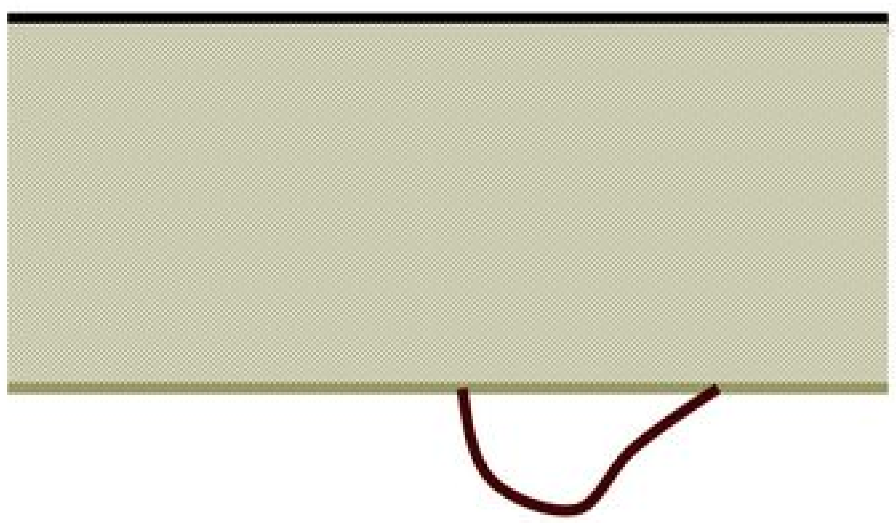
Black hole embeding



(Kobayashi et. al; Karch and O'Bannon),

More Black hole embeddings:

Minkowski Embedding

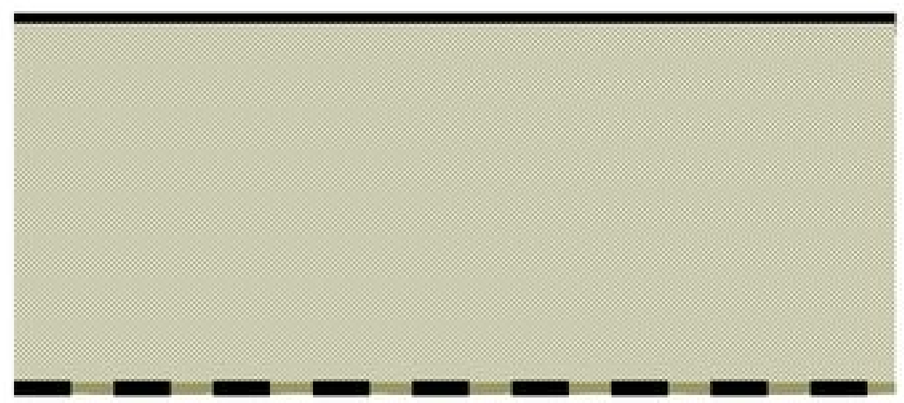


stable mesons

low T horizon

(Babington, Erdmenger, Evans, Kirsch)

