Quantum oscillations & black hole ringing

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Plan of talk

Motivation – unconventional phases at finite density

1. Low temperature and finite density
2. Probing states with magnetic fields

Magnetic susceptibility at weak and strong coupling

1. Free fermions and bosons
2. Strongly coupled matter

Philosophical interlude

Quantum oscillations in strongly coupled theories

1. $1/N$ corrections to the free energy
2. Black hole ringing

Motivation – unconventional phases at finite density

1. Low temperature and finite density
2. Probing matter with magnetic fields
3. Example: Quantum oscillations in High - T_c superconductors

Low temperature and finite density

- Effective field theories in condensed matter physics often have a finite charge density.
- Weak coupling intuition at low temperatures and finite density:
  - Charged fermions: Fermi surface is built up.
  - Charged bosons: condensation instabilities (e.g. superconductivity).
- Weakly interacting low energy excitations about a condensate or Fermi surface are very well characterised.
- There seem to be materials where these descriptions do not work.
- Perspective of this talk: AdS/CFT gives a tractable theory with an exotic finite density ground state.

Probing matter with magnetic fields

- de Haas - van Alphen effect (1930): a Fermi surface leads to oscillations in the magnetic susceptibility as a function of $1/B$.
  - In a magnetic field

$$[P_x, P_y] \sim iB \quad \Rightarrow \quad \oint P_x dP_y \sim 2\pi(\ell + \tfrac{1}{2})B \,.$$

  - When the area of the orbit is a cross section of the Fermi surface there is a sharp response. I.e. at

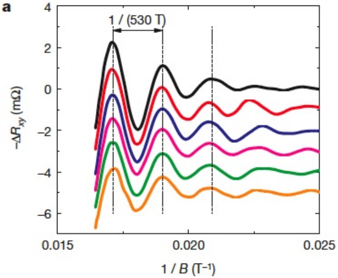
$$1/B \sim \ell/A_F \sim \ell/k_F^2 \sim \ell/\mu^2 \,.$$

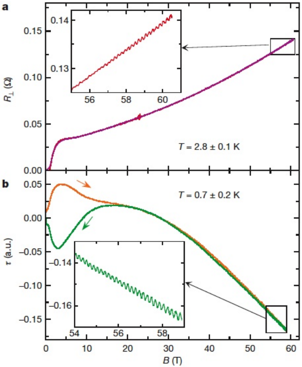
- (Also: Large magnetic field suppresses superconducting instabilities.)

Quantum oscillations in High - T_c superconductors

Doiron-Leyraud et al. 2007 (Nature), Vignolle et al. 2008 (Nature).

- de Haas - van Alphen oscillations in underdoped and overdoped cuprates.





- In underdoped region, carrier density much lower than naïve expectation: "small Fermi surface".

Magnetic susceptibility at weak and strong coupling

1. Free fermions and bosons
2. Strongly coupled matter
3. Large N magnetic susceptibility

Free fermions

- Free bosons or fermions in magnetic fields have Landau levels

$$\varepsilon_\ell = \sqrt{m^2 + 2|qB|(\ell + \tfrac{1}{2} \pm \tfrac{1}{2})}\,.$$

- Free energy for fermions (D=2+1)

$$\Omega = -\frac{|qB|AT}{2\pi} \sum_{\ell} \sum_{\pm} \log\left(1 + e^{-(\varepsilon_\ell \pm q\mu)/T}\right)\,.$$

- Zero temperature limit

$$\lim_{T\to 0} \Omega = -\frac{|qB|A}{2\pi} \sum_{\ell} (q\mu - \varepsilon_\ell)\theta(q\mu - \varepsilon_\ell)\,.$$

