DRSTP - String Theory and the Swampland

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DRSTP - String Theory and the Swampland

See reviews 1903.06289, 2102.01111
References in here.

1) Introduction

Physics is built using effective theories, valid up to a certain energy scale

e.g. Theory of Blackbody radiation

Power emitted as a function of wavelength

$$B(\lambda)_{classical} = \frac{2kT}{\lambda^2}$$

Valid for $\lambda \gg l_{cutoff} \sim \frac{\hbar c}{kT}$.

$$B(\lambda)_{quantum} = \frac{2hc}{\lambda^3}\left(e^{\frac{\hbar c}{\lambda kT}} - 1\right)^{-1}.$$

Cutoff scale: $\Lambda_{cutoff} \sim \frac{1}{l_{cutoff}}$.

Natural units:
- $\hbar = c = 1$
- Sometimes, $M_p = 1$.

Modern Wilsonian Effective Theories are built using certain expectations

- Can guess/estimate necessary cutoff from the EFT

for example, gravity becomes strongly-coupled, so highly quantum in nature, at

$$E \sim M_p \sim 10^{19}\,\text{GeV} \sim 10^{-35}\,\text{m}$$

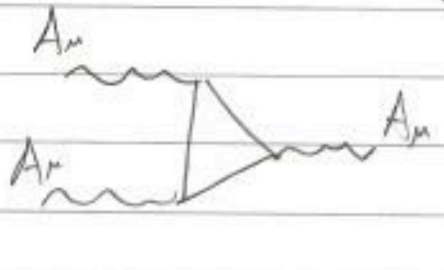
So M_p serves as a cutoff for any effective theory (that is not a full-fledged theory of QG).

- Write Lagrangian operators consistent with symmetries of the theory, and suppress/enhance operators by the cut-off scale:

eg. $$\mathcal{L}(\phi) \sim \frac{1}{2}(\partial\phi)^2 + \Lambda^2\phi^2 + \lambda\phi^4 + \frac{1}{\Lambda^2}\phi^6.$$

These are <u>expectations</u>, there are also <u>consistency rules</u>

e.g. Anomaly cancellation:



The Swampland Program aims to supplement the consistency rules for any effective theory that is coupled to gravity.

These new rules also often go against the Wilsonian expectations! That is when they are the most interesting.

- How could we have missed such general consistency constraints until now?

Inconsistency does not manifest in the EFT, it is a requirement for an ultraviolet completion to Quantum Gravity.

Definition:

- EFTs that potentially have an ultraviolet completion to Quantum Gravity are in the Landscape
- EFTs that have no possible ultraviolet completion to Quantum Gravity are in the Swampland

Methodology of the Swampland Program

The key idea behind the Swampland are two expectations of physics at sufficiently high energies:

1) Unification
2) Uniqueness

} Initiated by Einstein.

We expect that at sufficiently high energies the laws of physics are uniquely fixed.

String theory manifests such features: at high energies there are only 5 string theories which have no constant parameters.

We now think of these as the possible weakly coupled theories of quantum gravity.

At even higher energies we expect a single unique theory, but which is strongly coupled, M-theory.

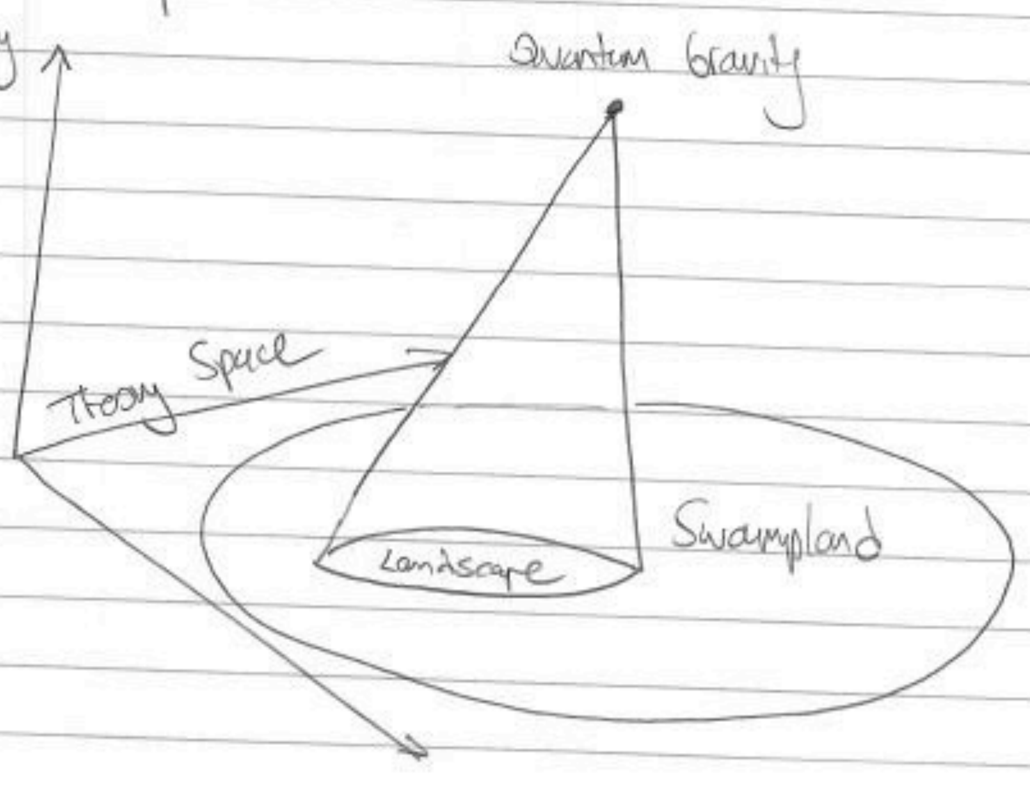
So, all effective theories coupled to gravity have a single, unique ultraviolet completion!