Black Hole Embedding



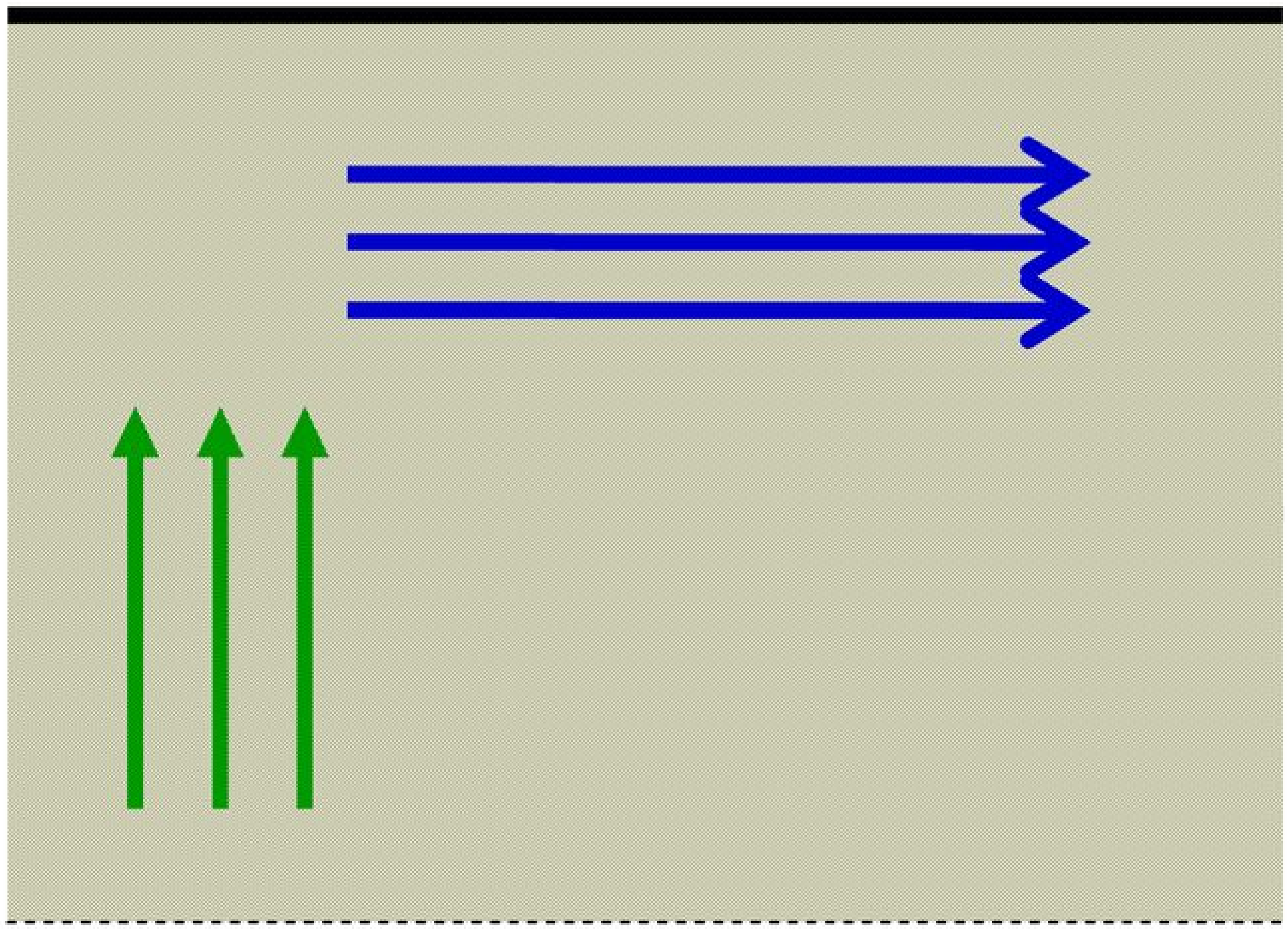
high T horizon

only unstable modes

1st order phase transition
chiral condensate jumps
generic (large N, large λ)

Goal: Give σ for any phase that is described by black hole embedding.

Need solution with E-field:

