When a Symmetry Breaks

–How to fix the Goldstone's theorem–

Arnold-Sommerfeld Colloquium, July 24, 2019

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Symmetry

Amalie Emmy Noether
1882-1935

Symmetry → Conservation law

Symmetry ← Conserved charge

parity

except for the weak interactions

rotational symmetry

100 trillion times faster than speed of light

moving in the 3D map of galaxies based on observations

$$\vec{F} = m\vec{a}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \longrightarrow \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

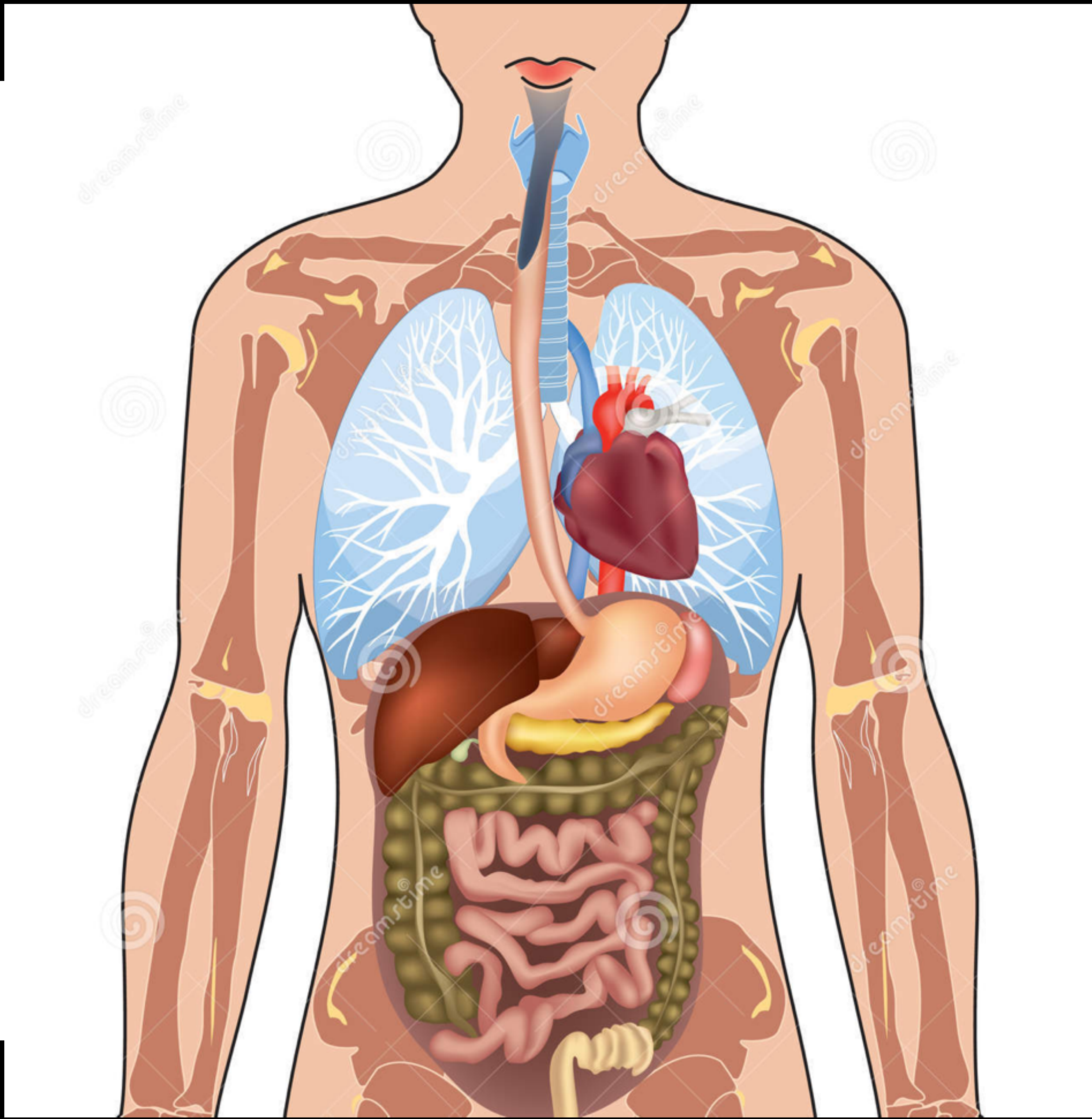
parity

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \longrightarrow \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

rotation

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \longrightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ a \end{pmatrix}$$

translation



If the laws of physics is symmetric, what is the origin of diversity?

Spontaneous Symmetry Breaking

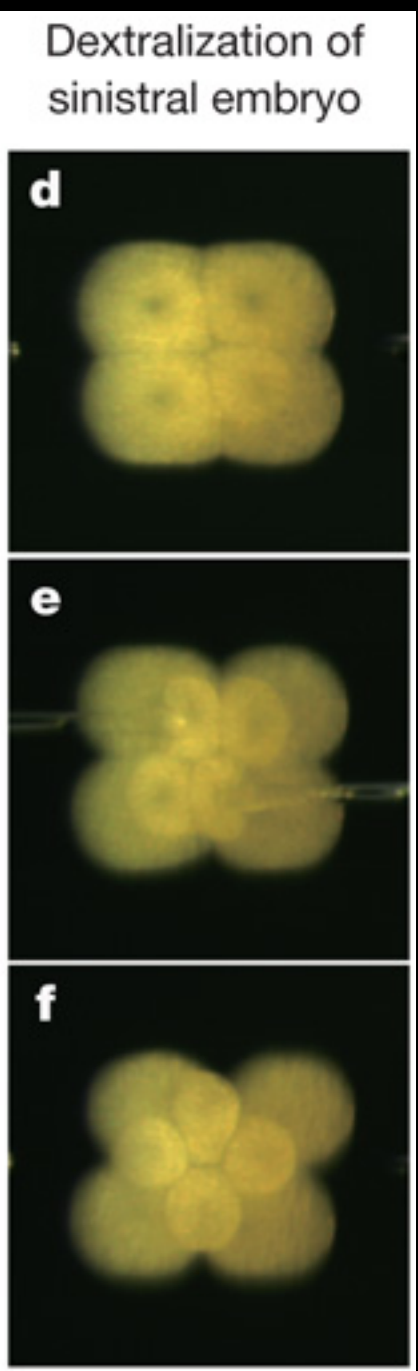
Spontaneous Symmetry Breaking

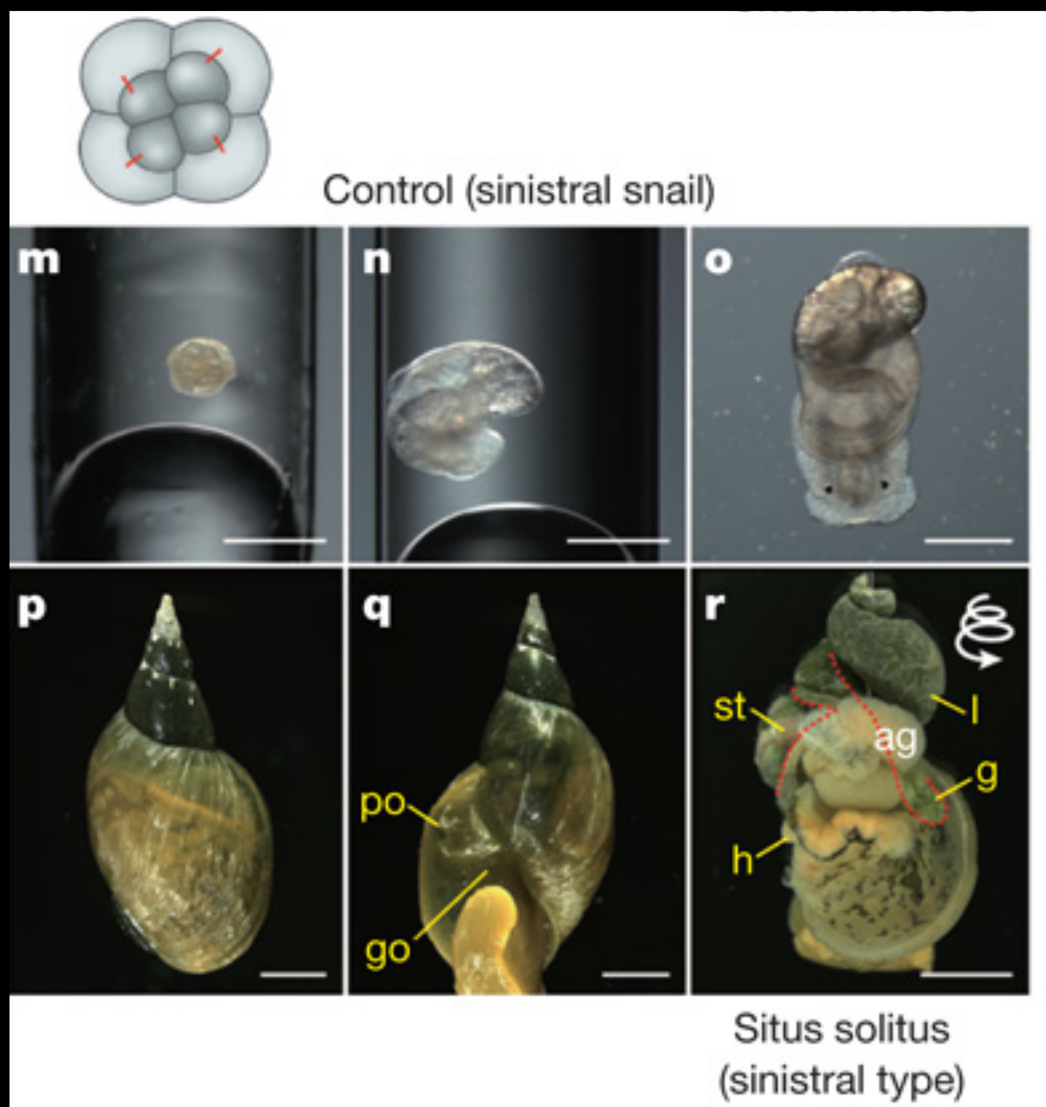
- System has a symmetry *G*
- But its ground state respects only the subset of symmetry *H*
- Then there are multiple ground states degenerate in energy *G/H*

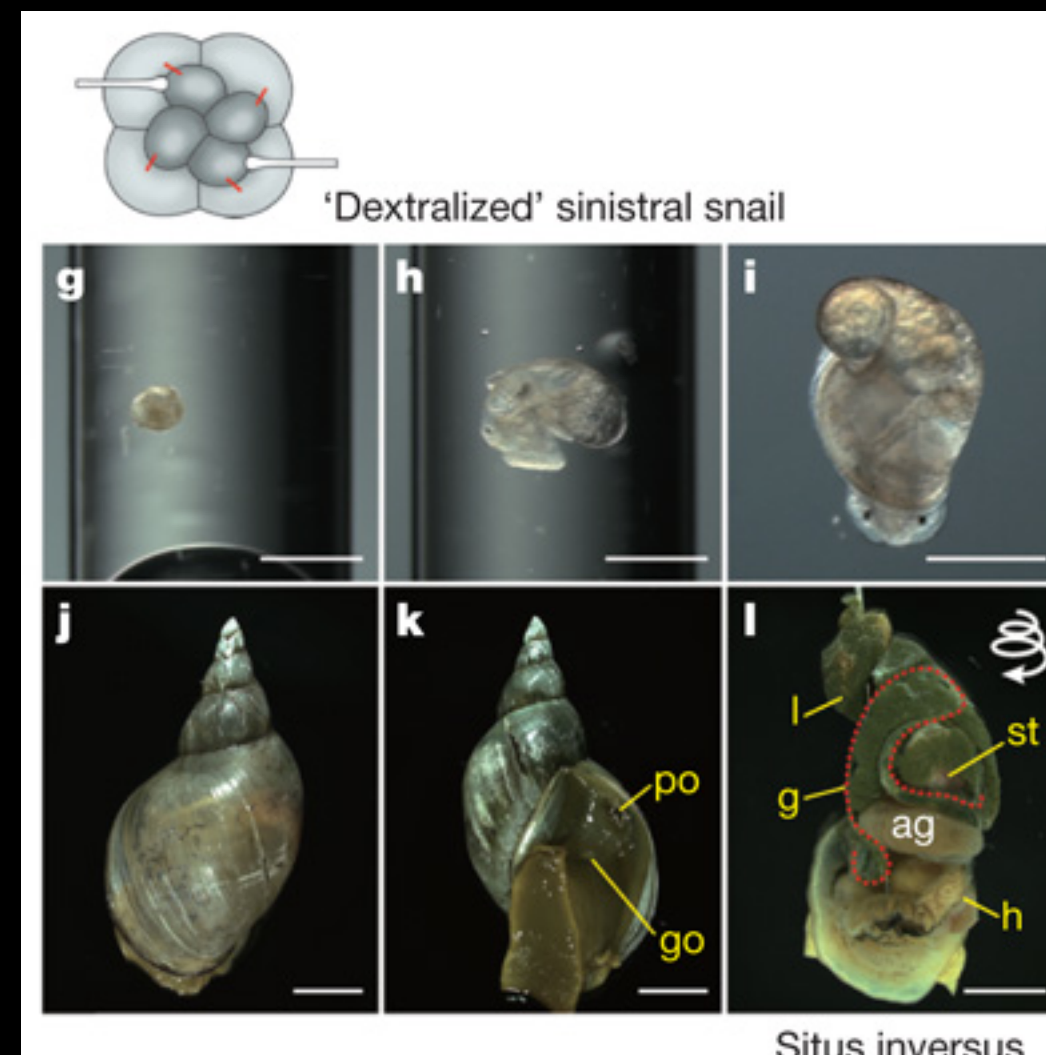
Halibut vs Flounder

Chirality


Reiko Kuroda


Dextralization of sinistral embryo
d
e
f


Control (sinistral snail)
m
n
o
p
q
r
po
go
st
ag
l
g
h
Situs solitus
(sinistral type)


'Dextralized' sinistral snail
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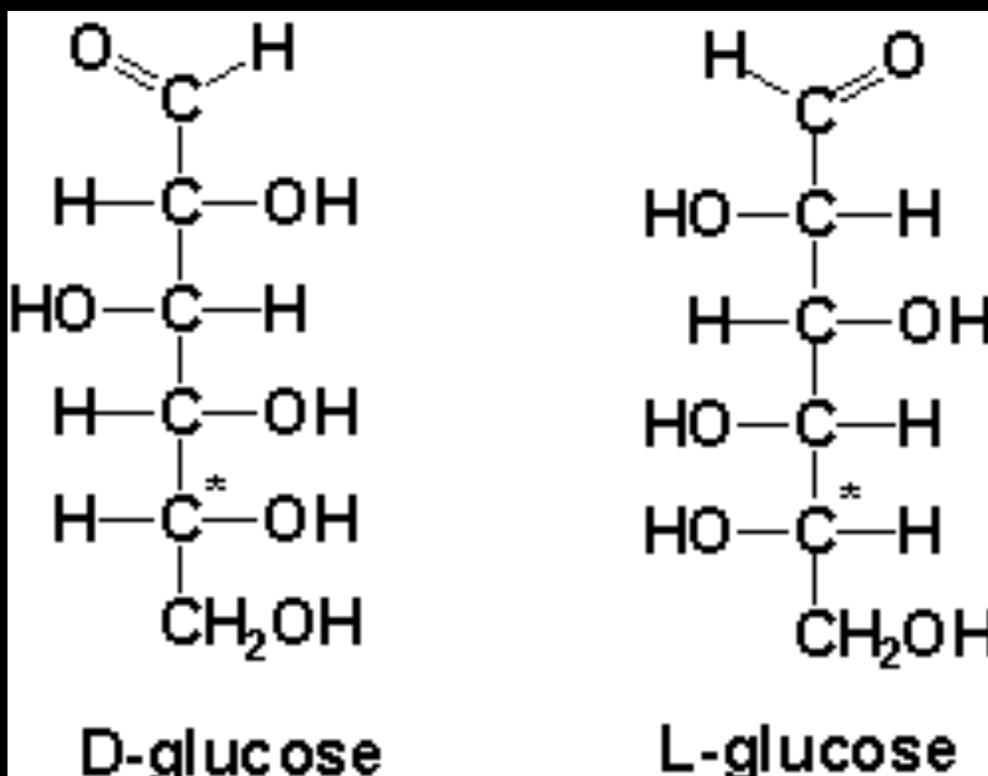
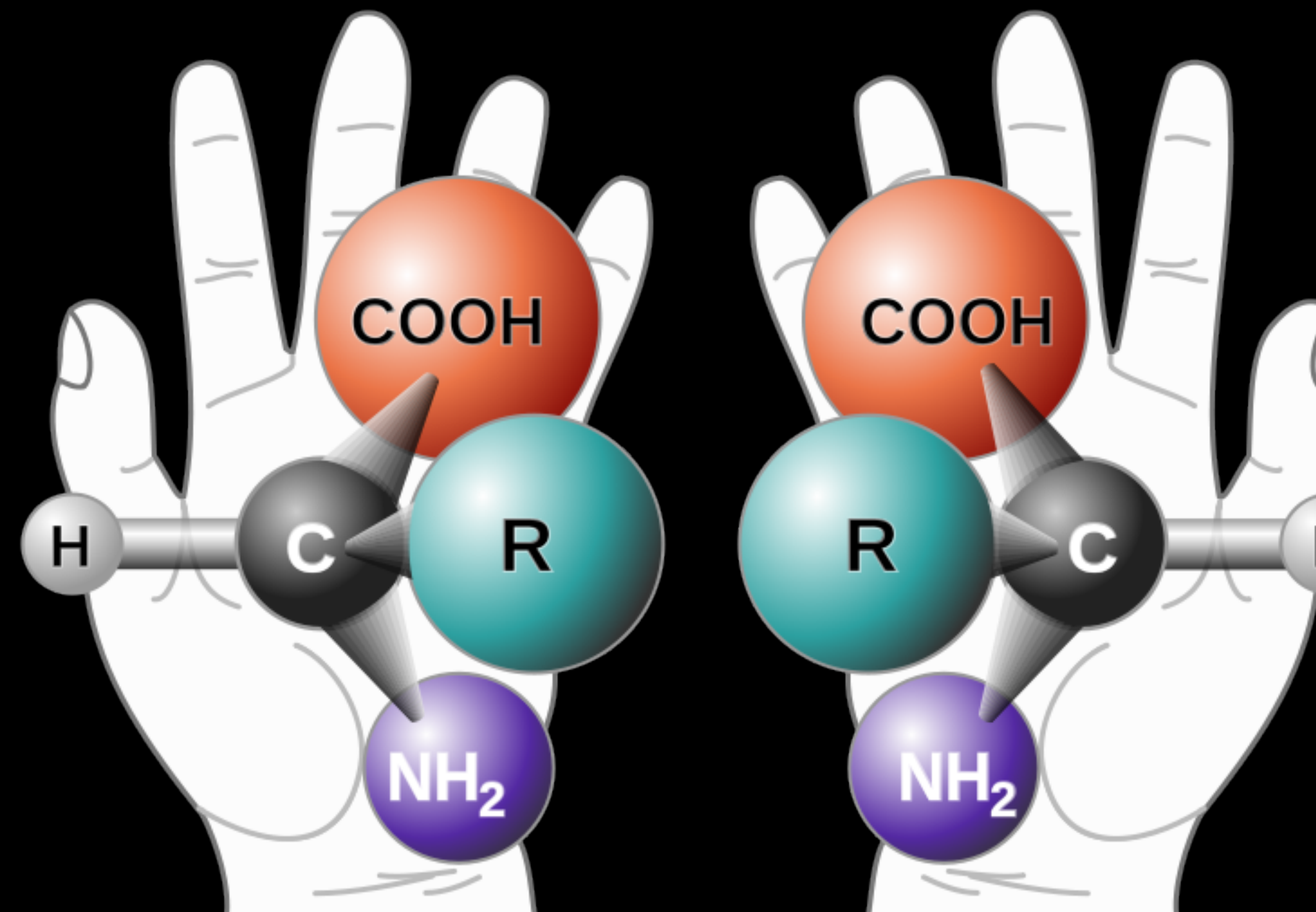
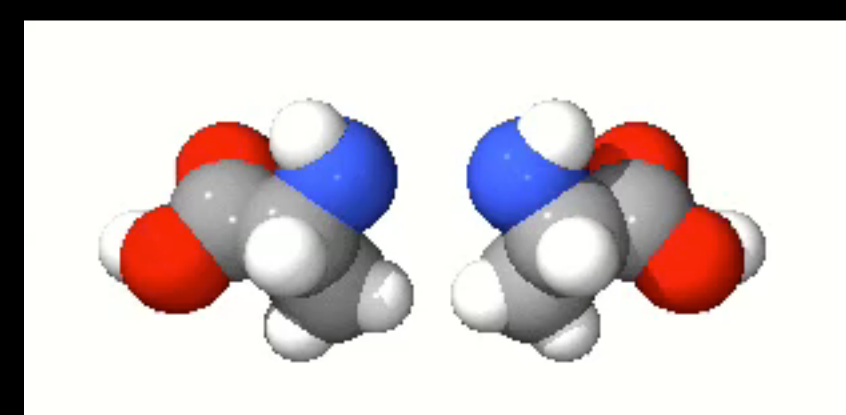
Situs inversus