(Not true without gravity.)

Different effective theories correspond to different solutions / configurations of this single theory.

e.g. Newtonian friction co-efficients vary between materials. But all Newtonian friction comes from electromagnetic interactions of electrons, described by a single theory: QED.

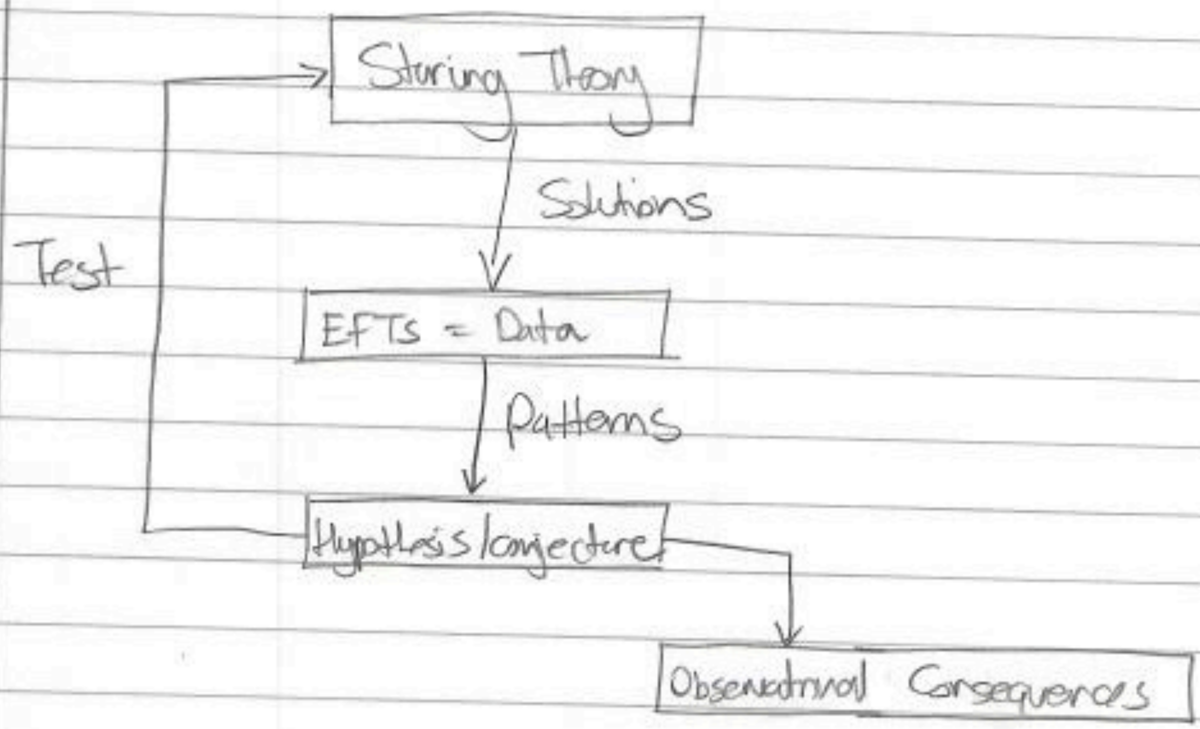
We have this picture:



Swampland constraints, or consistency rules, correspond to universal properties of solutions to the unique theory of quantum gravity

How do we extract such properties?

Scientific Method - Look for patterns in the data
Set of ST solutions



This is supplemented by input from:

- Black Hole physics
- Holography (Lecture 4)
- Amplitudes

} under developed !

2) Swampland intuition - Weak-coupling limits

Let us build some intuition for Swampland constraints by looking at general aspects of String Theory

Ten-dimensional type II String Theory has only two parameters:

String tension, $[M_s] = 1$,

String Coupling, $[g_s] = 0$.

It has a massless graviton mode, so the low-energy effective action includes gravity

$$S_{10D} = \int d^{10}x \left(\frac{M_s^8}{g_s^2}\right) R + \cdots$$

The Planck mass in d-dimensions is always defined as

$$S = \int d^dx \, \frac{1}{2} \left(M_p^{(d)}\right)^{d-2} R + \cdots$$

So we see, $\boxed{M_s \sim g_s^{1/4} M_p}$

The spectrum of the string contains an infinite tower of oscillators,

$$M_n \sim \sqrt{n}\, M_s, \quad n \in \mathbb{N}$$

In the weak-coupling limit, an infinite tower of states become parametrically lighter than the Planck scale

$$M_{oscillators} \sim M_s \sim g_s^{1/4} M_p$$

Surprising: Weakly-coupled quantum gravity has an infinite tower of states far below M_p