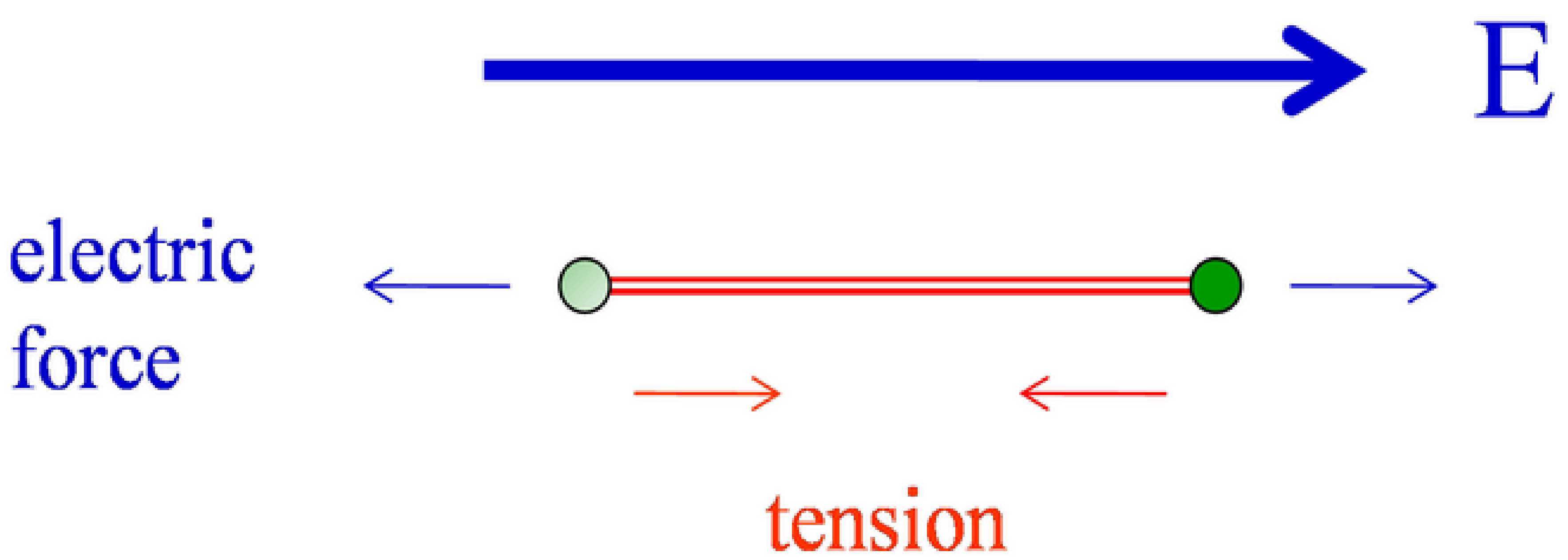


Where does conductivity come from?

Born Infeld Instability.

$$S_{D7} = -N_f T_{D7} \int d^8\xi \sqrt{-det\left(g_{ab} + (2\pi\alpha')F_{ab}\right)}$$

In flat space: $S = \sqrt{1 - E^2}$



For E>1 string gets ripped apart. Pair creation instability.