- Magnetic susceptibility has oscillations

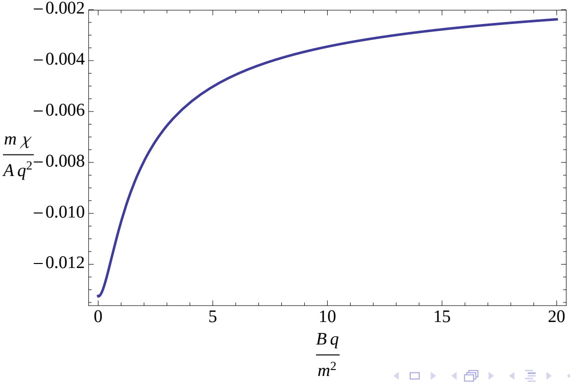
$$\chi \equiv -\frac{\partial^2\Omega}{\partial B^2} = -\frac{|qB|A}{2\pi} \sum_{\ell} \frac{q^2(\ell + \frac{1}{2})^2}{\varepsilon_\ell^2} \delta(q\mu - \varepsilon_\ell) + \cdots\,,$$

Free bosons

- Free energy for bosons – unstable if $\varepsilon_0 < |q\mu|$

$$\Omega = \frac{|qB|A}{2\pi} \sum_{\ell} \sum_{\pm} \log\left(1 - e^{-(\varepsilon_\ell \pm q\mu)/T}\right) + \Omega|_{T=0} \,.$$

- Magnetic susceptibility at T=0 if stable (Hurwitz zeta function)



The normal state

- The minimal ingredient is Einstein-Maxwell theory

$$S_E[A,g] = \int d^4x\sqrt{g}\left[-\frac{1}{2\kappa^2}\left(R+\frac{6}{L^2}\right)+\frac{1}{4g^2}F^2\right] .$$

- The 'normal state' is dual to a dyonic black hole

$$ds^2 = \frac{L^2}{r^2}\left(f(r)d\tau^2+\frac{dr^2}{f(r)}+dx^i dx^i\right) ,$$

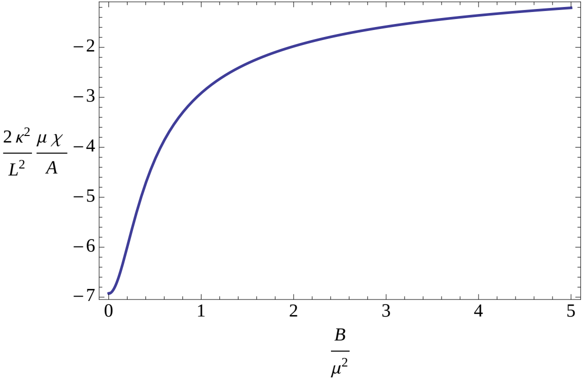
$$A = i\mu\left[1-\frac{r}{r_+}\right]d\tau + B\,x\,dy .$$

- Free energy is the action evaluated on shell

$$\Omega_0 = -\frac{AL^2}{2\kappa^2 r_+^3}\left(1+\frac{r_+^2\mu^2}{\gamma^2}-\frac{3r_+^4B^2}{\gamma^2}\right) .$$

Large N magnetic susceptibility

- Easy to compute $\chi \equiv -\frac{\partial^2 \Omega_0}{\partial B^2}$
- Plot result:



- Looks just like free bosons.... (but massless!)

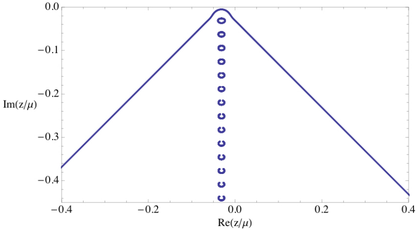
But...

Faulkner, Liu, McGreevy and Vegh '09

- At zero temperature: peak (not a quasiparticle!) in the fermion spectral function: $\mathrm{Im}\langle\Psi\Psi\rangle^R(\omega, k)$.
- Dispersion relation

$$\frac{\omega}{v_F} + h e^{i\theta}\omega^{2\nu} = k - k_F \,.$$

- Looks like a (non-Landau) Fermi surface!