Can see similar behaviour in classical gravity: dimensional reduction on a circle of pure gravity

$$dS_D^2 = \eta_{\mu\nu} dx^\mu dx^\nu + R^2 (dy + A_\mu dx^\mu)^2 \quad , D = d+1$$

A_μ graviphoton , R = radius of circle

$$g_{KK} = \frac{1}{2\pi R} \left(\frac{1}{2\pi R} \right)^{\frac{1}{d-2}} \quad \text{(in Planck units } M_p^{(d)} = 1\text{)}$$

There is an infinite tower of Kaluza-Klein states

$$M_n = \left(\frac{n}{R}\right) \left(\frac{1}{2\pi R}\right)^{\frac{1}{d-2}} \quad , \quad q_n = 2\pi n \; .$$

So can write:

$$\boxed{M_n = g_{KK} q_n M_p}$$

Weak coupling $g_{KK} \to 0$, brings down an infinite tower of states.

The lightest tower of states provides the degrees of freedom for a description of the theory:

- $g_s \to 0$: A string theory in d dimensions
- $g_{KK} \to 0$: A higher dimensional field theory in d+1 dim

Emergent string conjecture: These are the only possible infinite light towers in $g \to 0$ limits. (1904.06344)

Example test: type IIA string theory in 10D has a graviphoton A_μ,

$$g_A \sim \frac{1}{g_s^{3/4}}$$

Weak gauge coupling $\Leftrightarrow$ strong string coupling

Have non-perturbative charged D0 branes,

$$M^{D0} \sim \frac{1}{g_s^{3/4}} \sim g_A$$

At strong string coupling they form bound states

$$M_n^{D0} \sim \frac{n}{g_s^{3/4}} \sim n\, g_A\, M_p$$

Same as KK tower with $g_A \Leftrightarrow g_{KK}$

Strongly-coupled IIA ⇔ 11-dimensional supergravity

3. The Weak Gravity Conjecture (4-dimensions)

Consider a theory with gravity and U(1) gauge field

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_p^2 R - \frac{1}{4g^2} F^2 + \dots \right]$$

- Electric WGC: there must exist a charged state

$$m \leq \sqrt{2}\, g q M_p .$$

- Magnetic WGC: There exists an infinite tower of charged states with

$$m_\infty \sim g M_p .$$

Electric-Magnetic Duality

In general: Weak Electric ⇔ Strong Magnetic

EM duality: $M_{monopole} \sim \frac{1}{g} M_p$

Monopole mass is at least the energy stored in it's magnetic field:

$$M_{monopole} \sim \frac{\Lambda_{cutoff}}{g^2} \Rightarrow \underline{\Lambda_{cutoff} \sim g M_p}$$

Relation to Black-Hole stability

Consider an object with mass M and charge Q, which decays to a number of objects with mass m_i, and charge q_i.

$$M \geq \sum_i m_i \quad , \quad Q = \sum_i q_i$$

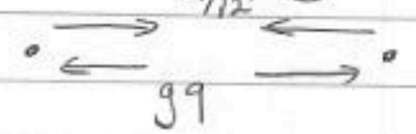
$$\frac{M}{Q} \geq \frac{1}{Q} \sum_i m_i = \frac{1}{Q} \sum_i \left(\frac{m_i}{q_i} \right) q_i \geq \frac{1}{Q} \left(\frac{m}{q} \right)\Big|_{min} \sum_i q_i$$

$$= \left(\frac{m}{q} \right)\Big|_{min}$$

So a decay can only occur to objects which have at least one with larger charge-to-mass ratio.

Extremal Black Holes have $M = \sqrt{2}\, Q M_p$, so WGC is the statement that BHs can decay.

Similarly, the forces acting on a particle with itself



WGC says there should be at least one particle on which gravity acts as the weakest force (so that it is <u>self-repulsive</u>).

WGC violation $\Rightarrow$ infinite number of stable states.

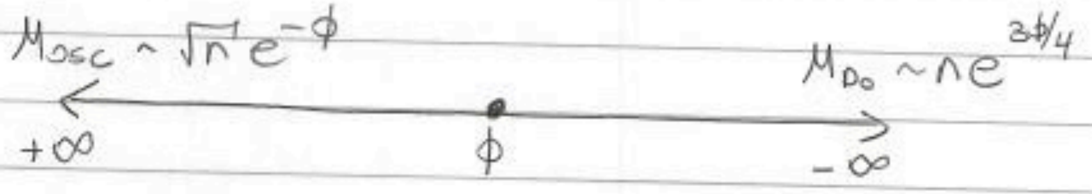
May be problematic – but no proof.

4) Distance conjecture

In string theory there are no dimensionless free constants, only expectation values of scalar fields

$$g_s = e^{-\phi} \quad , \quad \phi = \text{dilaton}$$

$$g_A \sim g_s^{-3/4} \sim e^{3/4\,\phi}$$



Distance conjecture:

For any ϕ, canonically normalised, for $|\phi| \gg M_p$,

$$M_\infty \sim e^{-\alpha(\phi/M_p)} M_p \qquad \alpha \sim O(1)$$

More precisely: Consider kinetic term

$$\mathcal{L} \supset P_{ij}(\phi)\, \partial\phi^i \partial\phi^j$$

Proper distance in field space between two points

$$\Delta(P,Q) = \int_\gamma \left(P_{ij} \frac{\partial\phi^i}{\partial\tau} \frac{\partial\phi^j}{\partial\tau} \right)^{1/2} d\tau$$

γ geodesic with parameter τ.