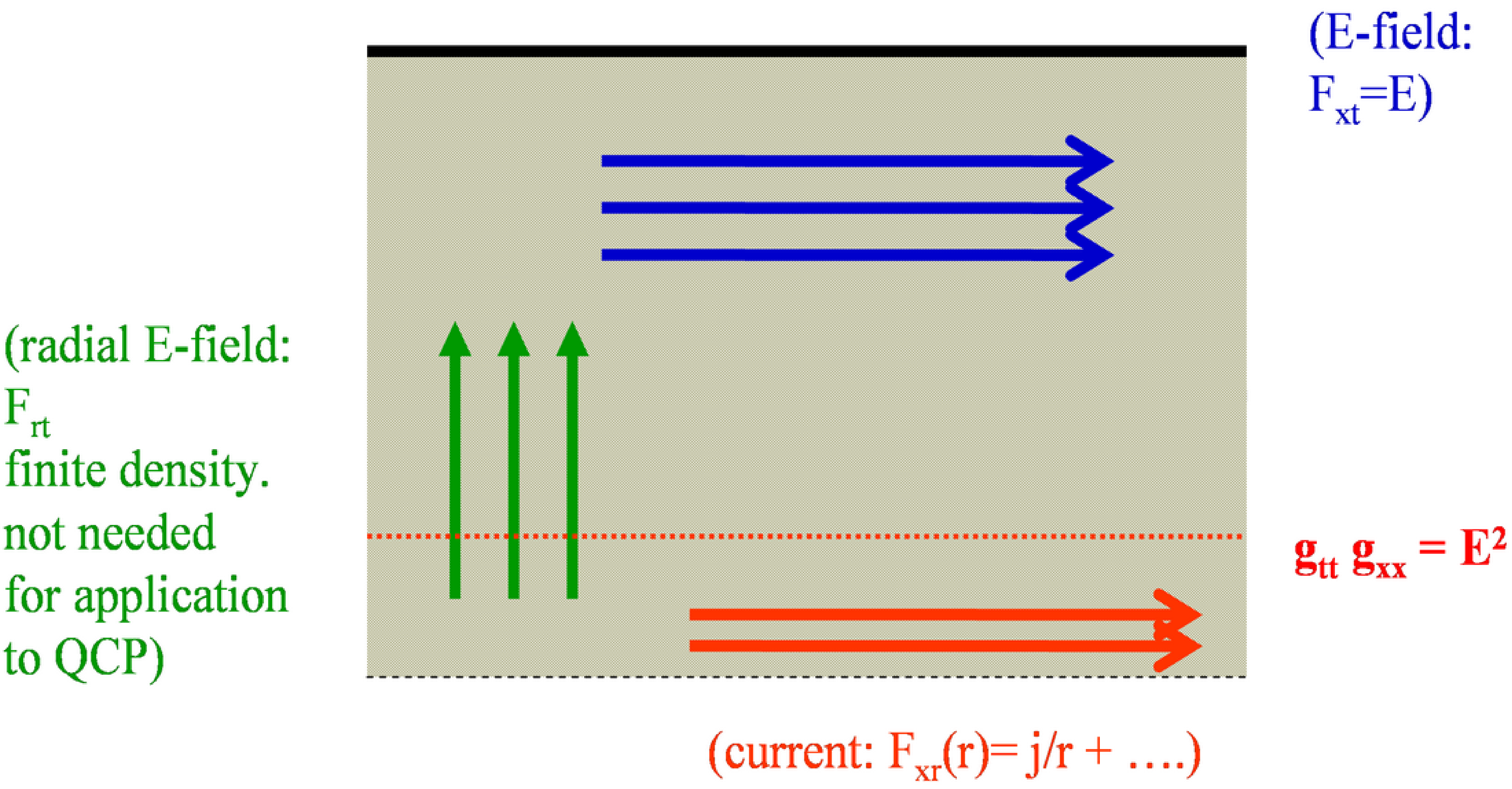
Born Infeld Instability.

$$S_{D7} = -N_f T_{D7} \int d^8\xi \sqrt{-det\left(g_{ab} + (2\pi\alpha')F_{ab}\right)}$$

In curved space: $S = \sqrt{g_{xx} g_{tt} - E^2}$

- Effective string tension position dependent.
- At bh horizon: g_{xx} finite, $g_{tt}=0$
- Black hole embedding always unstable against pair creation close to horizon!

Real solution with E-field+current:



Reality gives unique answer:

F_{rt} and hence j (and) are uniquely fixed by requiring that the solution is real for all values of r between horizon and infinity!

$$\sigma = \sqrt{\frac{N_f^2 N_c^2 T^2}{16\pi^2}\sqrt{e^2+1}\cos^6\theta(z_*) + \frac{d^2}{e^2+1}}$$

(D3/D7)

$$e = \frac{E}{\frac{\pi}{2}\sqrt{\lambda}T^2} \qquad d = \frac{\langle J^t \rangle}{\frac{\pi}{2}\sqrt{\lambda}T^2}$$

Lessons:

$$\sigma = \sqrt{\frac{N_f^2 N_c^2 T^2}{16\pi^2}\sqrt{e^2+1}\cos^6\theta(z_*) + \frac{d^2}{e^2+1}}$$

Two contributions add in quadrature.