Philosophical interlude

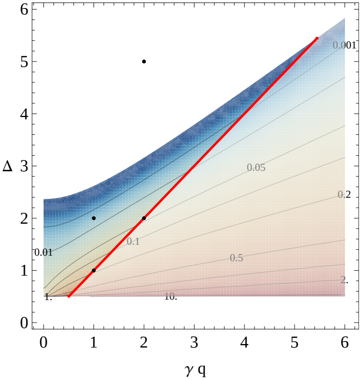
1. The string landscape is a blessing
2. Universality obscures physics
3. Bulk quantum effects are crucial

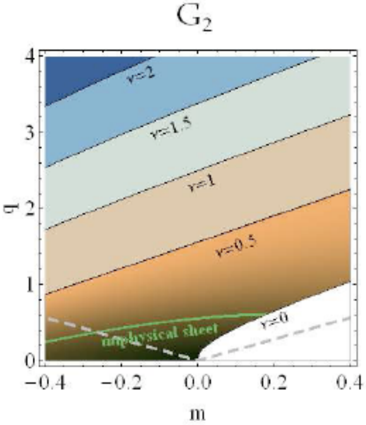
The string landscape is a blessing

- There are many, many, many.... asymptotically AdS_4 solutions to string theory.
- These all have dual field theories.
- $\mathcal{N} = 8$ SYM is unlikely to be representative.
- Landscape $\Rightarrow$ quantum gravity UV completion is not a strong constraint on effective field theories.
- Helps legitimize effective field theory approach to AdS/CFT.
- E.g. scan the space of possible behaviour as a function of mass m and charge q of scalar and fermion.

Criterion for superconductivity and Fermi surfaces

Denef-SAH '09; Faulkner, Liu, McGreevy and Vegh '09





Universality obscures physics

- Universality is the fact that at leading order in 'N' many quantities only depend on the Einstein-Maxwell action

$$S_E[A,g] = \int d^4x\sqrt{g}\left[-\frac{1}{2\kappa^2}\left(R+\frac{6}{L^2}\right)+\frac{1}{4g^2}F^2\right].$$

- e.g. shear viscosity, electric conductivity, heat capacity, magnetic susceptibility.
- Physically, the existence or not of a Fermi surface should effect the conductivity!
- Universality shows that the large N limit is washing out physics we care about.

Bulk quantum effects are crucial

- Some '$1/N$' effects are captured by higher derivative terms.
- These are not the most dramatic ones, loops of heavy modes.
- Loops of light modes give 'nonlocal' $1/N$ effects.
- These effects couple e.g. the Maxwell field and the charged matter. I.e. the charged matter runs in loops in the Maxwell propagator.
- New physical effects!

Quantum oscillations in strongly coupled theories

1. $1/N$ corrections to the free energy
2. Black hole ringing
3. Quantum oscillations

1/N corrections to the free energy

- Nontrivial Landau-level structure subleading in $1/N$?

  $\Rightarrow$ Quantum contribution from charged matter:

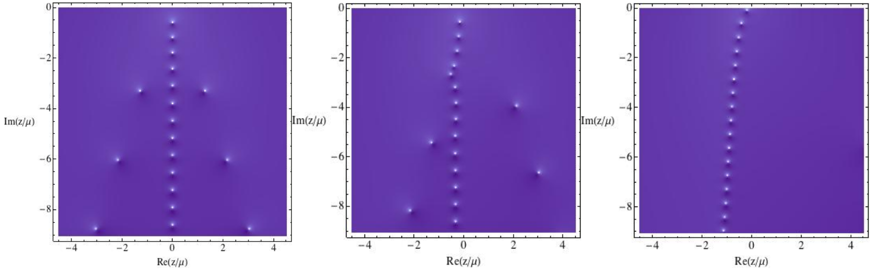
$$\Omega_{\text{1-loop}} = T \operatorname{tr} \log \left[-\hat{\nabla}^2 + m^2 \right] - T \operatorname{tr} \log \left[\Gamma \cdot \hat{D} + m \right] + \cdots .$$

- It is difficult to compute determinants in black hole backgrounds and it is hardly ever done...