Then,

$$M_\infty(Q) \sim M_\infty(P)\, e^{-\alpha \Delta(P,Q)}$$

as $\Delta(P,Q) \to \infty$.

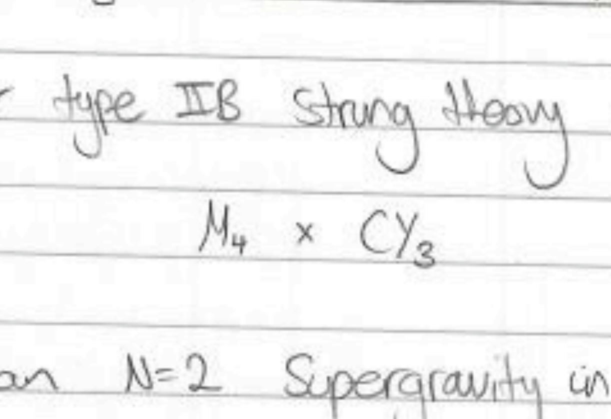
<u>Distance conjecture in Calabi-Yau moduli spaces</u>

Consider type IIB string theory compactified on

$$M_4 \times CY_3$$

Gives an $N=2$ Supergravity in four dimensions

The geometry of 3-cycles in the CY is controlled by the complex-structure moduli, z^i:



, $i = 1, \ldots, O(100)$

z^i are complex scalars with kinetic term:

$$\mathcal{L} \supset P_{i\bar{j}}\, \partial z^i \partial \bar{z}^{\bar{j}}$$

The geometry of the (Moduli) field space is determined by the period vector:

$$\Pi_I(z) \quad , I = a, i.$$

$$P_{i\bar{j}} = \partial_{z^i} \partial_{\bar{z}^{\bar{j}}} K \quad , \quad K = -\log\left[i \Pi^T \cdot \eta \cdot \bar{\Pi} \right]$$

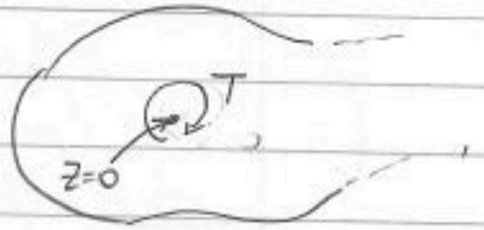
η is the symplectic intersection matrix of 3-cycles

$$\eta_{IJ} = C_I \cdot C_J$$

Consider a locus at infinite proper distance in the moduli space, denoted by:

$$z \to 0$$

<u>Theorem</u>: Such a locus is a singularity on the moduli space with a monodromy about it.



$$, \quad \Pi(z e^{2\pi i}) = T \cdot \Pi(z)$$

T is the monodromy matrix.

Define,
$$N = \log T$$

Then, N is Nilpotent : $N^{n+1} = 0 \; , \; n \leq 3$.

<u>Theorem</u>: $$\Pi(z) = \exp(N \log z) \underbrace{(a_0 + a_1 z + \dots)}_{\text{Holomorphic}}$$

<u>Theorem</u>: Infinite distance $\Leftrightarrow N^{d+1} a_0 \neq 0 \; , \; d > 0$.

To calculate the proper distance, change variables:

$$t = \frac{\log z}{2\pi i} \quad , \quad t \to +i\infty$$

$$\Pi(t) = \exp[Nt]\,(a_0 + O(e^{2\pi i t}))$$

$$\sim \quad t^d (N^d a_0)$$

$$K = -\log[i\,\Pi(t)\cdot\eta\cdot\bar{\Pi}(\bar{t})]$$

$$\sim -\log\,(t-\bar{t})^d$$

$$\Rightarrow \quad P_{t\bar{t}} \sim \frac{d}{4(\mathrm{Im}\,t)^2}$$

$$\Rightarrow \quad \Delta\phi \sim \int_{\mathrm{Im}\,t_i}^{\mathrm{Im}\,t_f} \frac{\sqrt{d}}{2}\,\frac{d(\mathrm{Im}\,t)}{\mathrm{Im}\,t} = \frac{\sqrt{d}}{2}\log\left(\frac{\mathrm{Im}\,t_f}{\mathrm{Im}\,t_i}\right).$$

The t^i control the sizes of 3-cycles, and these can be wrapped by D3 branes. The mass of such states is:

$$M(t_i, q) = |Z(t_i, q)| = \left|\frac{q\cdot\eta\cdot\Pi(t)}{[i\,\Pi^T(t)\cdot\eta\cdot\bar{\Pi}(\bar{t})]^{1/2}}\right|$$

$$\simeq \frac{|q\cdot\eta\cdot a_0|}{(\mathrm{Im}\,t)^{d/2}} + \frac{\mathrm{Im}\,t\;|q\cdot\eta\cdot N\cdot a_0|}{(\mathrm{Im}\,t)^{d/2}}$$

Lightest states have $M(t) \sim (\mathrm{Im}\,t)^{-d/2}$

Therefore,

$$\frac{M(t_f)}{M(t_i)} \sim \left(\frac{\operatorname{Im} t_i}{\operatorname{Im} t_f}\right)^{d/2} \sim e^{-\frac{2\Delta\phi}{\sqrt{d}}}$$