Chirality

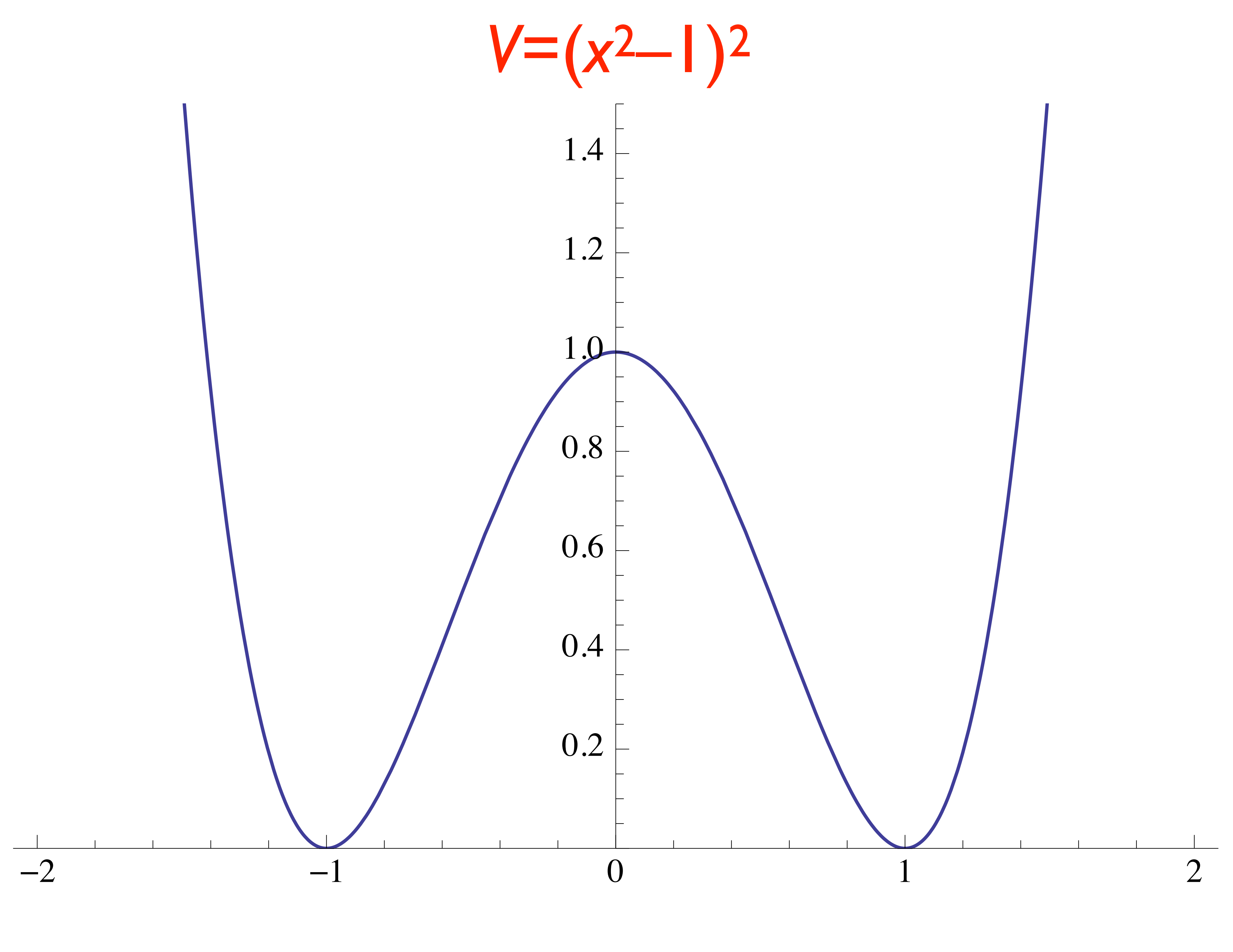




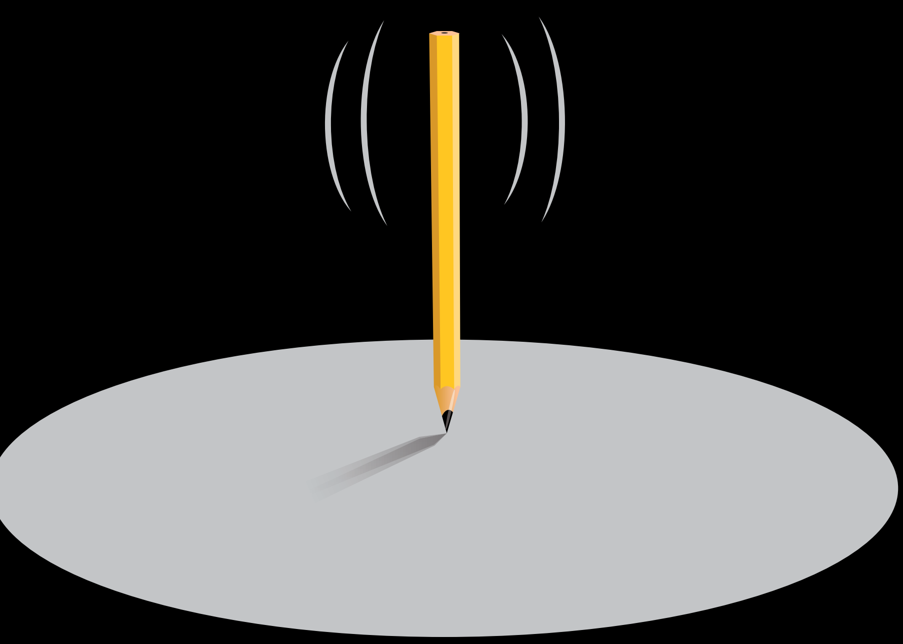
D-glucose is sugar
L-glucose cannot be digested

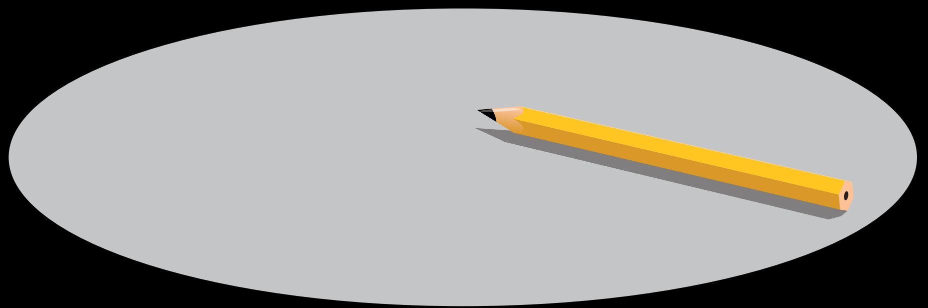


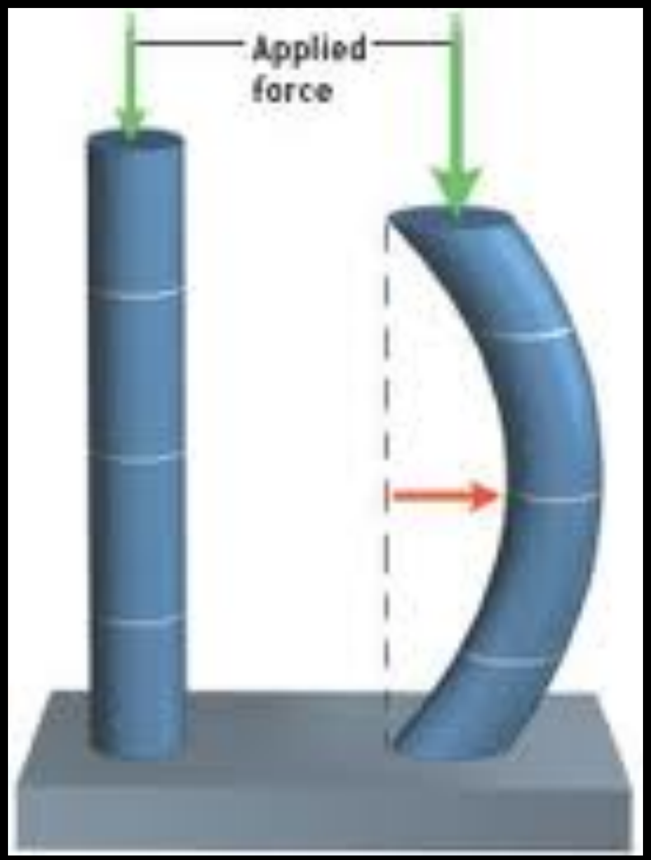
Potential Energy



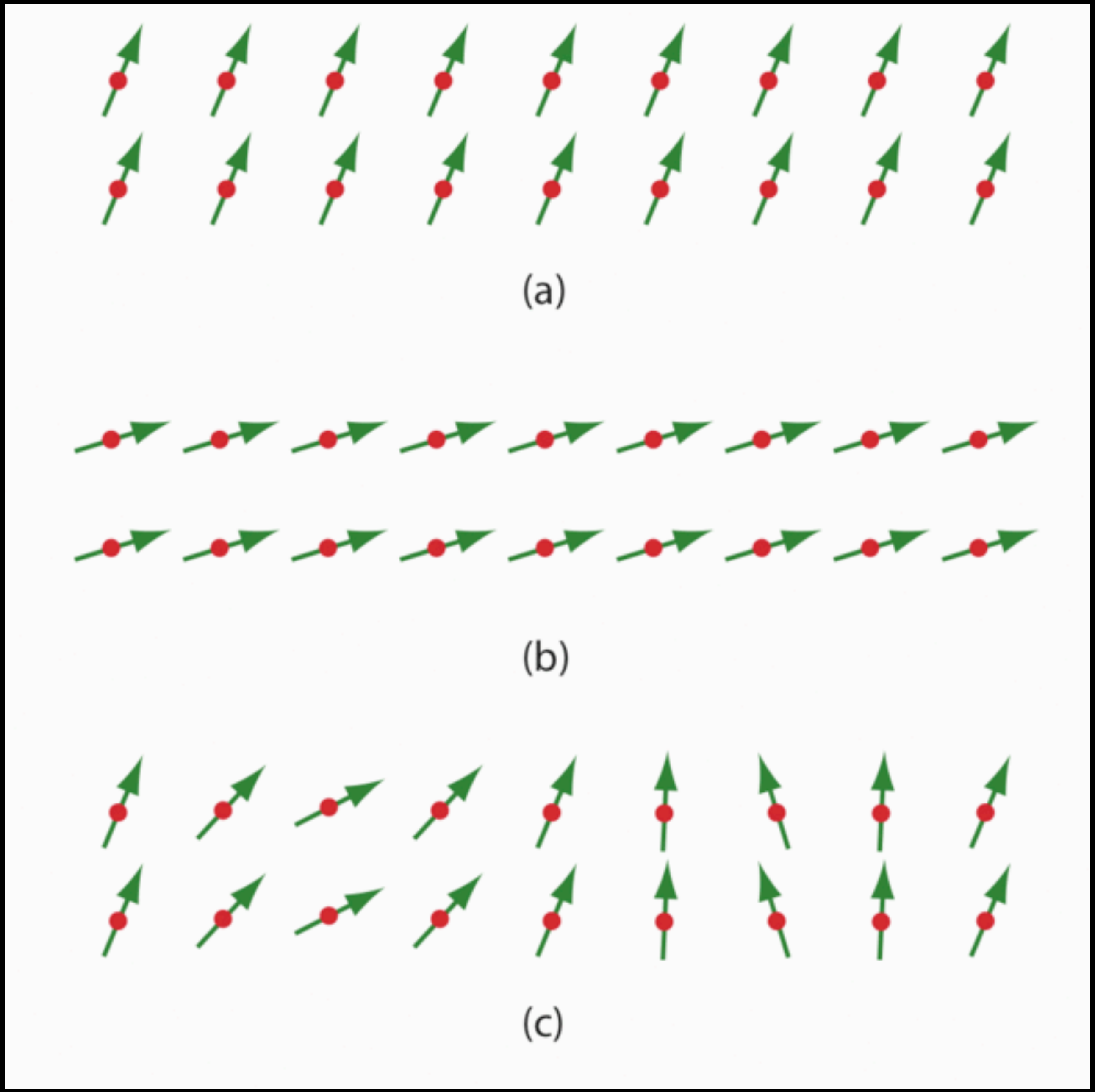
Rotational Symmetry

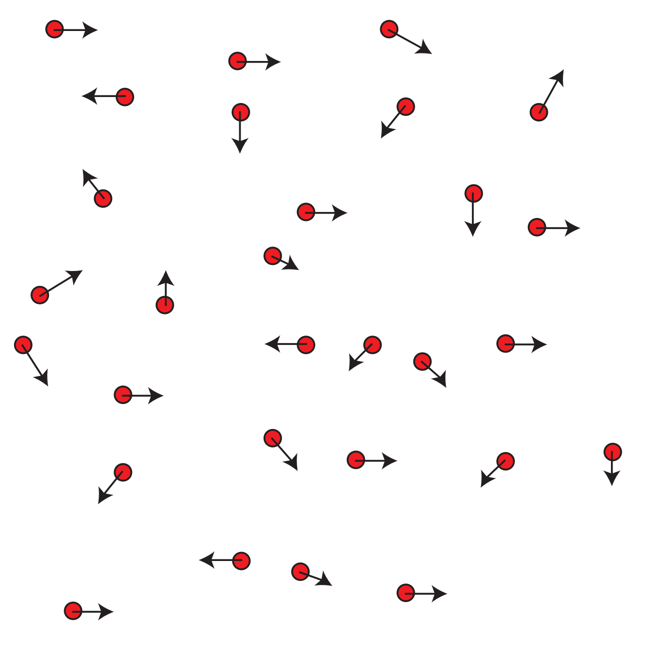




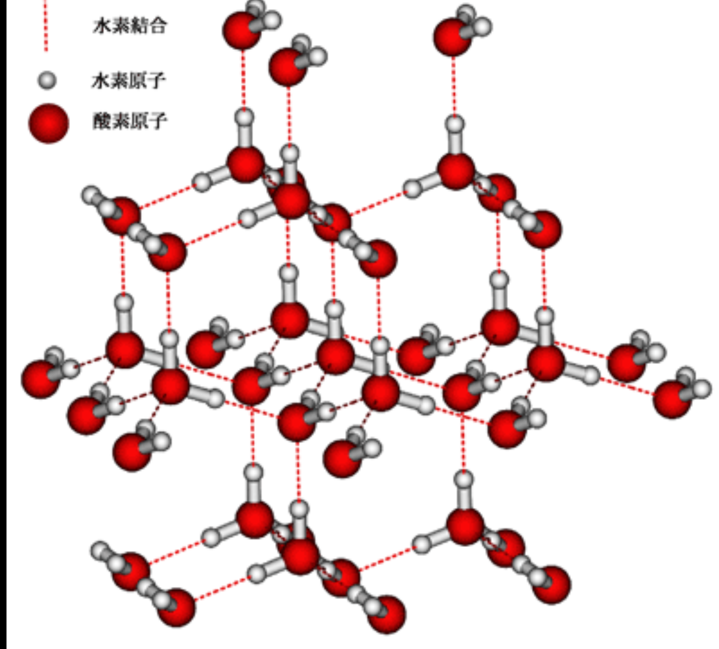


Magnet





Frozen



Phase Transition ⇒ Translational symmetry is broken

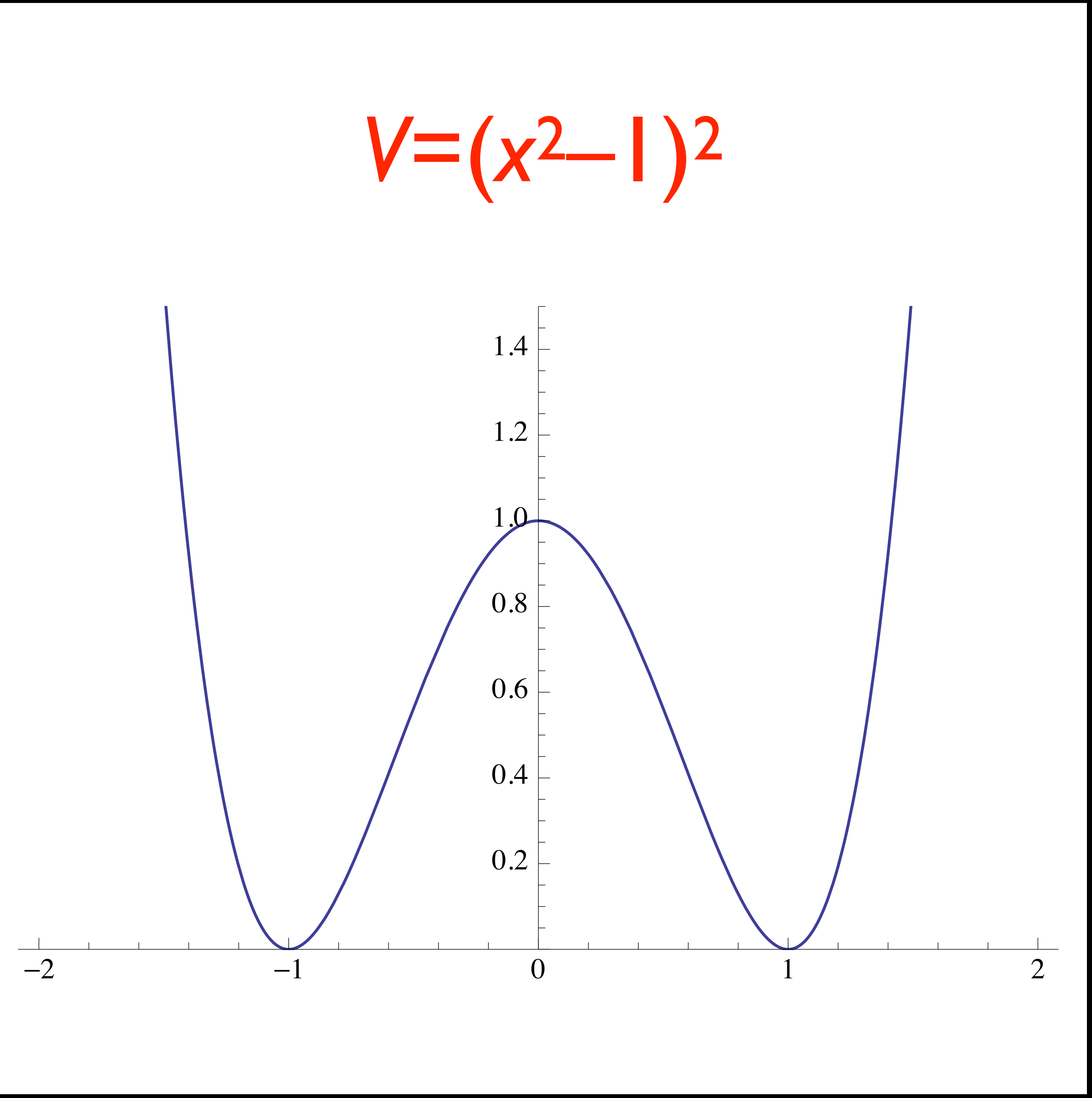
$$\psi(x) \longrightarrow e^{i\theta}\psi(x)$$