Lessons:

$$\sigma = \sqrt{\frac{N_f^2 N_c^2 T^2}{16\pi^2}\sqrt{e^2+1}\,\underline{\cos^6\theta(z_*)} + \frac{d^2}{e^2+1}}$$

Embedding of the brane only enters via it's value at one value of z. z* = horizon-radius at small E-field, but samples all of the geometry as we increase E.

at z*: $g_{tt}\, g_{xx} = E^2$

Lessons:

$$\sigma = \sqrt{\frac{N_f^2 N_c^2 T^2}{16\pi^2}\sqrt{e^2+1}\cos^6\theta(z_*) + \frac{d^2}{e^2+1}}$$

All orders in the field-strength E are included. DBI sums up all powers in E.

The e=0 version of this formula has been reobtained in linearized response. (Mas, Shock, Tarrio, Zoakos)

Lessons:

$$\sigma = \sqrt{\frac{N_f^2 N_c^2 T^2}{16\pi^2}\sqrt{e^2+1}\cos^6\theta(z_*) + \frac{d^2}{e^2+1}}$$

Conductivity of the neutral finite temperature plasma due to thermally created electron/hole pairs. The cos term vanishes for heavy charge carriers and is equal to 1 for massless charge carriers.

Lessons:

$$\sigma = \sqrt{\frac{N_f^2 N_c^2 T^2}{16\pi^2}\sqrt{e^2+1}\cos^6\theta(z_*) + \frac{d^2}{e^2+1}}$$

Drude-like contribution of the finite density of charge carriers. Reproduces conductivity due to a density d of charge carries experiencing the drag force from the trailing string.

Can we see non-commuting limits?

For simplicity, let's focus on d=0 for the remainder of the talk:

$$\sigma = \sigma_0 (E^2 + T^4)^{1/4}$$

Linear Response: $$\sigma = \sigma_0 T \xrightarrow{T \to 0} 0$$

T=0 QCP: $$\sigma = \sigma_0 E^{1/2}$$

Greene and Sondhi were right!

What about D3/D5 (2+1 dim)?

For d=0: $$\sigma = \sigma_0$$

Correlator $C(\omega/T, \omega/E^{1/2})=C(\omega/T,E/T^2)$ is actually **constant** as a function of E/T^2 in theories with a probe brane dual.

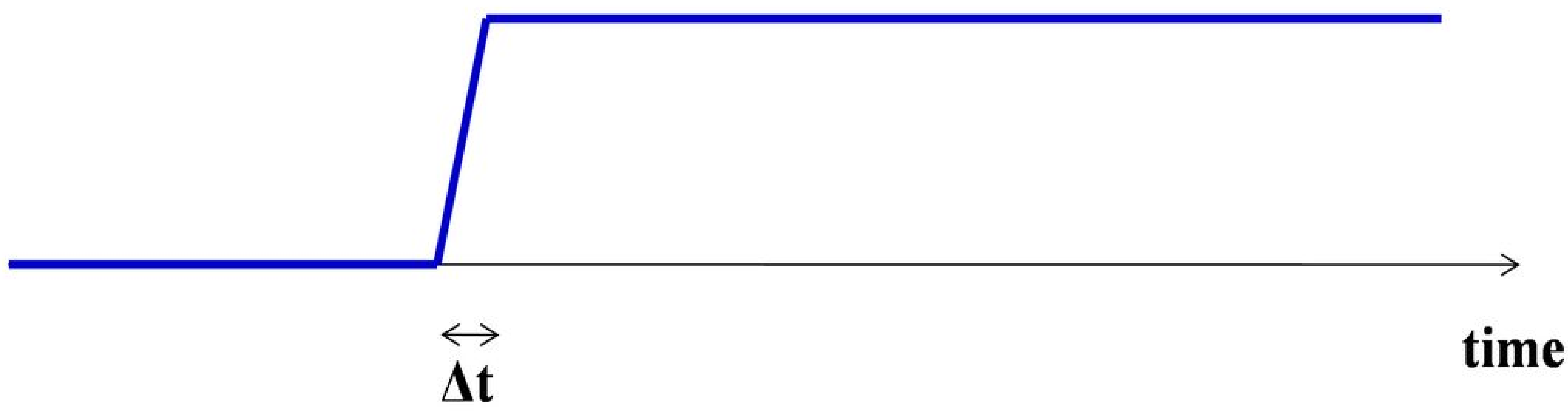
Independence of ω/T was argued by Herzog, Kovtun, Sachdev and Son. This was a consequence of S-duality of 4d Maxwell. S-duality also underlies E/T^2 independence (O' Bannon).