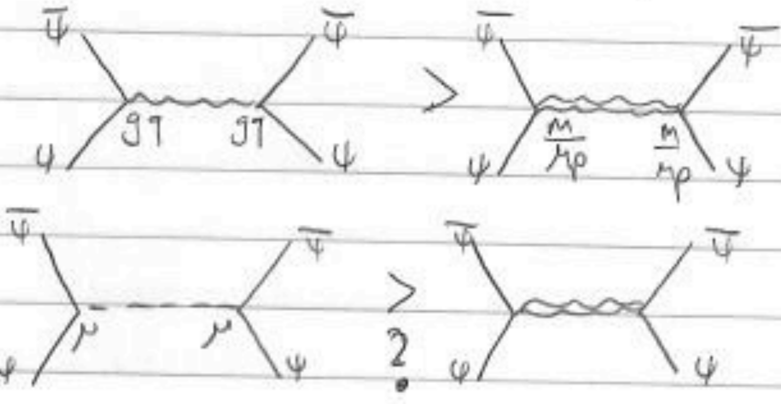
So, distance conjecture holds for any infinite distance locus in any Calabi-Yau moduli space.

($\mathcal{O}(10^{10})$ known CY's, having typically $\mathcal{O}(100)$ dimensional moduli spaces, each with $\mathcal{O}(10^{100})$? different infinite distance loci.)

5) Relations and underlying physics

Distance conjecture and gravity as weakest force

A massless scalar field can mediate a Coulomb self-force (attractive), which can be compared to gravity:



$$\mathcal{L} \supset m(\phi)\bar{\Psi}\Psi = (\partial_\phi m)\,\phi\,\bar{\Psi}\Psi$$

$$\Rightarrow \mu = (\partial_\phi m)$$

True, generally (also for scalar particles)

Gravity as weakest force:

$$\mu = (\partial_\phi M) > M \qquad \text{(Planck units)}$$

For $\phi \to \infty \Rightarrow M \sim e^{-\alpha\phi}$ → Distance conjecture!

(e.g. $M \sim \phi^p \Rightarrow p > \phi$, violated for $\phi \to \infty$)

WGC and Distance Conjecture from unification

The Species scale

Are the infinite towers of states really infinite?

Consider dimensional reduction on a circle : $D = d+1$

$$(M_p^{(D)})^{D-2} \int_{M_D} R^{(D)} \sqrt{-G^{(D)}} = (M_p^{(D)})^{D-2} (2\pi R) \int_{M_d} R^{(d)} \sqrt{-g^{(d)}}$$

$$\Rightarrow (M_p^{(d)})^{d-2} = (M_p^{(D)})^{D-2} \, 2\pi R$$

So, in d-dimensional Planck units:

$$M_p^{(D)} \sim \frac{1}{R^{\frac{1}{D-2}}}$$

This is the true scale of strong coupling of gravity.

The Kaluza-Klein tower mass scale

$$M_{KK} \sim \frac{1}{R}$$

So the number of states between M_{KK} and $M_p^{(D)}$ is

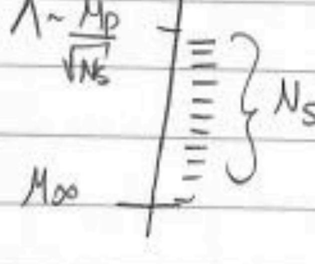
$$N_s \sim \frac{M_p^{(D)}}{M_{KK}} \sim R^{\frac{d-2}{D-2}}$$

$$\Rightarrow M_p^{(D)} \sim \frac{1}{N_s^{\frac{1}{d-2}}}$$

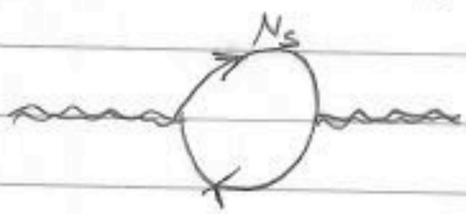
An example of a general relation:

$$\boxed{\Lambda_s \sim \frac{M_p}{N_s^{\frac{1}{d-2}}}}$$ Λ_s = species scale

In four-dimensions $\Lambda_s \sim \frac{M_p}{\sqrt{N_s}}$, so the towers:



Can also understand it from 1-loop renormalisation:



$\sim \Lambda^2 N_s$

Strong-coupling is the scale at which the 1-loop corrections are of order the tree-level result

$$M_p^2 \sim \Lambda_s^2 N_s \Rightarrow \Lambda_s \sim \frac{M_p}{\sqrt{N_s}}.$$

What if we insist that any gauge field must become strongly-coupled at the same scale as gravity? (so they can unify at that scale)

1-loop running from a single charged particle

$$\frac{1}{g_{IR}^2} = \frac{1}{g_{UV}^2} + q^2 \log\left(\frac{m}{\Lambda}\right)$$

Strongly coupled at $\Lambda \sim e^{1/g_{IR}^2} m \gg M_p$ (Landau pole)

To unify need a <u>tower</u> of charged states:

$$\frac{1}{g_{IR}^2} = \frac{1}{g_{UV}^2} + \sum_n^N (q_n)^2 \log\left(\frac{m_n}{\Lambda}\right)$$

How many states in tower:

$$\left.\begin{aligned} M_n &\sim n M_\infty \\ \Lambda &\sim N M_\infty \\ \Lambda \sim \Lambda_s &\sim \frac{M_p}{\sqrt{N}} \end{aligned}\right\} N \sim \left(\frac{M_p}{M_\infty}\right)^{2/3}$$

Strong-coupling: $\frac{1}{g_{IR}^2} \sim \sum_n^N n^2 \log\left(\frac{M_n}{\Lambda}\right) \sim N^3 \sim \left(\frac{M_p}{M_\infty}\right)^2$

$$\Rightarrow \quad \underline{m_\infty = g_{IR} M_p}$$

This is the magnetic Weak Gravity Conjecture!