Superfluid

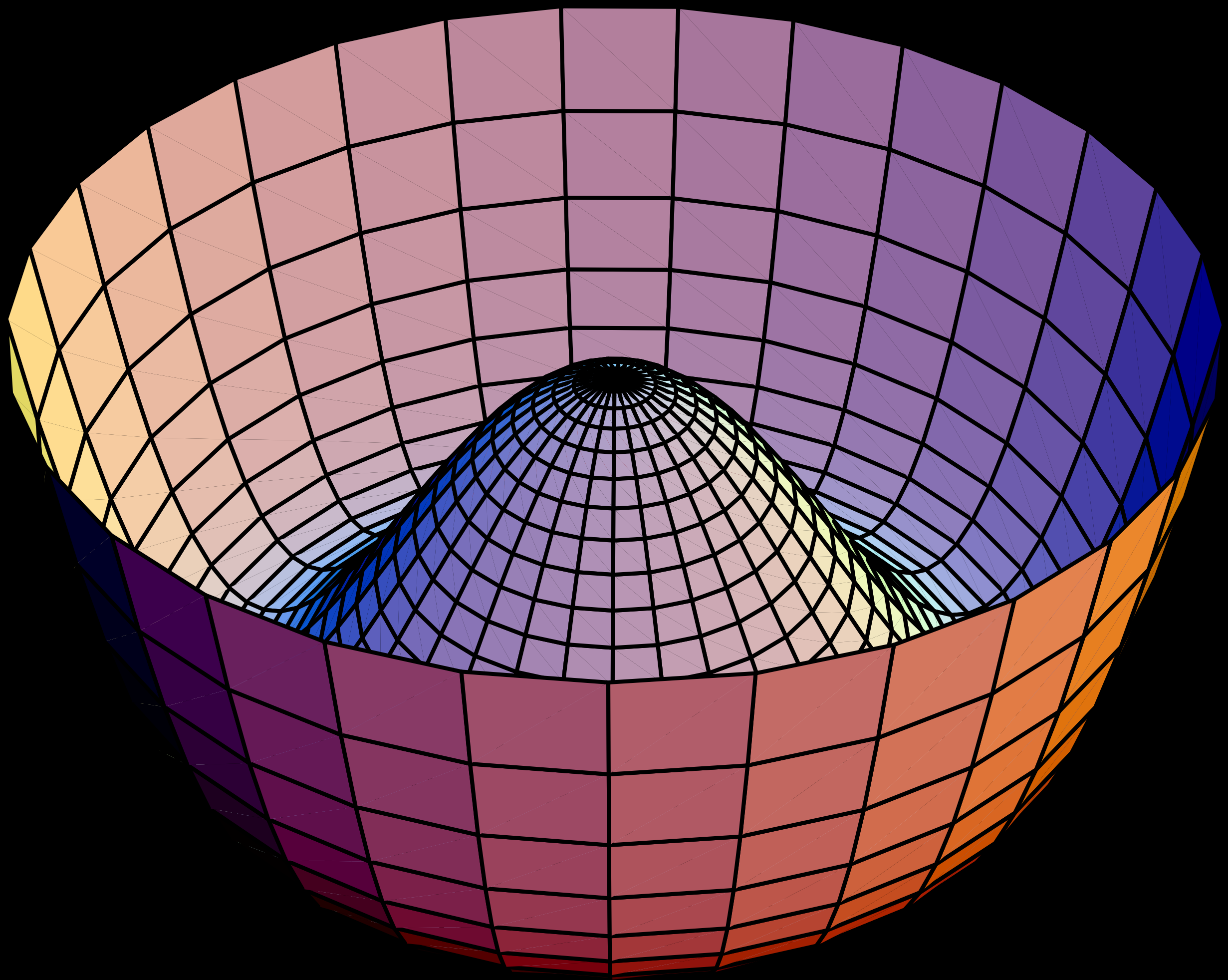
$$\psi(x) \longrightarrow e^{i\theta(x)}\psi(x)$$

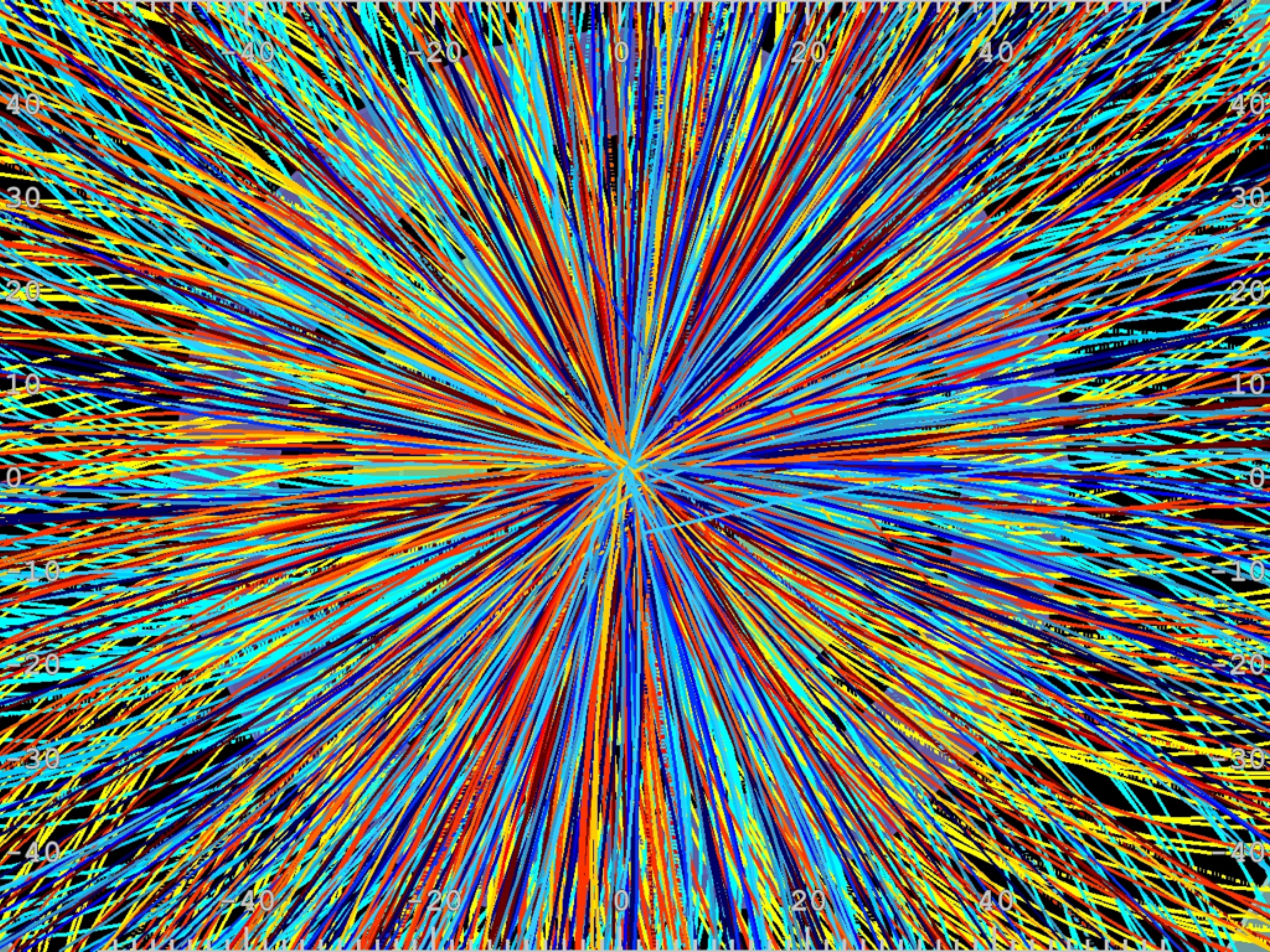
Superconductor

Potential Energy



$$V = (x^2 + y^2 - 1)^2$$

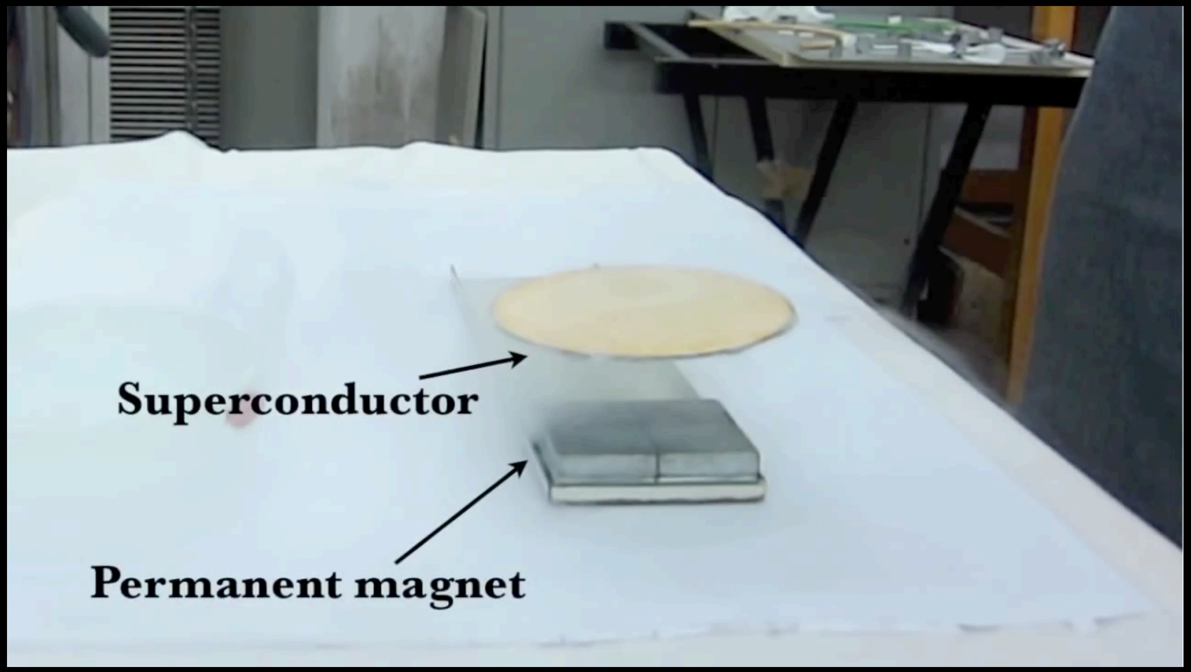


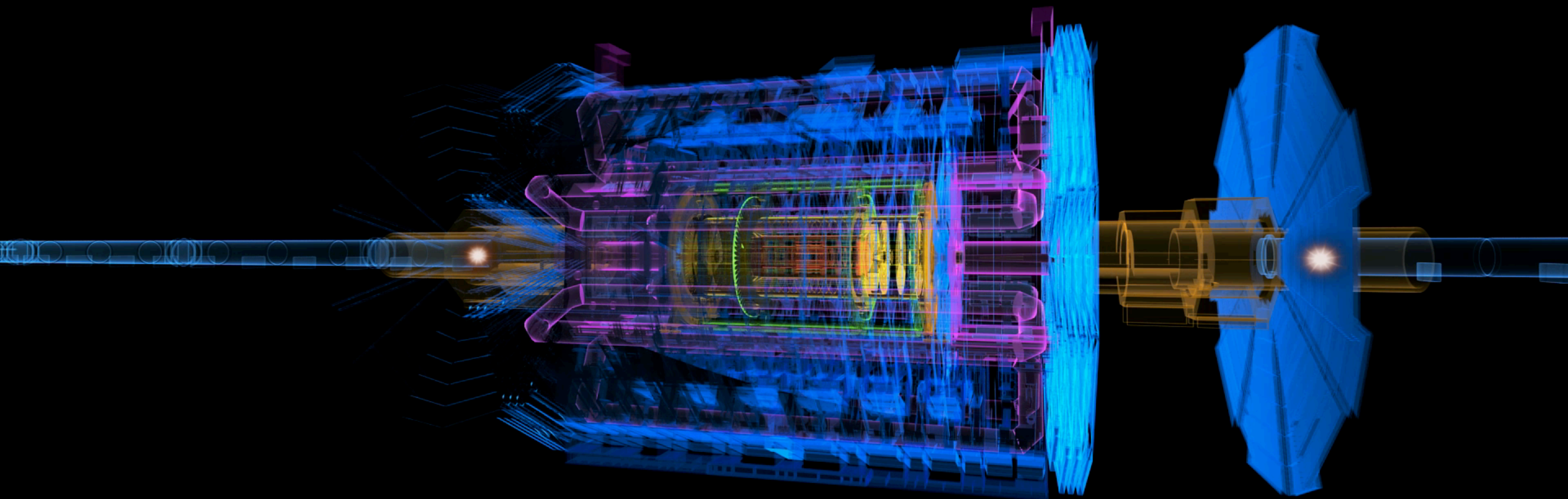


-40 -20 0 20 40
40
30
20
10
0
-10
-20
-30
-40

Mystery

- Weak force is basically the same kind as the electromagnetism
- But then why is its range much shorter than the size of nuclei?

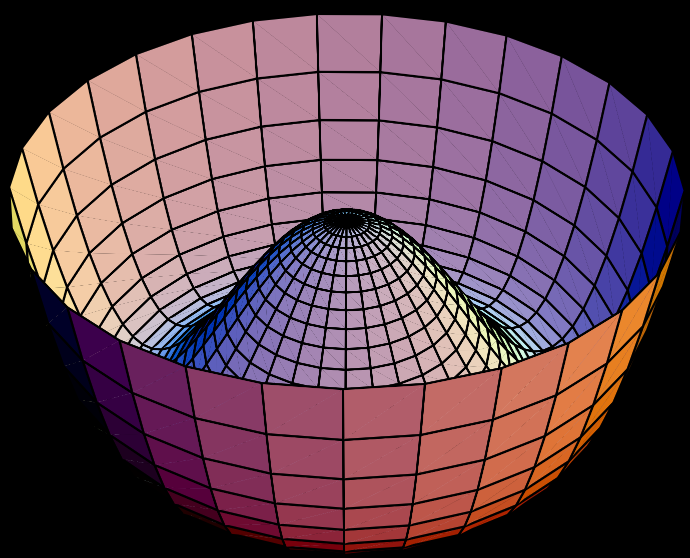


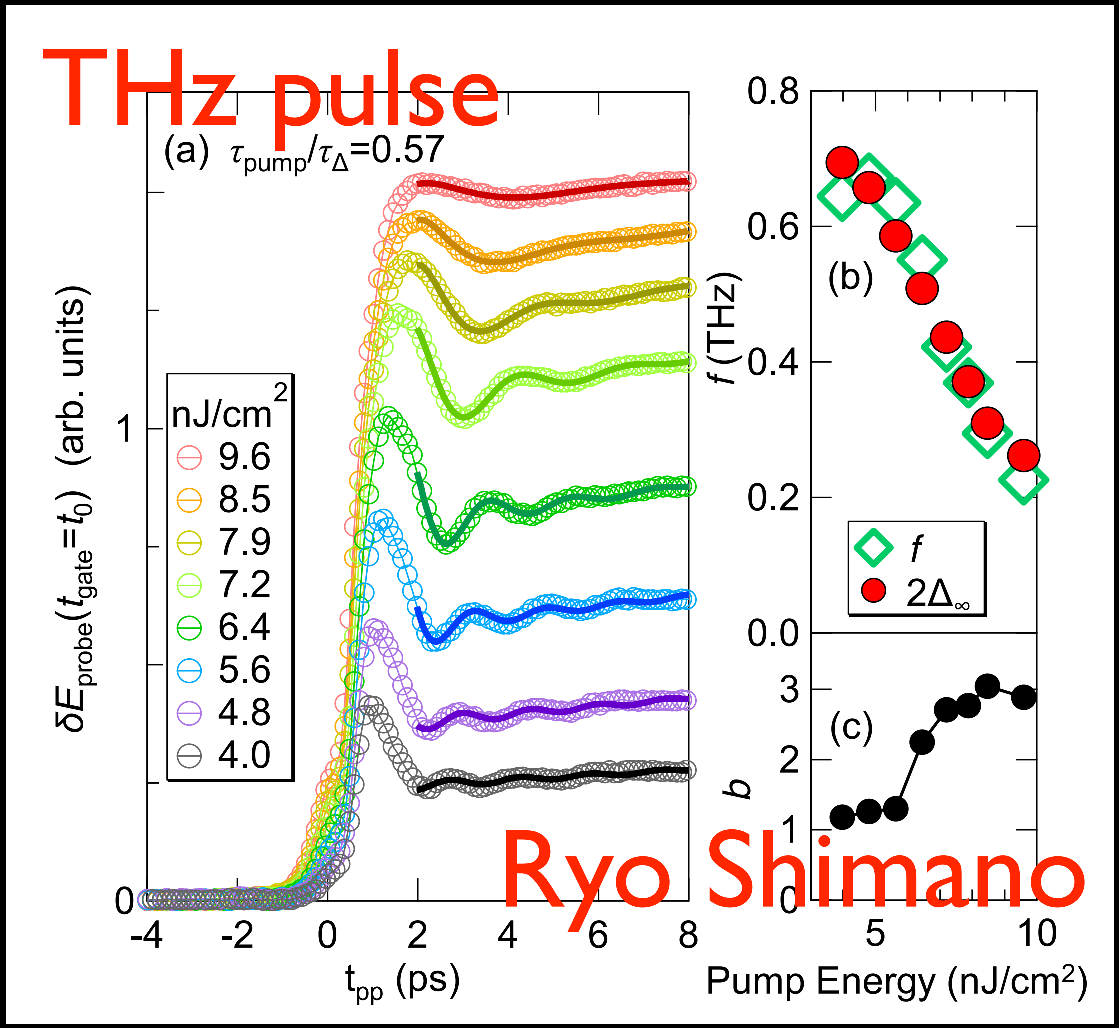


Higgs boson decays into two photons


Higgs
Independence Day
July 4, 2012

superconductors





Goldstone's theorem

- When a continuous symmetry is spontaneously broken, there appear the same number of massless particles (gapless excitations) as the number of broken symmetries