Result from static analysis.

For the generic case (including finite T, d, B, $E \cdot B$, any dimension) the zero frequency and zero E-field limits indeed do not commute at zero temperature, as predicted by Sondhi and Greene.

In 2+1 however conductivity is completely independent of ω, E and T (does depend on B, d).

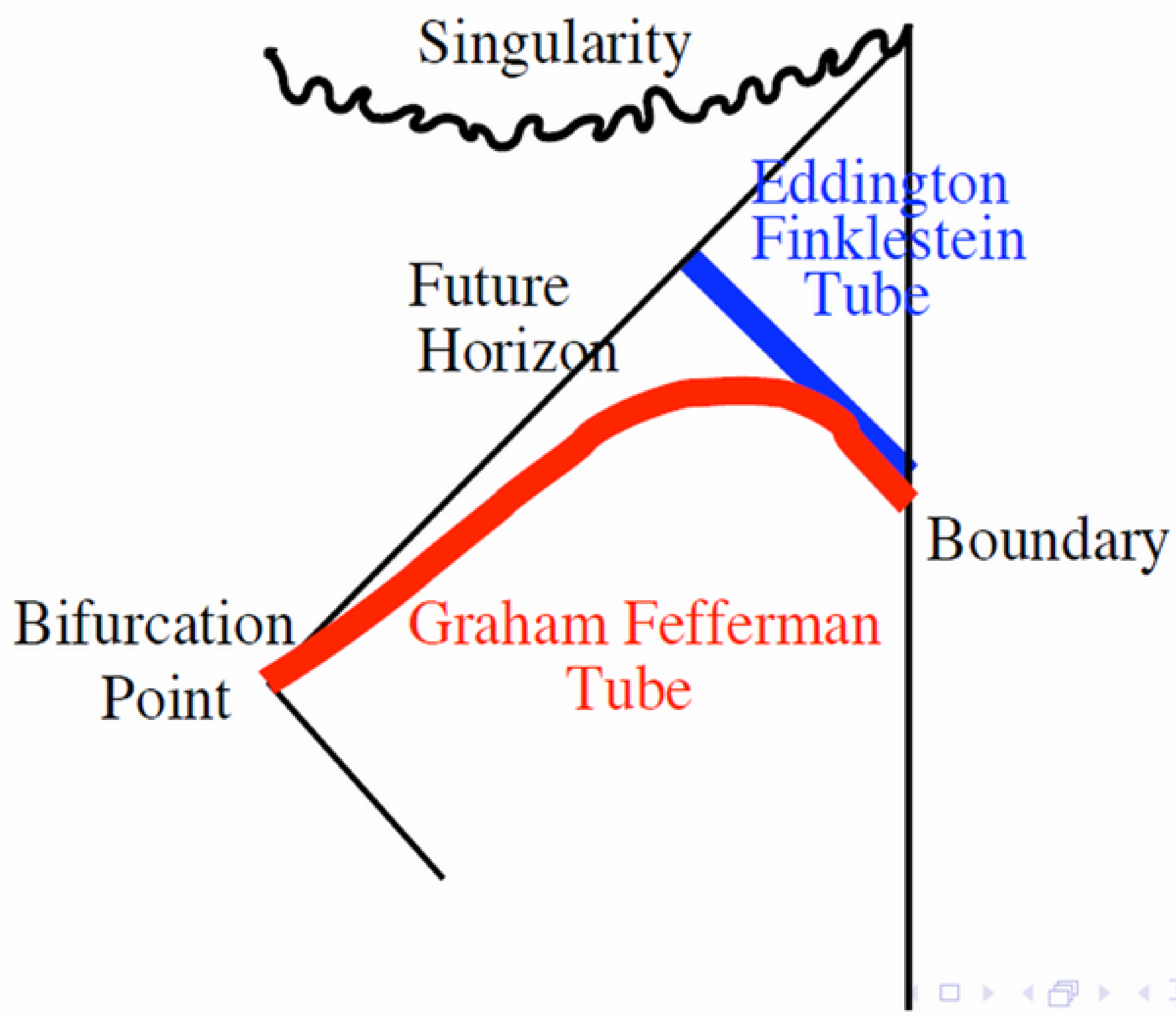