- Reformulate the problem using quasinormal modes.

Black hole ringing

- Late times: a perturbed black hole 'rings' with characteristic frequencies.
- Quasinormal modes: poles of the retarded Green's function (bulk or boundary).
- Some typical quasinormal for charged AdS black holes at low temperature (not trivial to make these plots!)



The free energy and quasinormal modes

- We derived (new to my knowledge) formulae for the determinant as a sum over quasinormal modes $z_\star(\ell)$ of the black hole

$$\Omega_{\text{1-loop, B}} = \frac{|qB|AT}{2\pi} \sum_{\ell} \sum_{z_\star(\ell)} \log \left(\frac{|z_\star(\ell)|}{2\pi T} \left| \Gamma \left(\frac{i z_\star(\ell)}{2\pi T} \right) \right|^2 \right) + \text{Loc}\,.$$

$$\Omega_{\text{1-loop, F}} = -\frac{|qB|AT}{2\pi} \sum_{\ell} \sum_{z_\star(\ell)} \log \left(\frac{1}{2\pi} \left| \Gamma \left(\frac{i z_\star(\ell)}{2\pi T} + \frac{1}{2} \right) \right|^2 \right) + \text{Loc}\,.$$

- For the BTZ black hole we did the sum explicitly and checked agreement with the known result (also did de Sitter).

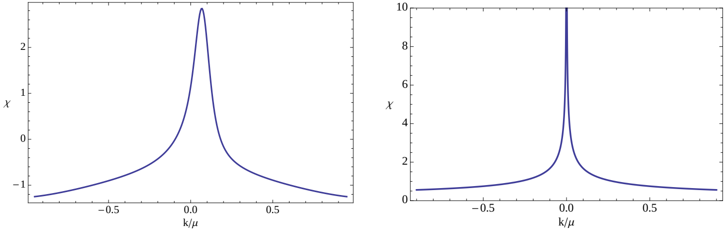
Quantum oscillations

- The power of these formulae is that if a set of quasinormal mode does something non-analytic, then this is directly identified.
- Faulkner-Liu-McGreevy-Vegh have shown that at $T = 0$ there is a fermion quasinormal mode that plane bounces off the real axis at $k = k_F$.
- At a finite magnetic field, this gives a bounce when $2B\ell = k_F^2$.
- At low temperature $T \ll \mu$

$$
\begin{aligned}
\Omega &= -\frac{|qB|A}{2\pi} \sum_{\ell} \sum_{z_\star(\ell)} \frac{1}{\pi} \mathrm{Im} \left[z_\star(\ell) \log \frac{i z_\star(\ell)}{2\pi T} \right] + \cdots . \\
&= \frac{|qB|A}{2\pi} \sum_{\ell} \frac{1}{\pi} \mathrm{Im} \, \frac{1}{2\pi i} \int_{-\infty}^{\infty} z \log \frac{iz}{2\pi T} \frac{\mathcal{F}'(z)}{\mathcal{F}(z)} dz \, .
\end{aligned}
$$

- Where $\mathcal{F}(z_\star) = 0$.

- Take two derivatives to get the susceptibility: $\chi = -\partial_B^2 \Omega$.
- Do it numerically at some low finite T:



- Analytically at $T = 0$:

$$\chi \sim +|qB| A \sum_{\ell} \ell^2 \Big| 2\ell |qB| - k_F^2 \Big|^{-2+1/2\nu} .$$

- Power law nonanalyticities with

$$\Delta\left(\frac{1}{B}\right) = \frac{2\pi q}{A_F} .$$

Conclusions

- There exist systems with finite charge density that are described as neither conventional Fermi liquids or superfluids.
- AdS/CFT provides model exotic stable finite density systems.
- Magnetic fields are an essential experimental and theoretical tool for probing such systems.
- There is interesting structure at $1/N$ in AdS/CFT related to Landau levels for fermions.
- Found a method for computing determinants about black holes using quasinormal modes.
- Fermionic loops are shown to give de Haas - van Alphen oscillations.