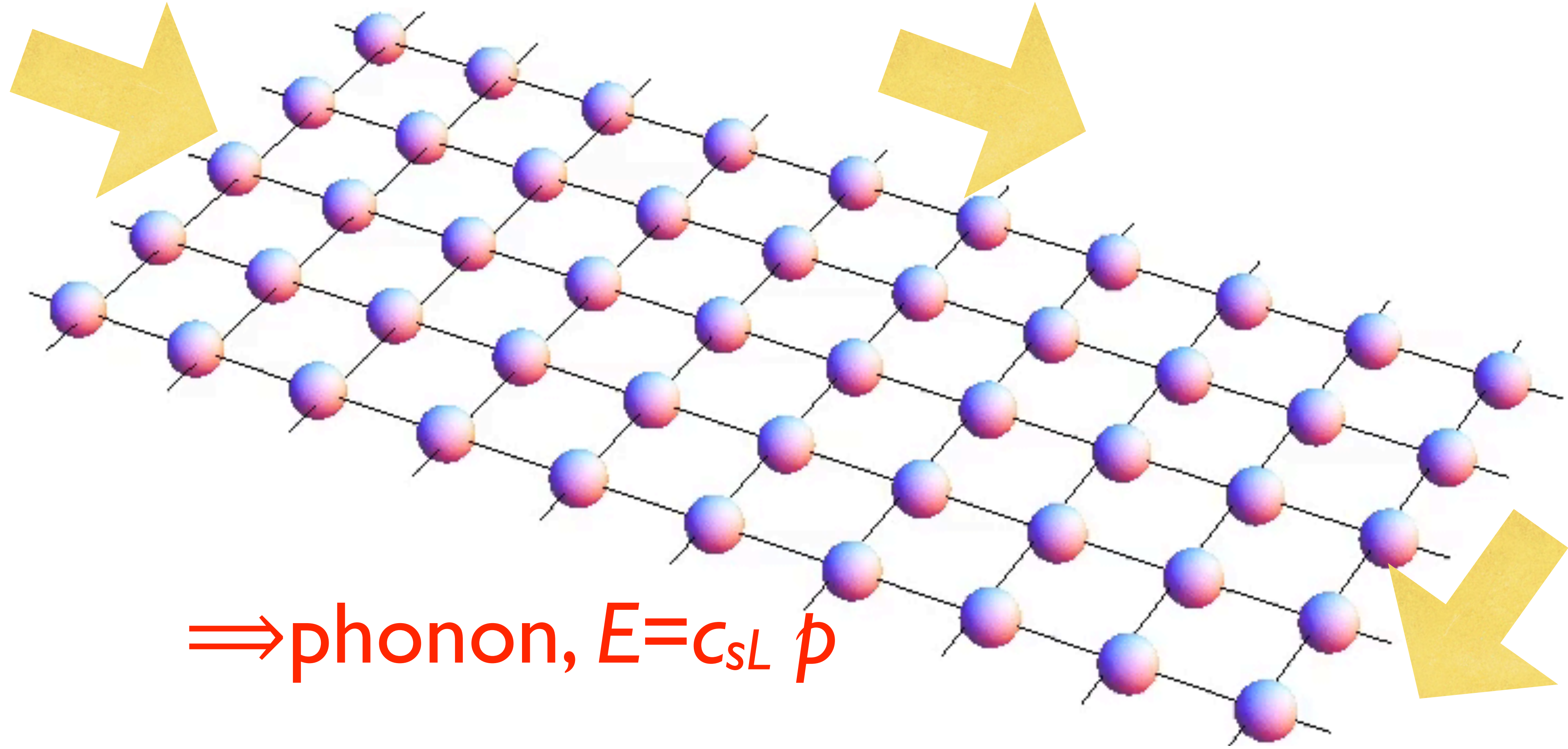
- Their dispersion relation is linear

$$E \propto p$$

Scanned at the American Institute of Physics

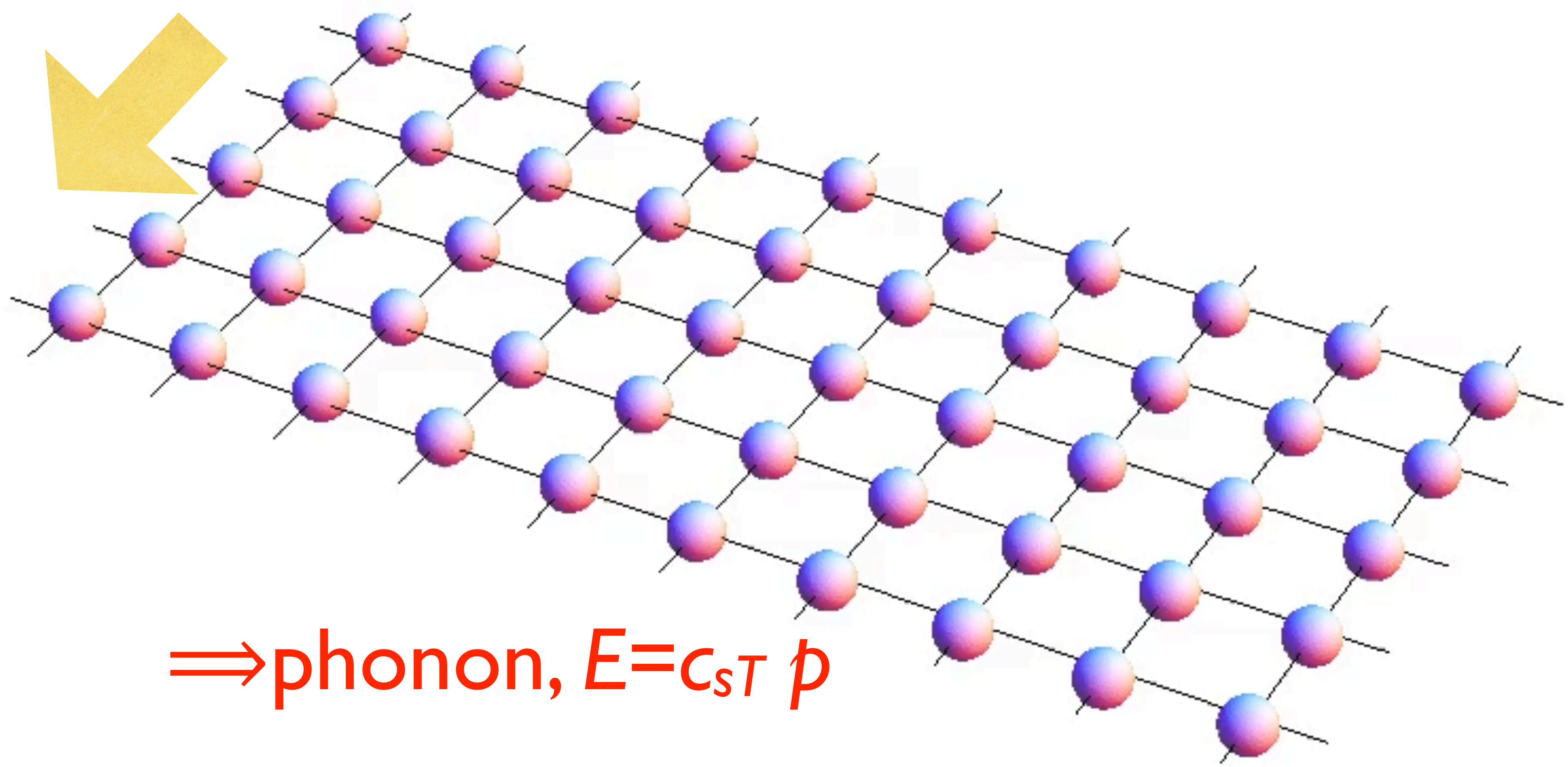
Nambu-Goldstone Bosons

longitudinal



⇒phonon, $E = c_{sL}\, p$

crystal



⇒phonon, $E = c_{sT}\, p$

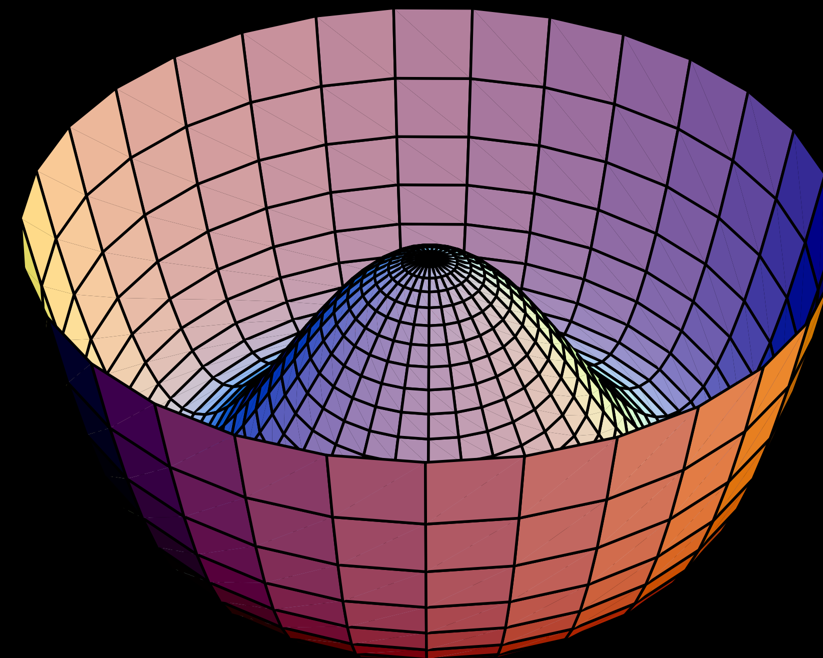
transverse

Particle numbers

- U(1) symmetry

$$\psi(x) \to e^{i\theta}\psi(x)$$

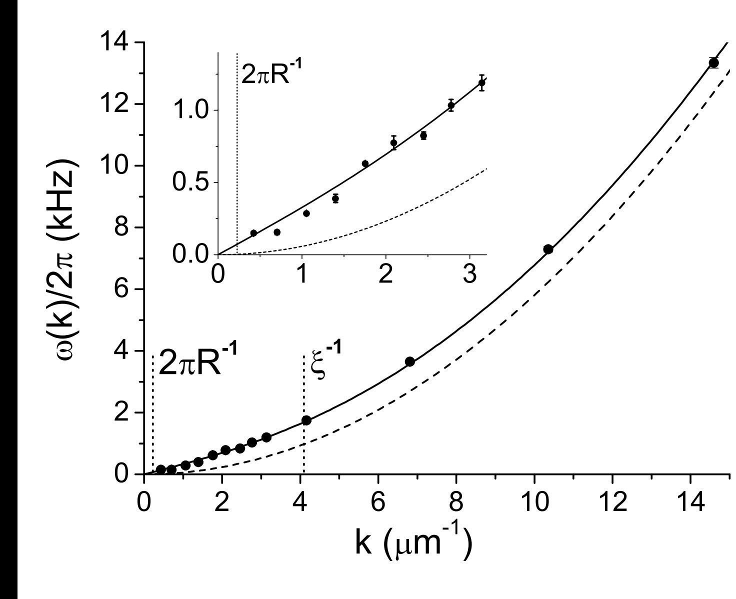
$$N = \int dx \psi^* \psi$$

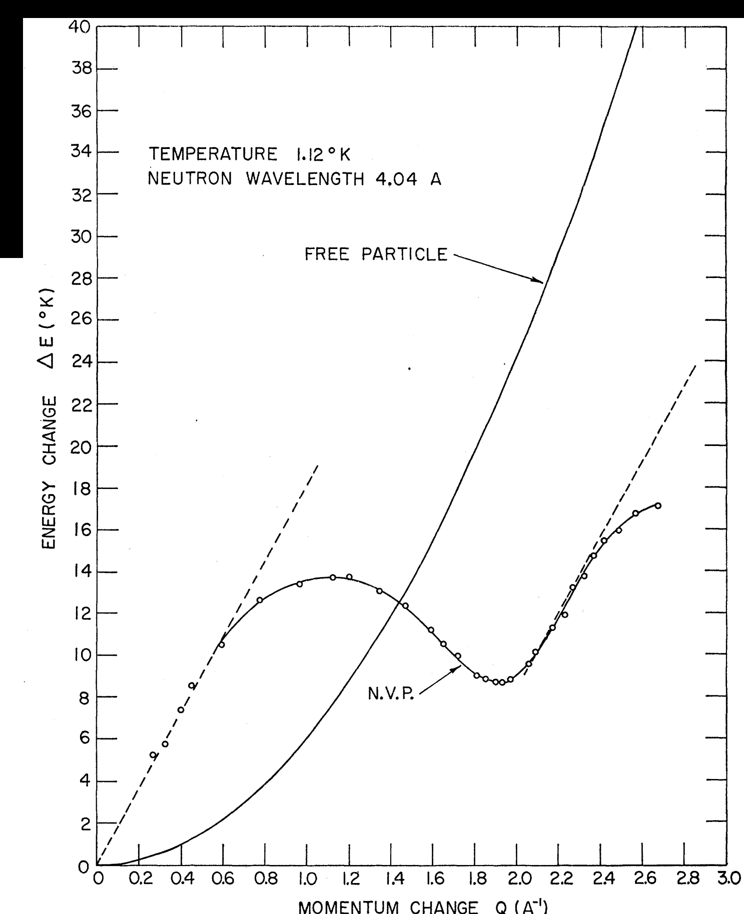


- Ginzburg-Landau theory

$$V = -\mu\psi^*\psi + \lambda(\psi^*\psi)^2 \qquad \langle 0|\psi|0\rangle \neq 0$$

- *G*=U(1), *H*=0
- ^{4}He superfluid
- scalar BEC





Supernova remnant (Crab Nebula)

Heisenberg models

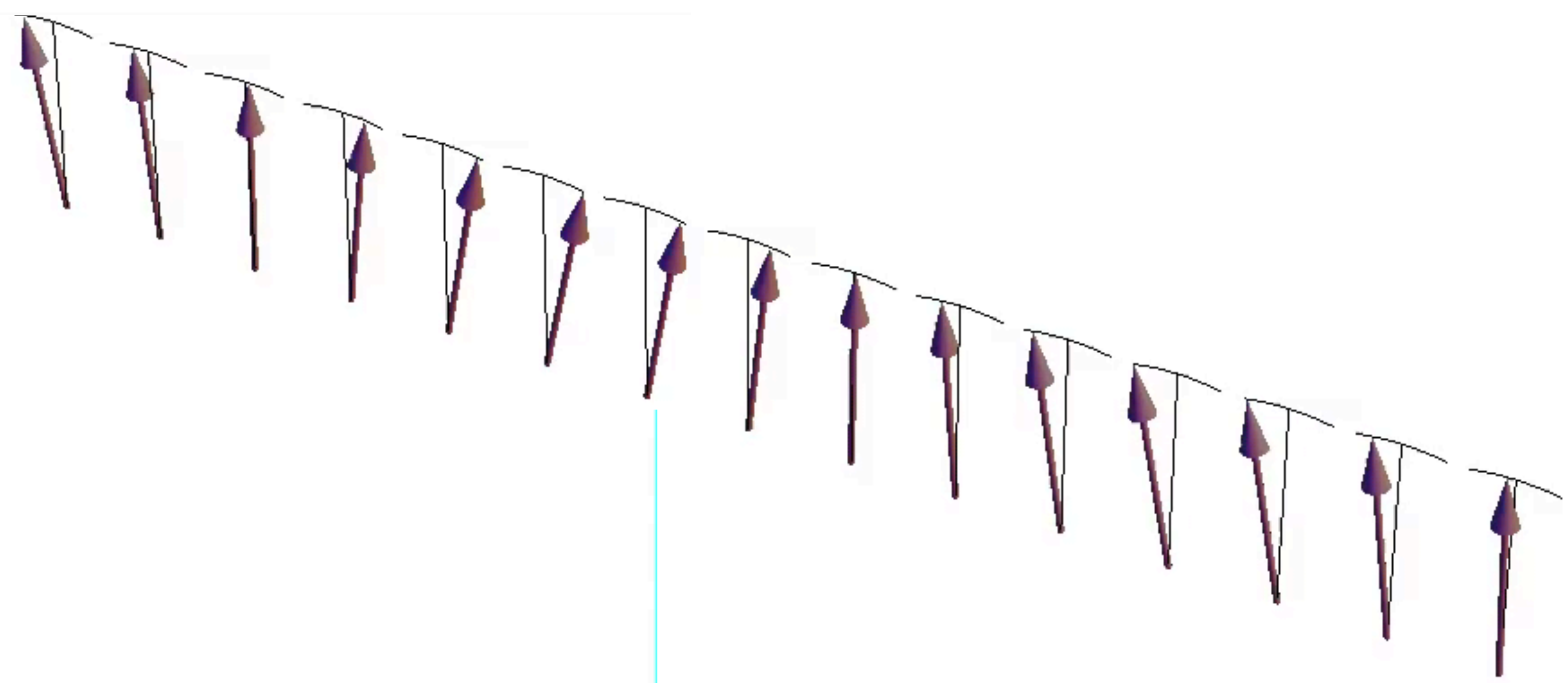
- Antiferromagnet $H = +J \sum_{\langle i,j \rangle} \vec{s}_i \cdot \vec{s}_j$ 2 NGBs $E \propto p$

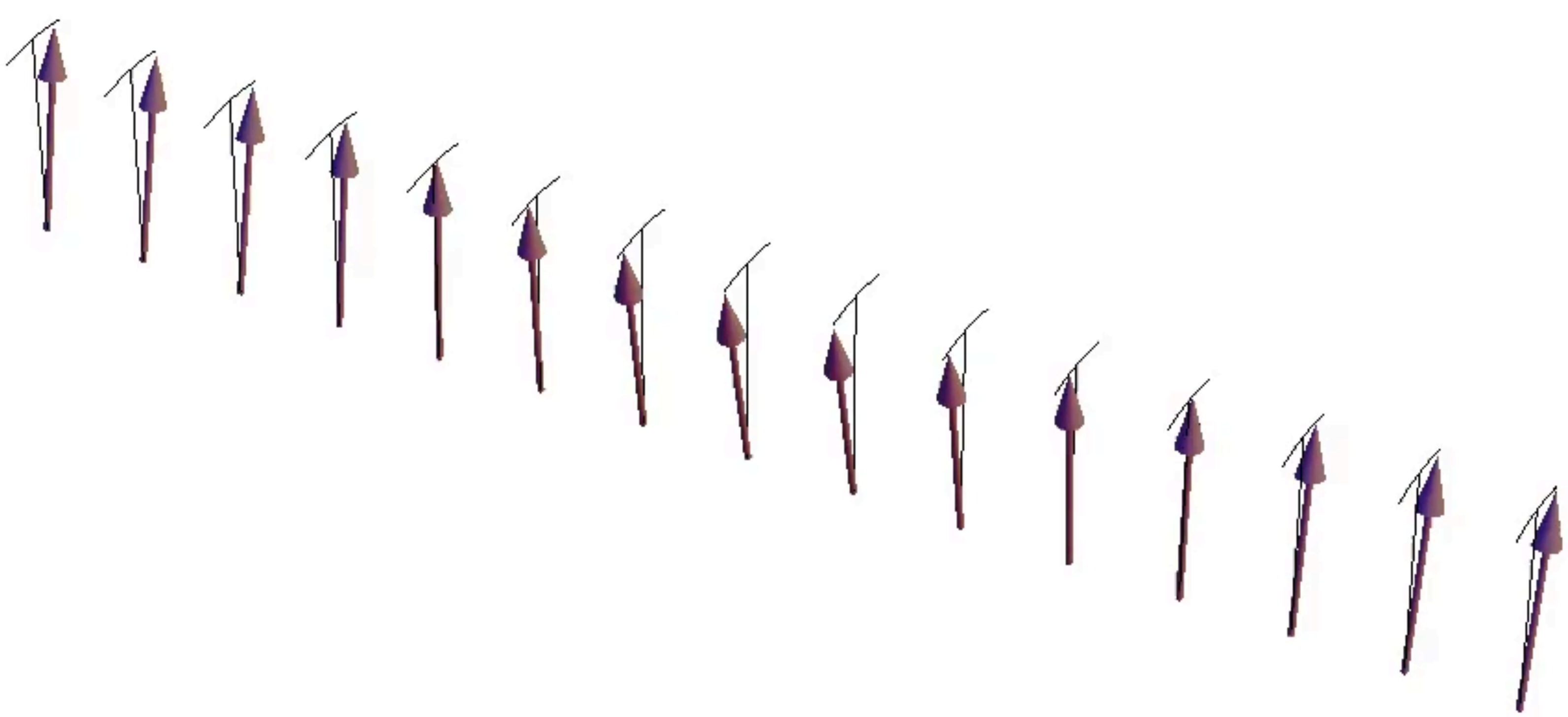
- Ferromagnet $H = -J \sum_{\langle i,j \rangle} \vec{s}_i \cdot \vec{s}_j$ 1 NGB $E \propto p^2$

Both G/H = SO(3)/SO(2) = S^2

the only mode

s?