Response to time dependent E-field



Can we find the unique real solution in the bulk with a boundary condition at infinity corresponding to a time-dependent electric field?

Infalling coordinate system:

$$ds^2 = 2dvdr - hdv^2 + r^2 d\vec{x}^2$$



BC: $F_{xt}=E(v)$

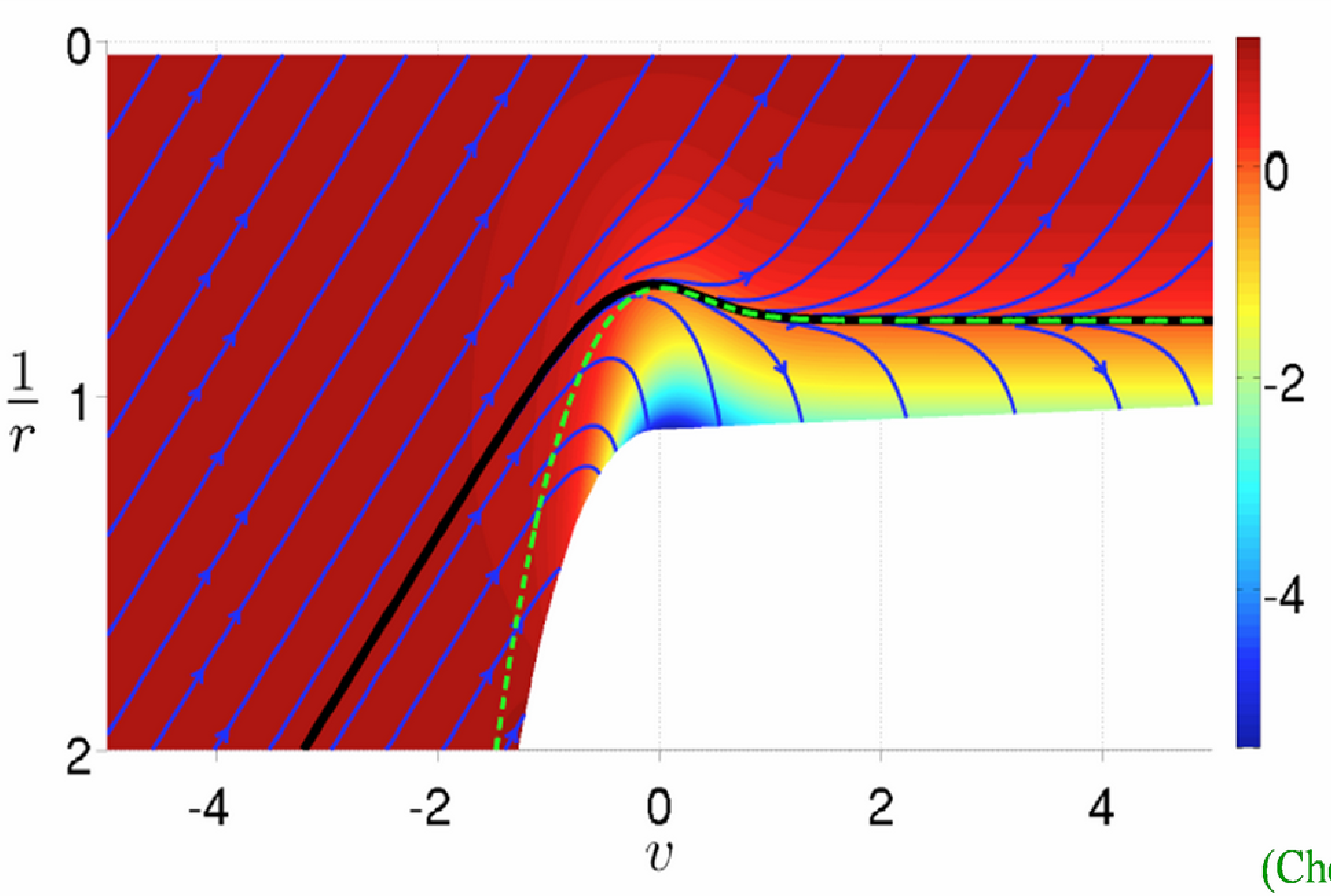
How to proceed?