What about scalar fields?

$$\mathcal{L} \supset P_{\phi\phi} (\partial\phi)^2$$

$$P_{\phi\phi}\big|_{IR} = P_{\phi\phi}\big|_{UV} + \sum_n^N (\partial_\phi m_n)^2 \log\left(\frac{m_n}{\Lambda}\right)$$

Strong coupling:

$$P_{\phi\phi}\big|_{IR} \sim \sum_n^N n^2 (\partial_\phi m_\infty)^2 \sim N^3 (\partial_\phi m_\infty)^2$$

$$\sim \left(\frac{\partial_\phi m_\infty}{m_\infty}\right)^2 M_p^2$$

Proper distance in field space:

$$\Delta\phi = \int_{\phi_i}^{\phi_f} \sqrt{P_{\phi\phi}\big|_{IR}}\, d\phi \sim M_p \int \partial_\phi (\log m_\infty)\, d\phi$$

$$\sim M_p \log\left(\frac{m_\infty(\phi_f)}{m_\infty(\phi_i)}\right)$$

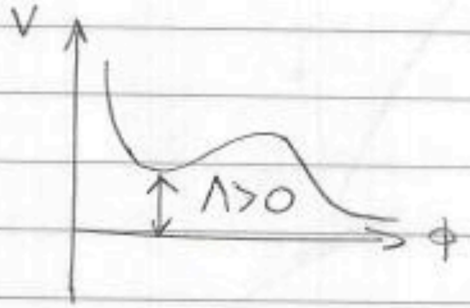
$$\Rightarrow \quad \underline{m_\infty(\phi_f) = m_\infty(\phi_i)\, e^{-\alpha\Delta\phi/M_p}}$$

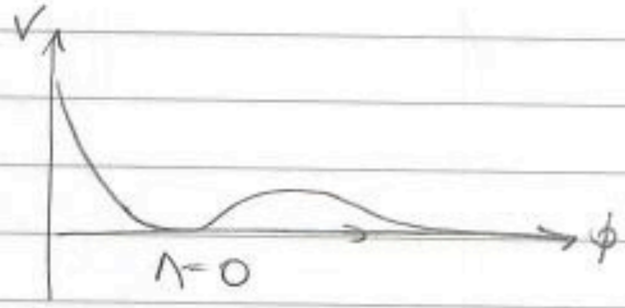
This is the Distance Conjecture!

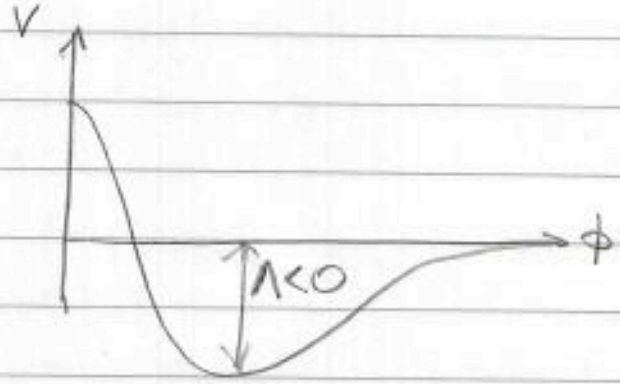
6) Swampland away from flat space

String theory is a framework in which we can calculate (for certain solutions) the cosmological constant Λ.

It corresponds to the value of the minimum of the full scalar potential:







$\Lambda = 0 \Rightarrow$ Minkowski space

$\Lambda < 0 \Rightarrow$ Anti-de Sitter space

$\Lambda > 0 \Rightarrow$ de Sitter space

6.1 $\Lambda > 0$: de Sitter

Difficult to extract information from String Theory: no fully understood de Sitter vacua are known.

Best candidate: KKLT, but many open questions.

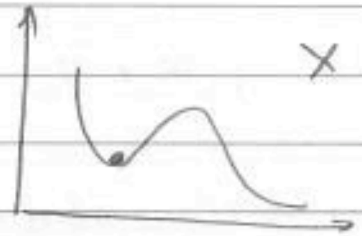
Possible that this is hinting at a Swampland obstruction to de Sitter.

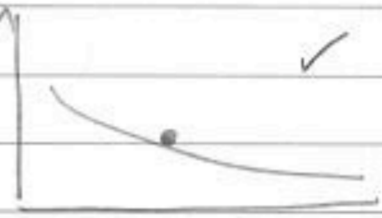
Refined de Sitter Conjecture

$$|\nabla V| \geq \frac{cV}{M_p} \quad \text{or} \quad \text{Min}(\nabla_i \nabla_j V) \leq -\frac{c'}{M_p^2} V,$$

with c, c' constants of order one.