No!

VOLUME 88, NUMBER 11 PHYSICAL REVIEW LETTERS 18 MARCH 2002

Spontaneous Symmetry Breaking with Abnormal Number of Nambu-Goldstone Bosons and Kaon Condensate

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PHYSICAL REVIEW D **70**, 014006 (2004)

Abnormal number of Nambu-Goldstone bosons in the color-asymmetric dense color superconducting phase of a Nambu−Jona-Lasinio−type model

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PHYSICAL REVIEW A **74**, 033604 (2006)

Superfluidity in a three-flavor Fermi gas with SU(3) symmetry

Lianyi He, Meng Jin, and Pengfei Zhuang
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(Received 26 April 2006; published 8 September 2006)

We investigate the superfluidity and the associated Nambu-Goldstone modes in a three-flavor atomic Fermi gas with SU(3) global symmetry. The *s*-wave pairing occurs in flavor antitriplet channel due to the Pauli principle, and the superfluid state contains both gapped and gapless fermionic excitations. Corresponding to the spontaneous breaking of the SU(3) symmetry to a SU(2) symmetry with five broken generators, there are only three Nambu-Goldstone modes, one is with linear dispersion law and two are with quadratic dispersion law. The other two expected Nambu-Goldstone modes become massive with a mass gap of the order of the fermion energy gap in a wide coupling range. The abnormal number of Nambu-Goldstone modes, the quadratic dispersion law, and the mass gap have significant effect on the low-temperature thermodynamics of the matter.

DOI: 10.1103/PhysRevA.74.033604 PACS number(s): 03.75.Ss, 05.30.Fk, 74.20.Fg, 34.90.+q

Spontaneous Breaking of Lie and Current Algebras

Yoichiro Nambu[1]

Received December 26, 2002; accepted January 29, 2003

The anomalous properties of Nambu–Goldstone bosons, found by Miransky and others in the symmetry breaking induced by a chemical potential, are attributed to the SSB of Lie and current algebras. Ferromagnetism, antiferromagnetism, and their relativistic analogs are discussed as examples.[2]

KEY WORDS: Symmetry breaking; Nambu–Goldstone boson; color superconductivity; chemical potential; ferromganetism; Lorentz symmetry; current algebra.

1. INTRODUCTION AND SUMMARY

In general the number of the Nambu–Goldstone (NG) bosons associated with a spontaneous symmetry breaking (SSB) $G \to H$ is equal to the number of symmetry generators Q_i in the coset G/H. In the absence of a gauge field, their energy ω goes as a power k^γ of wave number. In a relativistic theory, $\gamma = 1$ necessarily unless Lorentz invariance is broken.

There are, however, exceptions to the above "theorem."[(1–5)] Recently

credit: Department of Physics Berkeley

Ask questions!

Haruki Watanabe

Heisenberg Magnets

- Antiferromagnet

2 NGBs

$$\langle 0|S_z|0\rangle = 0$$

- Ferromagnet

1 NGB

$$\langle 0|S_z|0\rangle = -i\langle 0|[S_x, S_y]|0\rangle \neq 0$$

Two rotations are "canonically conjugate" cf. $[x, p]=i\hbar$

two operators describe one degree of freedom

π^a: Nambu-Goldstone field

lives on space of ground states: *G/H*

$$\mathcal{L}_{\rm eff} = c_a(\pi)\dot{\pi}^a + \bar{g}_{ab}(\pi)\dot{\pi}^a\dot{\pi}^b - g_{ab}(\pi)\nabla_i\pi^a\nabla_i\pi^b$$

$$L = p\dot{q} \qquad\qquad\qquad\qquad -H(p,q)$$

two NG fields are canonically conjugate to each other

a *pair* describes *one* degree of freedom

$$\rho_{ab} = \frac{-i}{V}\langle 0|[Q^a, Q^b]|0\rangle$$

$$n_{NGB} = n_A + n_B = n_{BG} - \frac{1}{2}\mathrm{rank}\rho$$

We could completely classify on what patterns of symmetry breaking allow for the first term

Applications

$$n_{NGB} = n_{BG} - \frac{1}{2}\text{rank}\rho$$

example	coset space	BG	NGB	rank ρ	theorem
anti-ferromagnet	O(3)/O(2)	2	2	0	2=2-0
ferromagnet	O(3)/O(2)	2	1	2	1=2-1
superfluid ^{4}He	U(1)	1	1	0	1=1-0
superfluid ^{3}He B phase	O(3)xO(3)xU(1)/O(2)	4	4	0	4=4-0
(in magnetic field)	O(2)xO(3)xU(1)/O(2)	4	3	2	3=4-1
BEC (F=0)	U(1)	1	1	0	1=1-0
BEC (F=1) polar	O(3)xU(1)/U(1)	3	3	0	3=3-0
BEC (F=1) ferro	O(3)xU(1)/SO(2)	3	2	2	2=3-1
3-comp. Fermi liquid	U(3)/U(2)	5	3	4	3=5-2
neutron star	U(1)	1	1	0	1=1-0
kaon cond. (μ=0)	U(2)/U(1)	3	3	0	3=3-0
kaon cond. (μ≠0)	U(2)/U(1)	3	2	2	2=3-1
crystal	$\mathbb{R}^3/\mathbb{Z}^3$	3	3	0	3=3-0
(in magnetic field)	$\mathbb{R}^3/\mathbb{Z}^3$	3	2	2	2=3-1

Physics
spotlighting exceptional research

PRL **108,** 251602 (2012) PHYSICAL REVIEW LETTERS week ending 22 JUNE 2012

Unified Description of Nambu-Goldstone Bosons without Lorentz Invariance

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(Received 3 March 2012; published 21 June 2012)

Using the effective Lagrangian approach, we clarify general issues about Nambu-Goldstone bosons without Lorentz invariance. We show how to count their number and study their dispersion relations. Their number is less than the number of broken generators when some of them form canonically conjugate pairs. The pairing occurs when the generators have a nonzero expectation value of their commutator. For non-semi-simple algebras, central extensions are possible. The underlying geometry of the coset space in general is partially symplectic.

presymplectic structure
on homogeneous spaces

Low-*E* Effective L

- consider $\pi^a(x)$ fields: $\mathbb{R}^{3,1} \rightarrow G/H$ ("*pions*")
- Write *action* $S = \int d^4x\, L(\pi, \partial\pi)$ which is *G*-invariant
- expand in powers of derivative, keep low orders (often up to the second order)

$$\mathcal{L}_{\rm eff} = g_{ab}(\pi)\partial_\mu\pi^a\partial^\mu\pi^b$$

$$\mathcal{L}_{\rm eff} = c_a(\pi)\dot{\pi}^a + \bar{g}_{ab}(\pi)\dot{\pi}^a\dot{\pi}^b - g_{ab}(\pi)\nabla_i\pi^a\nabla_i\pi^b$$

Leutwyler

General formula

$$\mathcal{L}_{\text{eff}} = c_a(\pi)\dot{\pi}^a + \bar{g}_{ab}(\pi)\dot{\pi}^a\dot{\pi}^b - g_{ab}(\pi)\nabla_i\pi^a\nabla_i\pi^b$$

- Define a commutator among broken generators

$$\rho_{ab} = \frac{-i}{V}\langle 0|[Q^a, Q^b]|0\rangle \qquad c_a\dot{\pi}^a \approx \frac{1}{2}\rho_{ab}\pi^b\dot{\pi}^a$$

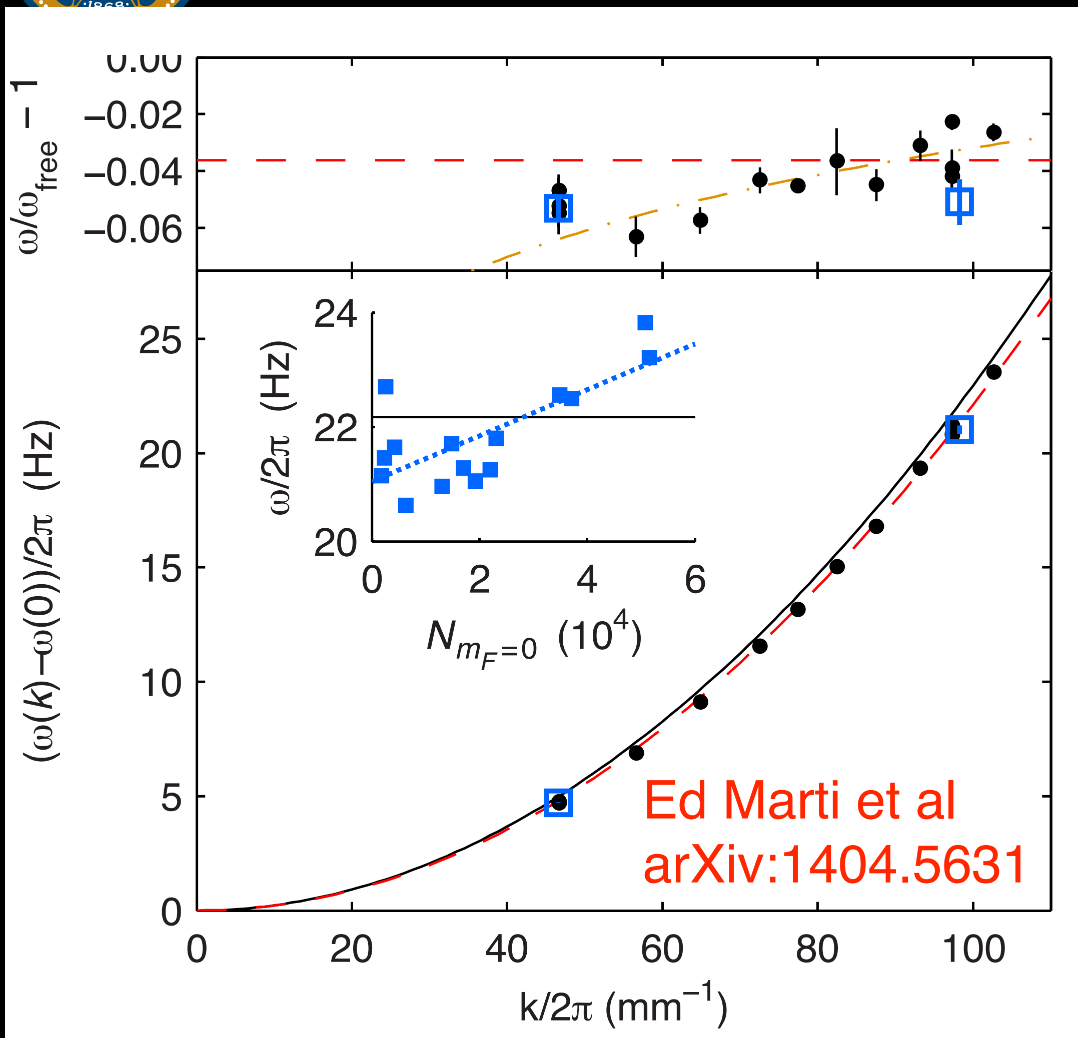
- n_B = 1/2 rank ρ counts the number of canonically conjugate pairs (Type-B) generically ⇒ $E \propto p^2$
  - each pair describes one d.o.f.
- the remainder $n_A = n_{BG} - 2n_B$
  - stand-alone NGB d.o.f. (Type-A) generically ⇒ $E \propto p$

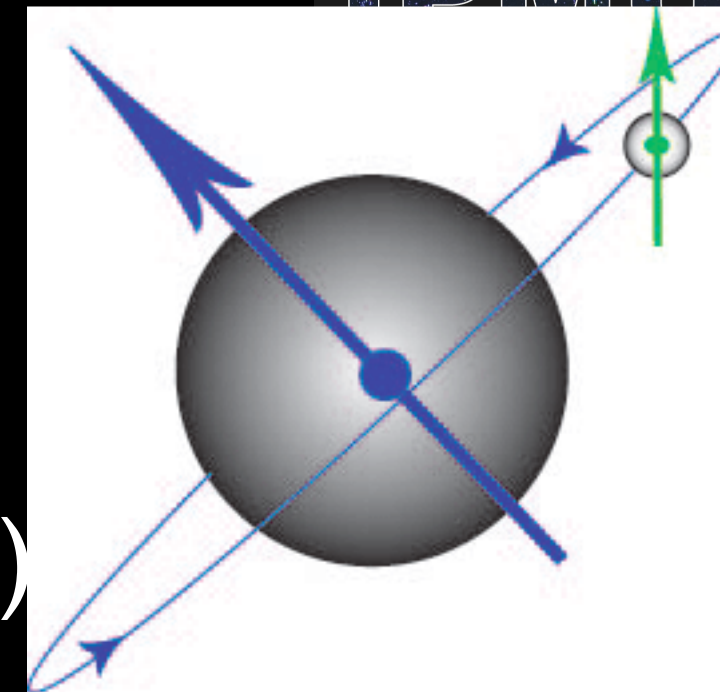
$$n_{NGB} = n_A + n_B = n_{BG} - \frac{1}{2}\text{rank}\rho$$

inor BEC

toms (ferromagnetic)

SO(2)





$$\psi = \begin{pmatrix} \psi_x \\ \psi_y \\ \psi_z \end{pmatrix} = \begin{pmatrix} R_x & I_x \\ R_y & I_y \\ R_z & I_z \end{pmatrix}$$

- 3 broken generators
- 1 NGB with $E \propto p$
- 1 NGB with $E \propto p^2$

$$\langle \psi \rangle = v \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} = v \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

physical origin

$$\mathcal{L}_{\rm eff} = c_a(\pi)\dot{\pi}^a + \bar{g}_{ab}(\pi)\dot{\pi}^a\dot{\pi}^b - g_{ab}(\pi)\nabla_i\pi^a\nabla_i\pi^b$$

- What is $c_a(\pi)$?
- it defines one-form $c_1 = c_a(\pi)\, d\pi^a$ on G/H
- L must be G-invariant up to a surface term

$$\mathcal{L}_{V_i} c = i_{V_i} dc + d(i_{V_i} c) = \boxed{de_i} + d(i_{V_i} c) \qquad de_i = i_{V_i}\omega$$

- the Noether current picks up surface term

$$j_i^0 = -\bar{g}_{ab} h_i^a \dot{\pi}^b + e_i$$

- in the ground state = stationary:

$$\langle 0 | j_i^0 | 0 \rangle = e_i(0)$$

- it is "charge density" of the ground state

Presymplectic Geometry

assumption: $H^2(\mathfrak{g})=0$

closed G-inv
$d\,c = \pi^*\omega_2$

$G/H \longleftarrow F$

$\pi \downarrow$

symplectic homogeneous ω_2 G/U

Type A
$E \propto p$

Type B
$E \propto p^2$

$$\omega_2 = \frac{1}{2}\rho_{ab} d\pi^a \wedge d\pi^b + O(\pi)^3$$

$$\rho_{ab} = \frac{-i}{V}\langle 0|[Q^a, Q^b]|0\rangle$$

NGBs for generators *a* and *b* are symplectic pairs and describe a single degree of freedom

$$\mathrm{dim}G - \mathrm{dim}H = n_A + 2n_B$$

allows for complete classification of possibilities

Bon-Yao Chu (1974)

Corollary 2

If the second dimension cohomology group $H^2(\mathfrak{g})$ of the Lie algebra $\mathfrak{g}$ for a connected Lie group G is trivial, then every left-invariant closed 2-form on G induces a symplectic homogeneous space.

$H^2(\mathfrak{g})=0$ for semi-simple groups

Classification of presymplectic structures

- Borel (1954): G compact semi-simple, $T \subset G$ a torus, U centralizer of T, then G/U Kähler
  - Note Kähler manifolds are symplectic
  - For a given G/H, find all $U \supset H$
  - Project G/H to G/U, with fiber U/H
  - pull back symplectic form on G/U to G/H
- If G is not semi-simple, it has $U(1)^k$ factors, and possible central extensions are

$$\dim H^2(\mathfrak{u}(1)^k) = \frac{1}{2}k(k-1)$$

Classification of presymplectic structures

- For example, G=SO(n)
- First consider flag manifold SO(n)/U(1)^r

$$\rho_{ab} = \mathrm{diag}(\overbrace{0,\cdots,0}^{m}, \overbrace{\alpha_1,\cdots\alpha_1}^{n_1}, \cdots \overbrace{\alpha_k,\cdots\alpha_k}^{n_k}) \otimes i\sigma_2$$

- ρ_{ab} generates a torus T
- ρ_{ab} breaks SO(n) to U=SO(m)xU(n_1)x...xU(n_k), $n=m+\Sigma_k 2n_k$
- SO(n)/U is Kähler and symplectic
- Type-B NGBs live on SO(n)/U
- Type-A NGBs live on U/U(1)^r
- For more general H, only consider U⊃H

no-go theorem

- Not every NGBs can be paired as Type-B
- SU(3)/U(1)^2: Kähler and symplectic

Type-A	Type-B	$n_A+2n_B=6$
6	0	6
4	1	6
2	2	6
0	3	6

n_A	n_B	U
30	0	.
20	5	$SU(5) \times U(1)$
14	8	$SU(4) \times SU(2) \times U(1)$
12	9	$SU(4) \times U(1)^2$
12	9	$SU(3)^2 \times U(1)$
8	11	$SU(3) \times SU(2) \times U(1)^2$
6	12	$SU(3) \times U(1)^3$
6	12	$SU(2)^3 \times U(1)^2$
4	13	$SU(2)^2 \times U(1)^3$
2	14	$SU(2) \times U(1)^4$
0	15	$U(1)^5$

TABLE III. Possible number of type-A and type-B NGBs for $SU(6)/U(1)^5$.

n_A	n_B	U
40	0	.
24	8	$SO(8) \times U(1)$
20	10	$U(5)$
14	13	$SO(6) \times U(2)$
12	14	$SO(6) \times U(1)^2$
12	14	$U(4) \times U(1)$
10	15	$SO(4) \times U(3)$
8	16	$U(3) \times U(2)$
6	17	$SO(4) \times U(2) \times U(1)$
6	17	$U(3) \times U(1)^2$
4	18	$SO(4) \times U(1)^3$
4	18	$U(2)^2 \times U(1)$
2	19	$U(2) \times U(1)^3$
0	30	$U(1)^5$

TABLE IV. Possible number of type-A and type-B NGBs for $SO(10)/U(1)^5$.

n_A	n_B	$U \subset SO(11)$	$U \subset Sp(5)$
50	0	.	.
32	9	$SO(9) \times U(1)$	$Sp(4) \times U(1)$
20	15	$SO(7) \times U(2)$	$Sp(3) \times U(2)$
20	15	$U(5)$	$U(5)$
18	16	$SO(7) \times U(1)^2$	$Sp(3) \times U(1)^2$
14	18	$SO(5) \times U(3)$	$Sp(2) \times U(3)$
14	18	$SO(3) \times U(4)$	$Sp(1) \times U(4)$
12	19	$U(4) \times U(1)$	$U(4) \times U(1)$
10	20	$SO(5) \times U(2) \times U(1)$	$Sp(2) \times U(2) \times U(1)$
8	21	$SO(5) \times U(1)^3$	$Sp(2) \times U(1)^3$
8	21	$SO(3) \times U(3) \times U(1)$	$Sp(1) \times U(3) \times U(1)$
8	21	$U(3) \times U(2)$	$U(3) \times U(2)$
6	22	$SO(3) \times U(2)^2$	$Sp(1) \times U(2)^2$
6	22	$U(3) \times U(1)^2$	$U(3) \times U(1)^2$
4	23	$SO(3) \times U(2) \times U(1)^2$	$Sp(1) \times U(2) \times U(1)^2$
4	23	$U(2)^2 \times U(1)$	$U(2)^2 \times U(1)$
2	24	$SO(3) \times U(1)^4$	$Sp(1) \times U(1)^4$
2	24	$U(2) \times U(1)^3$	$U(2) \times U(1)^3$
0	25	$U(1)^5$	$U(1)^5$

TABLE V. Possible number of type-A and type-B NGBs for $SO(11)/U(1)^5$ and $Sp(5)/U(1)^5$.