Minwalla et al approaches:

Hydrodynamics: Start with static case.
Promote J to slowly time varying J.
Solve (peturbatively) for corrections.

Small Perturbation: Start with vacuum.
Solve (peturbatively) using the smallness
of the external source as expansion parameter.

Let the computer do it:



(Chesler and Yaffe)

Flavor Branes.

For flavor branes (dual to a 2+1 CFT) we can just write down the full non-linear solution to arbitrary E(t):

$$F_{xv} = E(v)$$

(all other components vanish)

Calculate current from this:

$$\langle j_x(t) \rangle = \sigma E(t)$$

(instantaneous response!!)

Interpretation?

Expected: $$\langle j_x(t)\rangle = \int d\tau \sigma(\tau) E(t-\tau)$$

Memory. Only constrained by causality (Kramers/ Kronig relations).

Find: $$\sigma(\tau) \sim \delta(\tau)$$

Interpretation?

Expected: $$\langle j_x(t)\rangle = \int d\tau \sigma(\tau) E(t-\tau)$$

Memory. Only constrained by causality (Kramers/ Kronig relations).

Find: $$\sigma(\tau) \sim \delta(\tau)$$

So: $\sigma(\omega)$ is ω-independent

We knew that!
(It is still remarkable that one can so easily find the full time dependent bulk solution.)

Energy loss?

(Karch, O'Bannon, Thompson)

The energy and momentum densities of the flavor sector can be obtained from the bulk w/o having to include backreaction.

$$(\text{Total energy})_{\text{bulk}} = (\text{Total energy})_{\text{boundary}}$$

here: $$\epsilon = \int dt\, j(t) E(t)$$

correct Ohmic heating;
FINITE!

Time dependent solution in 3+1

$$\partial_r \left(\frac{F_{xv} r + h r F_{xr}}{\bar{\mathcal{L}}} \right) + \partial_v \left(\frac{F_{xr} r}{\bar{\mathcal{L}}} \right)$$

No longer simple analytic solution.

E(t) = const.: $j = \sigma_0 \, E^{3/2}$

small E, small time derivative: $j = \sigma_0 \, (\dot{E} + E^{3/2})$

Result from dynamical analysis.

Dynamical analysis supports picture of Sondhi and Greene that postulates an intermediate stationary state for QCP up to $t \sim 1/E^{1/2}$.

In 2+1 analytic solution for any E(t) can be found. Confirms expecation of frequency independence.

Approximate results in 3+1 available. Hopefully more can be learned.

Conclusion.

Flavor branes are a great laboratory to explore interesting fluids and condensed matter systems.