If true, would predict Dark Energy is dynamical





Maybe true, but more likely holds as an asymptotic statement in parametric limits:

$$\phi \to \infty \quad , \quad g \to 0$$

dilaton field is exactly a flat direction.

Taking $N \to \infty$ allows the $g_s \to 0$ limit in controlled regime

3) Flat space limit and light towers

The flat space limit $\Lambda \to 0$, $R \to \infty$ is accompanied by an infinite tower of states becoming light

$$M_{KK}^{S^5} \sim \frac{1}{R}$$

Infact, the radius of curvature of AdS_5 and the S^5 are equal. So the background is not 5-dimensional.

This is called no scale-separation.

All fully-understood AdS solutions of string theory have these properties. This motivates an AdS Distance Conjecture:

- The flat space limit of AdS is accompanied by an infinite tower of states which behave as

$$M_\infty \sim \left|\frac{\Delta}{M_p^2}\right|^\alpha M_p, \quad \alpha \sim O(1).$$

The strong version implies no scale separation:

· (Strong ADC) : Supersymmetric AdS vacua are such that $\alpha = 1/2$

$$\frac{1}{R_{KK}} \sim M_\infty \sim \left|\frac{\Lambda}{M_p^2}\right|^{1/2} M_p \sim \Lambda^{1/2} \sim \frac{1}{R_{AdS}}$$

Relation to Distance Conjecture

The ADC views the flat space limit of AdS as a type of infinite distance limit, in analogy with the distance conjecture.

We can make this a little more precise: consider the Distance Conjecture for Calabi-Yau moduli space. The moduli are variations of the CY metric;

$$\Delta = \int_{\tau_i}^{\tau_f} \left(P_{ij} \frac{\partial \phi^i}{\partial \tau} \frac{\partial \phi^j}{\partial \tau} \right)^{1/2} d\tau$$

$$= \underbrace{\int_{\tau_i}^{\tau_f} \left(\frac{1}{V_{CY}} \int_{CY} \sqrt{g}\, g^{MN} g^{OP} \frac{\partial g_{MO}}{\partial \tau} \frac{\partial g_{NP}}{\partial \tau} \right)^{1/2} d\tau}$$

Distance in the space of metrics (De-Witt)

Apply to the AdS metric for variation $R \to \infty$.

Since R is an overall factor

$$\Delta \sim -\int_{R_i}^{R_f} \frac{dR}{R} \sim -\ln R$$

Then take,

$$M_\infty \sim e^{-\beta|\Delta|} \sim \left(\frac{1}{R}\right)^{d/2} \sim |\Lambda|^\alpha$$

Which is the ADC.

<u>Note</u>: there is an (infamous) negative sign for the distance.

<u>6.3. The Weak Gravity conjecture in AdS</u> (2108.04594)

Consider a 5D action with a negative cosmological constant:

$$S = \int d^5x \sqrt{-g} \Big[\tfrac{1}{2} R + 6 - \frac{1}{4g^2} F^2 - |D\phi|^2 - M^2|\phi|^2$$

$$- a|\phi|^4 + b|\phi|^2 |D\phi|^2 + \cdots \Big]$$

We have set the AdS scale to 1: $R_{AdS} = 1$.

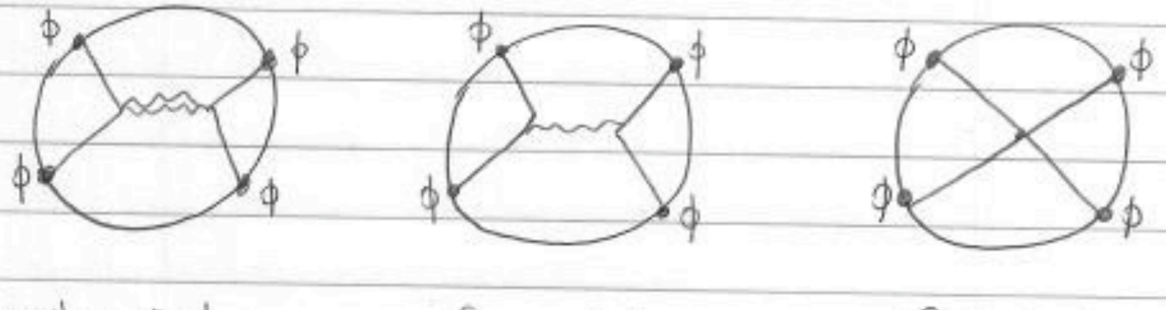
ϕ is charged field, with $q = 1$

In flat space we would require:

$$m \leq \sqrt{2} g M_p$$

However, if we are motivated by the absence of stable bound states we need to consider all the contributions to the possible binding energy.