List of possible *U* for *G* with rank=5

Anthony Leggett

It has long been appreciated that an important consequence of the phenomenon of spontaneously broken symmetry, whether occurring in particle physics or in the physics of condensed matter, is the existence of the long-wavelength collective excitations known as Nambu-Goldstone (NG) bosons. However, while in particle physics the constraints imposed by Lorentz invariance make the enumeration and classification of these bosons a relatively simple matter, in the condensed matter area the situation has been more obscure; while in any given case one can usually work out their nature and spectra, a generally applicable technique has been lacking. In their paper Watanabe and Murayama have now derived a beautiful general relation between the number of broken generators, the rank of the matrix of commutators of the generators and the number of NG bosons. This relation reproduces the relevant results for all known cases and gives a simple framework for discussing any currently unknown form of ordering which may be discovered in the future.

stability@T=0 in d+1 dim

- Type A: $$\mathcal{L}_{\text{eff}} = \bar{g}_{ab}\dot{\pi}^a\dot{\pi}^b - g_{ab}\nabla_i\pi^a\nabla\pi^b$$
  - scaling $$\vec{x}' = a\vec{x},\ t' = at$$ $$\pi'^a(a\vec{x}, at) = a^{(1-d)/2}\pi^a(\vec{x}, t)$$
  - interaction $$\nabla_i\pi^a\nabla_i\pi^b\pi^c \sim a^{-(d-1)/2}$$
  - IR free for $d \geq 2$ (no SSB in d=1)
- Type B: $$\mathcal{L}_{\text{eff}} = \rho_{ab}\pi^a\dot{\pi}^b - g_{ab}\nabla_i\pi^a\nabla\pi^b$$
  - scaling $$\vec{x}' = a\vec{x},\ t' = a^2 t$$
  - interaction $$\pi'^a(a\vec{x}, a^2 t) = a^{-d/2}\pi^a(\vec{x}, t)$$
  - IR free for $d \geq 1$ $$\nabla_i\pi^a\nabla_i\pi^b\pi^c \sim a^{-d/2}$$

Apparent Violation of Coleman's theorem

gapped (pseudo) NGB

- If the symmetry is broken by external fields etc, we can predict the gap exactly

$$\tilde{H} = H - \mu Q$$

$$n_{mNGB} = \frac{1}{2}(\text{rank}\rho - \text{rank}\tilde{\rho})$$

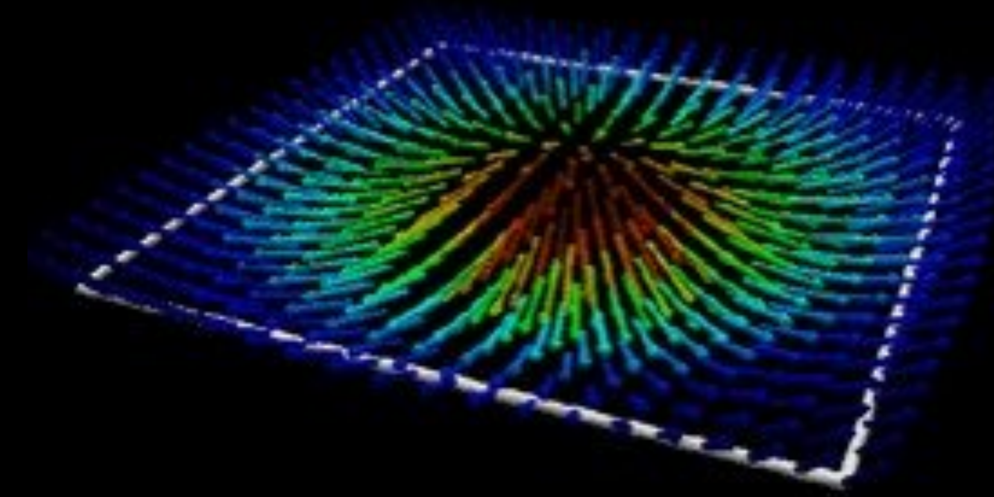
$$\rho_{ab} = \frac{-i}{V}\langle 0|[Q_a, Q_b]|0\rangle \qquad [Q_a, H] = 0$$

$$\tilde{\rho}_{ab} = \frac{-i}{V}\langle 0|[\tilde{Q}_a, \tilde{Q}_b]|0\rangle \qquad [\tilde{Q}_a, \tilde{H}] = [\tilde{Q}_a, H - \mu Q] = 0$$

$$\tilde{H}(E_\alpha|0\rangle) = \mu\alpha(E_\alpha|0\rangle)$$

ferromagnet and anti-ferromagnet in a constant magnetic field
relativistic BECs, kaon condensation
QCD with chemical potential for isospin

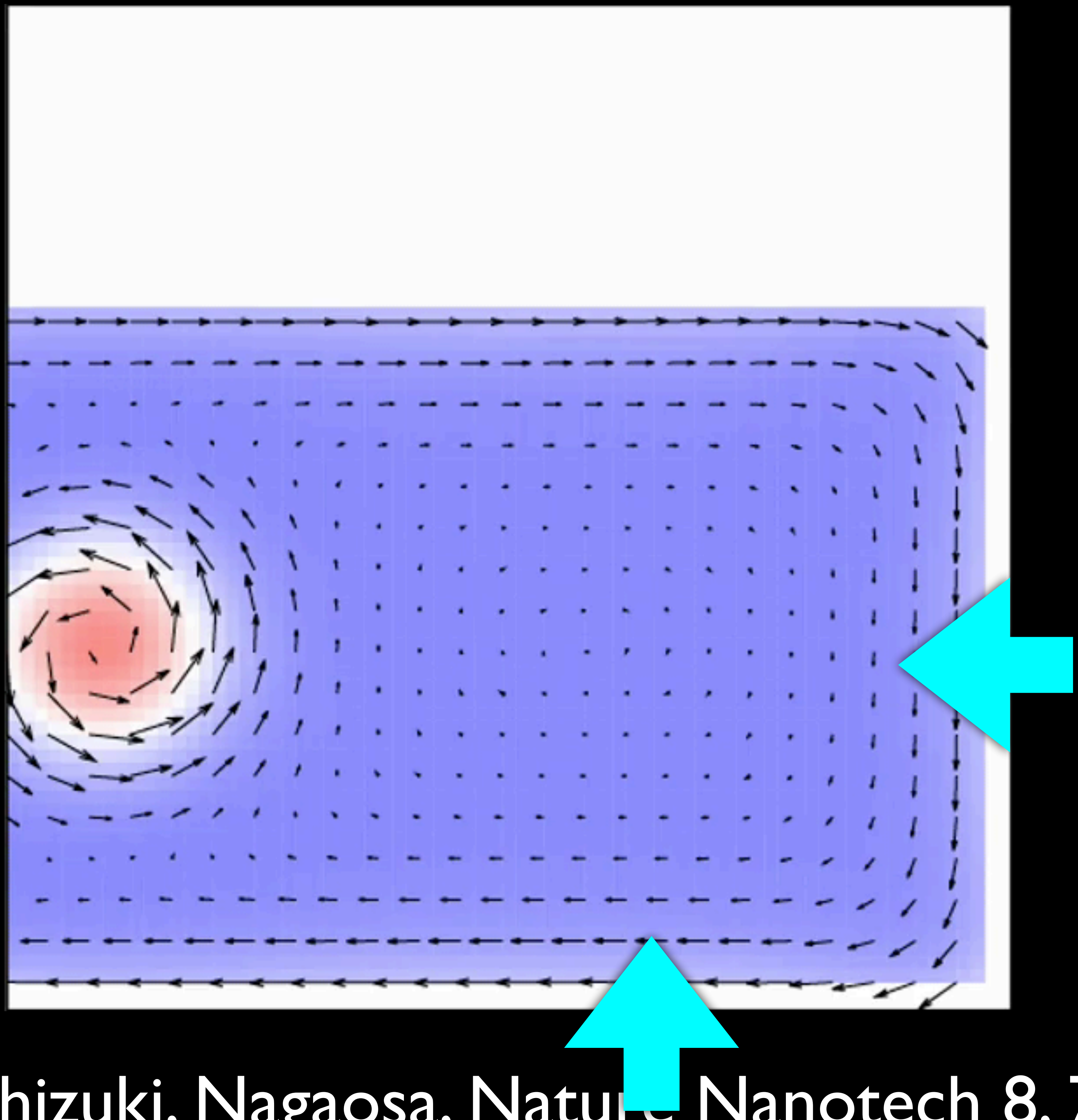
skyrmion



- Consider a Heisenberg ferromagnet
- On a two-dimensional plane, non-trivial maps $\mathbb{R}^2 \to S^2$ classified by $\pi_2(S^2) = \mathbb{Z}$
- skyrmion has moduli:
  - translations in *x* and *y* directions
  - dilation
  - rotation
- derive effective Lagrangian for moduli
- momenta don't commute!

$$[P_x, P_y] = i\hbar\, 4\pi s N_{\rm skyrmion}$$

possible for general holomorphic maps $\mathbb{C}\to$ *Kähler*

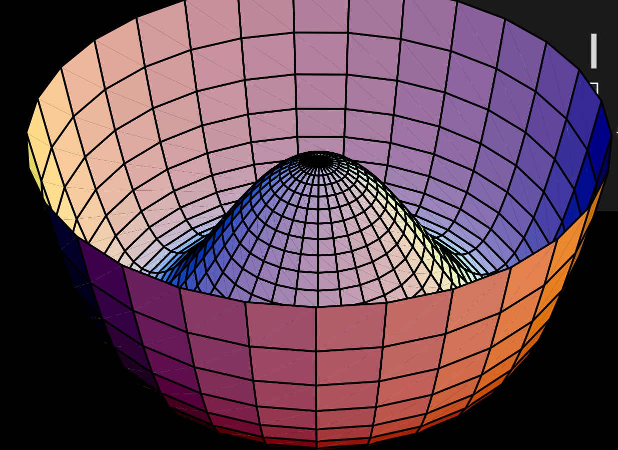


Iwasaki, Mochizuki, Nagaosa, Nature Nanotech 8, 742 (2013)

Space-Time Symmetry

- When a symmetry has to do with space-time, the number of NGBs are reduced
- crystal: translations and rotations are both spontaneously broken $R^{0i} = \epsilon_{ijk} x^j T^{0k}$
- they are both generated by the energy-momentum tensor
- would-be NGBs for rotations are the same excitations as those for translations (phonons)

Noether constraints



Examples

- Ginzburg-Landau theory

$$V = -\mu\psi^*\psi + \lambda(\psi^*\psi)^2$$

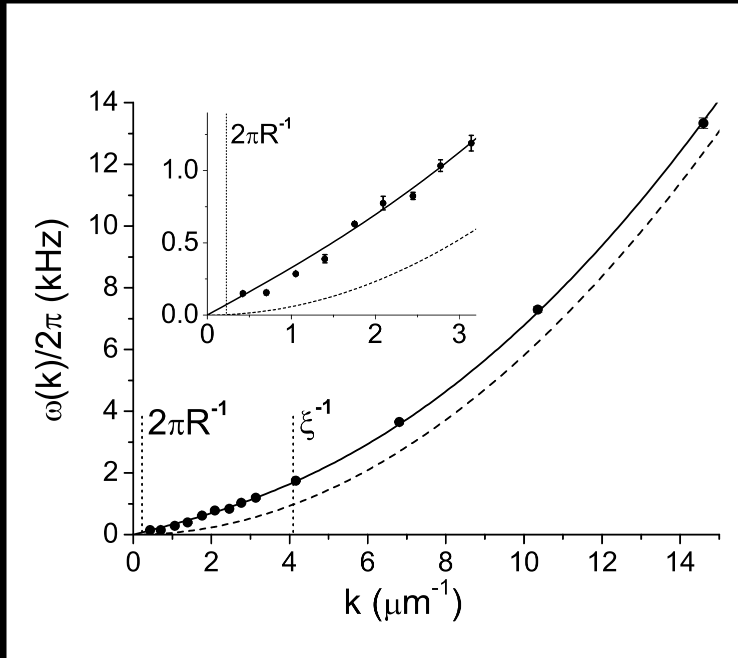
- G=U(1), H=0
- ^{4}He superfluid
- scalar BEC $\langle 0|\psi|0\rangle \neq 0$
- U(1) $\psi(\vec{x},t) \to e^{i\theta}\psi(\vec{x},t)$
- Galilean boost

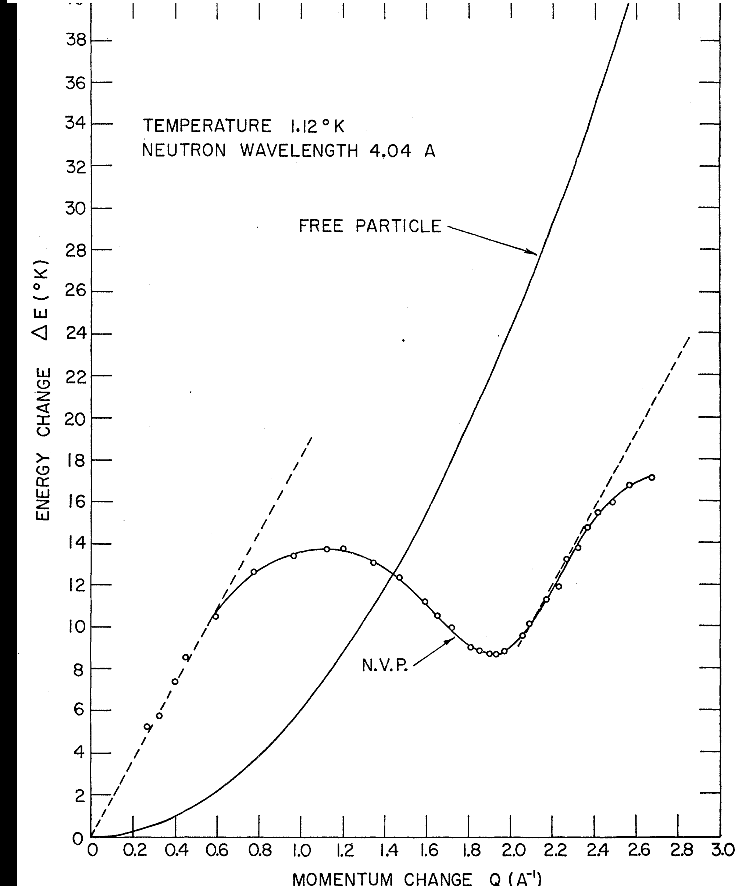
$$\psi(\vec{x},t) \to e^{i(m\vec{x}\cdot\vec{x}-\frac{1}{2}m\vec{v}^2t)}\psi(\vec{x}-\vec{v}t,t)$$

- both broken n_{BG}=1+3=4

$$B^{i\mu} = tT^{i\mu} - mx^i j^\mu$$

⇒ no separate NGBs for Galilean boosts

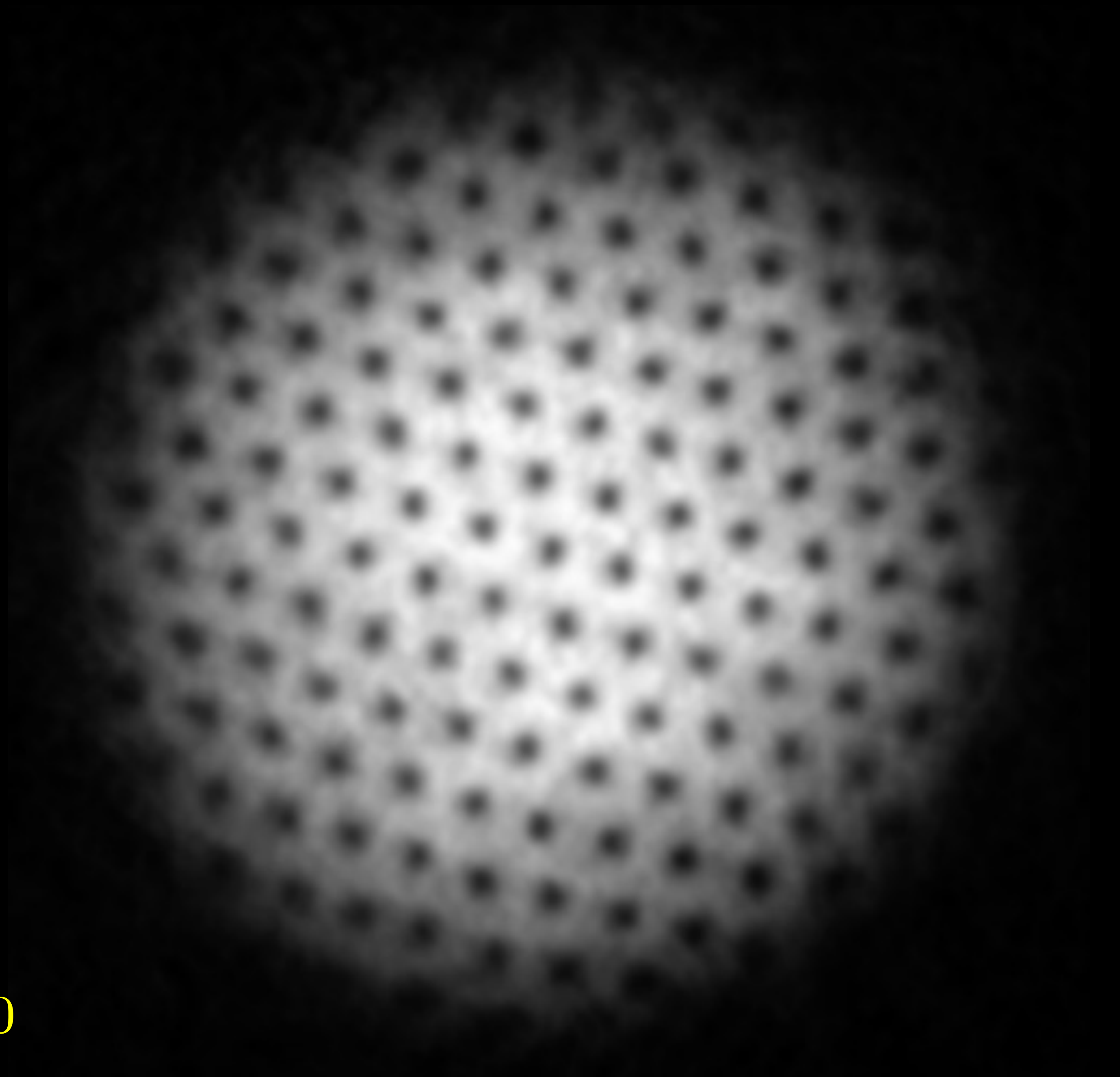




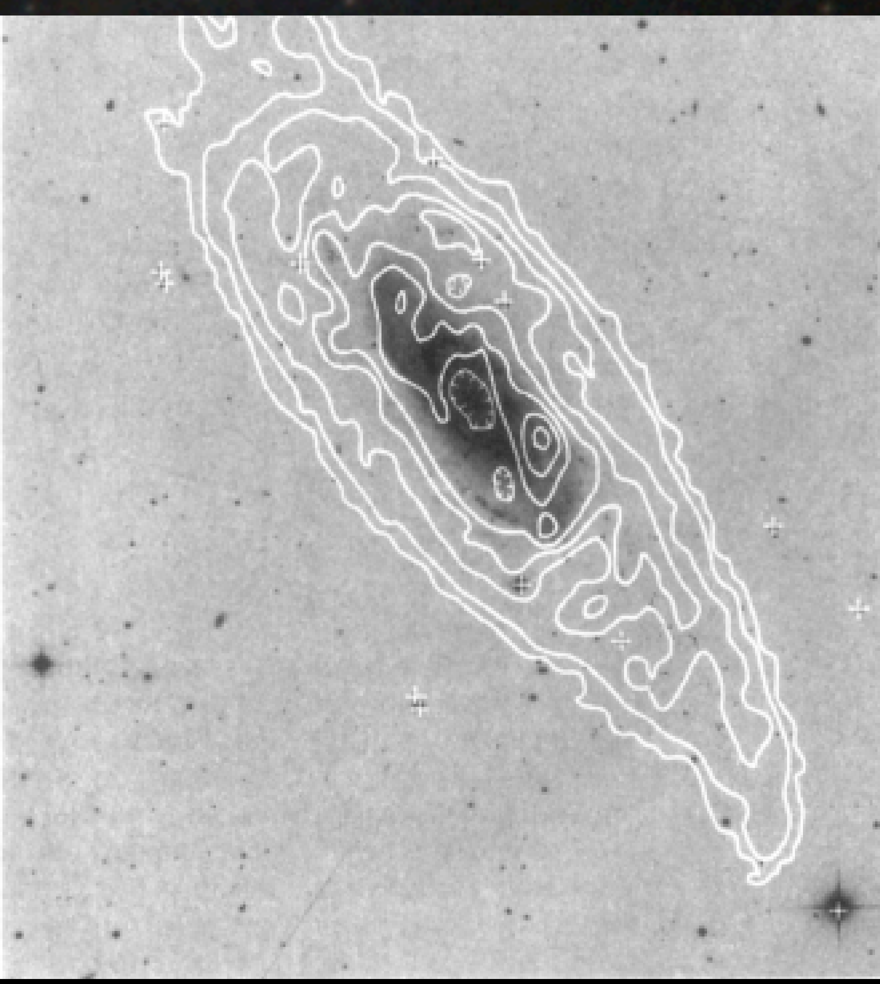
vortex lattice

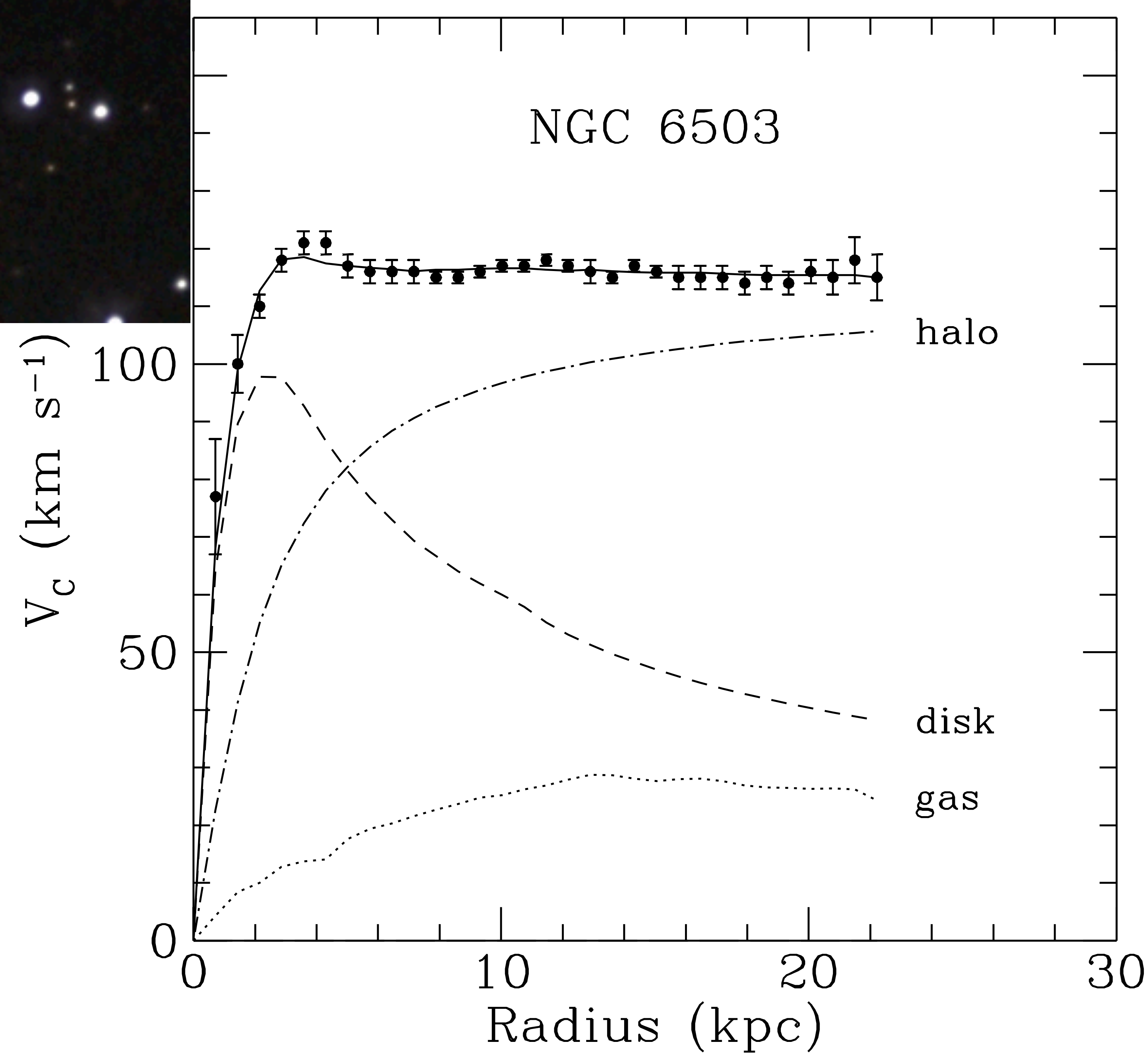
- rotate a (2d) BEC
- vortices form a triangular lattice
- broken: U(1), $P_{x,y}$, J_z
- only one Type-A NGB with $E \propto p^2$
- called Tkachenko mode

$$T^{0i} = mj^i - 2m\Omega\epsilon^{ij}x^j j^0$$



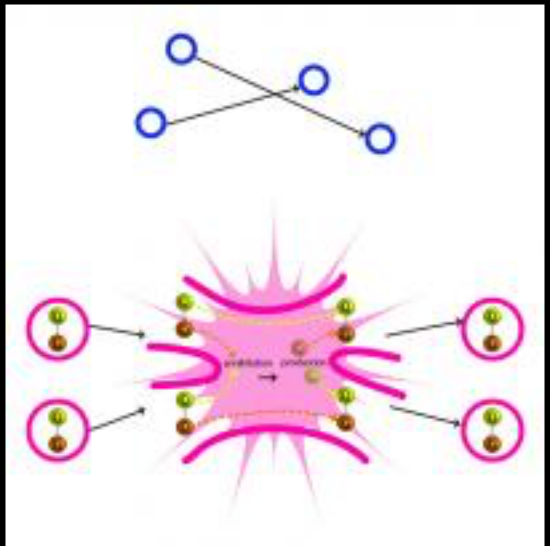
we have a precise effective Lagrangian for this

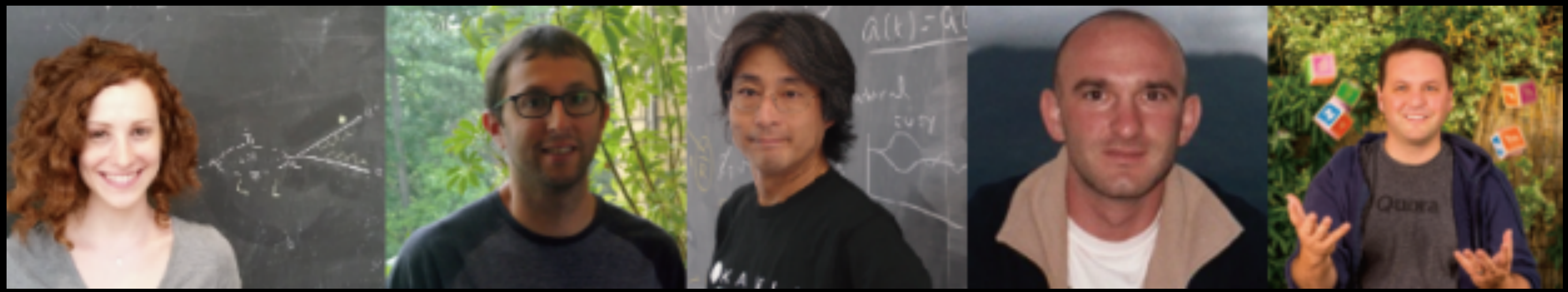



NGC 6503
halo
disk
gas
V_c (km s^{-1})
0
50
100
0
10
20
30
Radius (kpc)

NGB as dark matter

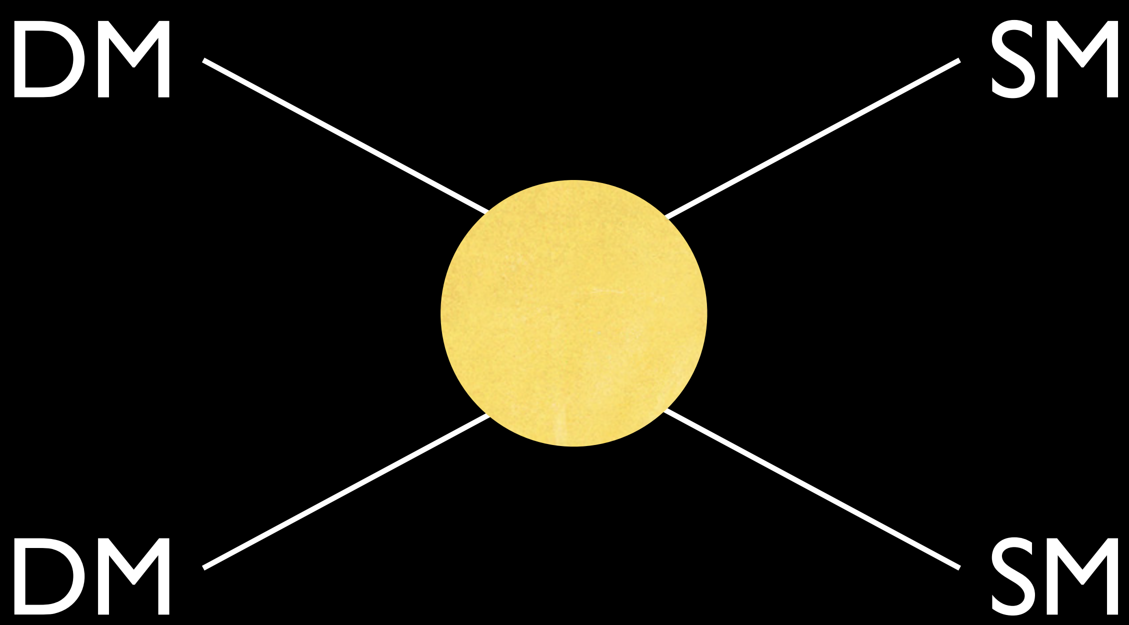
- Nambu proposed pions are light because of spontaneous symmetry breaking
- Perhaps dark matter is also just like pions?
- Then it would interact with itself!





Miracles

$$\frac{n_{\rm DM}}{s} = 4.4 \times 10^{-10} \frac{\rm GeV}{m_{\rm DM}}$$



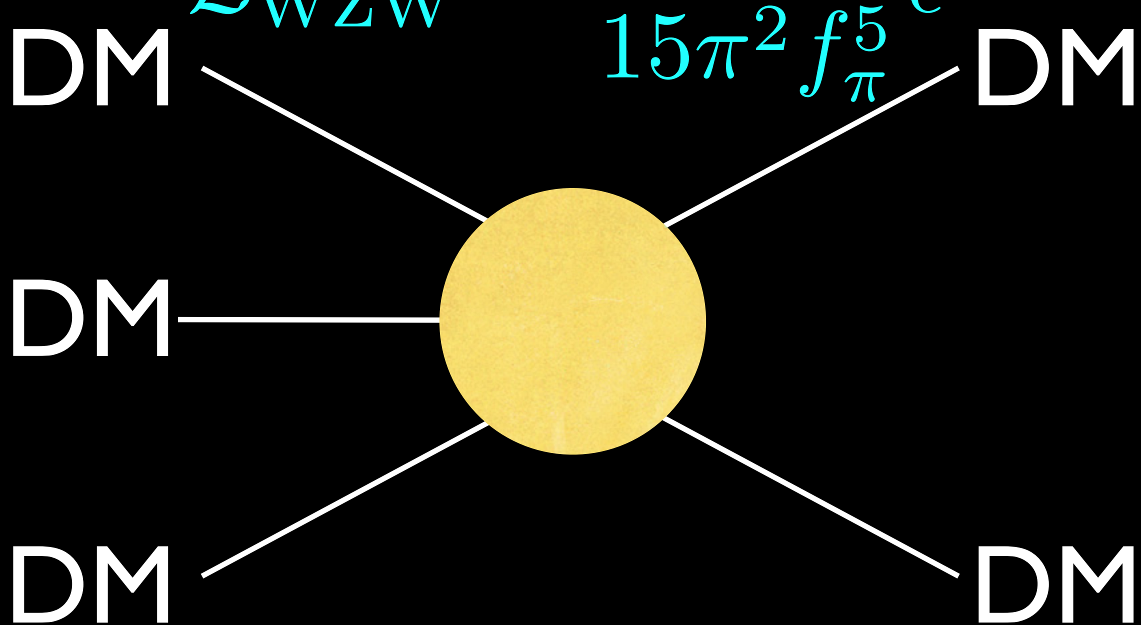
$$\langle \sigma_{2\to 2} v \rangle \approx \frac{\alpha^2}{m^2}$$

$$\alpha \approx 10^{-2}$$

$$m \approx 300 \text{ GeV}$$

WIMP miracle!

$$\mathcal{L}_{\rm WZW} = \frac{-2iN_c}{15\pi^2 f_\pi^5} \epsilon^{\mu\nu\rho\sigma} {\rm Tr} \pi \partial_\mu \pi \partial_\nu \pi \partial_\rho \pi \partial_\sigma \pi \qquad \pi_5(G/H) = \mathbb{Z}$$



$$\langle \sigma_{3\to 2} v^2 \rangle \approx \frac{\alpha^3}{m^5}$$

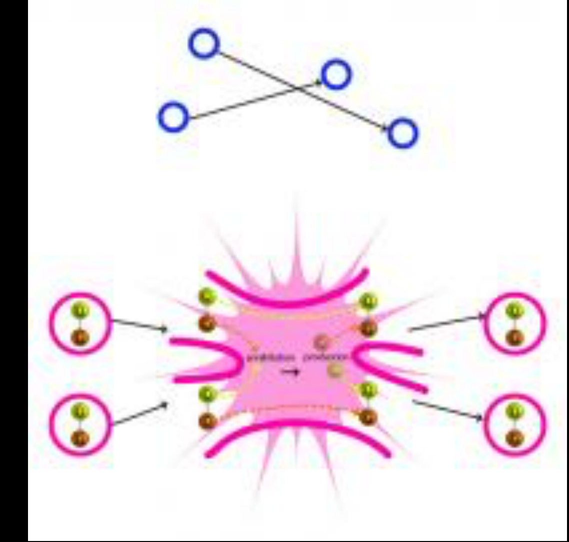
$$\alpha \approx 4\pi$$

$$m \approx 300 \text{MeV}$$

Hochberg, Kuflik, Volansky, Wacker arXiv:1402.5143

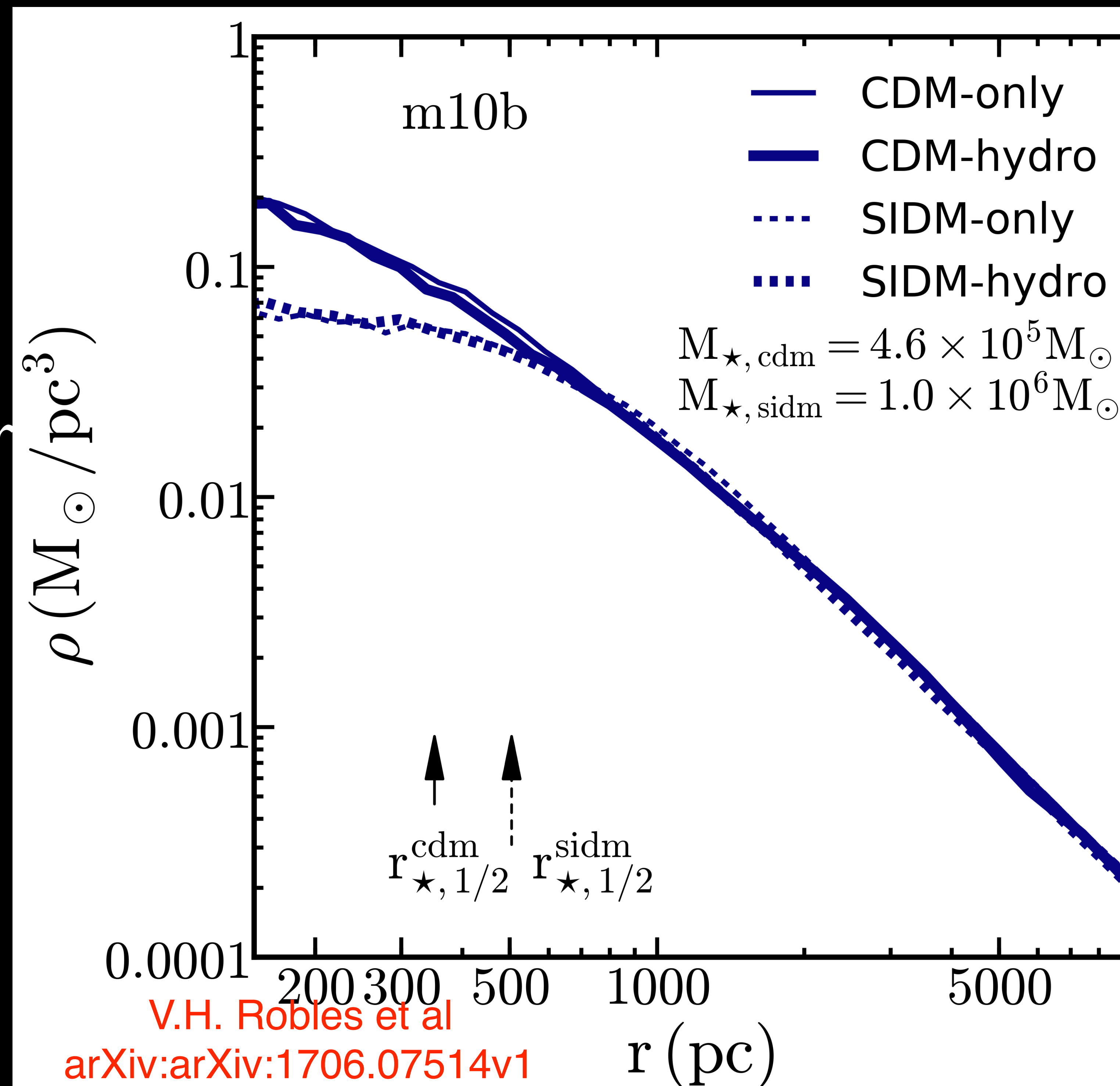
SIMP miracle!