AdS exchange diagrams:



Graviton Exchange Gauge Exchange Contact terms

Binding energy $\gamma_{\phi^2} = \gamma_{\phi^2}^{photon} + \gamma_{\phi^2}^{graviton} + \gamma_{\phi^2}^{quartic}$

$$\gamma_{\phi^2}^{photon} = \frac{\pi^2 N_\Delta^4 g^2 q^2}{2\Delta - 1}$$

$$M^2 = \Delta(\Delta-4),$$

$$\gamma_{\phi^2}^{graviton} = -\frac{2\pi^2 N_\Delta^4 \Delta^2(\Delta-2)}{3(\Delta-1)(2\Delta-1)}$$

$$N_\Delta = \sqrt{\frac{\Delta-1}{2\pi^2}}$$

$$\gamma_{\phi^2}^{quartic} = \frac{\pi^2 N_\Delta^4 (a + b\Delta(2-\Delta))}{(\Delta-1)(2\Delta-1)}$$

In SUSY theories, when ϕ is BPS: $-g^2q^2 = a + b\Delta(2-\Delta) = -\frac{2}{3}\Delta^2$

$\Rightarrow \underline{\gamma_{\phi^2} = 0}$.

In AdS contact terms are not sub-leading, because cannot separate the particle from its copy by an infinite distance.

Positive Binding Conjecture: $\gamma \geq 0$ for some particle

7) Holography and the Swampland

String theory / Quantum Gravity on AdS spaces is expected to be dual to a Conformal Field Theory on the boundary of AdS.

eg. Type IIB on $AdS_5 \times S^5 \Leftrightarrow N=4$ Super Yang Mills on $Mink_4$.

There Swampland constraints in AdS should have dual constraints on CFTs.

7.1. Scale separation on CFT gaps (1908.05225, 2201.03660)

The masses of states on the gravity side are mapped to the conformal dimensions of dual operators on the CFT:

$$M^2 R_{AdS}^2 = \Delta(\Delta - d) \qquad (d = \text{CFT dim})$$

So no scale-separation implies a tower of states with mass scale

$$M_{KK} R_{AdS} \sim \frac{R_{AdS}}{R_{KK}} \sim O(1)$$

In the CFT dual this predicts an infinite tower of operators with dimensions of order one:

$$\Delta_n^{KK} \sim O(1)\, n$$

So the operator spectrum has no gap; where we have a few operators with dimensions $O(1)$, and then an infinite number of operators with parametrically separated dimensions.

Conjecture: true for any CFT, not only "holographic" ones.

(e.g. weakly-coupled : $\phi, \phi^2, \phi^3 \ldots$)

Aside : Holographic Versus Non-Holographic CFTs

In holography, the dual to the radius of AdS, in Planck units, is the central charge of the CFT:

$$(R_{AdS} M_p)^3 \sim C \qquad (\text{for } CFT_4/AdS_5)$$

The central charge (roughly) counts the number of degrees of freedom of the CFT.

Holographic CFTs are those which have gravitational duals that behave like Einstein gravity, so they require (at least) : $C \gg 1$.

Nonetheless, one can say that every (local) CFT defines a quantum gravity theory in AdS. Because:

- CFTs are UV/IR complete
- Local CFTs have an energy-momentum tensor, which is the operator dual to the graviton (massless spin-2 field).

Often, such "quantum gravities" look nothing like Einstein gravity, say they may have an infinite number of massless higher spin fields. But, they are all expected to be part of string theory.

7.2 CFT Distance Conjecture (2011.03583, 2011.10040)

CFTs can come in families with exactly marginal operators:

$$\mathcal{L}_{CFT} \supset \sum_i t^i O_i$$

(t^i: deformation parameters; O_i: exactly marginal)

These are dual to moduli spaces on the gravity side. We therefore expect a dual to the distance conjecture.

Can define a metric on the deformation space (conformal manifold):

$$g_{ij}(t) = |x-y|^{2d} \langle O_i(x) O_j(y) \rangle$$

We can use the metric to define a distance on the conformal manifold : $d(t_1, t_2)$.

Then the conjecture is that as $d \to \infty$, there is an infinite tower of operators whose dimension goes to zero.

More precisely, if the minimal dimension of the tower, Δ_{min}, is bounded by ε : $\Delta_{min} \geq \varepsilon$. Then, $d \sim \ln \varepsilon$.

Further, it was proposed that all infinite distance loci in CFT's are Higher-Spin (HS) theories. That means that the tower of operators have increasing spin J's

$$\Delta_J = d - 2 + J.$$

7.3 Charge convexity Conjecture (2108.04594, 2206.06703)

Recall that in AdS we formulated a version of the WGC called the Positive Binding Conjecture, which demands the existence of a charged particle, say of charge q, with positive self-binding energy, $\gamma \geq 0$.

This implies that on the gravity side we have a state of energy E_q, and charge q under U(1) gauge symmetry, such that

$$E_{2q} - 2E_q \geq 0.$$

In the dual CFT we have a $U(1)$ global symmetry, and a bound on the dimension of charged operators:

$$\Delta(2q) - 2\Delta(q) \geq 0 .$$

More generally, the Charge Convexity Conjecture proposes:

$$\Delta(n_1 q_0 + n_2 q_0) \geq \Delta(n_1 q_0) + \Delta(n_2 q_0)$$

where $\Delta(q)$ is the lowest dimension operator of charge q, n_1 and n_2 are any positive integers, and $q_0 \sim O(1)$ (more precisely, cannot be made parametrically large).

Proposed to hold for $d > 2$.

In $d=2$, q_0 can be forced to be parametrically large, but in a very specific way.

Current level of the global symmetry current:

$$J(z) J(0) \sim \frac{k}{z^2}$$

Then can prove the conjecture, with $q_0 \leq k$.