self interaction

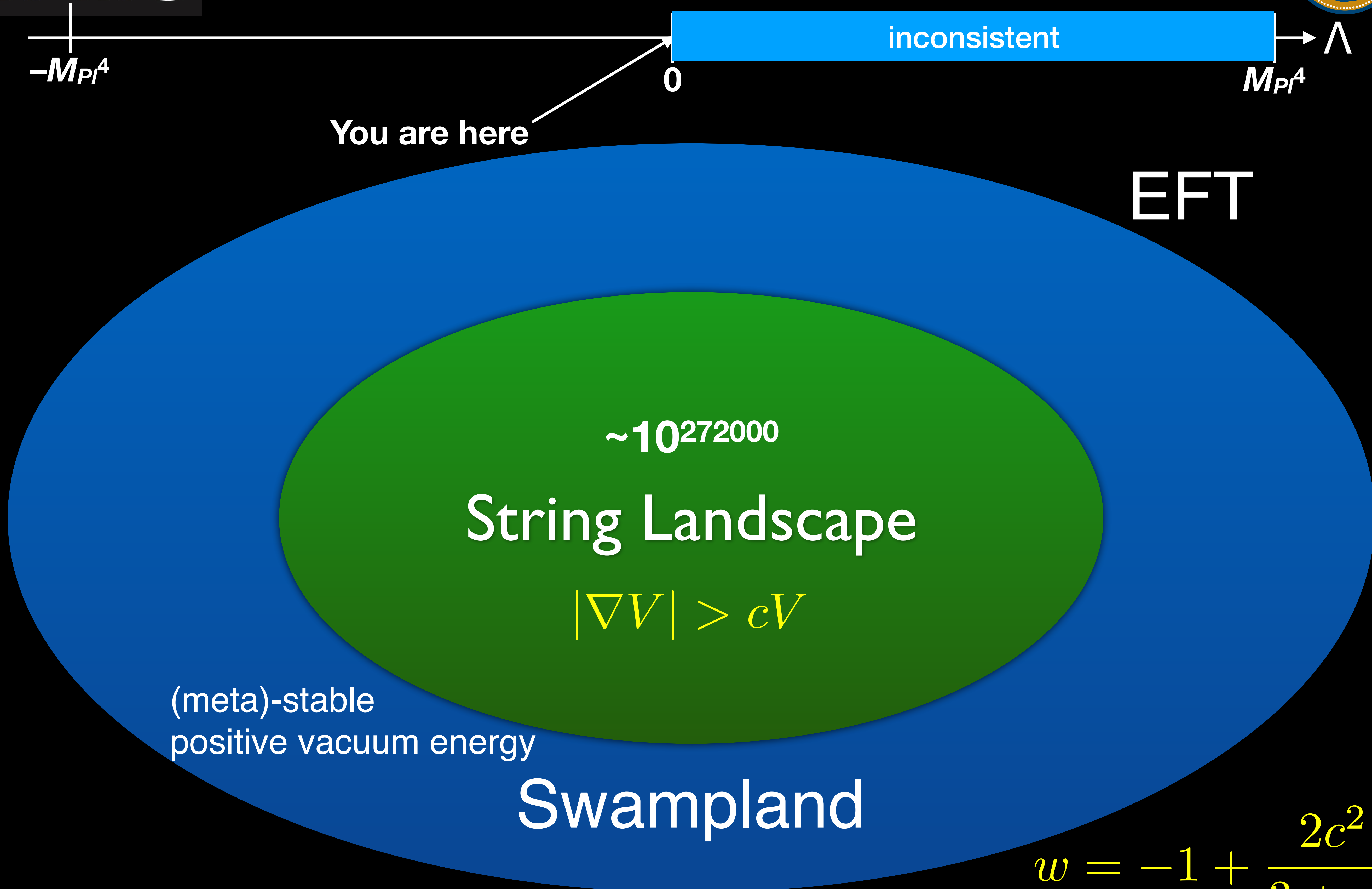


- $\sigma/m \sim cm^2/g$
  $\sim 10^{-24} cm^2$ / 300MeV
- flattens the cusps in NFW profile
- suppresses substructure
- actually desirable for dwarf galaxies?

SIDM
Spergel & Steinhardt (2000)
now complete theory



V.H. Robles et al
arXiv:arXiv:1706.07514v1



$$w = -1 + \frac{2c^2}{3 + c^2}$$

Obied, Ooguri, Spodyneiko, Vafa, arXiv:1806.08362

Swampland

Need $m_Q \sim H_0 \sim 10^{-33}$ eV

shift symmetry

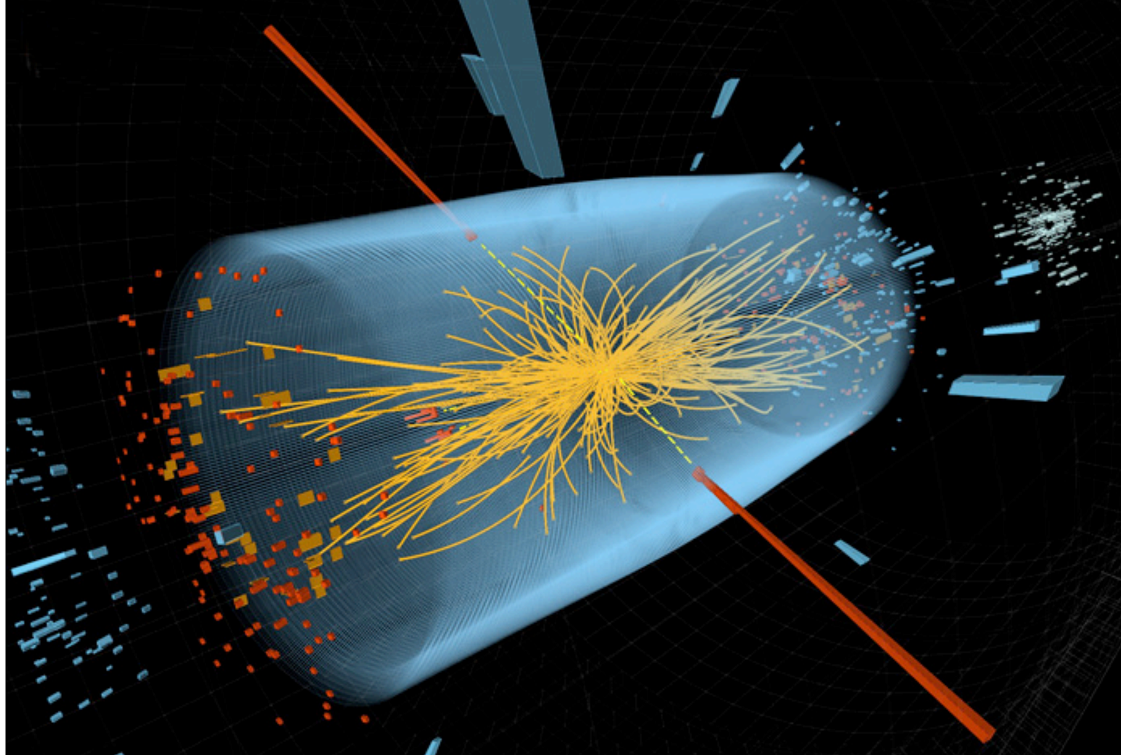
- incorporate into supergravity
- shift symmetry (monodromy) in Kähler
  - $Q \to Q + i\,\alpha$
  - $K(Q,Q^*) = K(Q+Q^*) \sim (Q+Q^*)^2/2$

$$V = e^K\left((K_i W + W_i)^* K_{\bar{i}}^{-1j}(K_j W + W_j) - 3|W|^2\right)$$

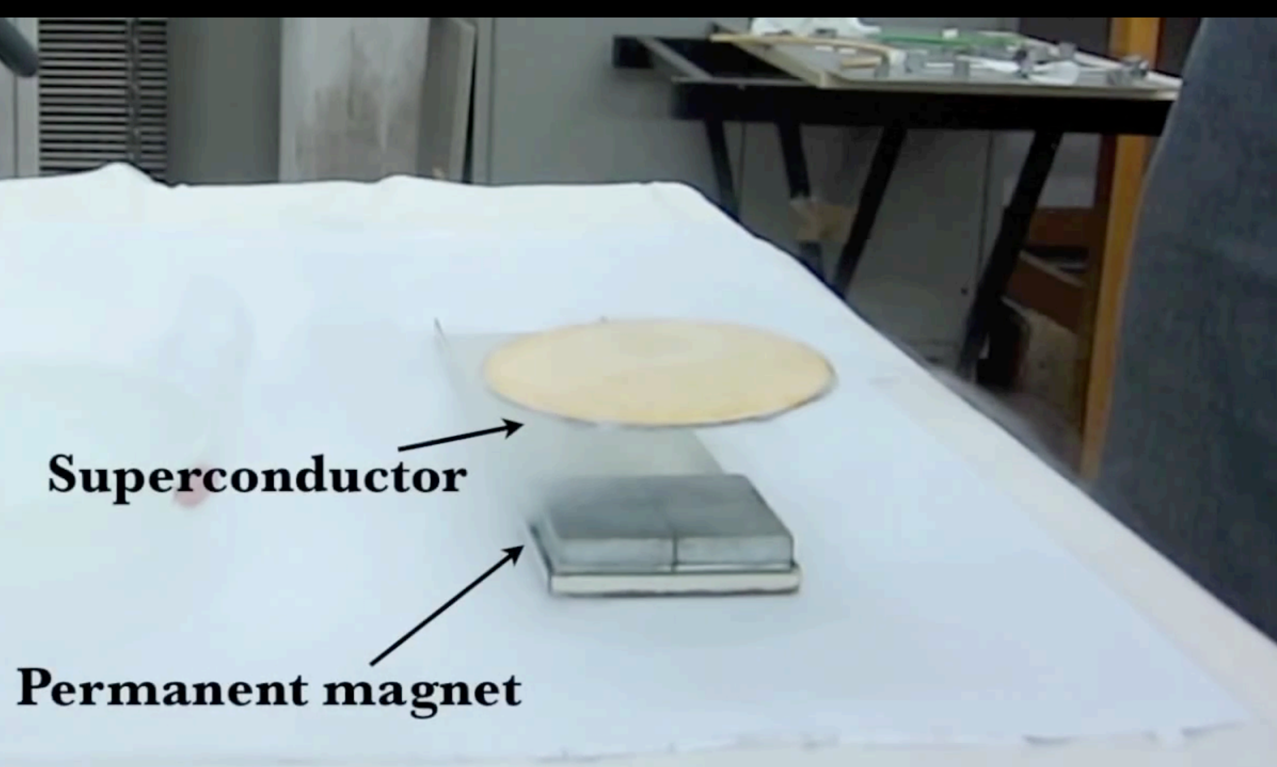
$$= \cancel{|W_Q|^2} - 3m_{3/2}(W(Q) + W^*(Q))$$

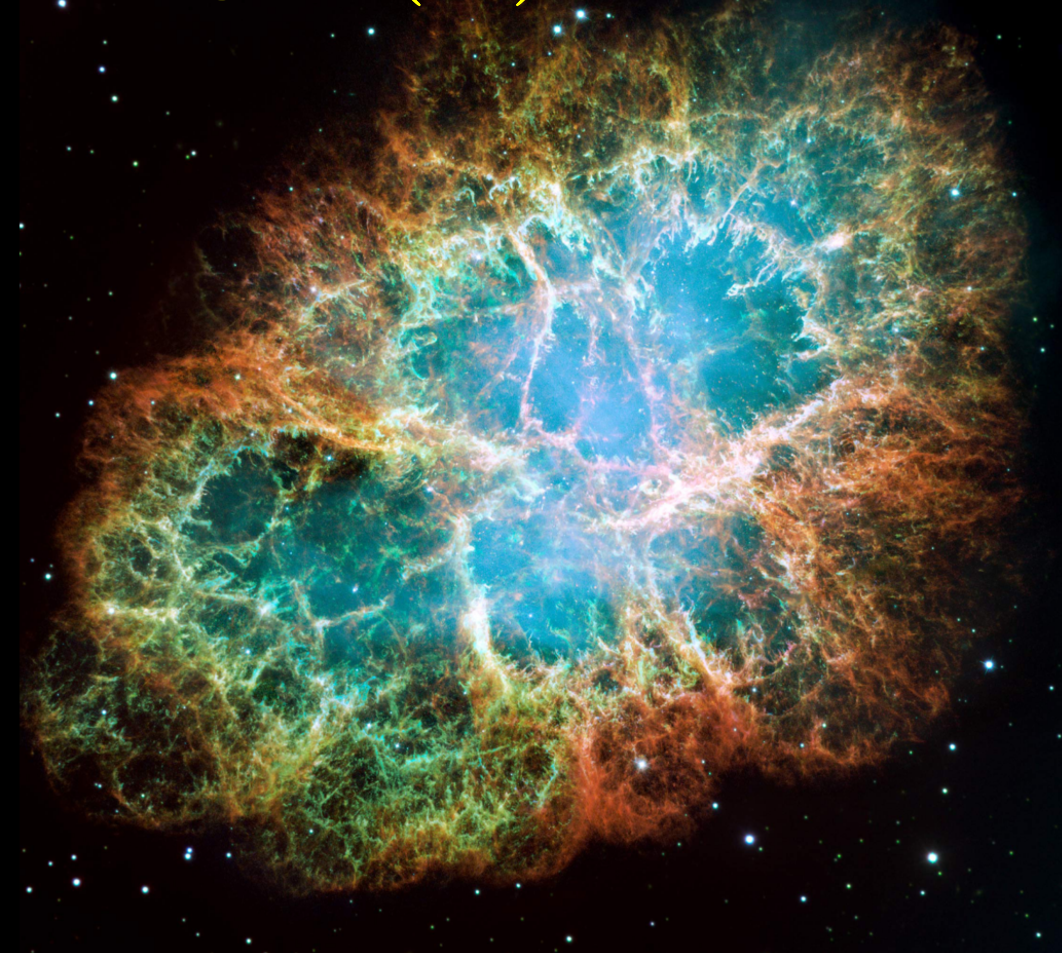
- need $m_{3/2}W(Q) \sim m_{3/2}\Lambda^3 \sim H_0^2$
- *any* potential can be lifted to supergravity
- also radiatively stable $\delta K \sim m_{3/2}^2\Lambda^6$
  $\delta m_Q^2 \sim m_{3/2}^4\Lambda^6$
- no fifth force through Q-Higgs mixing

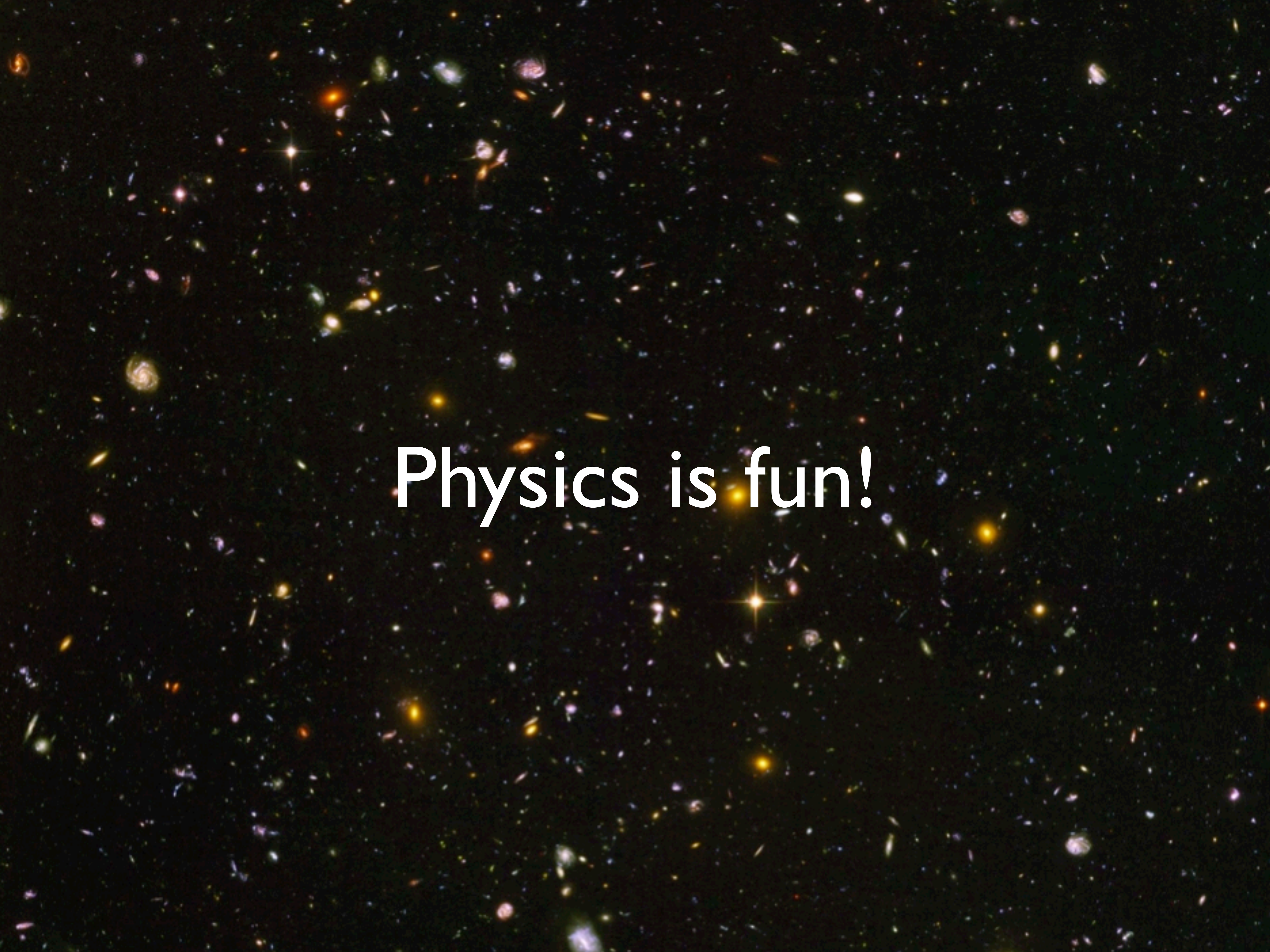
Chien-I Chiang, HM, arXiv:1808.02279



$$\mathcal{L}_{\text{eff}} = c_a(\pi)\dot{\pi}^a + \bar{g}_{ab}(\pi)\dot{\pi}^a\dot{\pi}^b - g_{ab}(\pi)\nabla_i\pi^a\nabla_i\pi^b$$






Physics is fun!